Homework 3

Exercise 1 (0.1 point). Let us consider modified MERGE SORT algorithm (cf. Section 2.3 in Cormen book)

function $A' = \text{MERGE\_SORT}(A)$
if $n = 1$ **then** $A' = A$
else $A' := \text{MERGE2}(\text{MERGE\_SORT}(A[1 : n/2]), \text{MERGE\_SORT}(A[n/2 + 1 : n]))$;
end;

We replace function MERGE by function MERGE2. This function differs from MERGE in the following sense: if in the process of merging two already sorted arrays into one new array one of initial arrays becomes empty, then we do not make any more comparisons but simply move remaining elements of the second array to the end of the new array.

Algorithm MERGE2 may be represented in the following way:

function $C = \text{MERGE2}(A, B)$
$i := 1$;
$j := 1$;
for $k := 1$ **to** $m + n$ **do**
  if $i \leq m$ **then**
    if $j \leq n$ **then**
      if $A[i] < B[j]$ **then**
        $C[k] := A[i]$;
        $i := i + 1$;
      else
        $C[k] := B[j]$;
        $j := j + 1$;
      end;
    else
      $C[k] := A[i]$;
      $i := i + 1$;
    end;
  else
    $C[k] := B[j]$;
    $j := j + 1$;
  end;
end;

From pseudocode we see that to merge two sorted arrays of length $n/2$ and $n/2$ into one new sorted array of length n, we will need minimum $n/2$ and maximum $n - 1$ comparison of array elements. Easily can be proved that modified MERGE_SORT algorithm makes minimum $(n \log_2 n)/2$ and maximum $n \log_2 n - n + 1$ comparisons of elements of initial array A (we do not count any other steps or program variable comparisons of kind $i \leq m$?). For example, when $n = 8$, the number of comparisons may be any number between 12 and 17.

An array $A = [a_1, a_2, \dots, a_8]$ is given. The task is to find how many comparisons of array A elements (of kind $A[i] < A[j]$) will make the modified algorithm MERGE_SORT and to enumerate all these comparisons.

Problems

1. $A = [3, 41, 52, 26, 38, 57, 9, 49]$;
2. $A = [1, 5, 8, 18, 99, 2, 6, 43]$;
3. $A = [5, 55, 25, 35, 1, 88, 46, 22]$;
4. $A = [92, 41, 15, 3, 6, 42, 55, 77]$;
5. $A = [47, 63, 51, 82, 17, 19, 26, 84]$;
6. $A = [12, 97, 16, 34, 97, 48, 76, 8]$;
7. $A = [95, 76, 84, 23, 56, 42, 31, 5]$;
8. $A = [3, 23, 33, 43, 51, 61, 72, 85]$;
9. $A = [66, 16, 26, 46, 76, 6, 86, 36]$;
10. $A = [17, 27, 43, 85, 16, 2, 9, 88]$;
11. $A = [1, 6, 8, 15, 76, 43, 98, 55]$;
12. $A = [32, 41, 23, 8, 49, 56, 67, 92]$;
13. $A = [11, 21, 92, 43, 65, 44, 33, 22]$;
14. $A = [12, 23, 34, 56, 41, 78, 56, 5]$;
15. $A = [16, 72, 43, 95, 18, 26, 13, 55]$;
16. $A = [66, 22, 10, 50, 80, 13, 49, 72]$;
17. $A = [95, 94, 68, 75, 34, 28, 19, 5]$;
18. $A = [2, 8, 10, 42, 56, 98, 63, 75]$;
19. $A = [96, 75, 84, 32, 1, 6, 7, 10]$;
20. $A = [1, 83, 5, 74, 2, 32, 46, 18]$;
21. $A = [10, 65, 43, 91, 28, 15, 16, 75]$;

22. $A = [13, 47, 60, 24, 29, 37, 81, 67]$;
23. $A = [15, 17, 28, 46, 92, 38, 16, 54]$;
24. $A = [80, 24, 56, 3, 97, 16, 15, 13]$;
25. $A = [1, 9, 7, 84, 35, 26, 75, 18]$;
26. $A = [95, 76, 13, 48, 52, 40, 92, 10]$;
27. $A = [66, 36, 28, 14, 57, 95, 80, 70]$;
28. $A = [11, 30, 4, 87, 56, 92, 18, 63]$;
29. $A = [17, 64, 82, 39, 47, 83, 96, 38]$;
30. $A = [7, 17, 71, 77, 11, 9, 19, 91]$;

Exercise 2 (0.1 point). Represent given integer N in three different ways:

(a) in a hexadecimal numeral system (see, e.g., http://en.wikipedia.org/wiki/Hexadecimal);

(b) in a mixed radix system with radices $60, 60, 24, 7, 52$ (see, e.g. http://en.wikipedia.org/wiki/Mixed_radix);

(c) in a residue number system defined by a set $\{92, 93, 95, 97\}$ (see, e.g., http://en.wikipedia.org/wiki/Residue_number_system).

Problems

1. 55555555;
2. 51525354;
3. 45678912;
4. 70000000;
5. 12345678;
6. 76543210;
7. 66666666;
8. 61626364;
9. 71727374;
10. 77777777;
11. 70707070;
12. 60606060;
13. 44444444;
14. 60000000;
15. 50000000;
16. 40000000;
17. 33333333;
18. 30000000;
19. 61626364;
20. 69696969;
21. 41424344;
22. 37383930;
23. 40414243;
24. 50515253;
25. 60616263;
26. 70717273;
27. 75757575;
28. 68686868;
29. 74747474;
30. 67676767.

Exercise 3 (0.1 point). Let us consider a rooted tree with root A and set of edges $E = \{e_1, e_2, \ldots, e_m\}$. Represent this tree in the following ways:

(a) graphically;

(b) by parent array;

(c) by children array;

(d) by leftmost child–right sibling array.

See page 29 of http://www.mif.vu.lt/ valdas/ALGORITMAI/Vadovelis/algoritmu_analize.pdf and http://en.wikipedia.org/wiki/Tree_(data_structure) Representing a tree graphically, sort children of each vertex from left to right in alphabetical order.

Problems

1. $E = \{(A, B), (A, C), (A, D), (B, E), (C, F)\}$;
2. $E = \{(A, C), (B, F), (B, D), (C, B), (B, E)\}$;
3. $E = \{(A, C), (A, F), (A, B), (A, E), (A, D)\}$;
4. $E = \{(A, F), (F, C), (C, B), (A, E), (B, D)\}$;
5. $E = \{(A, B), (B, C), (D, F), (C, E), (E, D)\}$;
6. $E = \{(A, C), (C, B), (B, D), (C, E), (C, F)\}$;
7. $E = \{(C, E), (B, D), (A, B), (B, C), (A, F)\}$;
8. $E = \{(A, F), (A, C), (F, B), (F, D), (C, E)\}$;
9. $E = \{(A, B), (B, C), (B, D), (A, E), (E, F)\}$;
10. $E = \{(A, B), (A, D), (E, C), (A, E), (A, F)\}$;
11. $E = \{(A, B), (C, D), (B, C), (C, E), ((D, F)\}$;
12. $E = \{(F, C), (A, B), (B, F), (B, D), (B, E)\}$;
13. $E = \{(C, B), (A, D), (C, E), (E, F), (D, C)\}$;
14. $E = \{(A, B), (B, E), (A, D), (A, F), (E, C)\}$;
15. $E = \{(A, B), (B, F), (A, D), (D, C), (F, E)\}$;
16. $E = \{(A, E), (B, C), (A, D), (B, F), (D, B)\}$;
17. $E = \{(A, D), (C, B), (D, E), (F, C), (A, F)\}$;

18. $E = \{(A, B), (B, D), (D, E), (B, C), (C, F)\}$;

19. $E = \{(A, F), (F, B), (C, D), (F, E), (B, C)\}$;

20. $E = \{(A, C), (C, D), (C, E), (C, F), (C, B)\}$;

21. $E = \{(A, D), (A, B), (B, E), (D, C), (C, F)\}$;

22. $E = \{(A, D), (A, C), (D, F), (C, E), (C, B)\}$;

23. $E = \{(A, F), (F, B), (B, E), (B, C), (B, D)\}$;

24. $E = \{(A, C), (C, F), (A, D), (B, E), (F, B)\}$;

25. $E = \{(A, B), (D, E), (B, C), (A, D), (A, F)\}$;

26. $E = \{(A, D), (C, E), (D, F), (B, C), (A, B)\}$;

27. $E = \{(C, E), (A, B), (A, F), (F, C), (B, D)\}$;

28. $E = \{(A, C), (B, E), (E, D), (C, B), (B, F)\}$;

29. $E = \{(A, C), (D, E), (A, D), (D, F), (D, B)\}$;

30. $E = \{(A, B), (A, F), (F, D), (B, E), (F, C)\}$.

Exercise 4 (0.1 point). A graph $G = (V, E)$ is given by an array of edges, i.e., by a vector $\vec{b} = (n, v_{11}, v_{12}, v_{21}, v_{22}, \ldots, v_{m1}, v_{m2})$, where $n = |V|$ is number of vertices and $e_i \in E \Rightarrow e_i = (v_{i1}, v_{i2})$ ($i = 1, \ldots, m$). Represent given graph in the following ways (see Section 22.1 in Cormen book):

(a) graphically;

(b) by adjacency matrix;

(c) by adjacency list;

(d) by incidence matrix.

Problems

1. $\vec{b} = (5, 1, 2, 2, 3, 1, 4, 1, 5, 2, 5)$;

2. $\vec{b} = (5, 1, 2, 1, 3, 1, 4, 2, 5, 3, 4)$;

3. $\vec{b} = (5, 1, 5, 1, 4, 4, 5, 2, 3, 2, 4)$;

4. $\vec{b} = (5, 1, 2, 2, 3, 4, 5, 3, 5, 3, 4)$;

5. $\vec{b} = (5, 1, 2, 2, 3, 3, 4, 4, 1)$;

6. $\vec{b} = (5, 1, 2, 1, 4, 1, 5, 4, 5, 5, 2)$;
7. $\vec{b} = (5, 2, 5, 4, 3, 2, 1, 4, 2, 4, 5)$;
8. $\vec{b} = (5, 1, 2, 2, 3, 3, 4, 4, 5, 5, 1)$;
9. $\vec{b} = (5, 1, 2, 2, 3, 3, 1, 4, 5)$;
10. $\vec{b} = (5, 1, 2, 1, 3, 1, 4, 2, 3, 3, 4)$;
11. $\vec{b} = (5, 1, 4, 2, 4, 4, 5, 2, 5, 5, 1)$;
12. $\vec{b} = (5, 2, 3, 2, 4, 3, 5, 4, 5, 3, 4)$;
13. $\vec{b} = (5, 1, 2, 1, 3, 3, 2)$;
14. $\vec{b} = (5, 1, 4, 2, 3, 4, 5, 5, 1)$;
15. $\vec{b} = (5, 1, 4, 2, 4, 4, 2)$;
16. $\vec{b} = (4, 1, 2, 1, 3, 3, 4, 2, 4, 1, 4)$;
17. $\vec{b} = (4, 1, 2, 1, 3, 1, 4, 2, 4)$;
18. $\vec{b} = (4, 1, 2, 1, 3, 1, 4, 2, 3)$;
19. $\vec{b} = (4, 1, 2, 1, 3, 2, 3)$;
20. $\vec{b} = (4, 1, 4, 2, 4, 3, 1, 3, 2, 1, 2)$;
21. $\vec{b} = (4, 3, 2, 3, 4, 3, 1, 4, 2)$;
22. $\vec{b} = (4, 2, 3, 2, 4, 1, 3, 1, 4)$;
23. $\vec{b} = (4, 1, 2, 2, 3, 3, 4, 4, 1)$;
24. $\vec{b} = (4, 1, 3, 2, 4, 1, 4)$;
25. $\vec{b} = (4, 1, 2, 3, 4, 1, 3)$;
26. $\vec{b} = (4, 1, 2, 1, 3, 1, 4)$;
27. $\vec{b} = (4, 1, 3, 3, 2, 2, 4)$;
28. $\vec{b} = (4, 3, 1, 3, 2, 3, 4, 2, 4)$;
29. $\vec{b} = (4, 1, 4, 2, 4, 3, 4, 2, 3, 1, 3)$;
30. $\vec{b} = (4, 2, 3, 2, 4)$.