Homework 5

Exercise 1 (0.2 point). Apply Floyd–Warshall algorithm and find shortest paths from all vertices to all if asymmetric distance matrix (weights of edges) is given. Additionally, find the shortest path (sequence of vertices) from vertex 1 to vertex 6. See Cormen book, Section 25.2.

Problems

1.
```
  0   7   9 Inf  14 Inf
  4   0  -2  10   1   7
  3   7   0 Inf   3  10
  1   8   9   0   1   9
  4 Inf   2   3   0 Inf
  2   2   5   4 Inf   0
```

2.
```
  0   8   6 Inf  15 Inf
  7   0  -2   9   9  13
  7   8   0   9   6   7
  9   8   7   0 Inf  15
  4 Inf   3   9   0 Inf
  4   8   2  10 Inf   0
```

3.
```
  0  13 Inf  10   9 Inf
  1   0  -2  10   8  12
  9  10   0   5 Inf  10
Inf   1   7   0   1 Inf
  5   4 Inf   9   0   9
  7   7   5 Inf   4   0
```

4.
```
  0 Inf  14  13  15 Inf
  4   0  -2   4  10   6
 10 Inf   0   9   5  10
Inf   8   1   0   6 Inf
Inf   7   7   4   0  15
  3   1   5 Inf  10   0
```

5.
```
  0  12   6 Inf   6 Inf
 10   0  -2   1   3   6
  7   3   0  10 Inf Inf
Inf   2   2   0   2  14
  6 Inf   5  10   0  10
  9   1 Inf   9   5   0
```

6.
```
  0   13  Inf   14   10  Inf
  1    0   -2    5    3   15
  5    8    0  Inf    7  Inf
  3  Inf    8    0    9    8
  9    8    4    6    0    9
  6   10  Inf    3    4    0
```

7.
```
  0    8    8  Inf   15  Inf
Inf    0   -2    5    8   11
  8    9    0    4    2    9
  5  Inf    9    0    4   13
  1    7    8  Inf    0  Inf
  1    6    4    8  Inf    0
```

8.
```
  0   12   14    7   12  Inf
  7    0   -2  Inf    1   12
Inf    6    0    8    6  Inf
  9    5    8    0  Inf    9
  2  Inf    9    3    0    6
  7    7  Inf    4    1    0
```

9.
```
  0    8   11    7    6  Inf
  1    0   -2    4    2   10
Inf    9    0    8    9  Inf
 10    4    3    0    6    9
  2  Inf    2    8    0    7
  2    5  Inf    4    2    0
```

10.
```
  0    7    8    6   15  Inf
 10    0   -2    5    8   14
  6   10    0    1    2   11
  2   10    7    0    7   11
  6    3    6  Inf    0    7
  6    9    6    6    2    0
```

11.
```
  0    6    8    9    9  Inf
  2    0   -2    2    6   11
Inf    7    0    7    8   11
  7   10    6    0    8    7
 10  Inf    6    6    0   10
  1    6    6    2    5    0
```

12.

```
  0   Inf    13     6    10   Inf
  8     0    -2    10    10     9
  9   Inf     0     3     7     9
  5   Inf     8     0     2   Inf
 10     5     1     4     0    15
  8     4     4   Inf    10     0
```

13.

```
  0   Inf     9     6     8   Inf
  7     0    -2     6    10    10
Inf     7     0     2     4     6
  6     9     1     0     5     8
 10     3   Inf     8     0     7
  5     5     1     3     2     0
```

14.

```
  0    11    15    12    14   Inf
  1     0    -2     4     8     9
  6    10     0   Inf     7   Inf
  6     9     6     0   Inf     8
  6    10    10     2     0    14
  4     4   Inf     3     9     0
```

15.

```
  0     8   Inf     9     7   Inf
  6     0    -2    10   Inf     8
  2     4     0     7     7    10
  5   Inf     3     0     3   Inf
Inf     9     8     3     0     6
  3     3     5   Inf     1     0
```

16.

```
  0   Inf    13     9     9   Inf
Inf     0    -2     5     7    12
  5     6     0   Inf     7    14
  9     1   Inf     0     1   Inf
Inf     7     6     3     0     9
  4     7     9   Inf     4     0
```

17.

```
  0    11    13    15     7   Inf
Inf     0    -2     6     4   Inf
  5     9     0   Inf     7    13
 10   Inf     5     0     9    12
  6     1     5   Inf     0     8
  7   Inf     8     7     3     0
```

18.

0	14	11	9	14	Inf
4	0	-2	7	6	6
9	Inf	0	10	10	10
8	7	9	0	Inf	Inf
7	7	9	Inf	0	15
8	1	5	Inf	10	0

19.

0	7	6	11	8	Inf
Inf	0	-2	5	8	8
8	Inf	0	7	4	12
7	5	8	0	6	6
6	7	6	8	0	Inf
5	3	7	1	Inf	0

20.

0	7	10	15	Inf	Inf
8	0	-2	Inf	10	9
6	3	0	Inf	2	Inf
Inf	8	10	0	5	7
3	2	4	1	0	10
6	4	Inf	2	5	0

21.

0	11	Inf	6	6	Inf
1	0	-2	2	7	7
8	8	0	6	7	13
1	7	Inf	0	3	Inf
8	9	5	6	0	15
5	2	8	Inf	10	0

22.

0	13	11	15	Inf	Inf
8	0	-2	8	9	15
Inf	3	0	5	10	6
6	Inf	8	0	8	9
1	2	8	4	0	Inf
3	10	1	4	Inf	0

23.

0	Inf	10	15	8	Inf
9	0	-2	Inf	6	6
3	6	0	8	Inf	10
5	1	5	0	3	6
2	1	9	6	0	6
Inf	1	5	1	1	0

24.

0	12	13	10	13	Inf
9	0	-2	5	Inf	10
3	6	0	4	Inf	15
8	Inf	10	0	8	13
3	1	6	Inf	0	10
Inf	5	10	8	5	0

25.

0	11	11	6	9	Inf
2	0	-2	10	4	12
7	4	0	4	2	12
3	4	1	0	Inf	12
2	10	1	9	0	7
4	7	7	7	2	0

26.

0	15	Inf	11	13	Inf
5	0	-2	10	6	12
4	5	0	Inf	4	Inf
4	3	10	0	8	11
2	Inf	7	1	0	13
2	7	Inf	6	8	0

27.

0	6	9	15	Inf	Inf
6	0	-2	8	5	Inf
8	2	0	9	4	9
Inf	6	10	0	10	14
9	9	10	5	0	14
6	Inf	4	9	9	0

28.

0	14	13	Inf	11	Inf
10	0	-2	Inf	2	9
Inf	6	0	1	10	8
3	8	10	0	9	7
2	5	3	10	0	7
Inf	4	3	2	2	0

29.

0	8	14	11	Inf	Inf
4	0	-2	9	Inf	11
4	6	0	9	1	15
6	Inf	10	0	6	Inf
7	6	4	Inf	0	13
5	6	10	Inf	8	0

30.

0	6	Inf	7	14	Inf
3	0	-2	1	10	Inf
3	Inf	0	8	8	15
3	4	8	0	5	6
3	7	8	Inf	0	15
8	Inf	10	1	10	0

Exercise 2 (0.2 point). Find minimum spanning tree in a given weighted graph using: (a) Kruskal's and (b) Prim's algorithms. The graph is represented by symmetric edge weight matrix. See Cormen book, Section 25.2.

Problems

1.

0	2	16	10	3	6	5	7
2	0	6	Inf	12	18	14	14
16	6	0	15	9	7	1	10
10	Inf	15	0	8	5	9	15
3	12	9	8	0	15	Inf	13
6	18	7	5	15	0	15	14
5	14	1	9	Inf	15	0	18
7	14	10	15	13	14	18	0

2.

0	1	10	Inf	11	5	17	16
1	0	14	19	17	Inf	Inf	4
10	14	0	15	Inf	20	15	5
Inf	19	15	0	20	20	10	Inf
11	17	Inf	20	0	4	18	10
5	Inf	20	20	4	0	2	10
17	Inf	15	10	18	2	0	5
16	4	5	Inf	10	10	5	0

3.

0	12	15	19	6	19	7	2
12	0	2	6	15	Inf	13	2
15	2	0	5	12	8	18	Inf
19	6	5	0	16	2	Inf	15
6	15	12	16	0	14	1	20
19	Inf	8	2	14	0	12	16
7	13	18	Inf	1	12	0	15
2	2	Inf	15	20	16	15	0

4.

0	2	4	1	18	1	15	12
2	0	13	Inf	-1	3	18	3
4	13	0	3	11	14	16	9
1	Inf	3	0	Inf	Inf	15	4
18	-1	11	Inf	0	18	Inf	Inf
1	3	14	Inf	18	0	3	20
15	18	16	15	Inf	3	0	1
12	3	9	4	Inf	20	1	0

5.

0	20	6	11	12	20	5	3
20	0	17	2	Inf	15	8	16
6	17	0	3	13	12	12	11
11	2	3	0	3	10	17	Inf
12	Inf	13	3	0	Inf	Inf	4
20	15	12	10	Inf	0	1	6
5	8	12	17	Inf	1	0	13
3	16	11	Inf	4	6	13	0

6.

0	12	2	5	16	19	19	Inf
12	0	14	3	Inf	15	1	12
2	14	0	6	2	4	10	15
5	3	6	0	20	6	9	15
16	Inf	2	20	0	11	Inf	2
19	15	4	6	11	0	7	6
19	1	10	9	Inf	7	0	5
Inf	12	15	15	2	6	5	0

7.

0	2	12	5	4	20	6	16
2	0	7	14	4	10	Inf	1
12	7	0	-1	8	11	16	9
5	14	-1	0	13	Inf	17	3
4	4	8	13	0	11	13	8
20	10	11	Inf	11	0	15	10
6	Inf	16	17	13	15	0	12
16	1	9	3	8	10	12	0

8.

0	4	3	1	Inf	16	11	5
4	0	10	13	5	-1	16	Inf
3	10	0	6	11	9	13	2
1	13	6	0	1	Inf	18	6
Inf	5	11	1	0	13	1	3
16	-1	9	Inf	13	0	9	13
11	16	13	18	1	9	0	4
5	Inf	2	6	3	13	4	0

9.

0	14	3	7	4	8	14	17
14	0	14	11	Inf	14	4	16
3	14	0	8	5	16	19	7
7	11	8	0	4	17	17	3
4	Inf	5	4	0	9	8	19
8	14	16	17	9	0	1	13
14	4	19	17	8	1	0	Inf
17	16	7	3	19	13	Inf	0

10.

0	5	15	11	8	12	15	9
5	0	13	12	8	6	17	6
15	13	0	9	13	14	Inf	20
11	12	9	0	Inf	7	12	2
8	8	13	Inf	0	13	10	11
12	6	14	7	13	0	4	18
15	17	Inf	12	10	4	0	14
9	6	20	2	11	18	14	0

11.

0	14	19	19	15	9	15	10
14	0	16	19	7	17	17	Inf
19	16	0	1	7	Inf	1	4
19	19	1	0	Inf	12	9	6
15	7	7	Inf	0	19	7	-1
9	17	Inf	12	19	0	Inf	17
15	17	1	9	7	Inf	0	16
10	Inf	4	6	-1	17	16	0

12.

0	10	15	16	20	3	17	18
10	0	13	17	16	5	-1	6
15	13	0	19	2	3	11	16
16	17	19	0	Inf	1	15	19
20	16	2	Inf	0	Inf	3	18
3	5	3	1	Inf	0	16	2
17	-1	11	15	3	16	0	Inf
18	6	16	19	18	2	Inf	0

13.

0	10	8	18	17	16	16	Inf
10	0	Inf	20	10	12	8	11
8	Inf	0	9	5	13	12	7
18	20	9	0	10	Inf	6	18
17	10	5	10	0	17	17	15
16	12	13	Inf	17	0	9	8
16	8	12	6	17	9	0	12
Inf	11	7	18	15	8	12	0

14.

0	4	3	Inf	14	3	Inf	19
4	0	13	12	13	Inf	18	1
3	13	0	16	4	2	2	6
Inf	12	16	0	10	7	20	20
14	13	4	10	0	4	14	16
3	Inf	2	7	4	0	7	11
Inf	18	2	20	14	7	0	14
19	1	6	20	16	11	14	0

15.

0	11	9	16	17	13	7	7
11	0	5	18	16	6	14	18
9	5	0	11	7	Inf	Inf	8
16	18	11	0	15	18	5	4
17	16	7	15	0	14	12	10
13	6	Inf	18	14	0	1	Inf
7	14	Inf	5	12	1	0	12
7	18	8	4	10	Inf	12	0

16.

0	2	20	Inf	17	18	Inf	7
2	0	15	18	15	6	8	16
20	15	0	13	13	5	13	14
Inf	18	13	0	7	Inf	6	4
17	15	13	7	0	14	17	18
18	6	5	Inf	14	0	15	Inf
Inf	8	13	6	17	15	0	9
7	16	14	4	18	Inf	9	0

17.

0	-1	8	19	17	11	3	Inf
-1	0	12	13	18	14	15	17
8	12	0	1	14	16	13	15
19	13	1	0	1	19	13	6
17	18	14	1	0	Inf	3	1
11	14	16	19	Inf	0	Inf	2
3	15	13	13	3	Inf	0	1
Inf	17	15	6	1	2	1	0

18.

0	10	-1	3	4	10	19	15
10	0	18	11	20	20	10	1
-1	18	0	Inf	10	11	5	Inf
3	11	Inf	0	1	7	2	6
4	20	10	1	0	5	Inf	8
10	20	11	7	5	0	17	7
19	10	5	2	Inf	17	0	13
15	1	Inf	6	8	7	13	0

19.

0	11	14	11	8	11	Inf	-2
11	0	17	6	1	16	6	Inf
14	17	0	14	15	2	9	9
11	6	14	0	Inf	10	1	5
8	1	15	Inf	0	5	11	17
11	16	2	10	5	0	19	4
Inf	6	9	1	11	19	0	17
-2	Inf	9	5	17	4	17	0

20.

0	Inf	16	Inf	7	10	12	5
Inf	0	10	15	11	14	11	5
16	10	0	3	16	19	14	14
Inf	15	3	0	6	8	14	Inf
7	11	16	6	0	Inf	15	2
10	14	19	8	Inf	0	14	9
12	11	14	14	15	14	0	15
5	5	14	Inf	2	9	15	0

21.

0	9	17	6	11	Inf	Inf	6
9	0	Inf	6	Inf	11	12	15
17	Inf	0	17	14	16	17	12
6	6	17	0	8	7	12	4
11	Inf	14	8	0	1	18	18
Inf	11	16	7	1	0	17	Inf
Inf	12	17	12	18	17	0	5
6	15	12	4	18	Inf	5	0

22.

0	12	12	17	5	3	10	6
12	0	Inf	1	19	5	16	Inf
12	Inf	0	3	Inf	11	1	11
17	1	3	0	2	19	12	12
5	19	Inf	2	0	10	Inf	19
3	5	11	19	10	0	19	18
10	16	1	12	Inf	19	0	20
6	Inf	11	12	19	18	20	0

23.

0	3	19	19	14	14	12	10
3	0	16	9	7	Inf	3	9
19	16	0	14	10	5	14	1
19	9	14	0	Inf	5	5	Inf
14	7	10	Inf	0	11	10	6
14	Inf	5	5	11	0	Inf	16
12	3	14	5	10	Inf	0	20
10	9	1	Inf	6	16	20	0

24.

0	Inf	4	Inf	20	2	9	Inf
Inf	0	13	14	19	5	4	17
4	13	0	5	14	3	7	11
Inf	14	5	0	7	Inf	6	18
20	19	14	7	0	18	1	12
2	5	3	Inf	18	0	Inf	16
9	4	7	6	1	Inf	0	5
Inf	17	11	18	12	16	5	0

25.

0	16	9	13	17	16	7	4
16	0	20	10	14	8	4	Inf
9	20	0	16	11	4	13	6
13	10	16	0	10	14	14	15
17	14	11	10	0	15	17	18
16	8	4	14	15	0	15	19
7	4	13	14	17	15	0	14
4	Inf	6	15	18	19	14	0

26.

0	15	16	3	14	9	12	7
15	0	1	16	Inf	9	13	10
16	1	0	19	12	4	6	20
3	16	19	0	2	15	15	13
14	Inf	12	2	0	Inf	4	Inf
9	9	4	15	Inf	0	Inf	12
12	13	6	15	4	Inf	0	19
7	10	20	13	Inf	12	19	0

27.

0	16	13	8	Inf	7	5	13
16	0	Inf	12	17	10	19	17
13	Inf	0	10	1	Inf	4	10
8	12	10	0	18	18	3	10
Inf	17	1	18	0	6	11	14
7	10	Inf	18	6	0	Inf	10
5	19	4	3	11	Inf	0	15
13	17	10	10	14	10	15	0

28.

0	19	8	17	3	3	11	13
19	0	4	11	7	6	11	1
8	4	0	Inf	Inf	3	20	12
17	11	Inf	0	8	4	3	4
3	7	Inf	8	0	5	5	4
3	6	3	4	5	0	11	11
11	11	20	3	5	11	0	15
13	1	12	4	4	11	15	0

29.

0	17	8	3	5	4	9	17
17	0	5	11	3	18	5	15
8	5	0	11	Inf	Inf	17	7
3	11	11	0	3	12	Inf	Inf
5	3	Inf	3	0	2	13	8
4	18	Inf	12	2	0	13	20
9	5	17	Inf	13	13	0	6
17	15	7	Inf	8	20	1	0

30.

0	9	13	13	17	10	7	9
9	0	11	10	9	5	18	20
13	11	0	8	15	5	5	Inf
13	10	8	0	Inf	19	5	2
17	9	15	Inf	0	Inf	17	16
10	5	5	19	Inf	0	Inf	6
7	18	5	5	17	Inf	0	10
9	20	Inf	16	2	6	10	0