Physics Notes

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Abstract

These notes are written by myself, which means they are prone to typos and errors. If you find errata, do contact me so I can remedy. or give you access to the Github repository for you to push any changes.

Use these notes with caution.

1 Physical Quantities, Units and Measurement

Preamble

Measurement is a tool that we use in physics a lot. It is difficult to get fully accurate measurements due to how well we can create instruments, control random errors, and other factors. Nonetheless we try to minimise these errors by practising proper measurement techniques. We use measurements to determine physical quantities, and these quantities are communicated with units.

1.1 Physical Quantities

Definition 1.1: Physical Quantity

A physical quantity is a quantity consisting of a **numerical magnitude** and a **unit**.

The numerical magnitude tells us the size of the quantity, and the unit tells us what the quantity is expressed in.
Physical quantities can be either a **basic quantity**):

Physical Quantity		SI Unit	
mass	m	kilogram	kg
time	t	second	s
temperature	T	kelvin	K
length	l	metre	m
current	I	ampere	A
amount	n	mole	mol

or a **derived quantity**, which are derived from basic quantities.

1.1.1 Dimensional Analysis

This is not explicitly taught in syllabus, but it is a very important tool to help you if you are stuck in a problem. The main idea is to treat units like **algebraic terms**, and manipulate them accordingly to get the right derived unit for the quantity. Usually, a single unit is written in square brackets [ ] to avoid confusion with units with multiple letters (*e.g.* [mol] and [m]).

1.2 Prefixes, Standard Form, and Order of Magnitude

If a number is too large or too small, it will get very annoying to write a lot of digits. That is what prefixes and standard form aim to solve. The former will be written with the unit, while the latter will be written with the numerical magnitude.
A number is expressed in standard form as

$$\underbrace{A}_{\text{base}} \times \underbrace{10^N}_{\text{factor}}$$

where $1 \leqslant A < 10$ and $N \in \mathbb{Z}$.
A unit can be rewritten with any of these prefixes preceding its symbol:

Prefix	Symbol	Factor	Order of Magnitude
tera	T	10^{12}	12
giga	G	10^{9}	9
mega	M	10^{6}	6
kilo	k	10^{3}	3
deci	d	10^{-1}	-1
centi	c	10^{-2}	-2
milli	m	10^{-3}	-3
micro	µ	10^{-6}	-6
nano	n	10^{-9}	-9
pico	p	10^{-12}	-12

1.3 Scalars and Vectors

Definition 1.2: Scalar Quantity

A scalar quantity has a magnitude, unit, but **no** direction.

Definition 1.3: Vector Quantity

A vector quantity has a magnitude, unit, and direction.

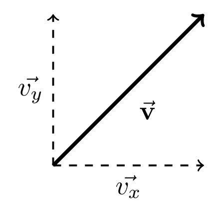
1.4 Vector Addition

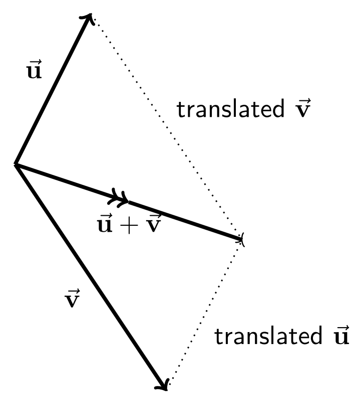
Vectors can be added by using the trigonometric method or the graphical method.

Equation 1.1: Magnitude of Vectors

The magnitude of a vector $\vec{\mathbf{v}}$ with components $\vec{v_x}$ and $\vec{v_y}$ is given by

$$|\vec{v}| = \sqrt{|\vec{v_x}|^2 + |\vec{v_y}|^2}$$





1.5 Measurement

1.5.1 Precision and Accuracy

Precision is how well a set of readings of the same physical quantity agree with each other.

Accuracy is how close the set of readings are to the true value.

1.5.2 Measurement of Lengths

Parallax error should be avoided when measuring lengths. In the case of a measuring tape or a metre rule, the object needs to be **in contact** with the measuring instrument.

Vernier Callipers

Accuracy: ± 0.01 cm

1. Check for zero error. This error is Δx.
2. Place the object to be measured at the appropriate measurement site (internal jaws, external jaws, or tail).
3. Slide the vernier scale so that the jaws or tail measure the entirety of the object.
4. On the main scale (with 0.1 cm subdivisions), take the reading that is on or left of the '0' mark of the vernier scale, x_{main}.
5. On the vernier scale (with 0.01 cm subdivisions), read the mark that coincides with a mark on the main scale, x_{vernier}.
6. The measurement is the sum of the reading on the main scale and vernier scale, and then subtracting the zero error, $x - \Delta x$.

Micrometer Screw Gauge

Accuracy: ± 0.001 cm

1. Check for zero error. This error is Δx.
2. Place the object in between the anvil and the spindle.
3. Close the jaws on the micrometer screw gauge until the object is in contact. Turn the ratchet until a 'click' sound is heard.
4. On the datum line (with 0.5 mm subdivisions), take the reading that is on the left of the circular scale, x_{datum}.
5. On the circular scale (with 0.01 mm subdivisions), take the reading that coincides with the datum line, x_{circular}.
6. The measurement is the sum of the reading on the datum line and circular scale, and then subtracting the zero error, $x - \Delta x$.

1.5.3 Simple Pendulum

A simple pendulum is one on the premises that the string is massless, and the bob is a point mass.

Equation 1.2: Period of Simple Pendulum

Using the approximation $\cos\theta \approx 1 - \frac{\theta^2}{2}$, for a reasonably small θ (angle of release),

$$T = 2\pi\sqrt{\frac{L}{g}}$$

2 Kinematics

Preamble

Kinematics is the study of the motion of objects. It can describe the way a thing moves in space over time. We will only cover one-dimensional motion in this chapter.

2.1 Distance and Displacement

Definition 2.1: Distance

The distance traversed by an object in some time is the entire distance **regardless of the direction of motion**. The SI unit of distance is the metre [m].

Distances are a *scalar* quantity.

Definition 2.2: Displacement

The displacement of an object is the **net change in position** of an object. The SI unit of displacement is the meter [m].

Displacements are a *vector* quantity. When reporting the displacement of an object, it is important to also state the **direction** from the origin point.

2.2 Average Speed, Average Velocity, and Instantaneous Velocity

Equation 2.1: Average Speed

The average speed of an object is given as

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

Soeed is a *scalar* quantity.

Definition 2.3: Average Velocity

The average velocity of an object is the change in **rate of change of displacement** of the object from the origin point. The SI unit of velocity is metre per second [$\mathrm{m\,s^{-1}}$].

Equation 2.2: Average Velocity

The average velocity of an object can be computed as

$$\langle v \rangle = \frac{\Sigma s}{\Sigma t}$$

Definition 2.4: Instantaneous Velocity

The instantaneous velocity of an object is the rate of change of displacement of the object at some specific time**some specific time**. Mathematically, it is the derivative of the displacement function.

Equation 2.3: Instantaneous Velocity

The instantaneous velocity at a time t is computed as

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

Velocity is a *vector* quantity. When reporting the velocity of an object, it is important to also state the **direction** from the origin point.

2.3 Acceleration

Definition 2.5: Acceleration

Acceleration is the **rate of change of velocity**.

Equation 2.4: Acceleration

The acceleration of an object is computed as

$$a = \frac{\Delta v}{\Delta t}$$

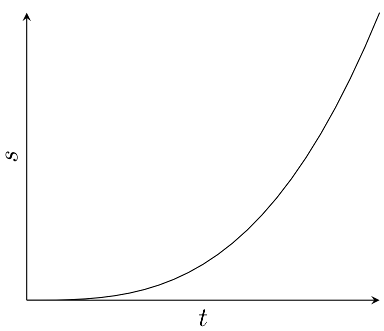
Acceleration is a *vector* quantity. When reporting the acceleration of an object, the direction from the origin point must be stated.

2.4 Kinematic Graphs

A kinematic graph is a visual representation of the state of motion of the object over a period of time. A kinematic graph is useful in many situations, and should be drawn when you are stuck in a kinematics problem.

2.4.1 Displacement-time Graph

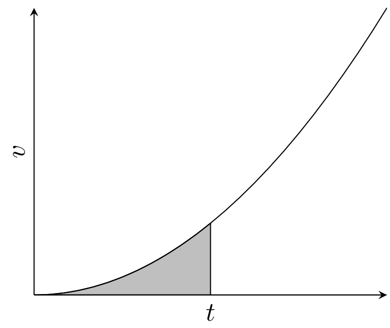
The displacement-time graph records the displacement of an object over a time period. The displacement is recorded on the vertical axis, the time is recorded on the horizontal axis.



The gradient of a displacement-time graph tells us its **velocity**.

2.4.2 Velocity-time Graph

The velocity-time graph records the velocity of an object over a time period. The velocity is recorded on the vertical axis, the time is recorded on the horizontal axis.



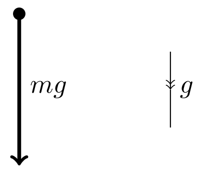
The gradient of a velocity-time graph tells us its **acceleration**; the area under a velocity-time graph tells us the **displacement**.

2.5 Freefall

Definition 2.6: Freefall

An object is in freefall when the only force acting on it is due to **gravity**.

This means that the acceleration due to freefall is always equal to the local acceleration g, and all other forces like air drag do not exist.



2.6 Air drag

In real situations, air drag, or air resistance, is a resistive force that works against the weight of an object when falling. Air drag is **proportional to the square of the velocity** of an object.

As an object falls, its velocity increases. Air drag then also increases. The acceleration of the object slowly decreases as the net force acting on the object is decreasing.

This continues until a point where the air drag is equal and opposite to the weight of the object. The object then experiences zero net force, and has zero acceleration, maintaining a constant velocity.

This constant velocity is **terminal velocity**.

3 Dynamics

Preamble

In physics, forces change the state of motion of an object. Studying forces allow us to talk about the effects on the object and predict the motions of the object. In this chapter, we will look at two-dimensional dynamics.

3.1 Forces

Definition 3.1: Force

A force is a push or pull on a body. The SI unit of force is the newton [N].

3.2 Newton's Laws of Motion

The three laws of motion are:

Definition 3.2: First Law

An object **does not change its state of motion** until being acted upon by a force. This is also known as the law of inertia.

Definition 3.3: Second Law

The net force F_{net} acting on a body is given by as

$$F = ma_{\text{net}}$$

where m is the mass of the body and a_{net} is the net acceleration of the body.

Definition 3.4: Third Law

When two bodies interact, the forces on the bodies from each other are always **equal in magnitude and opposite in direction**.

3.3 Effects of Forces

From the first law, we know that a force can accelerate a body (*i.e.* change velocity). This can be done by either changing the magnitude or direction of the velocity vector of the body.

3.3.1 Static System

Definition 3.5: Translational equilibrium

A body is said to be in translational equilibrium if the **net force on the body is zero**. This is sometimes called a static system, where no net acceleration takes place.

When resolving statics problems, it is important to ensure all force vectors add up to zero. Graphically, all these vectors when placed tip to tail should end where they started.

3.3.2 Unbalanced System

If the net force on a body is not zero, the object is not in translational equilibrium, and that means its velocity is changing.

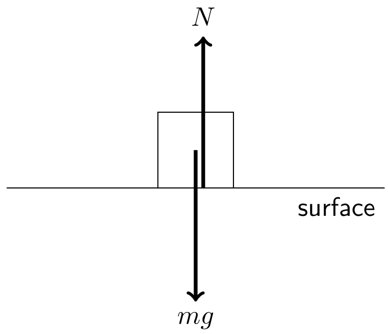
3.4 Types of Forces

It is not sufficient to just describe forces as "push" and "pull" forces. Different names for forces are designated for different contexts. In this syllabus, only friction is required, but I will add common forces as well. Refer to chapter 4 for weight.

3.4.1 Normal Force

Definition 3.6: Normal Force

The normal force is the force perpendicular to a surface that the surface applies to a body due to its compression.



3.4.2 Tension

Definition 3.7: Tension

Tension is the force exerted in a body when it is pulled on.

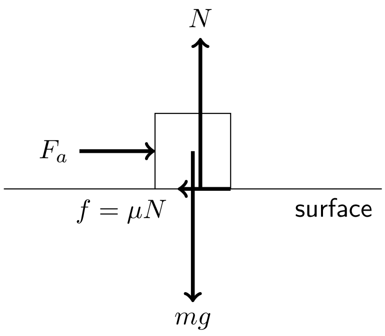
On a massless string, the tension on the two ends are equal.

3.4.3 Friction

Definition 3.8: Friction

Friction is the force parallel to a surface that a surface applies to an body due to its roughness.

Friction is a resistive force, that works against a force applied. There are two types of friction: kinetic and static friction. Kinetic friction deals with two objects moving on each other, and exists when an object is moving, while static friction deals with two objects that are stationary. The maximum static friction is the minimum force to be applied to allow an object to start moving on a surface.



4 Mass, Weight, and Density

Preamble

Matter is anything that takes up space and has mass. The three quantities we are exploring today will allow us to describe matter in different ways.

4.1 Mass

Definition 4.1: Mass

Mass is the **amount of substance** in a body. The SI unit of mass is the kilogram [kg].

The magnitude of mass depends on the number of atoms in the body.
Mass is a **scalar** quantity. It can be measured with an **electronic mass balance**.

Definition 4.2: Inertia

Inertia is a body's **resistance to changing its state of motion**. Inertia is dependent on the body's mass.

4.2 Weight

Definition 4.3: Weight

The weight of an object is defined as the **gravitational force acting on it** due to gravity. The weight of an object w with mass m is equal to

$$w = mg$$

where g is the local gravitational field strength. The SI unit of weight is the newton [N].

Weight is a force, therefore it is a **vector** quantity. It can be measured with a **spring balance**.

Definition 4.4: Gravitational Field

A gravitational field is a region of space where a mass experiences a force. The gravitational field strength is the gravitational force acting on a body per unit mass. On Earth, is equal to

$$g = 10\,\mathrm{m\,s^{-2}} = 10\,\mathrm{N\,kg^{-1}}$$

4.3 Density

Definition 4.5: Density

The density of an object is its mass per unit volume. The density of an object ρ with mass m and volume V is equal to

$$\rho = \frac{m}{V}$$

The SI unit of density is kilogram per cubic metre [$\mathrm{kg\,m^{-3}}$].

When an object is placed in a liquid,

$$\text{the object will} \begin{cases} \text{float} & \rho_{\text{object}} < \rho_{\text{liquid}} \\ \text{suspend} & \rho_{\text{object}} = \rho_{\text{liquid}} \\ \text{sink} & \rho_{\text{object}} > \rho_{\text{liquid}} \end{cases}$$

5 Turning Effect of Forces

Preamble

Objects do not only move in a straight line, they can also move in curves and circles and all kinds of funny shapes. In this chapter we will explore how we can make an object turn by applying a force.

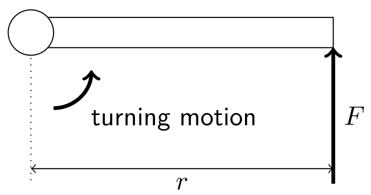
5.1 Moment

Definition 5.1: Moment

A moment is the **turning effect** of a force **around a pivot**. It is equal to the numerical product of the force applied F and the perpendicular distance r from the pivot

$$M_O = r \times F$$

The SI unit of moment is newton metre [N m].



Definition 5.2: Rotational Equilibrium

An object is said to be in rotational equilibrium if the sum of **anticlockwise moments** about a pivot point is **equal** to the sum of **clockwise moments** around the same pivot point.

5.2 Centre of Gravity

Definition 5.3: Centre of Gravity

The centre of gravity, or centre of mass, is a point where the weight of an object seems to be acting on. The centre of gravity can lie outside an object.

5.3 Stability

An object can be in stable, unstable, or neutral equilibrium.

Type of equilibrium	Stable	Unstable	Neutral
Centre of gravity	Low	High	
Base area	Large	Narrow	A line of contact points with surface
Slight displacement	Return to equilibrium	Topple over	Stay in new position

An object's stability can be increased by lowering the height of the centre of gravity, or increasing the base area of the object.

6 Pressure

Preamble

These preambles are feeling more dreadful to write because pressure is building up.

6.1 Pressure

Definition 6.1: Pressure

Pressure is defined as the amount of force per unit area. It is given as

$$p = \frac{F}{A}$$

The SI unit of pressure is the pascal [Pa].

Equation 6.1: Pressure due to a Fluid Column

Fluids of a density ρ can exert pressure p at a height h equal to

$$p = \rho g h$$

Equation 6.2: Transfer of Pressure

Pressure is constant in an incompressible liquid,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Equation 6.3: Work Done

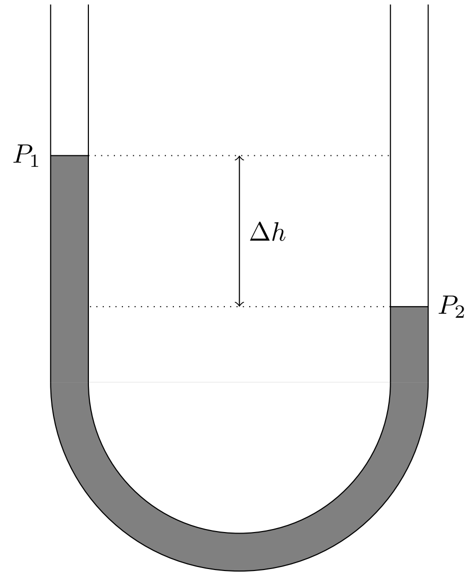
Energy is conserved by the first law of thermodynamics (which is useful to keep in mind when solving hydraulic press problems):

$$F_1 d_1 = F_2 d_2$$

Equation 6.4: Pressure Difference

A manometer can be used to measure pressure differences. It measures a Δh which corresponds to a pressure difference of

$$\Delta p = \rho g \Delta h$$



$$\Delta P = |P_2 - P_1| = \rho g \Delta h$$

7 Energy, Work, and Power

Preamble

The study of energy and matter form the basis of physics. In this chapter we will look at the concept of energy, work done, power, and other relevant quantities.

7.1 Energy

Definition 7.1: First Law of Thermodynamics

Energy cannot be created or destroyed, it is always conserved in a closed system, i.e.

$$\Delta E_T = 0$$

Definition 7.2: Kinetic Energy

Kinetic energy is the energy an object possesses when it is moving. It is given as

$$E_K = \frac{1}{2} m v^2$$

The SI unit of kinetic energy is the joule [J].

Definition 7.3: Gravitational Potential Energy

Gravitational potential energy is defined as how much work can be done by the gravitational force from a height h away. It is given as

$$E_P = mgh$$

The SI unit of gravitational potential energy is the joule [J].

Definition 7.4: Mechanical Energy

The mechanical energy of an object is the sum of its kinetic energy and its gravitational potential energy:

$$E_T = E_K + E_P$$

7.2 Work

Definition 7.5: Work Done

The work done by a force is the energy transferred by a force to an object. It is given as the numerical product of the force and the displacement in the direction of the force

$$W = Fs$$

The SI unit of work done is the joule [J].

Equation 7.1: Efficiency

Efficiency is calculated by

$$\eta = \frac{\text{output}}{\text{input}} \times 100\%$$

7.3 Power

Definition 7.6: Power

Power is defined as the rate of work done. It is calculated as

$$P = \frac{W}{t}$$

The SI unit of power is the watt [W].

8 Kinetic Model of Matter

Preamble

Matter is made up of small particles that behave in certain ways under different conditions. In this chapter we will accurately describe the particulate nature of matter and how it behaves under different temperature and pressure conditions.

8.1 Three States of Matter

The three most common states of matter are solid, liquid, and gas.

Solids have a fixed shape, and have a fixed volume. They have strong forces of attraction between particles. The particles vibrate around a fixed point in the solid.

Liquids have a shape that follows the container, and have a fixed volume. They have slightly weaker forces of attraction between particles compared to solids. The particles flow and slide past each other within the liquid.

Gases have do not have shape, and do not have a fixed volume. They have very weak forces of attraction between particles. The particles move freely.

Definition 8.1: Brownian Motion

Particles are in constant random motion. Brownian motion arises due to these random motions of particles in a fluid.

8.2 Gas Laws

There are three gas laws.

Definition 8.2: Ideal Gas Law

As a result of the three gas laws to be presented below, the relationship for an ideal gas between its temperature, pressure, and volume can be expressed as

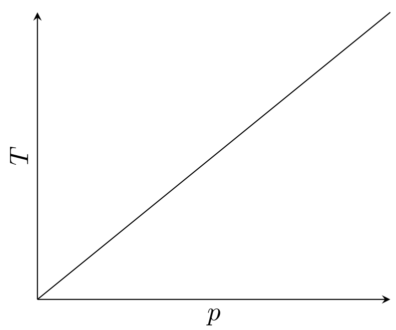
$$pV = nRT$$

where nR is some constant.

Equation 8.1: Charles Law

Charles Law states that the volume of a gas is directly proportional to its temperature. Mathematically,

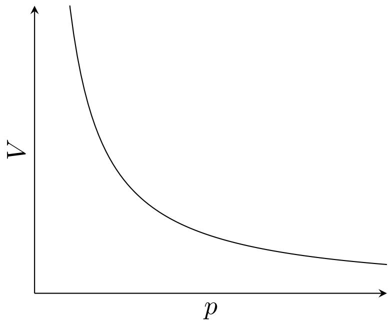
$$p \propto T$$



Equation 8.2: Boyle's Law

Boyle's law states that the pressure of a gas is inversely proportional to the volume of the gas. Mathematically,

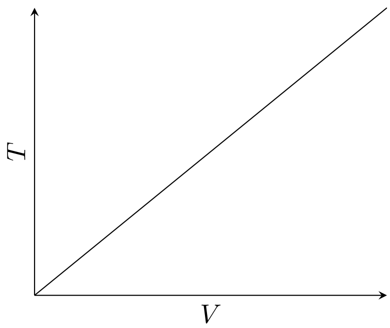
$$p \propto \frac{1}{V}$$



Equation 8.3: Gay-Lussac's Law

Gay-Lussac's Law states that the volume of a gas is directly proportional to its temperature. Mathematically,

$$V \propto T$$



Equation 8.4: Avogadro's Law

(*This is not in this syllabus but it is in O-Level Chemistry so I'll put it here.*) Avogadro's law states that the amount of gas is directly proportional to the volume of the gas. Mathematically,

$$n \propto V$$

9 Transfer of Thermal Energy

Preamble

Heat can be transferred in multiple ways. In this chapter we will look at three different methods for heat transfer.

Heat travels from a region of higher temperature to lower temperature.

Definition 9.1: Conduction

Conduction is the process whereby particles within a medium without the movement of the medium itself.

Particles collide with neighbouring particles and that energy gets transferred down the entire object, causing the object to increase in temperature.
Metals can conduct heat better due to **electron diffusion**.

Definition 9.2: Convection

Convection is the transfer of heat within a fluid due to differences in density.

Definition 9.3: Radiation

Radiation is the process where a body exchanges energy as heat through electromagnetic waves.

Factors that affect the rate of radiation include:

- **Colour:** darker objects radiate heat better than lighter objects (see emissivity)

- **Surface:** rougher surfaces radiate heat better than smoother surfaces (due to higher surface area)

Further reading: Radiation is modelled by the Stefan-Boltzmann Law:

$$P = A\varepsilon\sigma T^4$$

where ε is the emissivity and σ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$.

10 Temperature

Preamble

In this chapter we will learn how to make a thermometer because you can't buy one in practical exam.

Definition 10.1: Thermometric Property

A thermometric property is a property of matter that varies continuously with temperature.

Some examples of this include the volume of an object, the electromotive force of a thermocouple, and the height of a liquid column.

Equation 10.1: Thermometry Formula

To make a thermometer, you need some thermometric property X at temperatures 0 °C, 100 °C, and some temperature θ °C. Then you plug them into this formula

$$\theta\,°\mathrm{C} = \frac{X_\theta - X_0}{X_{100} - X_0} \times 100\,°\mathrm{C}$$

Equation 10.2: Temperature Conversion

To convert from degrees celsius [°C] to kelvin [K],

$$[\mathrm{K}] = [°\mathrm{C}] + 273.15$$

11 Thermal Properties of Matter

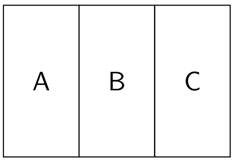
Preamble

Matter has some properties when it comes to heat. These preambles are also getting difficult to write because I'm running out of ideas.

11.1 Heat Energy

Definition 11.1: Zeroth Law of Thermodynamics

(*This isn't in syllabus.*) The zeroth law of thermodynamics states that if object A, B, and C are in thermal contact with each other, and if the temperature of object A is equal to that of B, and the temperature of object B is equal to that of C, then the temperature of object A must equal to that of C.



if $T_A = T_B$ and $T_B = T_C$ then

$$T_A = T_B = T_C$$

Definition 11.2: Heat Capacity

Heat capacity C is the amount of heat energy required to raise the temperature of an object by 1 K. Its relationship can be expressed as

$$\Delta Q = C\Delta T$$

The SI unit of heat capacity is joule per kelvin [$\mathrm{J\,K^{-1}}$].

Definition 11.3: Specific Heat Capacity

Specific heat capacity c is the amount of heat energy required to raise the temperature of a unit mass of an object by 1 K. Its relationship can be expressed as

$$\Delta Q = mc\Delta T$$

The SI unit of heat capacity is joule per kelvin per kilogram [$\mathrm{J\,K^{-1}\,kg^{-1}}$].

Definition 11.4: Latent Heat

Latent heat is the amount of heat energy required to allow a unit mass of an object to transition from one state from another. In general,

$$Q_{f/v} = ml_{f/v}$$

where $l_{f/v}$ is the specific latent heat of fusion/vaporisation, the heat energy required to melt or freeze/vaporise or condense a unit mass. The SI unit of specific latent heat is joule per kilogram [$\mathrm{J\,kg^{-1}}$].

11.2 Vaporisation

Definition 11.5: Evaporation

Evaporation is the process whereby a liquid vaporises at the surface because it has the energy equal or more than that of the latent heat of vaporisation, allowing it to escape into the atmosphere.

Evaporation can happen at any temperature.

Definition 11.6: Boiling

Boiling is the process where a liquid reaches boiling point and the particles have enough energy to vaporise.

Boiling only happens at boiling point.

12 General Wave Properties

Preamble

Waves are a fundamental method of describing the nature of matter and how it interacts with energy. In this chapter we will be covering general wave properties that would be helpful.

12.1 Definitions

Definition 12.1: Wave

A wave is the transfer of energy without the transfer of matter.

Definition 12.2: Transverse Wave

A transverse wave is when the particles oscillate perpendicular to the direction of propagation.

An example of a transverse wave is electromagnetic waves.

Definition 12.3: Longitudinal Wave

A longitudinal wave is when the particles oscillate parallel to the direction of propagation.

An example of a longitudinal wave is sound waves.

12.2 Parts of a Wave

Definition 12.4: Wavelength

The wavelength of a wave is the displacement between two successive in-phase points. It is usually represented by the Greek letter λ. The SI unit for wavelength is the metre [m].

Definition 12.5: Period

The period of a wave is the time taken for a particle to complete one oscillation. It is usually represented by the letter T. The SI unit for period is the second [s].

Definition 12.6: Frequency

The frequency of a wave is the number of times a particle completes one oscillation in one second. It is usually represented by the letter f. The SI unit for frequency is the hertz [Hz].

Equation 12.1: Period and Frequency

Period and frequency are reciprocals of each other,

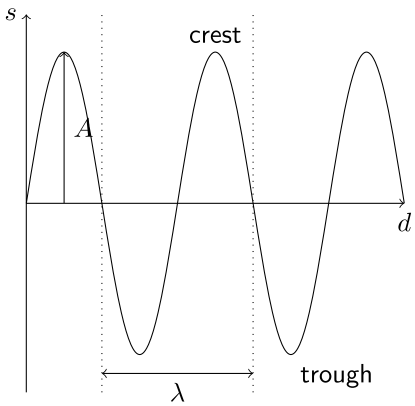
$$f = \frac{1}{T} \Leftrightarrow T = \frac{1}{f}$$

Definition 12.7: Amplitude

The amplitude of a wave is the maximum displacement of a particle in a wave. It is usually represented by the letter A. The most common unit for amplitude is the metre [m]; though keep in mind other physical quantities like voltage can exhibit periodic wave-like behaviour.

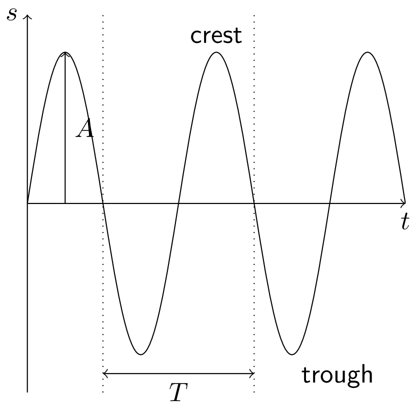
12.2.1 Displacement-distance Graph

This is also known as a snapshot graph.



12.2.2 Displacement-time Graph

This is also known as a history graph.



12.3 Wave Equation

Equation 12.2: Wave Equation

For a wave with frequency f and wavelength λ, the velocity v it is travelling at is equal to

$$v = f\lambda$$

13 Light

Preamble

Light can be studied as a wave. In this chapter we will look at how light interacts with matter.

Definition 13.1: Normal

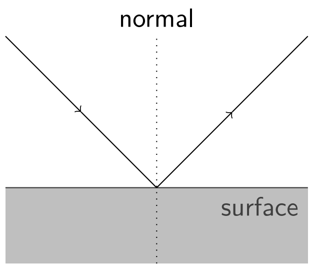
The normal is an imaginary line draw perpendicular to the surface that reflection is taking place at.

Definition 13.2: Angle of Incidence

The angle of incidence is the angle between the incident ray and the normal.

Definition 13.3: Angle of Reflection

The angle of reflection is the angle between the reflected ray and the normal.



13.1 Reflection

Definition 13.4: Law of Reflection

In reflection, the angle of incidence is equal to the angle of reflection. The incident ray, reflected ray, and the normal are coplanar.

$$\theta_1 = \theta_2$$

I have chosen to name the angles θ_1 and θ_2 due to the reversible nature of light. It does not matter which way the light goes; the angles will be preserved.