Homework 2

Exercise 1 (0.2 point). (a) Fill in the table indicating by plus (respectively minus) that we can write (respectively cannot write) $A = C(B)$, where A and B are functions and C is a notation from set $\{O, o, \Omega, \omega, \Theta\}$.

A	B	O	o	Ω	ω	Θ
$f(n)$	$g(n)$					
$g(n)$	$f(n)$					

In some cases l'Hopital's rule may be helpful as well as formulas

$$(a^x)' = a^x \ln a \quad \text{and} \quad (\log_a x)' = \frac{1}{x \ln a}.$$

(b) Find maximal n values such that functions f and g satisfy inequalities $f(n) \leq 1000000$ and $g(n) \leq 1000000$, i.e., indicate numbers N_f and N_g such that $f(N_f) \leq 1000000$ and $g(N_g) \leq 1000000$, but $f(N_f + 1) > 1000000$ and $g(N_g + 1) > 1000000$.

See Section 3.1 in Cormen book and Homework2_tutorial.

Problems

1. $f(n) = 100 \log_2 \log_2 n, \quad g(n) = \sqrt{2 \log_2 n}.$
2. $f(n) = 10 \log_2(2^{\sqrt{n}}), \quad g(n) = 100 \sqrt[3]{n}.$
3. $f(n) = \log_2^2 n, \quad g(n) = 2^{\log_2 \log_2 n^2}.$
4. $f(n) = 100n \log_2 n, \quad g(n) = \frac{n^2}{\log_2^2 n}.$
5. $f(n) = 100n \log_2 n, \quad g(n) = 2^{1.2 \log_2 n}.$
6. $f(n) = 10n \log_2 n + 0.5n^2, \quad g(n) = 100n\sqrt{n}.$
7. $f(n) = 4^{\sqrt{n}}, \quad g(n) = 2^{n/\log_2 n}.$
8. $f(n) = 10n^2 + 100n \log_2^2 n, \quad g(n) = n^2 \log_2 n.$
9. $f(n) = \frac{n}{100 \log_2^2 n}, \quad g(n) = n^{1/2}.$
10. $f(n) = \frac{n}{100 \log_2 n}, \quad g(n) = 10n^{2/3}.$
11. $f(n) = 10n\sqrt{n}, \quad g(n) = n \cdot (2^{\log_2 \sqrt{n}} + 128).$
12. $f(n) = 10n, \quad g(n) = n + \log_2^2 n^4.$
13. $f(n) = 10n, \quad g(n) = n + 100n^{2/3}.$

14. $f(n) = 10n\log_2 n, \quad g(n) = n^2 \cdot 2^{-\log_2\log_2 n}$.

15. $f(n) = \sqrt{n^3 \log_2 n}, \quad g(n) = 10n\sqrt{n} + 100n$.

16. $f(n) = 2^{\log_2^2 n}, \quad g(n) = 10n^{10}$.

17. $f(n) = (\sqrt{2})^{\log_2 n}, \quad g(n) = \frac{1}{2}\sqrt{n} + \sqrt[3]{n}$.

18. $f(n) = 10n^2 \log_2^2 n, \quad g(n) = n^2 \cdot (\sqrt{2})^{\log_2 n}$.

19. $f(n) = 10n^{\log_2 3}, \quad g(n) = n\sqrt{n} + 100n$.

20. $f(n) = 10n^{\log_2 7}, \quad g(n) = n^2 \cdot 2^{0.5\log_2 n}$.

21. $f(n) = 2\sqrt{2^n}, \quad g(n) = 4^{n/4} + n^2$.

22. $f(n) = \frac{2^n}{n^2}, \quad g(n) = 10n^2\sqrt{2^n}$.

23. $f(n) = 10n^2 \cdot 2^{\sqrt{n}}, \quad g(n) = \sqrt{2^n}$.

24. $f(n) = 2^n, \quad g(n) = n^2\sqrt{3^n}$.

25. $f(n) = 10n\log_2 n + 100n, \quad g(n) = n \cdot 4^{\log_2\log_2 n}$.

26. $f(n) = n\log_2 n, \quad g(n) = 100n^{1.1}$.

27. $f(n) = n^2 + n \cdot 2^{0.5\log_2 n}, \quad g(n) = 10n^2 + \frac{n^2}{\sqrt[3]{n}}$.

28. $f(n) = \sqrt[3]{n}\log_2 n, \quad g(n) = n \cdot 2^{-0.5\log_2 n}$.

29. $f(n) = \sqrt{n^3 \log_2 n}, \quad g(n) = 4^{\log_2 n}$.

30. $f(n) = 10n\log_2^2 n, \quad g(n) = \frac{n^2}{\log_2 n}$.

Exercise 2 (0.2 point). (a) Sort given functions f_1, f_2, f_3, f_4, f_5 in the increasing (nondecrerasing) order of their growth (each function should be O(next function)). Additionally indicate the functions that have the same growth order (each function is Θ of another function).

(b) Sort in the increasing (nondecreasing) order the values $f_1(n), f_2(n), f_3(n), f_4(n), f_5(n)$ for $n = 16$.

(c) Sort in the increasing (nondecreasing) order the values $f_1(n), f_2(n), f_3(n), f_4(n), f_5(n)$ for $n = 2^{16} = 65536$.

Problems

1. $f_1(n) = (\sqrt{2})^{\log_2 n}$, $f_2(n) = \frac{2^n}{100n}$, $f_3(n) = n^2 + 10n$, $f_4(n) = 10n^{1/2}$ ir $f_5(n) = 100\log_2^2 n$.

2. $f_1(n) = \sqrt[4]{2^n}$, $f_2(n) = 2^{\log_2 \log_2 n}$, $f_3(n) = \frac{n}{\log_2 n}$, $f_4(n) = \sqrt{10n}$ ir $f_5(n) = n^{\log_4 3}$.

3. $f_1(n) = 10n + \log_2^2(4^n)$, $f_2(n) = 10n\log_2 n$, $f_3(n) = \sqrt{2^n}$, $f_4(n) = n^{\log_2 5}$ ir $f_5(n) = \frac{n^3}{10\log_2 n}$.

4. $f_1(n) = 100n$, $f_2(n) = 2^{\log_2 \log_2 n}$, $f_3(n) = n^{4/3}$, $f_4(n) = n\log_2 n + 10n$ ir $f_5(n) = 2^{\sqrt[4]{n}}$.

5. $f_1(n) = 4^{\sqrt{n}}$, $f_2(n) = 100n^2\log_2 n$, $f_3(n) = n^2 \cdot 2^{n/2}$, $f_4(n) = n^{5/2}$ ir $f_5(n) = n^2(\log_2 n + \sqrt{n})$.

6. $f_1(n) = \sqrt{n^3 \log_2 n}$, $f_2(n) = 2^{\log_2^2 n}$, $f_3(n) = 100n\sqrt{n}$, $f_4(n) = \frac{n^2}{\log_2 n}$ ir $f_5(n) = \log_2^2 n$.

7. $f_1(n) = \frac{n^2}{\log_2^2 n}$, $f_2(n) = 10n\sqrt{n}$, $f_3(n) = 2^{n/\log_2 n}$, $f_4(n) = n^{\log_2 3}$ ir $f_5(n) = n + 10\sqrt{n}$.

8. $f_1(n) = 10n^2$, $f_2(n) = 2\sqrt{2^n}$, $f_3(n) = 100n\log_2^2 n$, $f_4(n) = \frac{n^3}{10\log_2 n}$ ir $f_5(n) = 4^{n/4}$.

9. $f_1(n) = 10n^{\log_2 3}$, $f_2(n) = n\sqrt{n}$, $f_3(n) = 2^{2\log_2 n}$, $f_4(n) = n^{\sqrt{2}}$ ir $f_5(n) = 1.1^n$.

10. $f_1(n) = 10n^{3/2}$, $f_2(n) = n\log_2 n + \frac{n^2}{2}$, $f_3(n) = \frac{n^2}{\log_2 n}$, $f_4(n) = 2^{\sqrt{n}}$ ir $f_5(n) = \sqrt{2^n}$.

11. $f_1(n) = n \cdot 2^{n/2}$, $f_2(n) = 4^{\log_2 \log_2 n}$, $f_3(n) = \sqrt{2^n}\log_2^2 n$, $f_4(n) = 2\log_2^2 n$ ir $f_5(n) = 10n$.

12. $f_1(n) = 10n$, $f_2(n) = \sqrt{2^{n/2}}$, $f_3(n) = \sqrt{n} \cdot 2^{\log_2 \sqrt{n}}$, $f_4(n) = \sqrt[3]{n^4}$ ir $f_5(n) = \log_2 2^{n\log_2 n}$.

13. $f_1(n) = \frac{n^3}{\sqrt{n}}$, $f_2(n) = 2^{n\log_2 n}$, $f_3(n) = n^2\sqrt{n} + 100n^2$, $f_4(n) = \frac{n^3}{\log_2 n}$ ir $f_5(n) = n^{\log_2 7}$.

14. $f_1(n) = 2^{1.2\log_2 n}$, $f_2(n) = 2^{\sqrt{n}}$, $f_3(n) = 100n\sqrt{n}$, $f_4(n) = n\log_2 n$ ir $f_5(n) = \frac{n^2}{\log_2 n}$.

15. $f_1(n) = 4^{\log_2 n}$, $f_2(n) = 4^{\sqrt{n}}$, $f_3(n) = \frac{n^3}{\log_2 n}$, $f_4(n) = 100n^2$ ir $f_5(n) = n\log_2^2 n$.

16. $f_1(n) = (n + \sqrt{n})^2$, $f_2(n) = \sqrt{2^n}$, $f_3(n) = 10n\log_2^2 n$, $f_4(n) = \frac{n^3}{10\log_2 n}$ ir $f_5(n) = n^{\log_2 5}$.

17. $f_1(n) = \frac{n}{\log_2 n}$, $f_2(n) = 10\log_2^2 n$, $f_3(n) = \frac{n}{10}$, $f_4(n) = 1.1^n$ ir $f_5(n) = 10\sqrt{n}\log_2 n$.

18. $f_1(n) = n + \log_2 n^4$, $f_2(n) = 2^{n/\log_2 n}$, $f_3(n) = \sqrt{n} + \sqrt[3]{n}$, $f_4(n) = 10n + \log_2 n$ ir $f_5(n) = \frac{n^2}{\log_2 n}$.

19. $f_1(n) = 4^{\log_2 n}$, $f_2(n) = 10n\log_2 n$, $f_3(n) = 2^{\sqrt[4]{n}}$, $f_4(n) = 10n^2 + \log_2^2 n$ ir $f_5(n) = n^{\log_2 3}$.

20. $f_1(n) = \sqrt{n\log_2 n}$, $f_2(n) = (\sqrt{2})^{\log_2 n}$, $f_3(n) = \frac{n}{\log_2 n}$, $f_4(n) = 2^{n/4}$ ir $f_5(n) = 10\sqrt{n}$.

21. $f_1(n) = \sqrt{2^n}$, $f_2(n) = 10n^2 + 100n\log_2^2 n$, $f_3(n) = n^2\sqrt{n}$, $f_4(n) = n^2\log_2 n$ ir $f_5(n) = n^{\log_2 7}$.

22. $f_1(n) = n + \log_2^2 n^4$, $f_2(n) = 10n\log_2 n$, $f_3(n) = 10n - 5$, $f_4(n) = \sqrt{2^n}$ ir $f_5(n) = n\sqrt{n}$.

23. $f_1(n) = 2^{\log_2^2 n}$, $f_2(n) = n^2 + 10n$, $f_3(n) = 10n^{10}$, $f_4(n) = 4^{\log_2 n}$ ir $f_5(n) = \frac{n^2}{\log_2 n}$.

24. $f_1(n) = n + 10\log_2 n$, $f_2(n) = n\log_2 n$, $f_3(n) = 2^{1.2\log_2 n}$, $f_4(n) = \sqrt{n}\log_2 n$ ir $f_5(n) = (1.2)^n$.

25. $f_1(n) = 10n^{\log_2 3}$, $f_2(n) = 10n\sqrt{n}$, $f_3(n) = \frac{n^2}{\log_2 n}$, $f_4(n) = n \cdot (\sqrt{2})^{\log_2 n}$ ir $f_5(n) = 2^{\sqrt{n}}$.

26. $f_1(n) = 2^{\sqrt[4]{n}}$, $f_2(n) = n\sqrt[4]{n}$, $f_3(n) = 10n\log_2 n$, $f_4(n) = \frac{n^2}{10\log_2 n}$ ir $f_5(n) = n^{5/4} + 10n$.

27. $f_1(n) = n^2 + n\sqrt{n}$, $f_2(n) = 2^{\sqrt{n}}$, $f_3(n) = \frac{n^2}{\log_2 n}$, $f_4(n) = 100n^{\log_2 3}$ ir $f_5(n) = 4^{\log_2 n}$.

28. $f_1(n) = 2^{n/2}$, $f_2(n) = 10n + (\frac{n}{2})^2$, $f_3(n) = n\log_2^2 n$, $f_4(n) = 4^{\log_2 n}$ ir $f_5(n) = \frac{n^3}{\log_2 n}$.

29. $f_1(n) = 2\log_2^2 n$, $f_2(n) = n\log_2 n$, $f_3(n) = 2\sqrt{n}$, $f_4(n) = \sqrt{n} + \log_2^2 n$ ir $f_5(n) = 2^{\log_2 \sqrt{n}}$.

30. $f_1(n) = \log_2 4^n$, $f_2(n) = n+2\sqrt{n}$, $f_3(n) = n\log_2 n$, $f_4(n) = 2^{\sqrt{n}}$ ir $f_5(n) = \frac{n^2}{\log_2 n}$.