

# 1 Problem

## 1.1 General Problem

We formulate our problem with the following five concepts:

- **State**( $\mathbb{S}$ ) including a **initial state** ( $s_0$ ) the agent begins with
- **Actions**( $\mathbb{A}$ ) available to agent at each State
- **Transition function** ( $F : \mathbb{S} \times \mathbb{A} \rightarrow \mathbb{S}$ ) that takes a (state, action) pair and return a new state
- **Goal Test**( $GT : S \rightarrow \{True, False\}$ ) that takes a state  $s$  and return true if  $s \in S_{goal}$ , the **Goal State** of the problem.
- **Path Cost** ( $C : \mathbb{S} \times \mathbb{S} \rightarrow \mathbb{R}$ ) That takes two states and return cost moving from one to another

## 1.2 Single-player Chexers

In this section, we formally describe how the above framework fits the Chexers game.

- Denote set of pieces with  $\mathbb{P} = \{(r, q, t)\}$ .  
 where

$r, q, -(r + q) \in [-3, 3]$  denotes the location on board.  
 $t \in \{red, blue, green, block\}$  stands for type of piece.

- $\mathbb{S} = \{p_i \in \mathbb{P}, t_b | i \leq n\}$  where  $n$  is number of pieces on board, where  $t_b$  is the searching type.
- $\mathbb{A} = \{(Move, p_i), (Jump, p_i), (Exit, p_i)\}, \forall p_i$  where  $t(p_i) = t_b, |\mathbb{A}_s| \leq 6n_p \forall s$ .
- $f(s, a) = s'$ . where  $s'$  differs exactly by one piece  $r(p_{i,s}), q(p_{i,s}) \neq r(p_{i,s'}), q(p_{i,s'})$  or  $p_i \notin s'$
- $c(s, s') = 1, \forall s, s'$  if  $\exists a$  such that  $f(s, a) = s'$

# 2 Search

## 2.1 preliminary

We define the following algorithm 1 General A\* search.

**Applying A\*** Based on our problem specification, we have all required operations except for H(node)

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**Algorithm 1** General A\* algorithm

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PRIORITY-QUEUE	▷ <b>Min</b> Priority Queue (min key)
ADD( $q$ , $key$ , $value$ ),	▷ Add ( $key$ , $value$ ) to $q$ . If $value$ exists, update the $key$
POP( $q$ )	▷ Pop the $value$ with <b>least</b> $key$
GET( $q$ , $value$ )	▷ Get the $key$ associated with a $value$
	▷ All above Queue operations can operate in $\Theta(1)$
NODE	▷ stores associated <b>state</b> and its <b>parent</b>
<b>Require:</b> EXPAND( $node$ )	▷ expand a <b>node</b> to get its <b>children</b>
<b>Require:</b> G( $node$ )	▷ get <b>total cost</b> arriving this $node$
<b>Require:</b> H( $node$ )	▷ Computes an <b>admissible estimation</b> of cost to goal state
<b>Require:</b> PATH( $node1$ , $node2$ )	▷ Path Cost arriving $node2$ from $node1$

  

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1: procedure A*( $problem$ ,  $initial$ )
2:    $openSet \leftarrow$  PRIORITY-QUEUE(H( $initial$ ),  $initial$ )
3:    $closedSet \leftarrow \{\}$ 
4:   while  $openSet$  is not empty do
5:      $node \leftarrow$  POP( $openSet$ )
6:     ADD( $closedSet$ ,  $node$ )
7:     if GOAL-TEST( $node$ ) is True then
8:       return  $node$ 
9:     end if
10:    for  $child$  in EXPAND( $node$ ) do
11:       $cost = G(node) + PATH(node, child) + H(child)$ 
12:      if  $child$  in  $closedSet$  then continue
13:      else if  $child$  not in  $openSet$  or  $cost < GET(openSet, child)$  then
14:        ADD( $openSet$ ,  $cost$ ,  $child$ )
15:      end if
16:    end for
17:  end while
18:  return no solution
19: end procedure
20: All operations  $O(1)$  unless explicitly stated
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