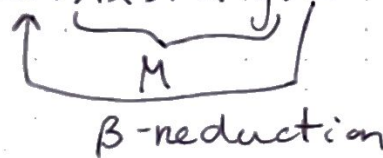


TP 4

λ -Calcul

- Variables: x, y, z, \dots
- Abstractions: $\lambda x. M$ représente la fonction $x \mapsto M$
"comme" $f(x) = M$ mais on laisse la fonction anonyme.
- Applications: MN c'est M appliqué sur N
"comme" $M(N)$
- α -réduction: Renommage
 $\lambda x. M \equiv \lambda y. M[x := y]$
ex.: $\lambda x. xx \equiv \lambda y. yy$
- β -réduction: $((\lambda x. M) N) \equiv M[x := N]$
"comme" si $f(x) = M$ et on évalue $f(N)$.
ex.: $(\lambda x. \lambda x. fx y)z = \lambda z. fz y$



- Notation / convention:
 - $MNR = (MN)R$
 - $\lambda xy. M = \lambda x. \lambda y. M$
 - $\lambda x. MN = \lambda x. (MN)$
- Encodage des booléens: VRAI et FAUX
- VRAI: $\lambda xy. x \quad (x, y) \mapsto x$
FAUX: $\lambda xy. y \quad (x, y) \mapsto y$

#1 λ -Calcul VRAI \wedge FAUX

ET := $\lambda p q. p q p$ ($p, q \mapsto p q p$ p, q vont être des fonctions)

Façon rapide:

$$\begin{aligned} & \text{ET VRAI FAUX} \\ &= (\lambda p q. p q p) (\text{VRAI}) (\text{FAUX}) \\ &= (\text{VRAI}) (\text{FAUX}) (\text{VRAI}) \\ &= (\lambda x y. x) (\text{FAUX}) (\text{VRAI}) \\ &= \text{FAUX} \end{aligned}$$

Façon longue:

$$\begin{aligned} & \text{ET VRAI FAUX} \\ &= (\lambda p q. p q p) (\lambda x y. x) (\lambda x y. y) \\ &= (\lambda p. \lambda q. p q p) (\lambda x y. x) (\lambda w z. z) \quad \downarrow \alpha\text{-reduc} \\ &= (\lambda q. (\lambda x y. x) q (\lambda x y. x)) (\lambda w z. z) \quad \uparrow \beta\text{-reduc} \\ &= (\lambda x y. x) (\lambda w z. z) (\lambda x y. x) \quad \uparrow \beta\text{-reduc} \\ &= (\lambda x. \lambda y. x) (\lambda w. \lambda z. z) (\lambda p q. p) \quad \downarrow \alpha\text{-reduc} \\ &= (\lambda y. (\lambda w. \lambda z. z)) (\lambda p q. p) \quad \uparrow \beta\text{-reduc} \\ &= (\lambda w. \lambda z. z) \\ &= \text{FAUX} \end{aligned}$$

Encodage de Church

$$0 := \lambda f. \lambda x. x \quad "f \mapsto (x \mapsto x)"$$

$$1 := \lambda f. \lambda x. f x \quad "f \mapsto (x \mapsto f(x))"$$

$$2 := \lambda f. \lambda x. f(f x) \quad "f \mapsto (x \mapsto f(f(x)))"$$

$$\vdots$$

$$n := \lambda f. \lambda x. f(f \dots (f x)) \quad "f \mapsto (x \mapsto f^{<n>}(x))"$$

#2 λ -Calcul

$$2.3 \quad \text{MULT} := \lambda m n. \lambda f. m(n f)$$

MULT 2 3

$$= (\lambda m n. \lambda f. m(n f)) (\lambda f. \lambda x. f(f x)) (\lambda f. \lambda x. f(f(f x)))$$

$$= (\lambda m. \lambda n. \lambda f. (m(n f))) (\lambda g. \lambda x. g(g x)) (\lambda h. \lambda y. h(h(h y)))$$

$$= (\lambda n. \lambda f. ((\lambda g. \lambda x. g(g x))(n f))) (\lambda h. \lambda y. h(h(h y)))$$

$$= (\lambda f. ((\lambda g. \lambda x. g(g x))((\lambda h. \lambda y. h(h(h y))) f)))$$

$$= \lambda f. ((\lambda g. \lambda x. g(g x))(\lambda y. f(f(f y))))$$

$$= \lambda f. (\lambda x. (\lambda y. f(f(f y))) (\lambda y. f(f(f y)) x))$$

$$= \lambda f. (\lambda x. (\lambda y. f(f(f y))) (f(f(f x))))$$

$$= \lambda f. (\lambda x. (f(f(f(f(f(f x)))))))$$

$$= \lambda f. \lambda x. f(f(f(f(f(f x)))))$$

$$= 6 \text{ selon l'encodage de Church}$$