# Heap Sort

Plan of attack

- First, we will learn how to turn a binary tree into a heap

- Next, we will learn how to turn a binary tree *back* into a heap after it has been changed in a certain way

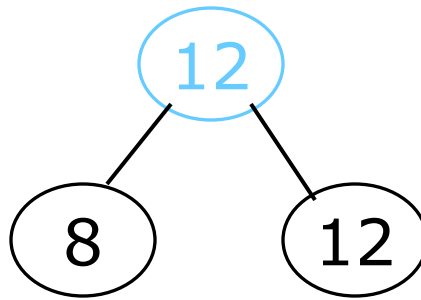- Finally (this is the cool part) we will see how to use these ideas to sort an *array*

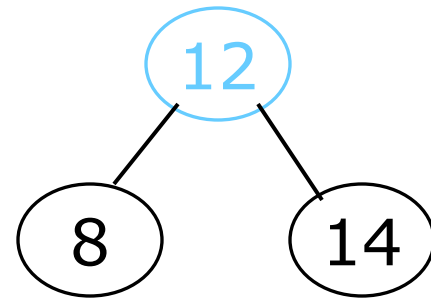All Examples with MaxHeap

The heap property

- A node has the heap property if the value in the node is as large as or larger than the values in its children
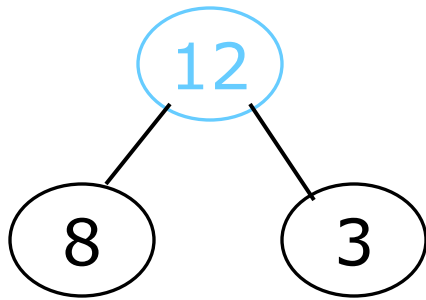

Blue node has heap property

Blue node has heap property

Blue node does not have heap property

- All leaf nodes automatically have the heap property
- A binary tree is a heap if *all* nodes in it have the heap property

siftUp

- Given a node that does not have the heap property, you can give it the heap property by exchanging its value with the value of the larger child

Blue node does not have heap property

Blue node has heap property
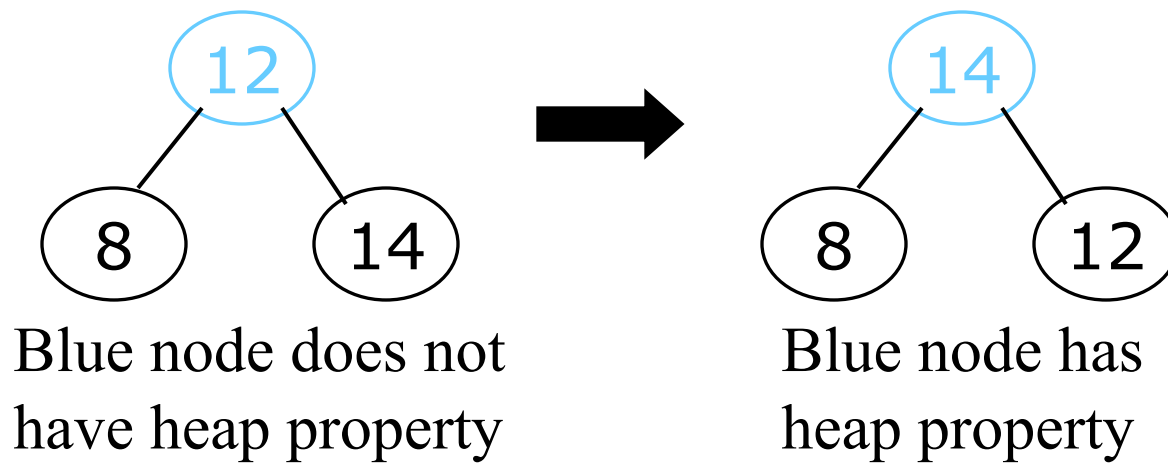
- This is sometimes called sifting up
- Notice that the child may have *lost* the heap property

Constructing a heap (Step I)

- A tree consisting of a single node is automatically a heap
- We construct a heap by adding nodes one at a time:
  - Add the node just to the right of the rightmost node in the deepest level
  - If the deepest level is full, start a new level

- Examples:

Add a new node here

Add a new node here
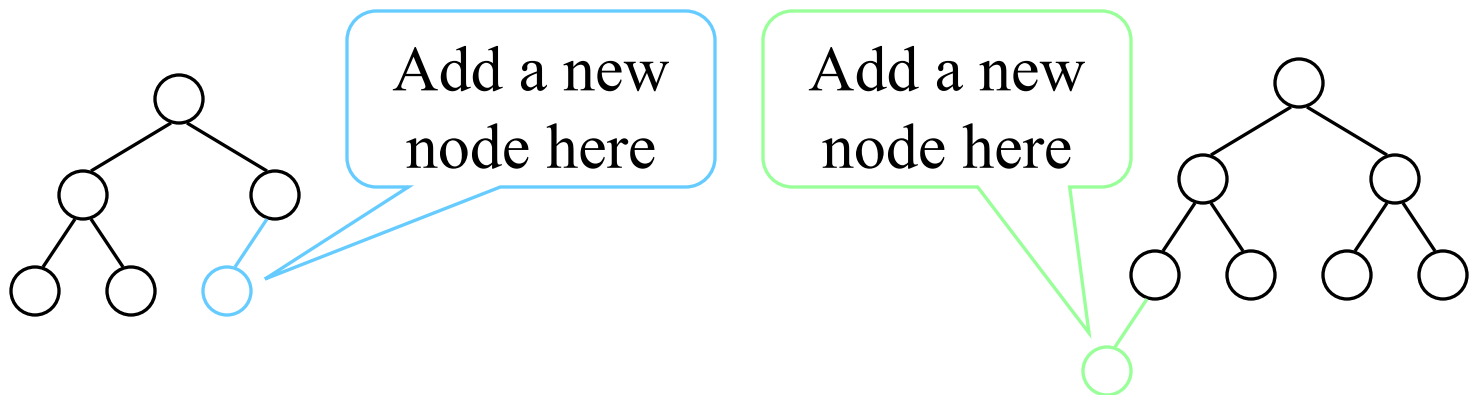
Constructing a heap (Step II)

- Each time we add a node, we may destroy the heap property of its parent node

- To fix this, we sift up

- But each time we sift up, the value of the topmost node in the sift may increase, and this may destroy the heap property of *its* parent node

- We repeat the sifting up process, moving up in the tree, until either

  – We reach nodes whose values don't need to be swapped (because the parent is *still* larger than both children), or

  – We reach the root

Constructing a heap III
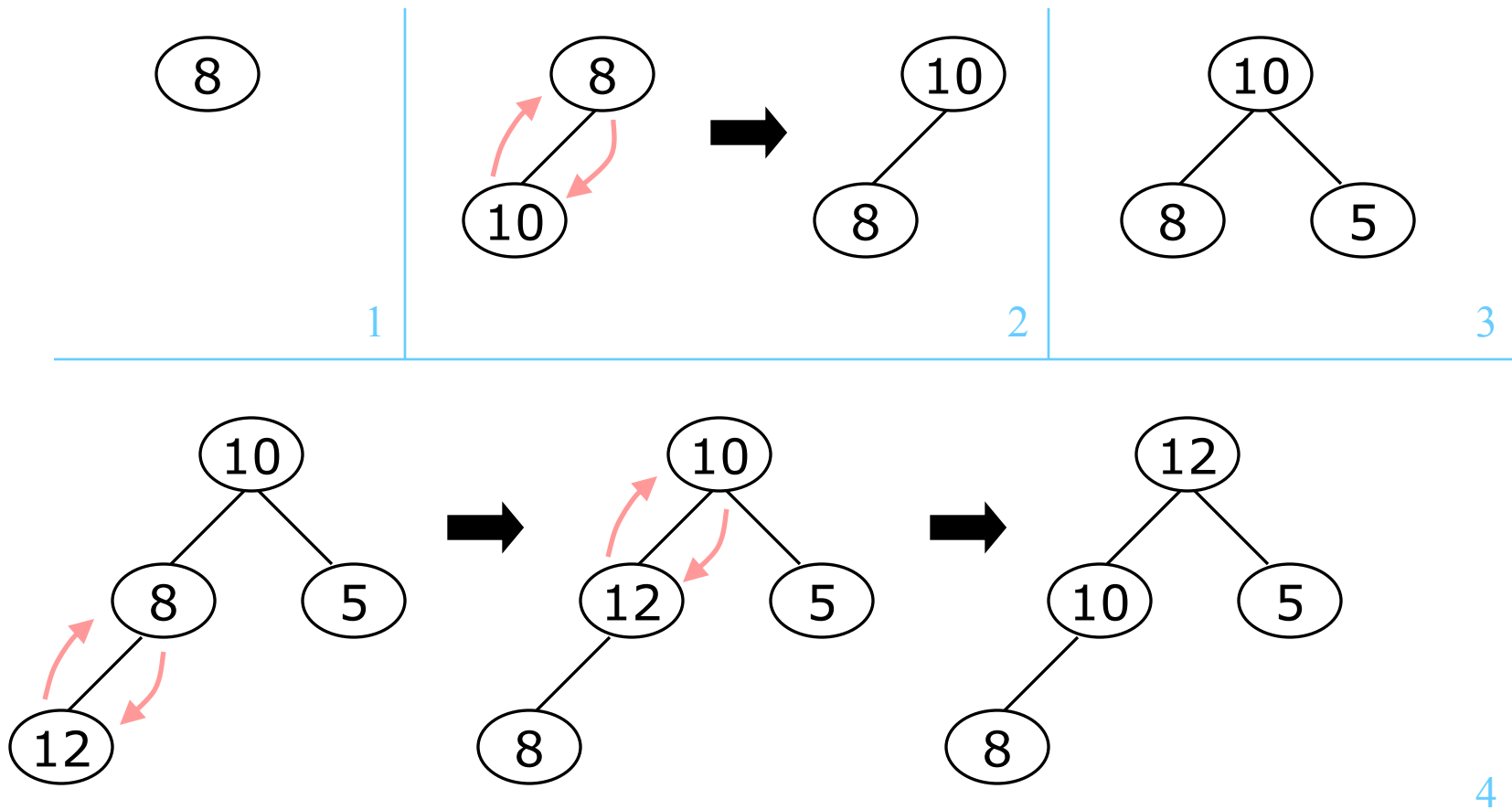
# Other children are not affected
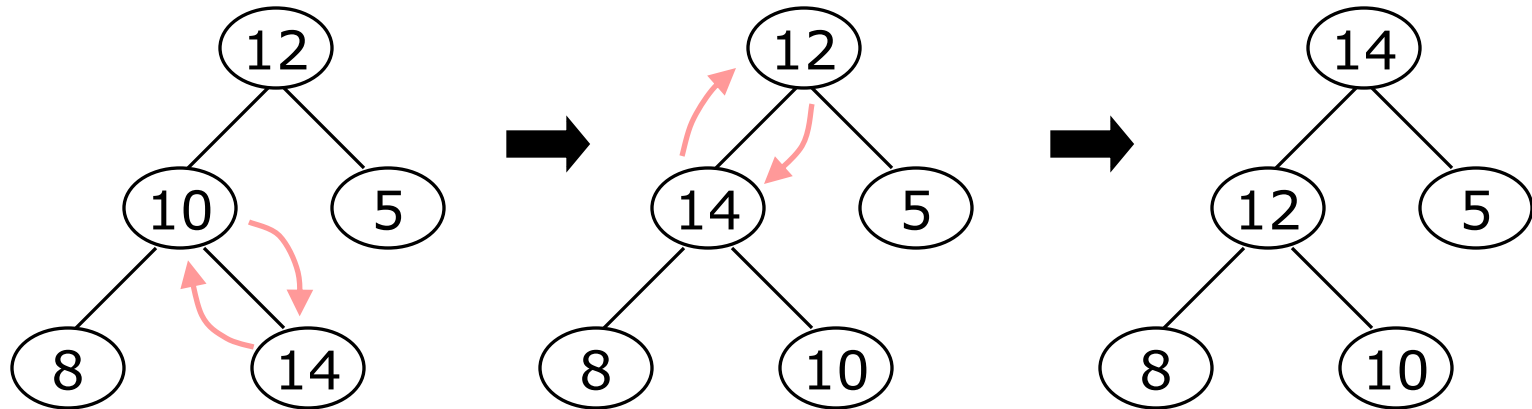
- The node containing 8 is not affected because its parent gets larger, not smaller
- The node containing 5 is not affected because its parent gets larger, not smaller
- The node containing 8 is still not affected because, although its parent got smaller, its parent is still greater than it was originally

A sample heap

- Here's a sample binary tree after it has been heapified
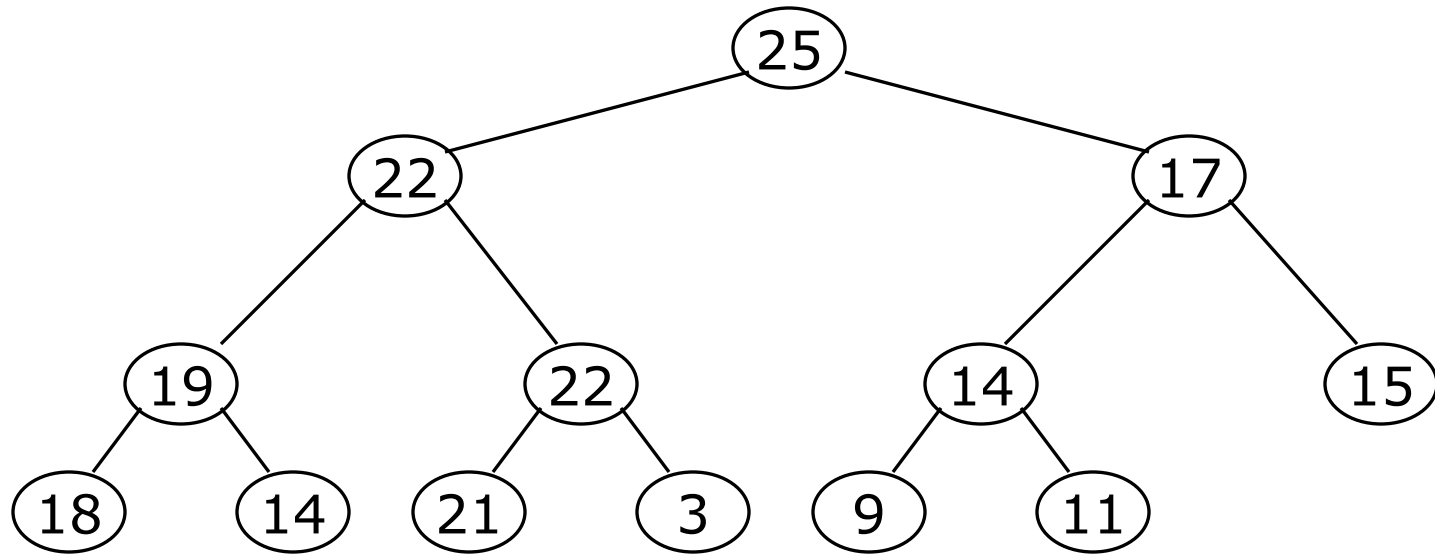
- Notice that heapified does *not* mean sorted
- Heapifying does *not* change the shape of the binary tree; this binary tree is balanced and left-justified because it started out that way

Removing the root (Step I)

- Notice that the largest number is now in the root
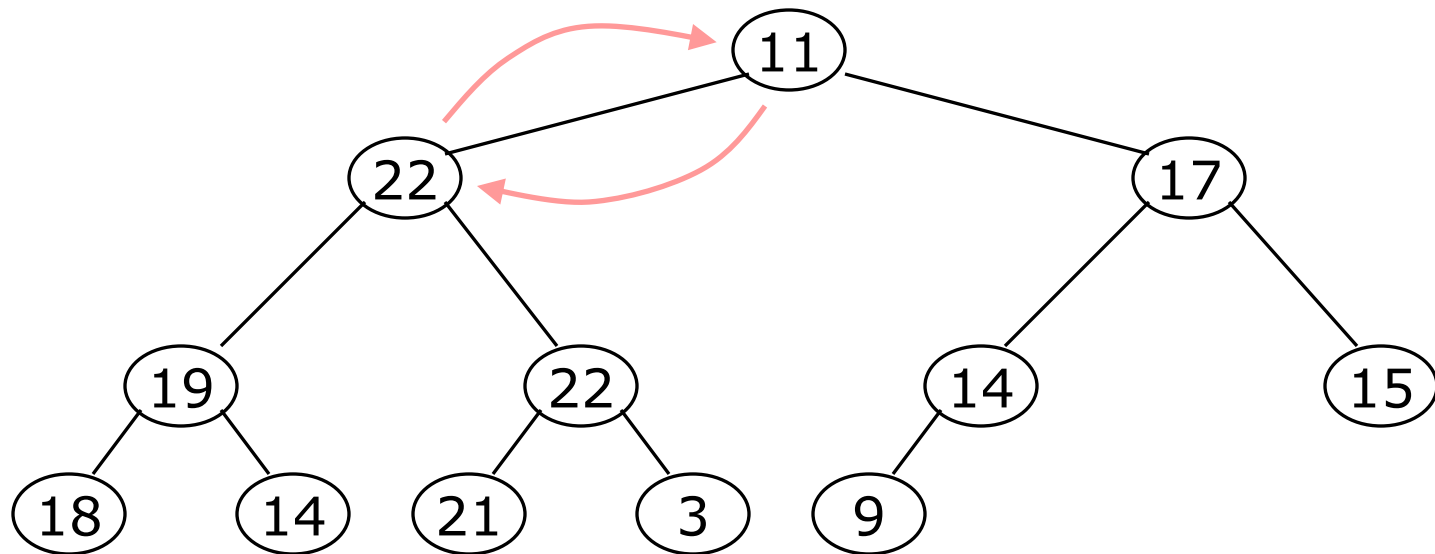- Suppose we *discard* the root:
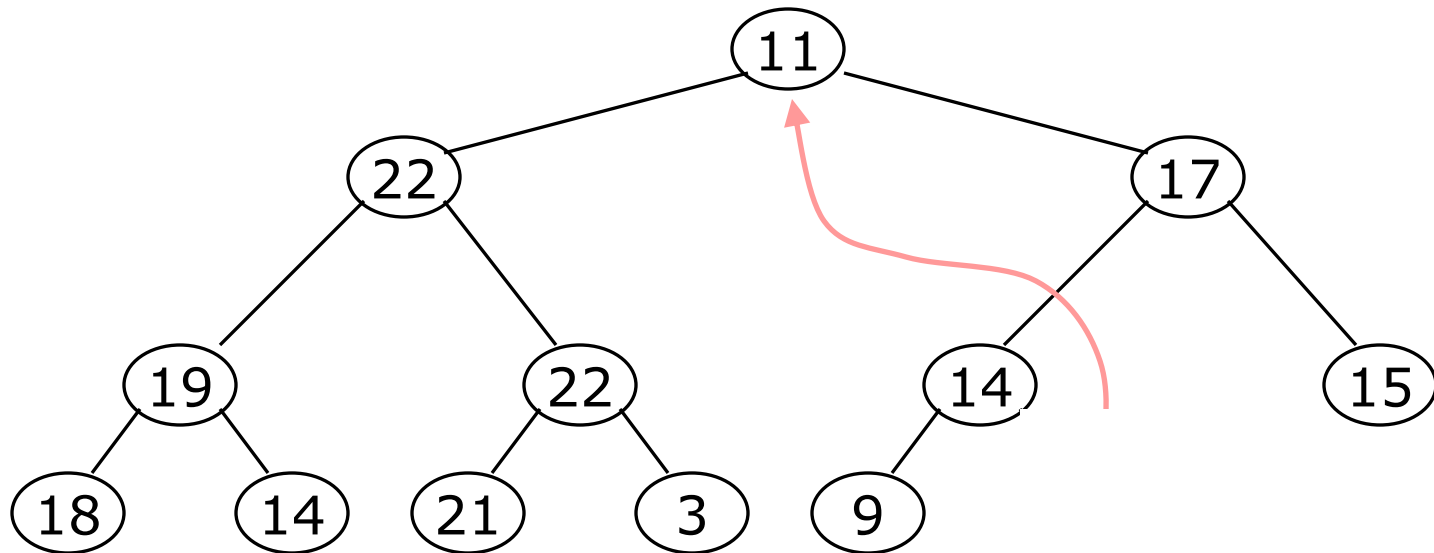
- Remove the rightmost leaf at the deepest level and use it for the new root

The reHeap method (Step II)

- Our tree is balanced and left-justified, but no longer a heap
- However, *only the root* lacks the heap property

- We can siftUp() the root
- After doing this, one and only one of its children may have lost the heap property

The reHeap method (Step II)

- Now the left child of the root (still the number 11) lacks the heap property
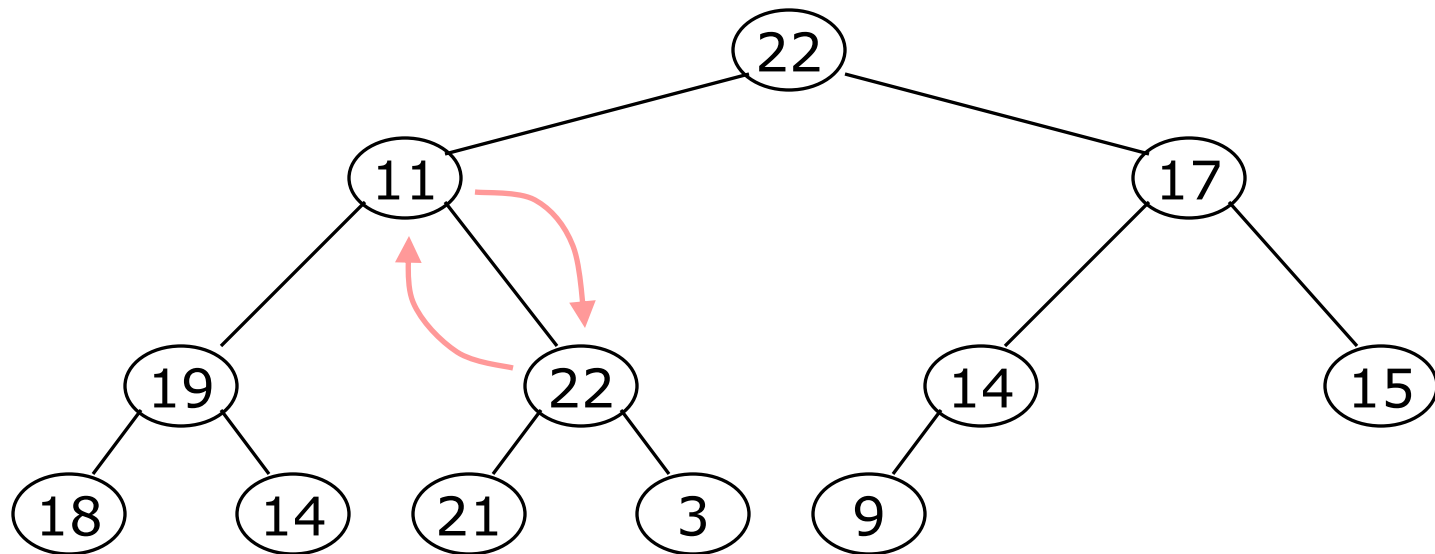
- We can siftUp() this node
- After doing this, one and only one of its children may have lost the heap property

The reHeap method (Step II)

- Now the right child of the left child of the root (still the number 11) lacks the heap property:
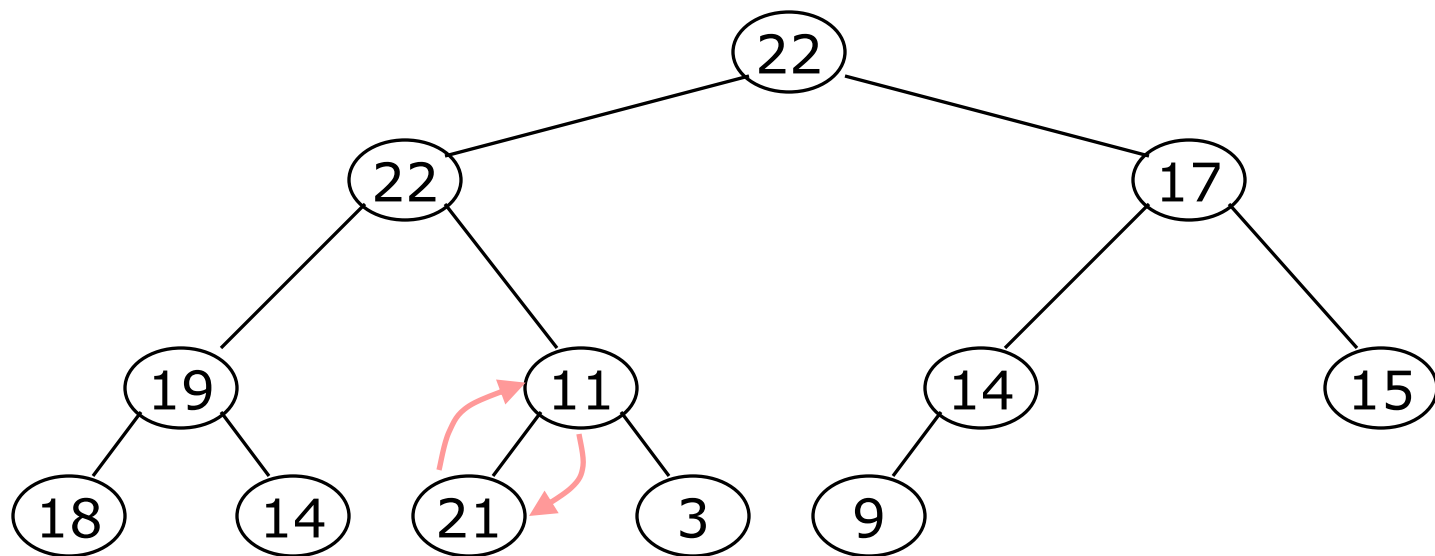
- We can siftUp() this node
- After doing this, one and only one of its children may have lost the heap property —but it doesn't, because it's a leaf

The reHeap method (Step II)

- Our tree is once again a heap, because every node in it has the heap property
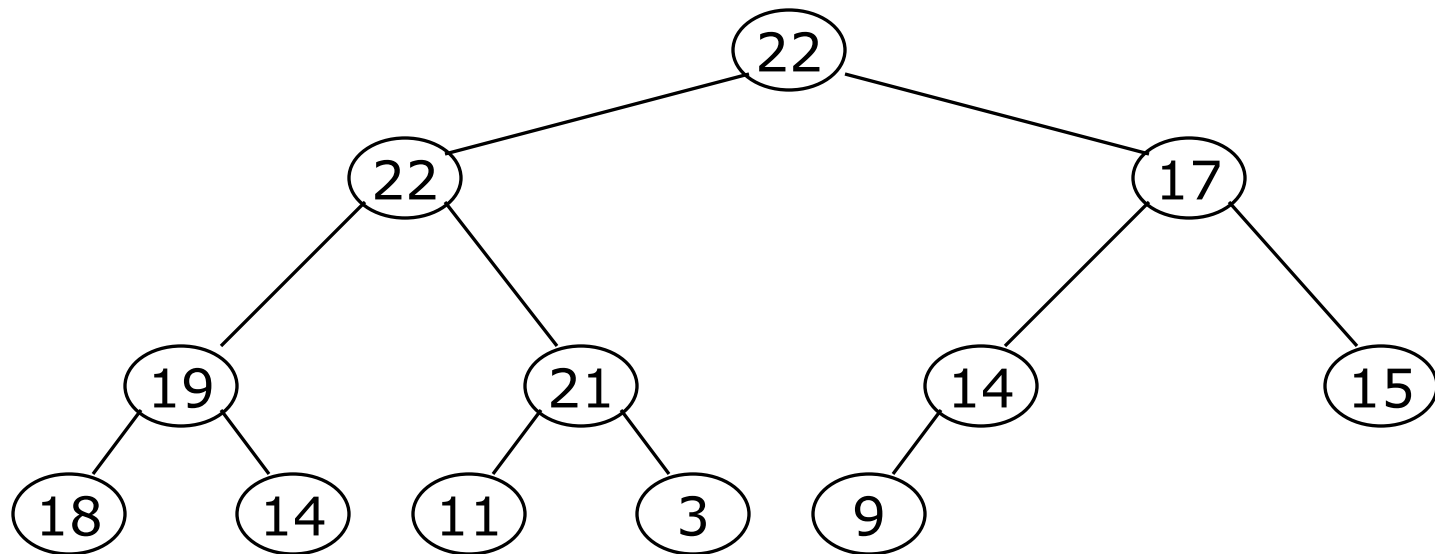
- Once again, the largest (or *a* largest) value is in the root
- We can repeat this process until the tree becomes empty
- This produces a sequence of values in order largest to smallest

Sorting

- What do heaps have to do with sorting an array?
- Here's the neat part:
  - Because the binary tree is *balanced* and *left justified,* it can be represented as an array
  - All our operations on binary trees can be represented as operations on *arrays*
  - To sort:

    heapify the array;
    while the array isn't empty {
        remove and replace the root;
        reheap the new root node;
    }

Mapping into an array

0	1	2	3	4	5	6	7	8	9	10	11	12
25	22	17	19	22	14	15	18	14	21	3	9	11
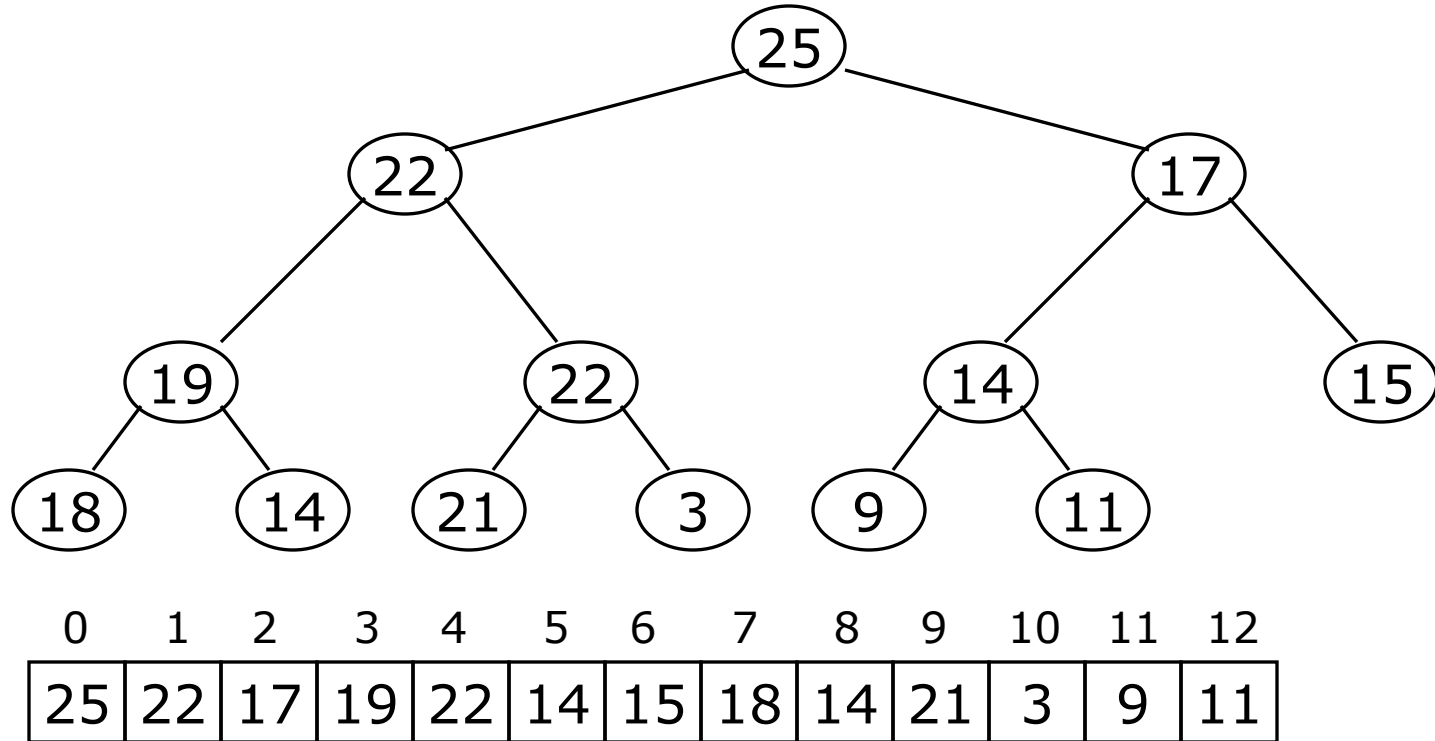
- Notice:
  - The left child of index i is at index 2*i+1
  - The right child of index i is at index 2*i+2
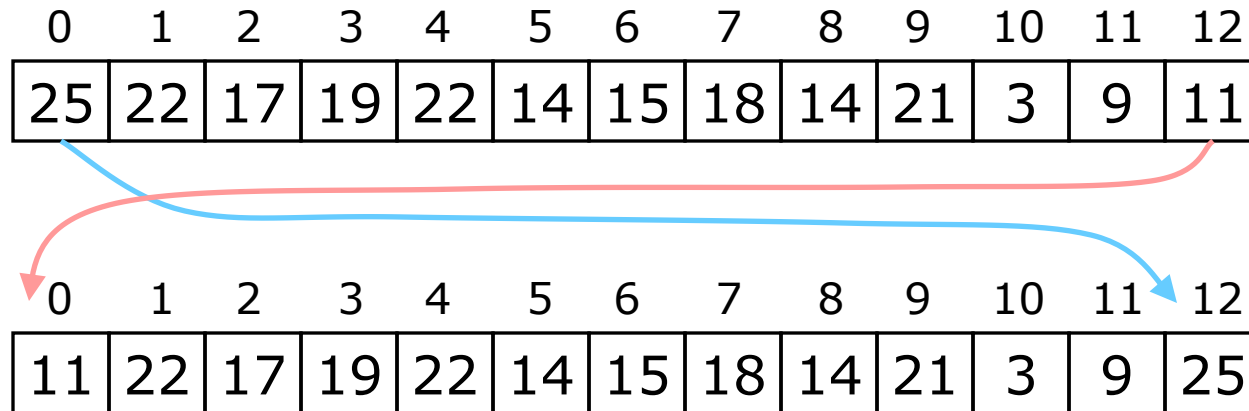 - Example: the children of node 3 (19) are 7 (18) and 8 (14)

Removing and replacing the root

- The "root" is the first element in the array

- The "rightmost node at the deepest level" is the last element

- Swap them...

0	1	2	3	4	5	6	7	8	9	10	11	12
25	22	17	19	22	14	15	18	14	21	3	9	11

0	1	2	3	4	5	6	7	8	9	10	11	12
11	22	17	19	22	14	15	18	14	21	3	9	25

- ...And pretend that the last element in the array no longer exists—that is, the "last index" is 11 (9)

Reheap and repeat

- Reheap the root node (index 0, containing 11)…

0	1	2	3	4	5	6	7	8	9	10	11	12
11	22	17	19	22	14	15	18	14	21	3	9	25

0	1	2	3	4	5	6	7	8	9	10	11	12
22	22	17	19	21	14	15	18	14	11	3	9	25

0	1	2	3	4	5	6	7	8	9	10	11	12
9	22	17	19	22	14	15	18	14	21	3	22	25
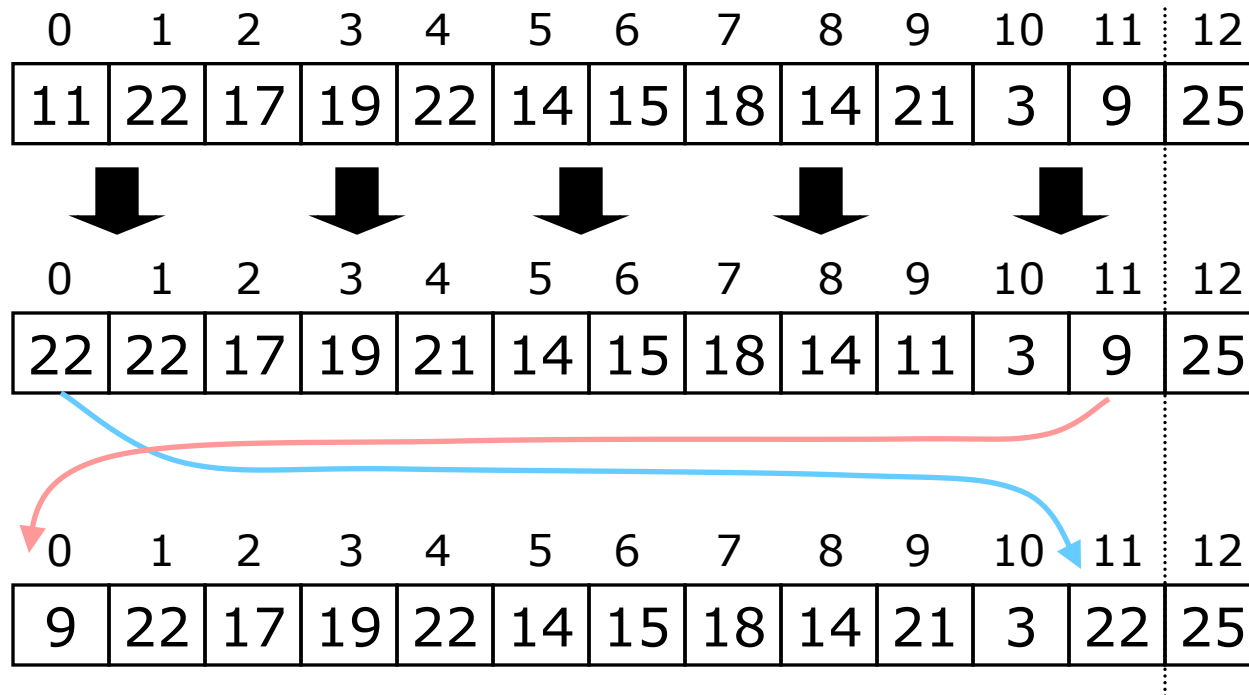
- …And again, remove and replace the root node
- Remember, though, that the "last" array index is changed
- Repeat until the last becomes first, and the array is sorted!