# Physics Notes

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**Abstract**

These notes are written by myself, which means they are prone to typos and errors. If you find errata, do contact me so I can remedy. or give you access to the Github repository for you to push any changes.

Some code (especially the `tcolorboxes`) are copied fron 4yn's a-lv-notes repository[1].

Use these notes with caution.

## Contents

# Part I

# Measurement

## 1 Physical Quantities, Units and Measurement

**Preamble**

Measurement is a tool that we use in physics a lot. It is difficult to get fully accurate measurements due to how well we can create instruments, control random errors, and other factors. Nonetheless we try to minimise these errors by practising proper measurement techniques. We use measurements to determine physical quantities, and these quantities are communicated with units.

### 1.1 Physical Quantities

**Definition 1.1: Physical Quantity**

A physical quantity is a quantity consisting of a **numerical magnitude** and a **unit**.

The numerical magnitude tells us the size of the quantity, and the unit tells us what the quantity is expressed in.

Physical quantities can be either a **basic quantity**):

| Physical Quantity | | SI Unit | |
|---|---|---|---|
| mass | $m$ | kilogram | kg |
| time | $t$ | second | s |
| temperature | $T$ | kelvin | K |
| length | $l$ | metre | m |
| current | $I$ | ampere | A |
| amount | $n$ | mole | mol |

or a **derived quantity**, which are derived from basic quantities.

#### 1.1.1 Dimensional Analysis

This is not explicitly taught in syllabus, but it is a very important tool to help you if you are stuck in a problem.

The main idea is to treat units like **algebraic terms**, and manipulate them accordingly to get the right derived unit for the quantity. Usually, a single unit is written in square brackets [ ] to avoid confusion with units with multiple letters (*e.g.* [mol] and [m]).

### 1.2 Prefixes, Standard Form, and Order of Magnitude

If a number is too large or too small, it will get very annoying to write a lot of digits. That is what prefixes and standard form aim to solve. The former will be written with the unit, while the latter will be written with the numerical magnitude.

A number is expressed in standard form as

$$\underbrace{A}_{\text{base}} \times \underbrace{10^N}_{\text{factor}}$$

where $1 \leqslant A < 10$ and $N \in \mathbb{Z}$.

A unit can be rewritten with any of these prefixes preceding its symbol:

| Prefix | Symbol | Factor | Order of Magnitude |
|---|---|---|---|
| tera | T | $10^{12}$ | 12 |
| giga | G | $10^{9}$ | 9 |
| mega | M | $10^{6}$ | 6 |
| kilo | k | $10^{3}$ | 3 |
| deci | d | $10^{-1}$ | −1 |
| centi | c | $10^{-2}$ | −2 |
| milli | m | $10^{-3}$ | −3 |
| micro | μ | $10^{-6}$ | −6 |
| nano | n | $10^{-9}$ | −9 |
| pico | p | $10^{-12}$ | −12 |

### 1.3 Scalars and Vectors

**Definition 1.2: Scalar Quantity**

A scalar quantity has a magnitude but **no** direction.

**Definition 1.3: Vector Quantity**

A vector quantity has a magnitude and direction.

### 1.4 Vector Analysis

Vectors can be added by using the trigonometric method or the graphical method.

**Equation 1.1: Components**

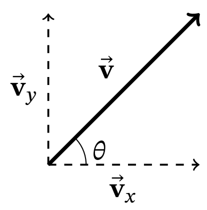
A two-dimensional vector $\vec{\mathbf{v}}$ can be broken down into components $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$, with magnitudes of

$$\vec{\mathbf{v}}_x = \mathbf{v}\cos\theta,\ \vec{\mathbf{v}}_y = \mathbf{v}\sin\theta$$

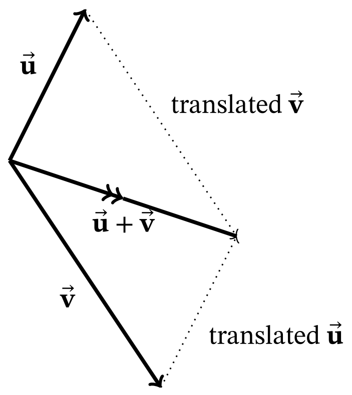
**Equation 1.2: Magnitude of Vectors**

The magnitude of a vector $\vec{\mathbf{v}}$ with components $\vec{\mathbf{v}}_x$ and $\vec{\mathbf{v}}_y$ is given by

$$|\vec{\mathbf{v}}| = \sqrt{|\vec{\mathbf{v}}_x|^2 + |\vec{\mathbf{v}}_y|^2}$$




Observer that when you add the two components together, they form the vector itself.




## 1.5 Measurement

### 1.5.1 Precision and Accuracy

**Precision** is how well a set of readings of the same physical quantity agree with each other.

**Accuracy** is how close the set of readings are to the true value.

### 1.5.2 Measurement of Lengths

Parallax error should be avoided when measuring lengths. In the case of a measuring tape or a metre rule, the object needs to be **in contact** with the measuring instrument.

#### Vernier Callipers

Accuracy: $\pm 0.01$ cm

1. Check for zero error. This error is $\Delta x$.
2. Place the object to be measured at the appropriate measurement site (internal jaws, external jaws, or tail).
3. Slide the vernier scale so that the jaws or tail measure the entirety of the object.
4. On the main scale (with 0.1 cm subdivisions), take the reading that is on or left of the '0' mark of the vernier scale, $x_{\text{main}}$.
5. On the vernier scale (with 0.01 cm subdivisions), read the mark that coincides with a mark on the main scale, $x_{\text{vernier}}$.
6. The measurement is the sum of the reading on the main scale and vernier scale, and then subtracting the zero error, $x - \Delta x$.

#### Micrometer Screw Gauge

Accuracy: $\pm 0.001$ cm

1. Check for zero error. This error is $\Delta x$.
2. Place the object in between the anvil and the spindle.
3. Close the jaws on the micrometer screw gauge until the object is in contact. Turn the ratchet until a 'click' sound is heard.
4. On the datum line (with 0.5 mm subdivisions), take the reading that is on the left of the circular scale, $x_{\text{datum}}$.
5. On the circular scale (with 0.01 mm subdivisions), take the reading that coincides with the datum line, $x_{\text{circular}}$.
6. The measurement is the sum of the reading on the datum line and circular scale, and then subtracting the zero error, $x - \Delta x$.

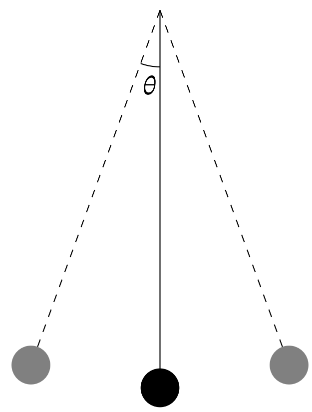
### 1.5.3 Simple Pendulum

A simple pendulum is one on the premises that the string is massless, and the bob is a point mass.

**Equation 1.3: Period of Simple Pendulum**

Using the approximation $\cos\theta \approx 1 - \frac{\theta^2}{2}$, for a reasonably small $\theta$ (angle of release),

$$T = 2\pi\sqrt{\frac{L}{g}}$$




# Part II

# Newtonian Mechanics

## 2 Kinematics

**Preamble**

Kinematics is the study of the motion of objects. It can describe the way a thing moves in space over time. We will only cover one-dimensional motion in this chapter.

## 2.1 Distance and Displacement

**Definition 2.1: Distance**

The distance traversed by an object in some time is the entire distance **regardless of the direction of motion**. The SI unit of distance is the metre [m].

Distances are a *scalar* quantity.

**Definition 2.2: Displacement**

The displacement of an object is the **net change in position** of an object. The SI unit of displacement is the meter [m].

Displacements are a *vector* quantity. When reporting the displacement of an object, it is important to also state the **direction** from the origin point.

## 2.2 Average Speed, Average Velocity, and Instantaneous Velocity

**Equation 2.1: Average Speed**

The average speed of an object is given as

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

Speed is a *scalar* quantity.

**Definition 2.3: Average Velocity**

The average velocity of an object is the change in **rate of change of displacement** of the object from the origin point. The SI unit of velocity is metre per second $[\mathrm{m\,s^{-1}}]$.

**Equation 2.2: Average Velocity**

The average velocity of an object can be computed as

$$\langle v \rangle = \frac{\Sigma s}{\Sigma t}$$

**Definition 2.4: Instantaneous Velocity**

The instantaneous velocity of an object is the rate of change of displacement of the object at **some specific time**. Mathematically, it is the derivative of the displacement function.

**Equation 2.3: Instantaneous Velocity**

The instantaneous velocity at a time $t$ is computed as

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

Velocity is a *vector* quantity. When reporting the velocity of an object, it is important to also state the **direction** from the origin point.

## 2.3 Acceleration

**Definition 2.5: Acceleration**

Acceleration is the **rate of change of velocity**.

**Equation 2.4: Acceleration**

The acceleration of an object is computed as

$$a = \frac{\Delta v}{\Delta t}$$

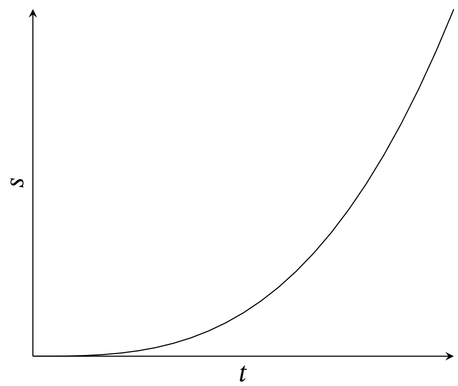
Acceleration is a *vector* quantity. When reporting the acceleration of an object, the direction from the origin point must be stated.

## 2.4 Kinematic Graphs

A kinematic graph is a visual representation of the state of motion of the object over a period of time. A kinematic graph is useful in many situations, and should be drawn when you are stuck in a kinematics problem.

### 2.4.1 Displacement-time Graph

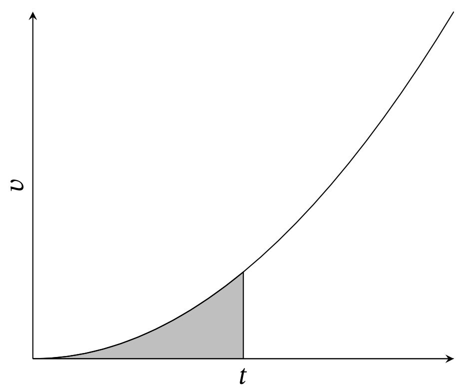
The displacement-time graph records the displacement of an object over a time period. The displacement is recorded on the vertical axis, the time is recorded on the horizontal axis.




The gradient of a displacement-time graph tells us its **velocity**.

### 2.4.2 Velocity-time Graph

The velocity-time graph records the velocity of an object over a time period. The velocity is recorded on the vertical axis, the time is recorded on the horizontal axis.



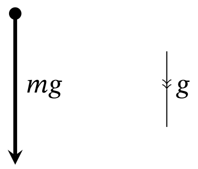

The gradient of a velocity-time graph tells us its **acceleration**; the area under a velocity-time graph tells us the **displacement**.

## 2.5 Freefall

**Definition 2.6: Freefall**

An object is in freefall when the only force acting on it is due to **gravity**.

This means that the acceleration due to freefall is always equal to the local acceleration $g$, and all other forces like air drag do not exist.




## 2.6 Air drag

In real situations, air drag, or air resistance, is a resistive force that works against the weight of an object when falling. Air drag is **proportional to the square of the velocity** of an object.

As an object falls, its velocity increases. Air drag then also increases. The acceleration of the object slowly decreases as the net force acting on the object is decreasing.

This continues until a point where the air drag is equal and opposite to the weight of the object. The object then experiences zero net force, and has zero acceleration, maintaining a constant velocity.

This constant velocity is **terminal velocity**.

# 3 Dynamics

**Preamble**

In physics, forces change the state of motion of an object. Studying forces allow us to talk about the effects on the object and predict the motions of the object. In this chapter, we will look at two-dimensional dynamics.

## 3.1 Forces

**Definition 3.1: Force**

A force is a push or pull on a body. The SI unit of force is the newton [N].

## 3.2 Newton's Laws of Motion

The three laws of motion are:

**Definition 3.2: First Law**

Newton's first law states that every object will continue in its state of rest or uniform motion in a straight line unless a resultant force acts on it.

**Definition 3.3: Second Law**

Newton's second law states that when a resultant force acts on an object of a constant mass, the object will accelerate in the direct ion of the resultant force. The product of the mass $m$ and acceleration $a_{\text{net}}$ of the object gives the resultant force.

$$F_{\text{net}} = ma_{\text{net}}$$

**Definition 3.4: Third Law**

Newton's third law states that if body A exerts a force $F_{AB}$ on body B, body B will exert an equal and opposite force $F_{BA}$ on body A.

## 3.3 Effects of Forces

From the first law, we know that a force can accelerate a body (*i.e.* change velocity). This can be done by either changing the magnitude or direction of the velocity vector of the body.

### 3.3.1 Static System

**Definition 3.5: Translational equilibrium**

A body is said to be in translational equilibrium if the **net force on the body is zero**. This is sometimes called a static system, where no net acceleration takes place.

When resolving statics problems, it is important to ensure all force vectors add up to zero. Graphically, all these vectors when placed tip to tail should end where they started.

### 3.3.2 Unbalanced System

If the net force on a body is not zero, the object is not in translational equilibrium, and that means its velocity is changing.

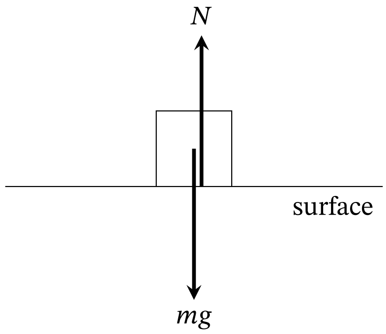
## 3.4 Types of Forces

It is not sufficient to just describe forces as "push" and "pull" forces. Different names for forces are designated for different contexts. In this syllabus, only friction is required, but I will add common forces as well. Refer to chapter 4 for weight.

### 3.4.1 Normal Force

**Definition 3.6: Normal Force**

The normal force is the force perpendicular to a surface that the surface applies to a body due to its compression.




#### 3.4.2 Tension

**Definition 3.7: Tension**

Tension is the force exerted in a body when it is pulled on.

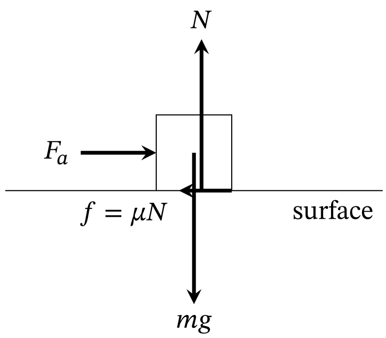
On a massless string, the tension on the two ends are equal.


#### 3.4.3 Friction

**Definition 3.8: Friction**

is the contact force that opposes or tends to oppose motion between surfaces in contact.

Friction is a resistive force, that works against a force applied. There are two types of friction: kinetic and static friction. Kinetic friction deals with two objects moving on each other, and exists when an object is moving, while static friction deals with two objects that are stationary. The maximum static friction is the minimum force to be applied to allow an object to start moving on a surface.




## 4 Mass, Weight, and Density

**Preamble**

Matter is anything that takes up space and has mass. The three quantities we are exploring today will allow us to describe matter in different ways.

### 4.1 Mass

**Definition 4.1: Mass**

Mass is the **amount of matter** in a body. The SI unit of mass is the kilogram [kg].

The magnitude of mass depends on the number of atoms in the body.

Mass is a **scalar** quantity. It can be measured with an **electronic mass balance**.

**Definition 4.2: Inertia**

The inertia of an object refers to the reluctance of the object to change its state of rest or motion, due to its mass.

### 4.2 Weight

**Definition 4.3: Weight**

The weight of an object is defined as the **gravitational force acting on it** due to gravity. The weight of an object $w$ with mass $m$ is equal to

$$w = mg$$

where $g$ is the local gravitational field strength. The SI unit of weight is the newton [N].

Weight is a force, therefore it is a **vector** quantity. It can be measured with a **spring balance**.

**Definition 4.4: Gravitational Field**

A gravitational field is a region in which a mass experiences a force due to gravitational attraction. The gravitational field strength is the gravitational force acting per unit mass. On Earth, is equal to

$$g = 10\,\mathrm{m\,s^{-2}} = 10\,\mathrm{N\,kg^{-1}}$$

### 4.3 Density

**Definition 4.5: Density**

The density of an object is its mass per unit volume. The density of an object $\rho$ with mass $m$ and volume $V$ is equal to

$$\rho = \frac{m}{V}$$

The SI unit of density is kilogram per cubic metre [$\mathrm{kg\,m^{-3}}$].

When an object is placed in a liquid,

$$\text{the object will} \begin{cases} \text{float} & \rho_{\text{object}} < \rho_{\text{liquid}} \\ \text{suspend} & \rho_{\text{object}} = \rho_{\text{liquid}} \\ \text{sink} & \rho_{\text{object}} > \rho_{\text{liquid}} \end{cases}$$

# 5 Turning Effect of Forces

**Preamble**

Objects do not only move in a straight line, they can also move in curves and circles and all kinds of funny shapes. In this chapter we will explore how we can make an object turn by applying a force.

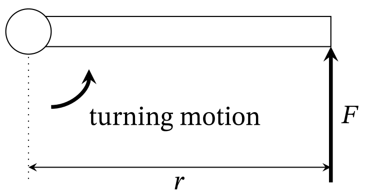
## 5.1 Moment

**Definition 5.1: Moment**

The moment of a force is the product of the force $F$ and the perpendicular distance from the pivot to the line of action of the force $r$

$$M_O = r \times F$$

The SI unit of moment is newton metre [N m].




**Definition 5.2: Principle of Moments**

The principle of moments states that when a body is in equilibrium, the sum of clockwise moments about a pivot is equal to the sum of anticlockwise moments about the same pivot..

## 5.2 Centre of Gravity

**Definition 5.3: Centre of Gravity**

The centre of gravity, or centre of mass, is a point where the weight of an object seems to be acting on. The centre of gravity can lie outside an object.

## 5.3 Stability

**Definition 5.4: Stability**

The stability of an object is a measure of its ability to return to its original position after it is s lightly displaced.

An object can be in stable, unstable, or neutral equilibrium.

| Type of equilibrium | Stable | Unstable | Neutral |
|---|---|---|---|
| Centre of gravity | Low | High | |
| Base area | Large | Narrow | A line of contact points with surface |
| Slight displacement | Return to equilibrium | Topple over | Stay in new position |

An object's stability can be increased by lowering the height of the centre of gravity, or increasing the base area of the object.

# 6 Pressure

**Preamble**

These preambles are feeling more dreadful to write because pressure is building up.

## 6.1 Pressure

**Definition 6.1: Pressure**

Pressure is defined as the amount of force per unit area. It is given as

$$p = \frac{F}{A}$$

The SI unit of pressure is the pascal [Pa].

**Equation 6.1: Pressure due to a Fluid Column**

Fluids of a density $\rho$ can exert pressure $p$ at a height $h$ equal to

$$p = \rho g h$$

**Equation 6.2: Transfer of Pressure**

Pressure is constant in an incompressible liquid,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

**Equation 6.3: Work Done**

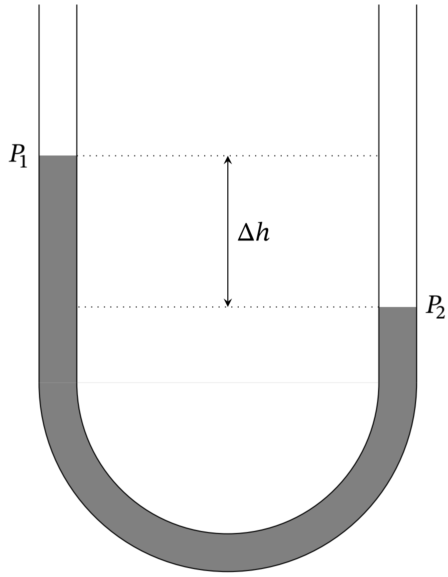
Energy is conserved by the first law of thermodynamics (which is useful to keep in mind when solving hydraulic press problems):

$$F_1 d_1 = F_2 d_2$$

**Equation 6.4: Pressure Difference**

A manometer can be used to measure pressure differences. It measures a $\Delta h$ which corresponds to a pressure difference of

$$\Delta p = \rho g \Delta h$$




$$\Delta P = |P_2 - P_1| = \rho g \Delta h$$

# 7 Energy, Work, and Power

**Preamble**

The study of energy and matter form the basis of physics. In this chapter we will look at the concept of energy, work done, power, and other relevant quantities.

## 7.1 Energy

**Definition 7.1: Energy**

Energy is the capacity to do work.

**Definition 7.2: Principle of Conservation of Energy**

Energy cannot be created nor destroyed, but can be converted from one form to another. The total energy in an isolated system is constant. i.e.

$$\Delta E_T = 0$$

**Definition 7.3: Kinetic Energy**

Kinetic energy is the energy an object possesses when it is moving. It is given as

$$E_K = \frac{1}{2}mv^2$$

The SI unit of kinetic energy is the joule [J].

**Definition 7.4: Gravitational Potential Energy**

Gravitational potential energy is defined as how much work can be done by the gravitational force from a height $h$ away. It is given as

$$E_P = mgh$$

The SI unit of gravitational potential energy is the joule [J].

**Definition 7.5: Mechanical Energy**

The mechanical energy of an object is the sum of its kinetic energy and its gravitational potential energy:

$$E_T = E_K + E_P$$

## 7.2 Work

**Definition 7.6: Work Done**

The work done by a constant force on an object is the product of the force $F$ and the distance moved by the object in the direction of the force (actually displacement $s$).

$$W = Fs$$

The SI unit of work done is the joule [J].

**Equation 7.1: Efficiency**

Efficiency is calculated by

$$\eta = \frac{\text{output}}{\text{input}} \times 100\%$$

## 7.3 Power

**Definition 7.7: Power**

Power is defined as the rate of work done or rate of energy conversion. It is calculated as

$$P = \frac{W}{t}$$

The SI unit of power is the watt [W].

# Part III

# Thermal Physics

# 8 Kinetic Model of Matter

**Preamble**

Matter is made up of small particles that behave in certain ways under different conditions. In this chapter we will accurately describe the particulate nature of matter and how it behaves under different temperature and pressure conditions.

## 8.1 Three States of Matter

The three most common states of matter are solid, liquid, and gas.

**Solids** have a fixed shape, and have a fixed volume. They have strong forces of attraction between particles. The particles vibrate around a fixed point in the solid.

**Liquids** have a shape that follows the container, and have a fixed volume. They have slightly weaker forces of attraction between particles compared to solids. The particles flow and slide past each other within the liquid.

**Gases** have do not have shape, and do not have a fixed volume. They have very weak forces of attraction between particles. The particles move freely.

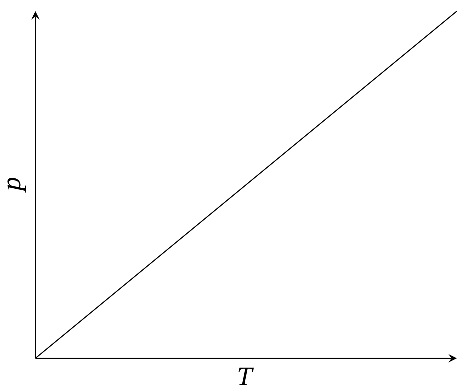
> **Definition 8.1: Brownian Motion**
>
> Particles are in constant random motion. Brownian motion arises due to these random motions of particles in a fluid.

## 8.2 Gas Laws

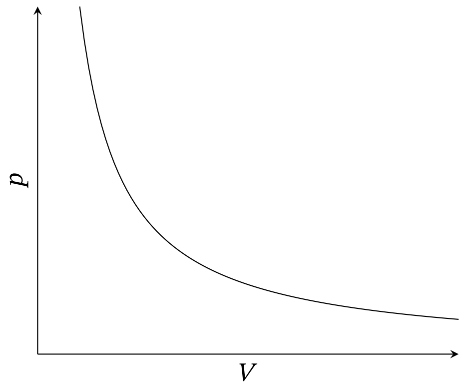
There are three gas laws.

> **Definition 8.2: Ideal Gas Law**
>
> As a result of the three gas laws to be presented below, the relationship for an ideal gas between its temperature, pressure, and volume can be expressed as
>
> $$pV = nRT$$
>
> where $nR$ is some constant.

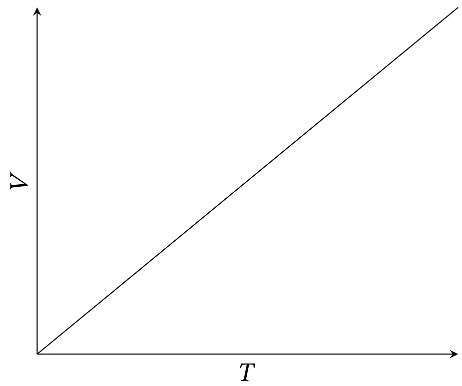
> **Equation 8.1: Charles Law**
>
> Charles Law states that the pressure of a gas is directly proportional to its temperature *if the volume stays constant (isochoric)*. Mathematically,
>
> $$p \propto T$$




> **Equation 8.2: Boyle's Law**
>
> Boyle's law states that the pressure of a gas is inversely proportional to the volume of the gas *if the temperature stays constant (isothermic)*. Mathematically,
>
> $$p \propto \frac{1}{V}$$




> **Equation 8.3: Gay-Lussac's Law**
>
> Gay-Lussac's Law states that the volume of a gas is directly proportional to its temperature *if the pressure stays constant (isobaric)*. Mathematically,
>
> $$V \propto T$$

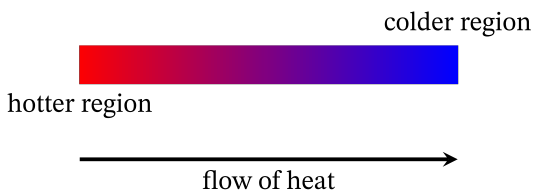



> **Equation 8.4: Avogadro's Law**
>
> (*This is not in this syllabus but it is in O-Level Chemistry so I'll put it here.*) Avogadro's law states that the amount of gas is directly proportional to the volume of the gas. Mathematically,
>
> $$n \propto V$$

# 9 Transfer of Thermal Energy

> **Preamble**
>
> Heat can be transferred in multiple ways. In this chapter we will look at three different methods for heat transfer.

Heat always flows from a region of higher temperature to a region of lower temperature. Net flow of thermal energy occurs only when there is a difference in temperature.



**Definition 9.1: Conduction**

Conduction is the process whereby particles within a medium without the movement of the medium itself.

Particles collide with neighbouring particles and that energy gets transferred down the entire object, causing the object to increase in temperature.

Metals can conduct heat better due to **electron diffusion**.

**Definition 9.2: Convection**

Convection is the transfer of thermal energy by means of convection currents in a fluid due to a difference in density.

**Definition 9.3: Radiation**

Radiation is the transfer of thermal energy in the form of electromagnetic waves such as infrared radiation without the aid of a medium.

Factors that affect the rate of radiation include:

- **Colour:** darker objects radiate heat better than lighter objects (see emissivity)
- **Surface:** rougher surfaces radiate heat better than smoother surfaces (due to higher surface area)

Further reading: Radiation is modelled by the Stefan-Boltzmann Law:

$$P = A\varepsilon\sigma T^4$$

where $\varepsilon$ is the emissivity and $\sigma$ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$.

# 10 Temperature

**Preamble**

In this chapter we will learn how to make a thermometer because you can't buy one in practical exam.

**Definition 10.1: Heat**

Heat is the amount of thermal energy that is being transferred from a hotter to a colder region.

**Definition 10.2: Ice Point**

The ice point is the temperature of pure melting ice at one atmosphere, and is assigned a value of 0 °C.

**Definition 10.3: Steam Point**

The steam point is the temperature of pure melting ice at one atmosphere, and is assigned a value of 100 °C.

**Definition 10.4: Thermometric Property**

A thermometric property is a property of matter that varies continuously with temperature.

Some examples of this include the volume of an object, the electromotive force of a thermocouple, and the height of a liquid column.

**Equation 10.1: Thermometry Formula**

To make a thermometer, you need some thermometric property $X$ at temperatures 0 °C, 100 °C, and some temperature $\theta$ °C. Then you plug them into this formula

$$\theta\,^{\circ}\mathrm{C} = \frac{X_\theta - X_0}{X_{100} - X_0} \times 100\,^{\circ}\mathrm{C}$$

**Equation 10.2: Temperature Conversion**

To convert from degrees celsius [°C] to kelvin [K],

$$[\mathrm{K}] = [^{\circ}\mathrm{C}] + 273.15$$

# 11 Thermal Properties of Matter

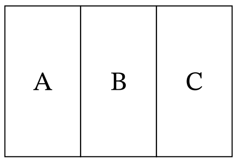
**Preamble**

Matter has some properties when it comes to heat. These preambles are also getting difficult to write because I'm running out of ideas.

## 11.1 Heat Energy

**Definition 11.1: Zeroth Law of Thermodynamics**

(*This isn't in syllabus.*) The zeroth law of thermodynamics states that if object A, B, and C are in thermal contact with each other, and if the temperature of object A is equal to that of B, and the temperature of object B is equal to that of C, then the temperature of object A must equal to that of C.




if $T_A = T_B$ and $T_B = T_C$ then

$$T_A = T_B = T_C$$

**Definition 11.2: Heat Capacity**

Heat capacity $C$ is the amount of heat energy required to raise the temperature of an object by 1 K. Its relationship can be expressed as

$$\Delta Q = C\Delta T$$

The SI unit of heat capacity is joule per kelvin [$\mathrm{J\,K^{-1}}$].

**Definition 11.3: Specific Heat Capacity**

Specific heat capacity $c$ is the amount of heat energy required to raise the temperature of a unit mass of an object by 1 K. Its relationship can be expressed as

$$\Delta Q = mc\Delta T$$

The SI unit of heat capacity is joule per kelvin per kilogram [$\mathrm{J\,K^{-1}\,kg^{-1}}$].

Definition 11.4: Latent Heat

Latent heat is the energy released or absorbed by a substance during a change of state, without a change in its temperature. In general,

$$Q_{f/v} = ml_{f/v}$$

where $l_{f/v}$ is the specific latent heat of fusion/vaporisation, the heat energy required to melt or freeze/vaporise or condense a unit mass. The SI unit of specific latent heat is joule per kilogram [$\mathrm{J\,kg^{-1}}$].

## 11.2 Vaporisation

Definition 11.5: Evaporation

Evaporation is the process whereby a liquid vaporises at the surface because it has the energy equal or more than that of the latent heat of vaporisation, allowing it to escape into the atmosphere.

Evaporation can happen at any temperature.

Definition 11.6: Boiling

Boiling is the process where a liquid reaches boiling point and the particles have enough energy to vaporise.

Boiling only happens at boiling point.

# Part IV

# Waves

# 12 General Wave Properties

Preamble

Waves are a fundamental method of describing the nature of matter and how it interacts with energy. In this chapter we will be covering general wave properties that would be helpful.

## 12.1 Definitions

Definition 12.1: Wave

A wave is the transfer of energy without the transfer of matter.

Definition 12.2: Transverse Wave

A transverse wave is when the particles oscillate perpendicular to the direction of propagation.

An example of a transverse wave is electromagnetic waves.

Definition 12.3: Longitudinal Wave

A longitudinal wave is when the particles oscillate parallel to the direction of propagation.

An example of a longitudinal wave is sound waves.

## 12.2 Parts of a Wave

### 12.2.1 Common Quantities

Definition 12.4: Amplitude

The amplitude of a wave is the maximum displacement of a particle in a wave. It is usually represented by the letter $A$. The most common unit for amplitude is the metre [m]; though keep in mind other physical quantities like voltage can exhibit periodic wave-like behaviour.

Definition 12.5: Wavelength

The wavelength of a wave is the displacement between two successive in-phase points. It is usually represented by the Greek letter $\lambda$. The SI unit for wavelength is the metre [m].

Definition 12.6: Wavefront

A wavefront is an imaginary line on a wave that joins all adjacent points that are in phase.

### 12.2.2 Time-based Quantities

Definition 12.7: Period

The period of a wave is the time taken for a particle to complete one oscillation. It is usually represented by the letter $T$. The SI unit for period is the second [s].

Definition 12.8: Frequency

The frequency of a wave is the number of times a particle completes one oscillation in one second. It is usually represented by the letter $f$. The SI unit for frequency is the hertz [Hz].

Equation 12.1: Period and Frequency

Period and frequency are reciprocals of each other,

$$f = \frac{1}{T} \Leftrightarrow T = \frac{1}{f}$$

### 12.2.3 Some Things Specific to Longitudinal Waves

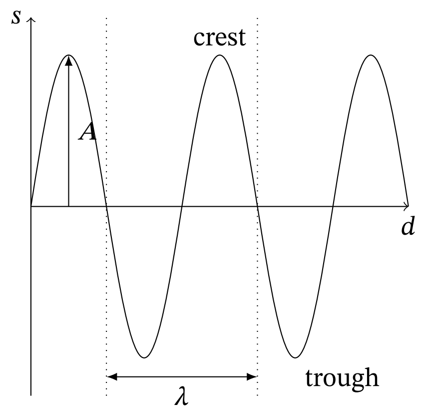
> **Definition 12.9: Compression**
>
> A compression in a longitudinal wave is where there are more particles around that region than in equilibrium.

> **Definition 12.10: Rarefaction**
>
> A rarefaction in a longitudinal wave is where there are less particles around that region than in equilibrium.

## 12.3 Graphs

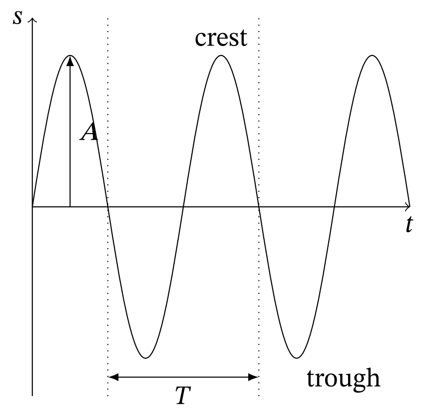
### 12.3.1 Displacement-distance Graph

This is also known as a snapshot graph.




### 12.3.2 Displacement-time Graph

This is also known as a history graph.




## 12.4 Wave Speed

> **Equation 12.2: Wave Speed**
>
> For a wave with frequency $f$ and wavelength $\lambda$, the velocity $v$ it is travelling at is equal to
>
> $$v = f\lambda$$

# 13 Light

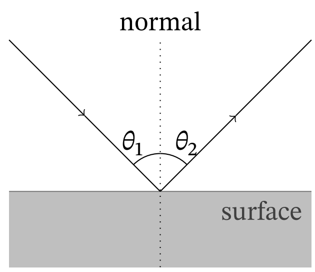
> **Preamble**
>
> Light can be studied as a wave. In this chapter we will look at how light interacts with matter.

## 13.1 Reflection

> **Definition 13.1: Normal**
>
> The normal is an imaginary line draw perpendicular to the surface that reflection is taking place at.

> **Definition 13.2: Angle of Incidence**
>
> The angle of incidence is the angle between the incident ray and the normal.

> **Definition 13.3: Angle of Reflection**
>
> The angle of reflection is the angle between the reflected ray and the normal.




> **Definition 13.4: First Law of Reflection**
>
> The incident ray, reflected ray, and the normal lie on the same plane.

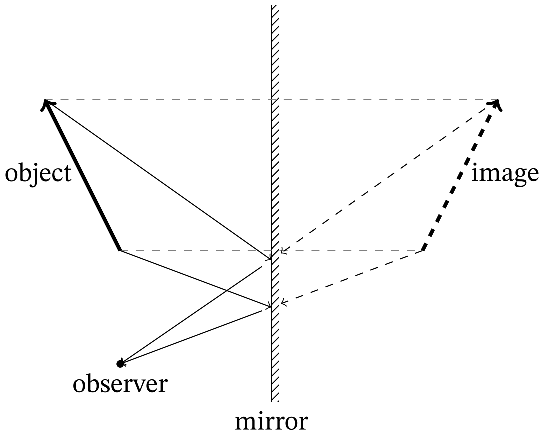
> **Definition 13.5: Second Law of Reflection**
>
> In reflection, the angle of incidence is equal to the angle of reflection.
>
> $$\theta_1 = \theta_2$$

I have chosen to name the angles $\theta_1$ and $\theta_2$ due to the reversible nature of light. It does not matter which way the light goes; the angles will be preserved.

> **Definition 13.6: Virtual Image**
>
> A virtual image is an image that cannot be cast on a screen.

The properties of an reflected image are:

- same shape and size
- same distance from the mirror
- laterally inverted
- upright
- virtual




## 13.2 Refraction

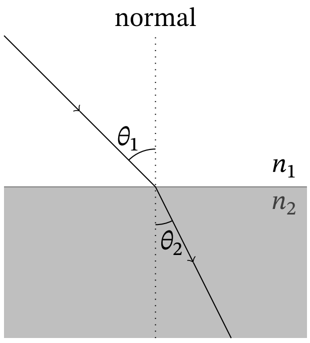
**Definition 13.7: Refraction**

Refraction is the phenomenon where travels from one medium to another medium with different optical densities and slows down.

### 13.2.1 Essentials

**Definition 13.8: Angle of Refraction**

The angle of refraction is the angle between the refracted ray and the normal.




**Definition 13.9: First Law of Refraction**

The incident ray, reflected ray, and the normal lie on the same plane.

**Definition 13.10: Second Law of Refraction**

For two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

**Equation 13.1: Refractive Index**

The refractive index of a medium is the ratio of the speed of light in vacuum to the speed of light in the medium

$$n = \frac{c}{v}$$

Sometimes it might also be

$$n = \frac{\text{real depth}}{\text{apparent depth}}$$

.

**Equation 13.2: Snell's Law**

Snell's Law is the same thing as the second law of refraction, mathematically expressed as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

**Definition 13.11: Critical Angle**

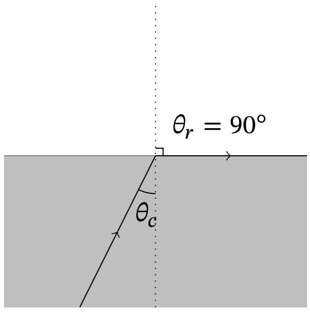
The critical angle is defined as the angle of incidence in an optically denser medium for which the angle of refraction in the optically less dense medium is 90°.

Derivation for critical angle formula for any refractive indices considering $n_1 > n_2$, from equation 13.2,

$$\begin{aligned} n_1 \sin\theta_c &= n_2 \sin 90^\circ \\ \sin\theta_c &= \frac{n_2(1)}{n_1} \\ \sin\theta_c &= \frac{n_2}{n_1} \\ \theta_c &= \sin^{-1}\left(\frac{n_2}{n_1}\right) \end{aligned}$$

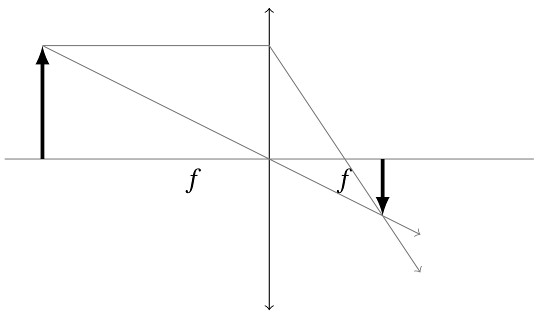
**Definition 13.12: Total Internal Reflection**

Total internal reflection is the complete reflection of a light ray inside an optically denser medium at its boundary with an optically less dense medium.




### 13.2.2 Lenses

For the most part of this section, we will consider a thin lens.




**Definition 13.13: Optical Centre**

The optical centre is the midpoint between the lens' surface on the principal axis.

Rays that travel through the optical centre are not deviated.

**Definition 13.14: Focal Length**

The focal length is the distance between the optical centre and the focal point.

**Equation 13.3: Thin Lens Equation**

(*This is not in syllabus.*) For a thin lens, the focal length and the distances between the object and its image is

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

# 14 Electromagnetic Spectrum

**Preamble**

The electromagnetic spectrum consists of electromagnetic waves of different frequencies. In this chapter we will explore these different frequencies and study some of their uses.

**Definition 14.1: Speed of Light**

All electromagnetic waves travel at the speed of light $c$ in a vacuum.

$$c = 3.0 \times 10^8 \text{ m s}^{-1}$$

## 14.1 Parts of the Electromagnetic Spectrum

In increasing frequency (i.e. decreasing wavelength), and their uses:

- Radio waves (e.g. radio and television communication)
- Microwaves (e.g. microwave oven and satellite television)
- Infra-red (e.g. infra-red remote controllers and intruder alarms)
- Visible light (e.g. optical fibres for medical uses and telecommunications)
- Ultra-violet (e.g. sunbeds and sterilisation)
- X-rays (e.g. radiological and engineering applications)
- Gamma rays (e.g. medical treatment)

## 14.2 Effects of the Electromagnetic Spectrum

When absorbing electromagnetic waves of various frequencies, different effects can be observed.

- Absorbing infrared rays can cause heating
- Higher frequencies such as x-rays can cause ionisation
- Overexposure to ultra-violet and higher frequency rays can lead to damage to living cells and tissue

# 15 Sound

**Preamble**

Sound is transferred in a form of a wave. In this chapter we will explore the different properties of sound and some of its applications.

## 15.1 Fundamentals

Some fundamental properties of sound:

- Sound is produced by a vibrating source.
- Sound exists in the form of a longitudinal wave.
- In different media, sound has different speeds. Generally, the higher the density, the faster the speed of sound.

gases liquids solids

$\longrightarrow$

increasing speed of sound

**Equation 15.1: Speed of Sound**

For a sound source from $d$ away from an observer and capturing it after a time $t$, the speed of sound can be calculated as

$$v = \frac{s}{t}$$

## 15.2 Properties of Sound

**Equation 15.2: Loudness**

The loudness of a sound wave is directly proportional to the square of its amplitude

$$\text{loudness} \begin{cases} \text{louder} & \text{higher } A \\ \text{softer} & \text{softer } A \end{cases}$$

**Equation 15.3: Pitch**

The pitch of a sound is directly proportional to its frequency

$$\text{pitch} \begin{cases} \text{higher} & \text{higher } f \\ \text{lower} & \text{lower } f \end{cases}$$

The human ear can hear sounds from between 20 Hz to 20 kHz.

## 15.3 Applications of Sound

**Definition 15.1: Echo**

An echo is the repetition of a sound due to the reflection of sound.

Echo is used in distance measurement systems such as SONAR in ships.

**Definition 15.2: Ultrasound**

Ultrasound is sound with frequencies above the upper limit of the human range of audibility (i.e. 20 kHz).

Ultrasound is used in product quality control and prenatal scanning.

# Part V

# Electricity and Magnetism

## 16 Static Electricity

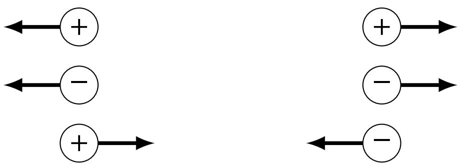
**Preamble**

Static electricity is the study of charges at rest. In this chapter we will explore that very concept.

**Definition 16.1: Charge**

Charge is measured in coulombs [C]. There are positive and negative charges.

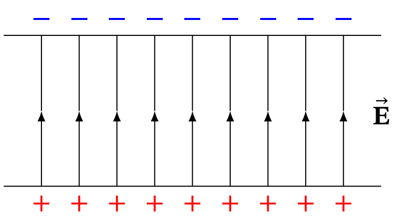
Like charges repel, unlike charges attract.



### 16.1 Electric Fields

**Definition 16.2: Electric Field**

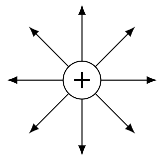
An electric field is a region of space whereby a charge experiences an electric force.



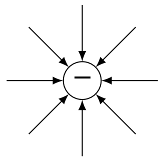

#### 16.1.1 Isolated Charges

Field lines are the path a test charge would take within that electric field. The tighter the field lines are, the stronger the electric field at that area, which means that the test charge would experience a stronger force.

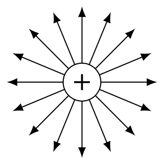
Field lines extend **out** from **positive** charges.



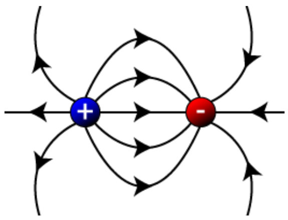
Field lines go **in** to **negative** charges.

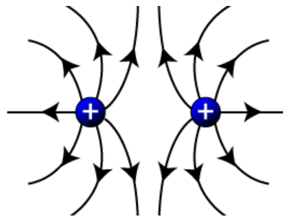


If a charge is stronger, it gets more field lines (e.g. this one has twice the charge as the one above, so it should get more)



Drawing these in TikZ was too difficult so take these from some online website.





### 16.2 Charging

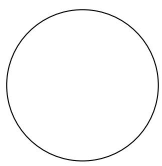
The two methods of charging are **rubbing** and **induction**.

#### 16.2.1 Rubbing

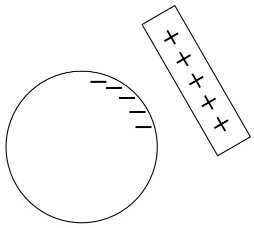
Electrons (negative charges) can be transferred from one object to another through rubbing. There are no movement of positive charges.

#### 16.2.2 Induction

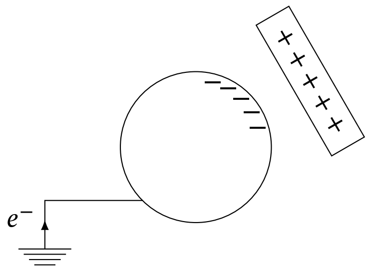
Charging with induction can be achieved for two conductors. The most classic example is the metal sphere case.



Suppose this sphere is overall neutral to begin with. Now a positively charged rod is brought to the sphere. This causes the electrons in the sphere to move towards the positively charged rod.



The sphere is then earthed. Electrons flow from earth up to the sphere.




The rod is then removed, leaving behind a negatively charged sphere.

## 16.3 Discharging

### 16.3.1 Insulators

Insulators can be discharged by **heating** or **providing humid conditions**.

### 16.3.2 Conductors

Conductors can be discharged through a process known as **earthing**. Earthing allows electrons to flow into (in the case of a positively charged object) and out of (in the case of a negatively charged object) the object.

## 16.4 Applications and Hazards of Electrostatic Charging

### 16.4.1 Applications

An application of electrostatics is in spray painting.

In spray painting, the object to be painted is charged. The paint will then be charged with the opposite charge, and allowing the paint to attract to the object's surface, allowing for a better coat and efficient painting.

### 16.4.2 Hazards

Lightning is a danger that is caused by electrostatic charging.

Charges build up in clouds due to friction between air and water molecules, which causes in ionised (charged) air which allows a conductive path between the charges built up in the clouds and ground, causing lightning.

This can be resolved by installing conductive lightning rods on high objects such as buildings to safely ground these large releases of electric energy.

# 17 Current of Electricity

**Preamble**

Current is the rate of flow of charge. When charges move there is current and hence we name this current electricity. In this chapter we will explore the fundamentals that govern current electricity.

## 17.1 Current

**Definition 17.1: Current**

Current is the rate of flow of charge.

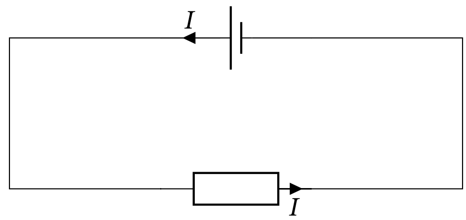
$$I = \frac{Q}{t}$$

The SI unit of current is ampere [A].

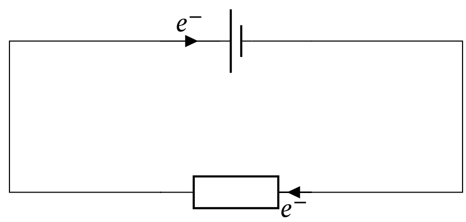
Current is measured with an ammeter.

### 17.1.1 Current Flow

Conventional current is where current flows from a higher voltage to a lower voltage.




Electron flow is the opposite of that.




## 17.2 Electromotive Force and Potential Difference

**Definition 17.2: Electromotive Force**

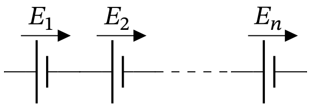
Electromotive force (e.m.f.) is the work done by a source in driving unit charge around a complete circuit.

$$E = \frac{W}{Q}$$

The SI unit of electromotive force is volt [V].

**Equation 17.1: Electromotive Forces in Series**

If multiple electromotive force sources are arranged in series




then the net electromotive force is

$$E_{\text{net}} = E_1 + E_2 + \cdots + E_n$$

**Definition 17.3: Potential Difference**

The potential difference (p.d.) (or voltage) across a component in a circuit as the work done to drive unit charge through the component.

$$V = \frac{W}{Q}$$

The SI unit of potential difference is volt [V].

## 17.3 Resistance

**Definition 17.4: Resistance**

The resistance of a component is the ratio of the potential difference across it to the current flowing through it.

$$R = \frac{V}{I}$$

The SI unit of resistance is ohm [Ω].

**Definition 17.5: Ohm's Law**

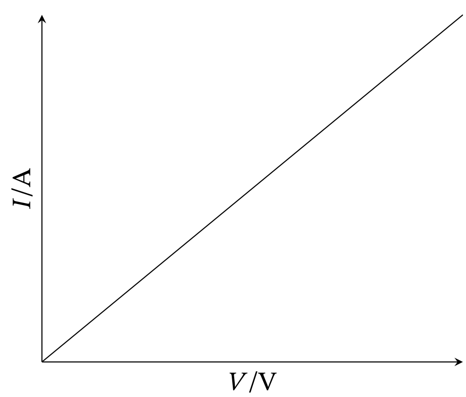
Ohm's Law states that the current passing through a metallic conductor is directly proportional lo the potential difference across it, provided that physical conditions (such as temperature) remain constant.

$$V = IR$$

**Definition 17.6: Ohmic Conductors**

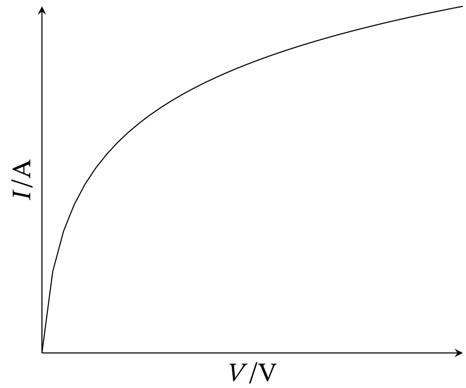
Ohmic conductors are conductors that obey Ohm's law.

An ohmic conductor might exhibit an $I-V$ graph as such:




Notice that the graph is **linear** and starts at the **origin**.

On the other hand, non-ohmic conductors may exhibit such a characteristic curve:




Notice that the graph is **not linear**.

## 17.4 Resistivity

**Definition 17.7: Resistivity**

Resistivity is the property of a material that determines its resistance when made into a wire or electrical component. The SI unit of resistivity is ohm metre [Ω m].

**Equation 17.2: Resistance of a Wire**

The resistance of the wire with length $l$, cross-sectional area $A$, and resistivity $\rho$ is equal to

$$R = \frac{\rho l}{A}$$

Rewritring this equation making $\rho$ the subject gives us

$$\rho = \frac{AR}{l}$$

Temperature affects resistance. The higher the temperature of a conductor, the higher its resistance.

$$R \begin{cases} \text{high} & \text{higher } T \\ \text{low} & \text{lower } T \end{cases}$$

This is not to be confused with the behaviour of a thermistor (chapter 18).

# 18 DC Circuits

**Preamble**

Most things at our homes run on direct current (DC). In this chapter we will explore how DC circuits behave and how it is used to make the many circuits and electronic devices around us.

**Equation 18.1: Kirchhoff's Current Law**

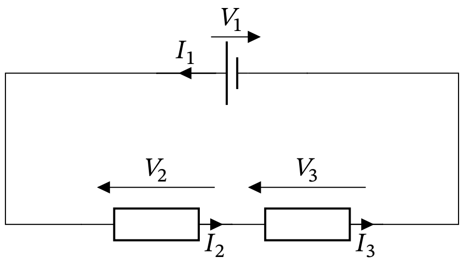
*(This isn't in syllabus.)* The current flowing in a junction must equal to the current flowing out of a junction.

$$\Sigma I_{\text{node}} = 0$$

> **Equation 18.2: Kirchhoff's Voltage Law**
>
> *(This isn't in syllabus.)* The algebraic sum of voltages in a loop/mesh is equal to zero.
>
> $$\Sigma V_{\text{mesh}} = 0$$

## 18.1 Series Circuits

We will look at this series circuit for this subsection.




### 18.1.1 Current

Current in a series circuit is always the same. In the case of the circuit above,

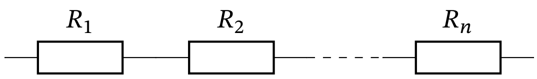
$$I_1 = I_2 = I_3$$

### 18.1.2 Voltage

The sum of voltages across components in a series circuit is equal to the voltage across the entire circuit. In the case of the circuit above,

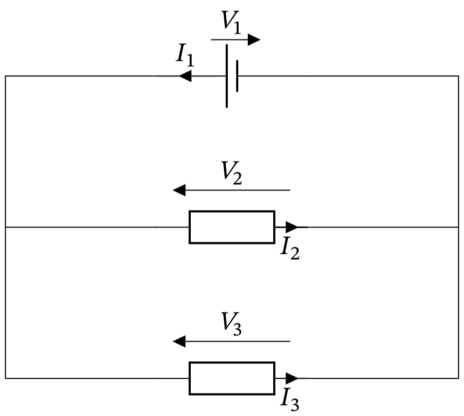
$$V_1 = V_2 + V_3$$

### 18.1.3 Resistance

> **Equation 18.3: Resistance in Series**
>
> If multiple resistors are arranged in series
>
> 
> 
>
> then the net resistance is
>
> $$R_{\text{net}} = R_1 + R_2 + \cdots R_n$$

## 18.2 Parallel Circuits

We will look at this series circuit for this subsection.




### 18.2.1 Current

The sum of individual currents in each parallel branch is equal to the main current flowing into or out of parallel branches. In the case of this circuit,

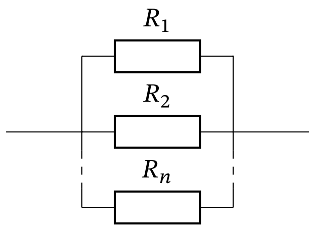
$$I_1 = I_2 + I_3$$

### 18.2.2 Voltage

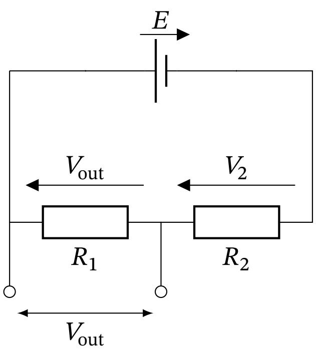
The voltages across parallel branches are the same. In the case of this circuit,

$$V_1 = V_2 = V_3$$

### 18.2.3 Resistance

> **Equation 18.4: Resistance in Parallel**
>
> If multiple resistors are arranged in parallel
>
> 
> 
>
> then the net resistance is
>
> $$R_{\text{net}} = \left(R_1^{-1} + R_2^{-1} + \cdots + R_n^{-1}\right)^{-1}$$

## 18.3 Voltage Divider




$$V_{\text{out}} = \frac{R_1}{R_1 + R_2} \times E$$

> **Equation 18.5: Voltage Divider**
>
> For a resistor $R_x$ in a series circuit with total resistance $R_T$, the voltage across the resistor $R_x$ is
>
> $$V_x = \frac{R_x}{R_T} \times E$$

## 18.4 Input and Output Transducers

**Definition 18.1: Input Transducer**

An input transducer is an electronic device that converts non-electrical energy into electrical energy.

We will look at two input transducers: (NTC-) thermistors and light dependent resistors (LDR).

Thermistors are devices which vary its resistance according to temperature. As the temperature increases, the resistance decreases.

$$R_{\mathrm{TH}} \begin{cases} \uparrow & T \downarrow \\ \downarrow & T \uparrow \end{cases}$$

Light-dependent resistors (LDR) varies its resistance according to the light intensity shining on it. As the light intensity shining on it increases, the resistance decreases.

$$R_{\mathrm{LDR}} \begin{cases} \uparrow & \text{light intensity} \downarrow \\ \downarrow & \text{light intensity} \uparrow \end{cases}$$