



Lecture 17: Solving Galactic Motion

Machine Learning Diffeq

- Recently within ML community :
 - The concept of Physics informed ML emerged

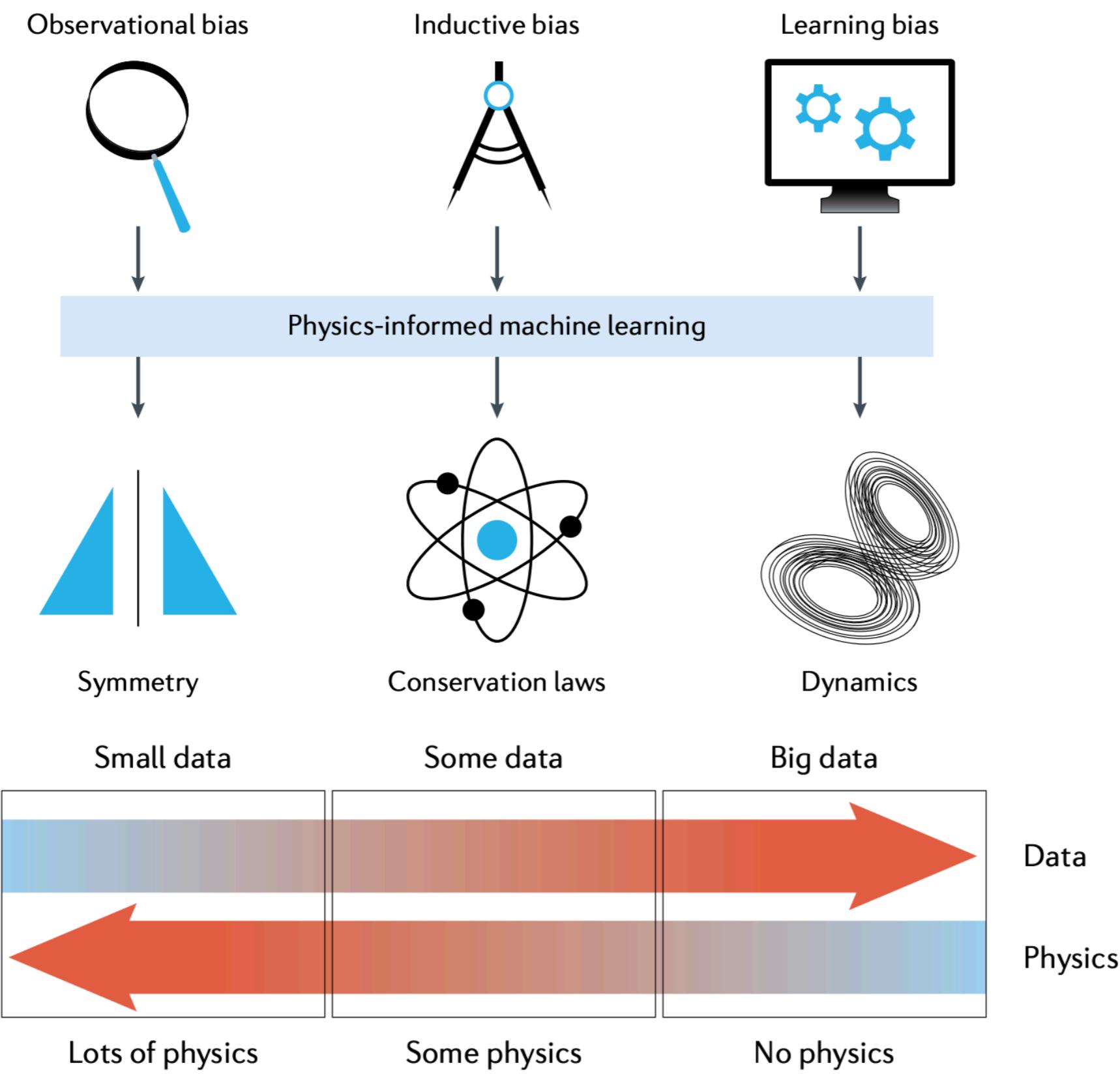
Strategy: $\mathcal{L}_{total} = \mathcal{L}_{NN} + \mathcal{L}_{Diffeq}$

$$\ddot{\theta} + \mu\dot{\theta} + k\theta = 0$$

$$\mathcal{L}_{Diffeq} = (\ddot{\theta} + \mu\dot{\theta} + k\theta)^2$$

Constraint on Differential Equation
Aim to approximate learning

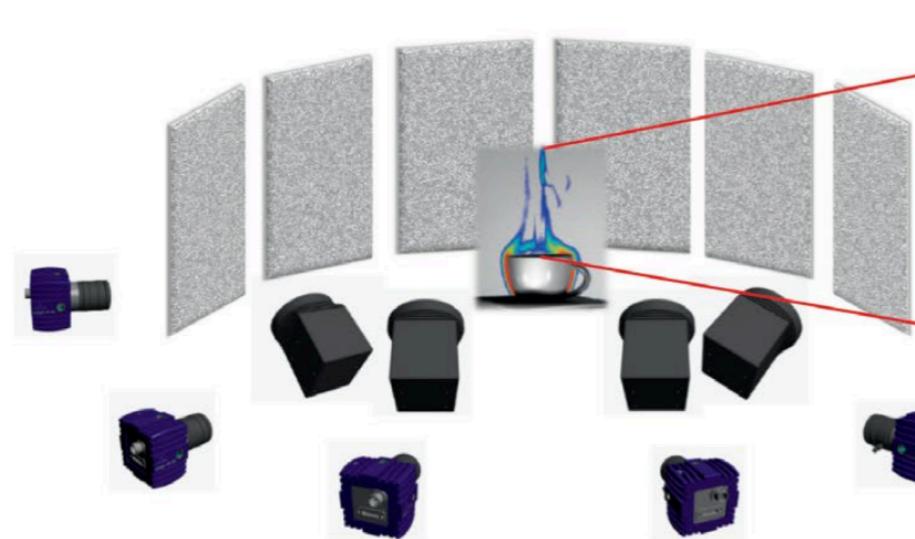
Physics Informed ML



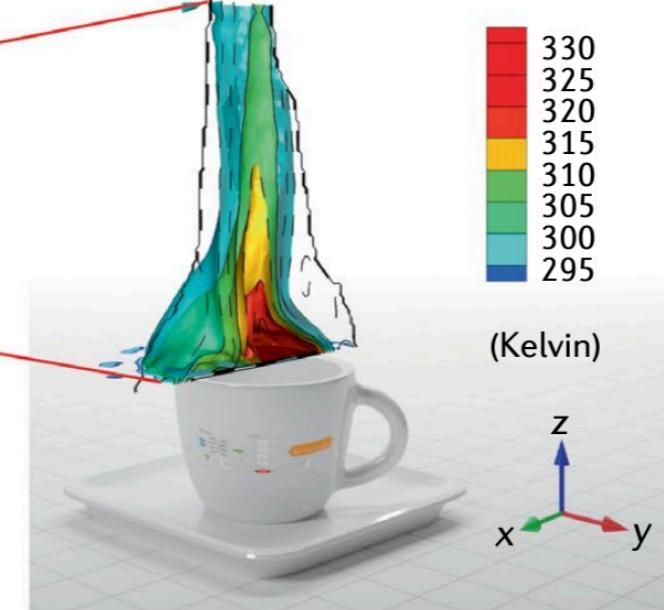
Physics Informed ML

a

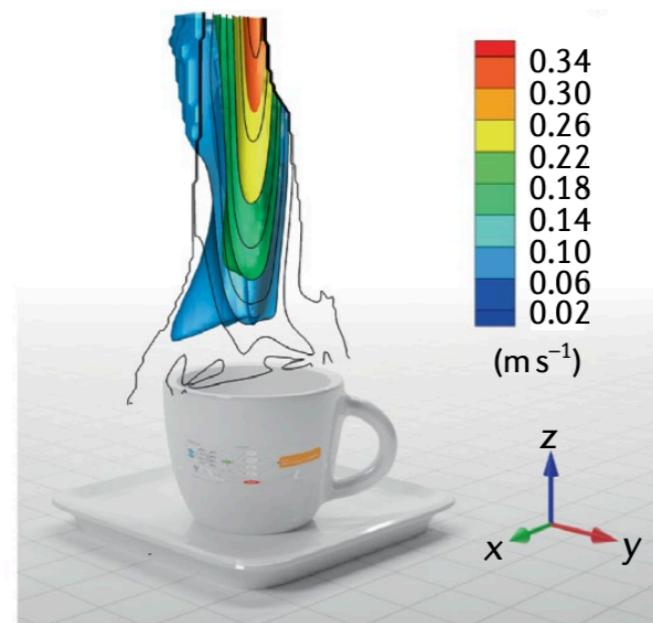
Tomo-BOS setup

**b**

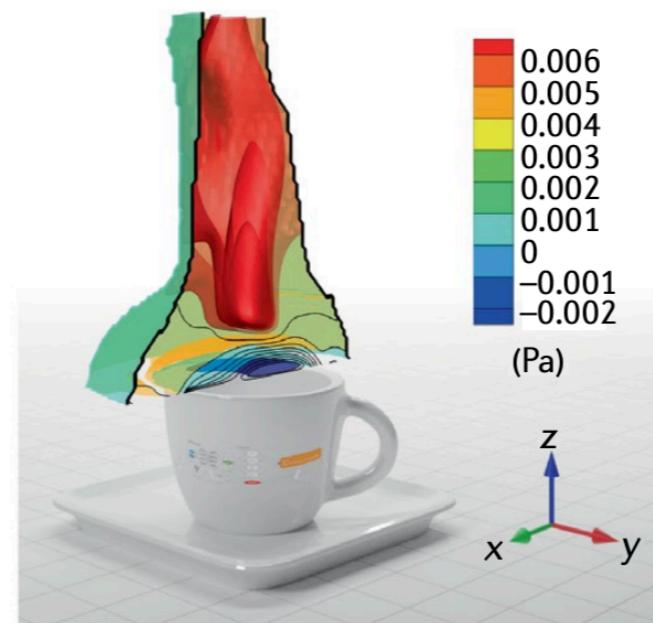
3D temperature data

**c**

3D velocity



3D pressure



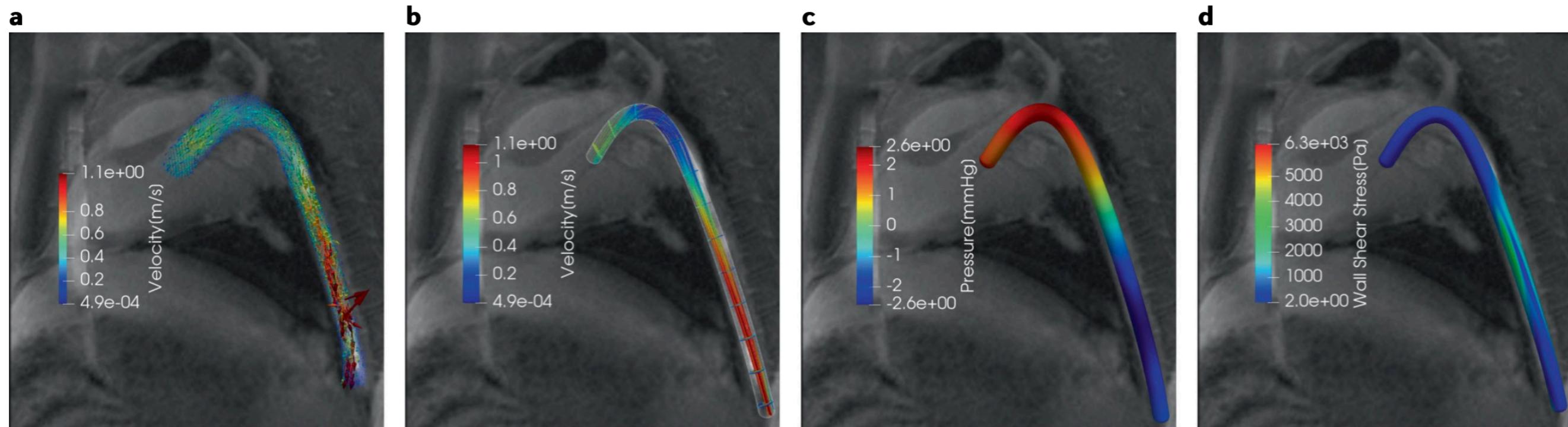
Given the
Laws of fluid flow

How do we model flow?

Physics-informed
neural network

These give us
Physics informed ML

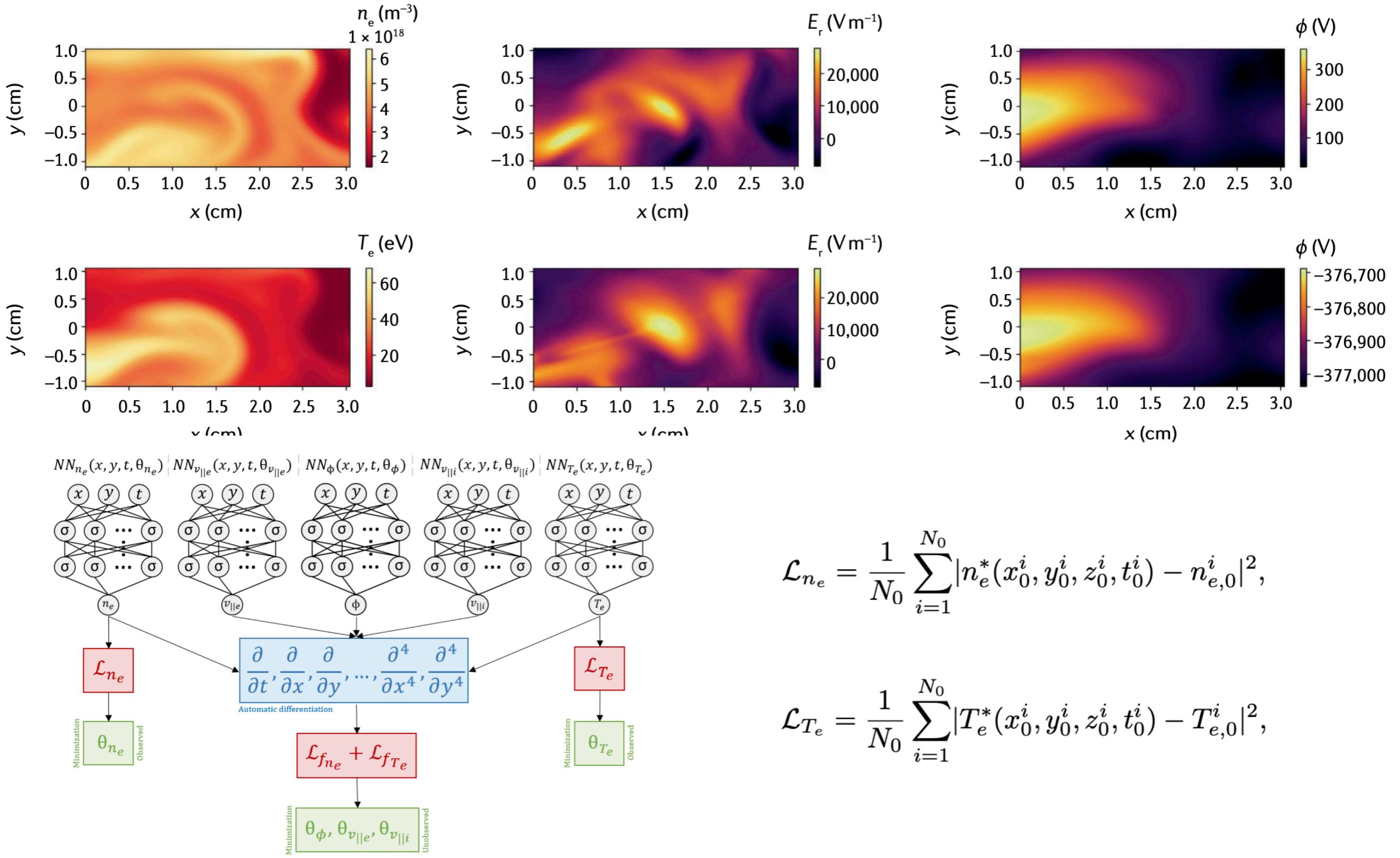
Physics Informed ML



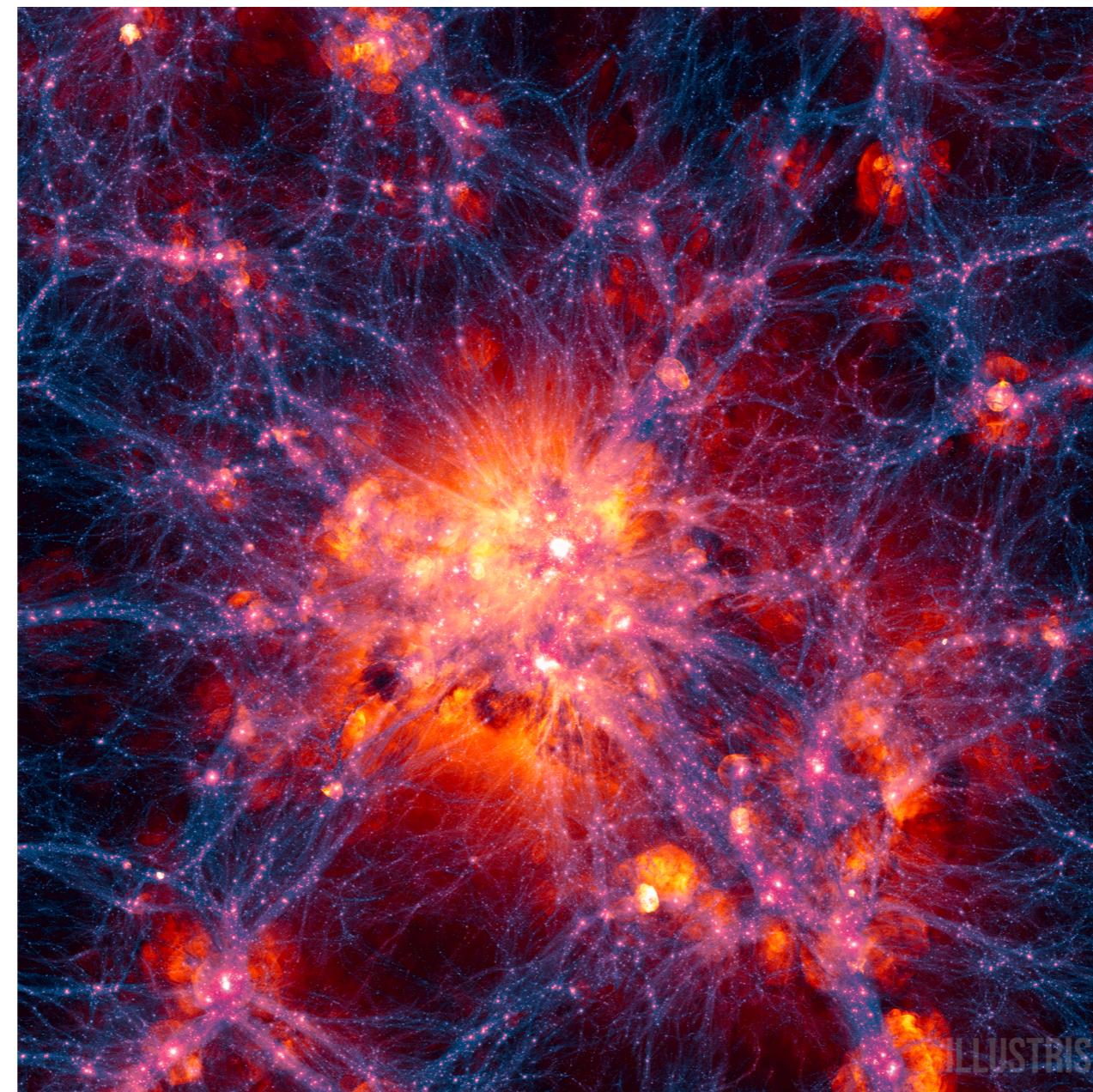
Navier Stokes equation to extrapolate blood flow in system

Navier Stokes equation to extrapolate blood flow in system

Physics Informed ML



Large Scale Simulations



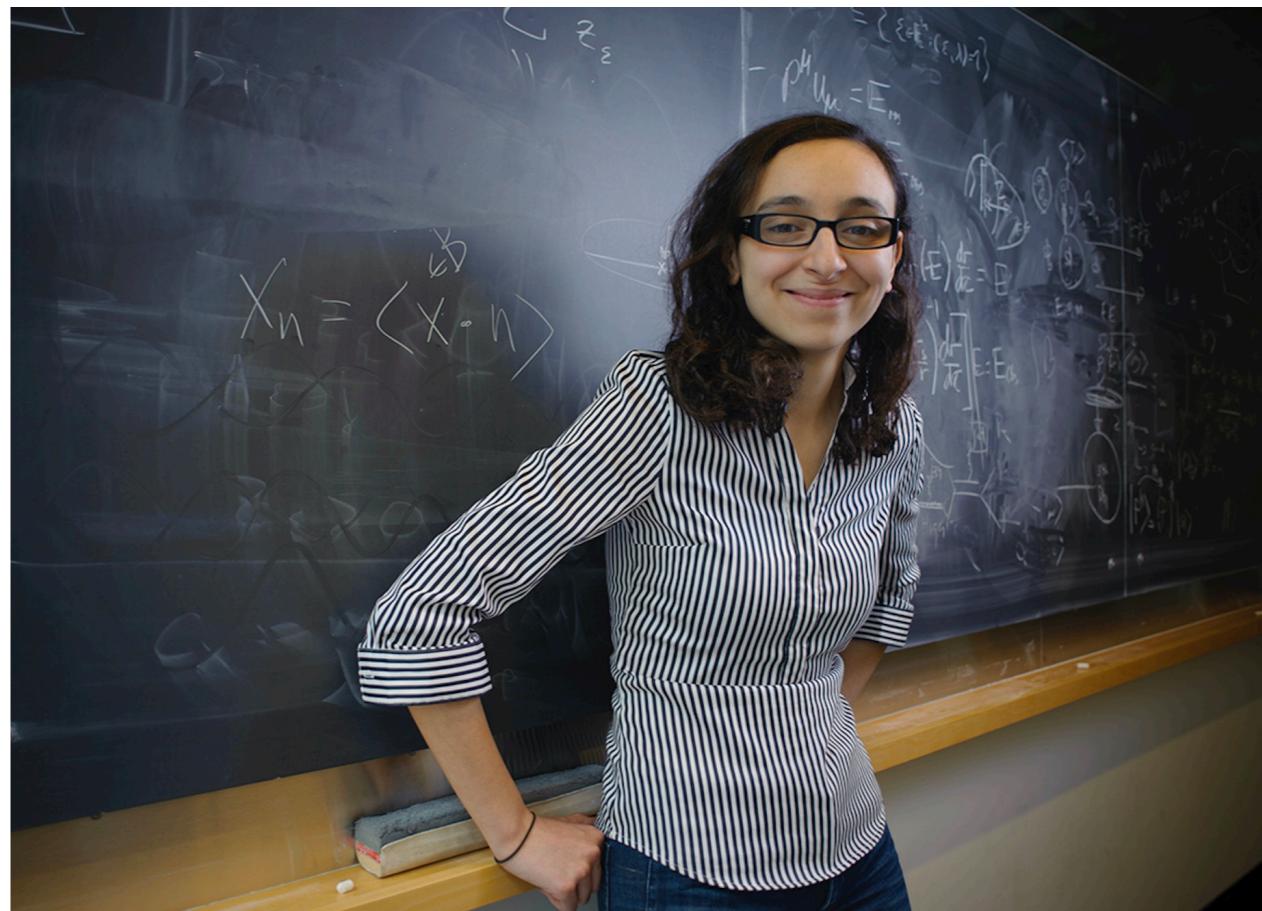
Our Goal In Lecture

- Going to build up the intuition for large-scale simulation
 - Focus on the simulation of galaxies to the universe
 - This lecture will give the tools to do this
- The devil is in the details
 - Our lecture use the core concepts in large scale sim
 - To get everything to work at scale is much harder
- We will touch on where the field is going

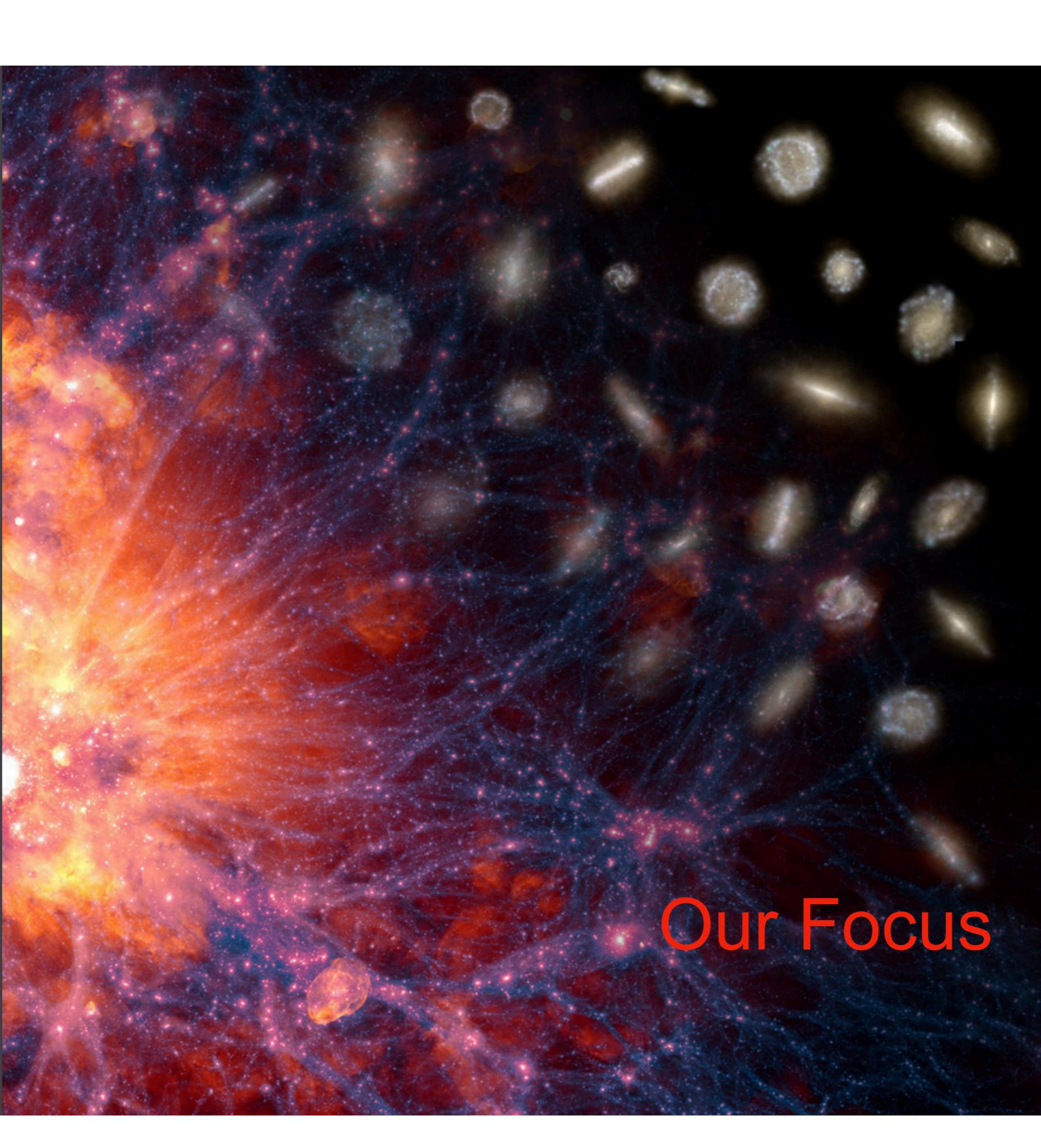
Lets Take a look at the¹⁰ Scale of Things

- <https://www.illustris-project.org/media/>

Prof. Mark Vogelsberg
Main Inspiration for talk



Prof. Lina Necib
Doing related work on smaller scales



Our Focus

Illustris/IllustrisTNG Model: - basic ingredients -

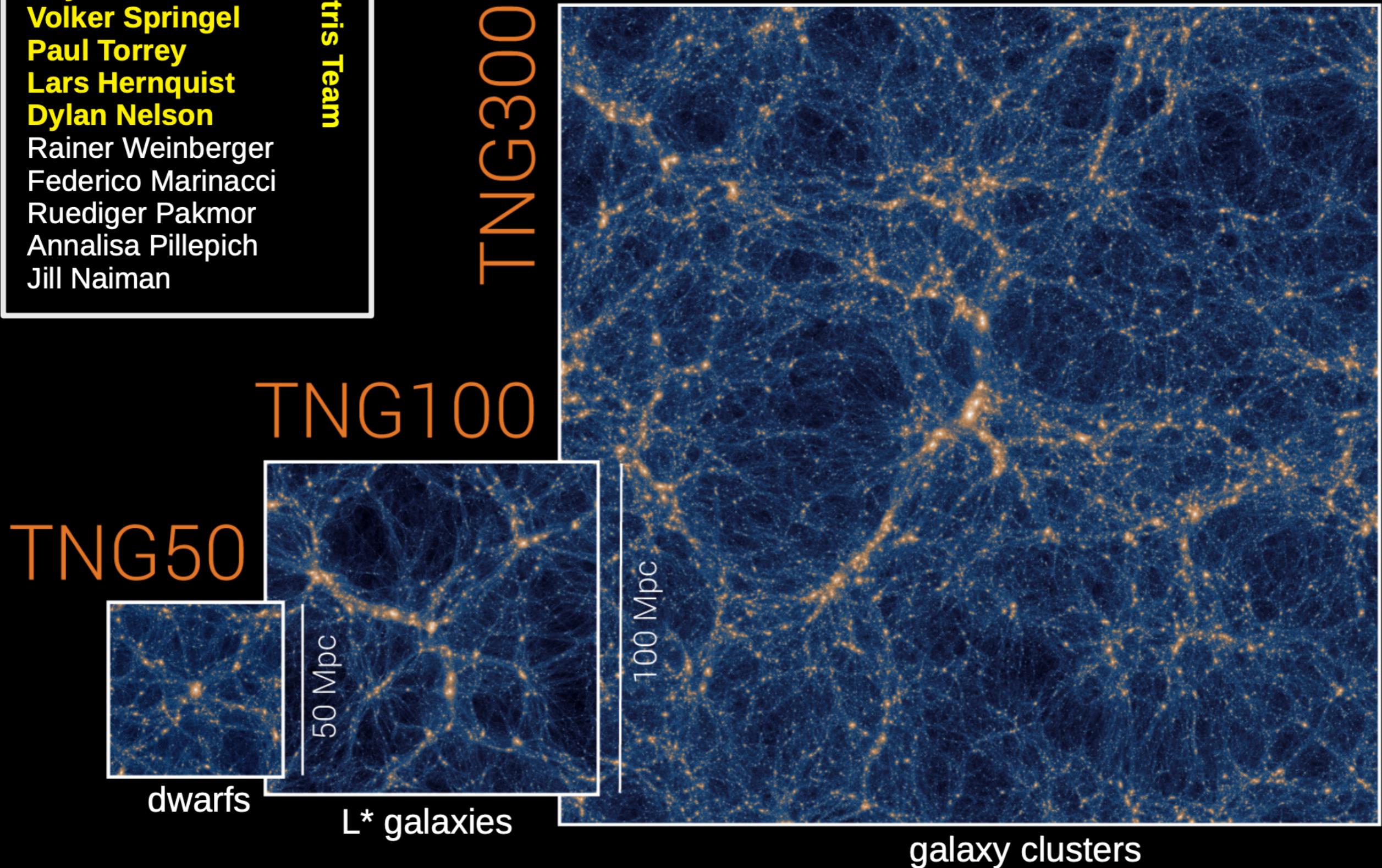
- hydrodynamics:
quasi-Lagrangian moving mesh
(Arepo, Springel 2010)
- heating / cooling:
primordial, metal line
- UV background:
with self-shielding correction
- star formation / ISM:
effective EOS
- chemical enrichment:
9 elements by SNIa, SNII, AGB
- supernova feedback:
kinetic SNII feedback
- supermassive black holes:
seeding, growth, merging
- AGN feedback:
quasar, radio mode, radiative

IllustrisTNG Team:

Mark Vogelsberger
Shy Genel
Volker Springel
Paul Torrey
Lars Hernquist
Dylan Nelson
Rainer Weinberger
Federico Marinacci
Ruediger Pakmor
Annalisa Pillepich
Jill Naiman

Illustris Team

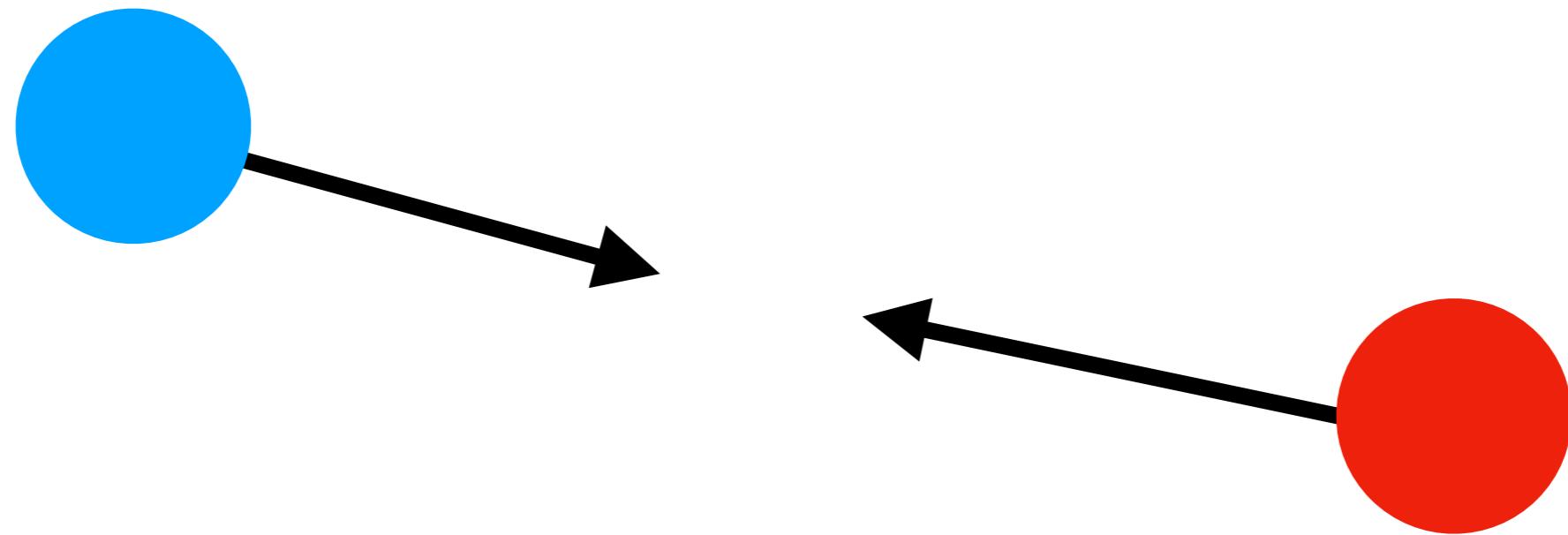
three boxes with different primary science focus
(~250 million CPUh)



Details

- This lecture is built on the following references:
 - <https://www.youtube.com/watch?v=Fo23ihGLPA0>
 - <https://td.lpi.ru/~eugvas/nbody/lectures.pdf>
 - https://www.tat.physik.uni-tuebingen.de/~schaefer/teach/f/chaos_english.pdf
 - <https://blbadger.github.io/3-body-problem.html>

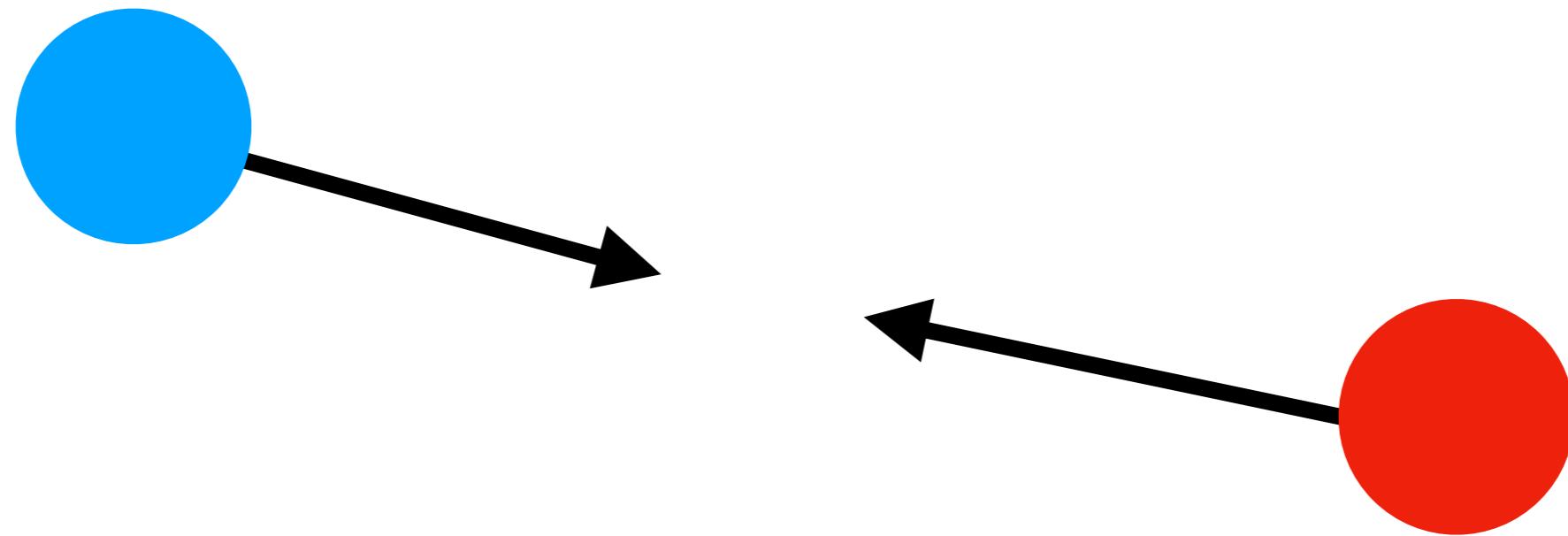
What do we need for Simulation?



$$F = \frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

- All Physics we need has been developed 100s of years ago
 - The two body problem has been solved since the 18th century

What do we need for Simulation?¹⁵

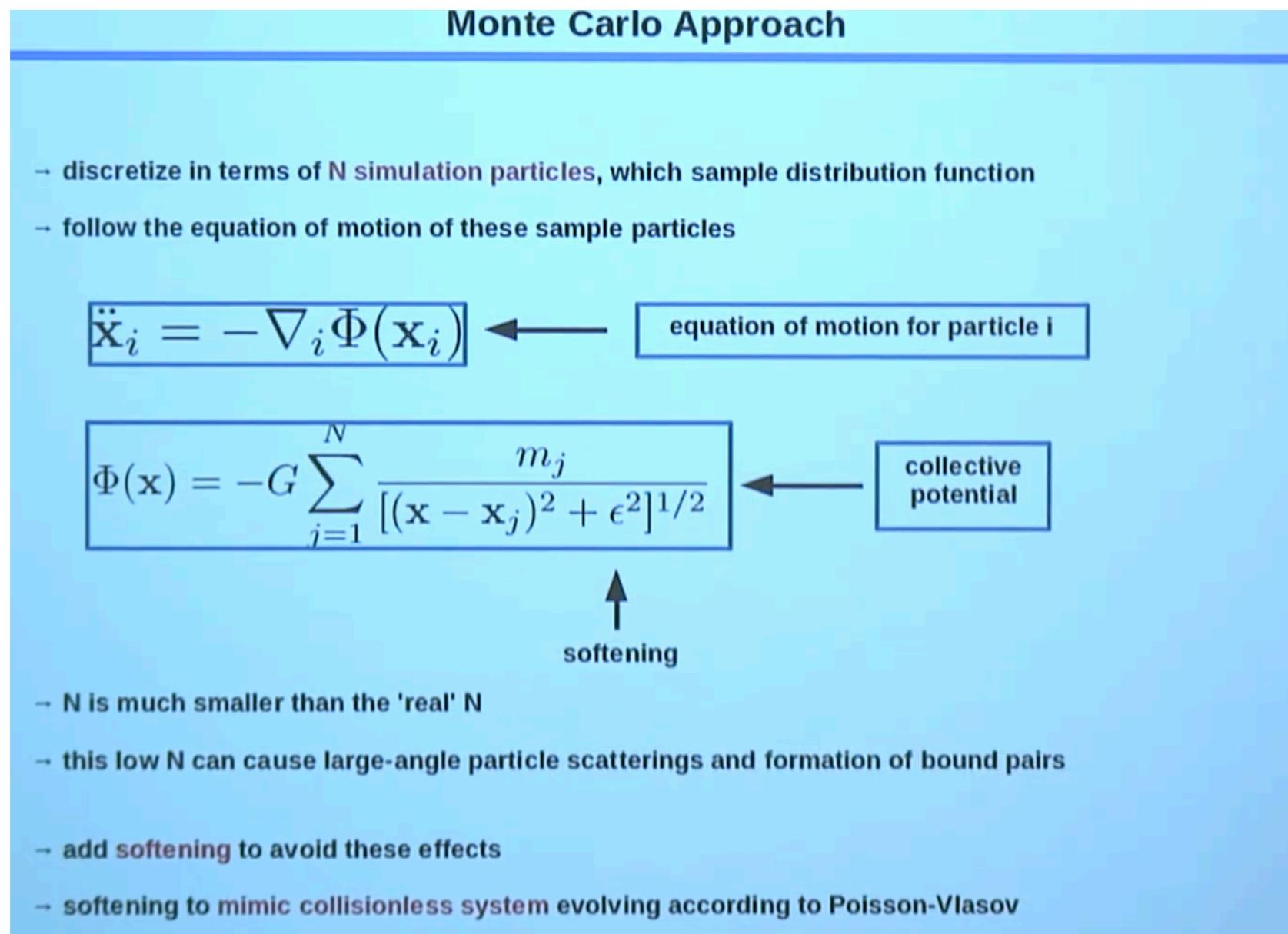


$$F = \frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2 + \epsilon|^3} (\vec{r}_1 - \vec{r}_2)$$

- To simulate this on a computer we will add a “softening” term

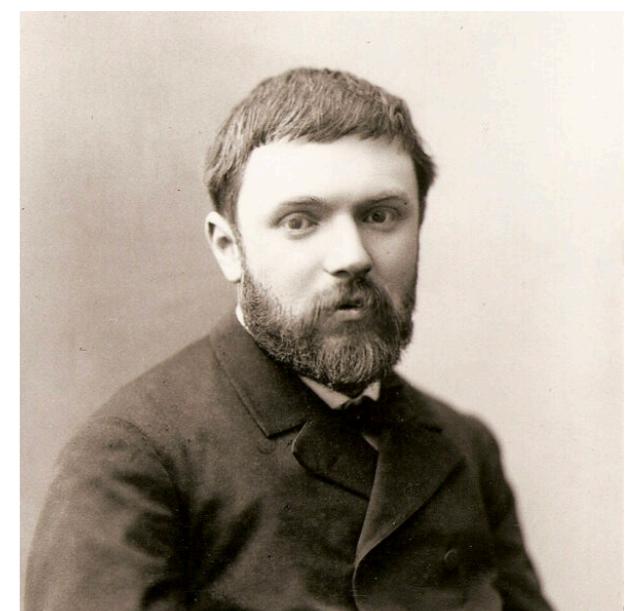
This is how DM/galaxies¹⁶ are modeled

- Instead of treating matter as a fluid
 - Discretize matter into chunks and solve n-body problem



3 body Problem: History

- The original two body problem was solved in 18th century
 - Work done by Netwon, Bernoulli Bros, Euler, Laplace,...
 - All started on the 3 body problem and built on the two body
- King Oscar II decided to make a competition:
 - For his 60th birthday he bestowed a prize on who could solve the n-body problem

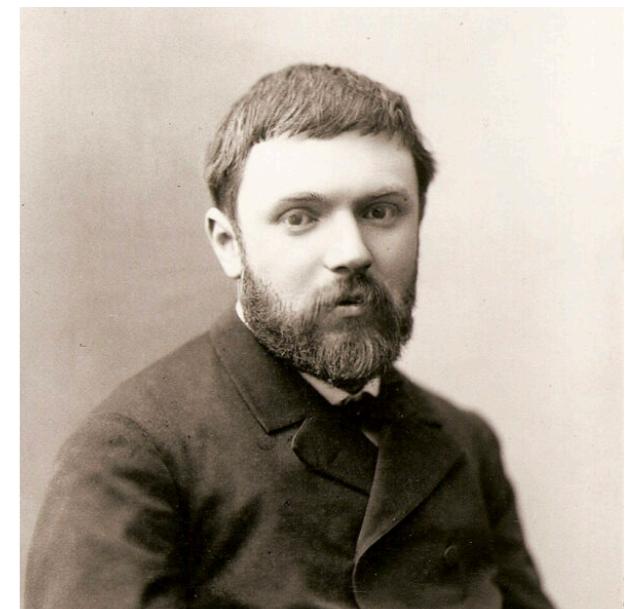


3 body Problem: History

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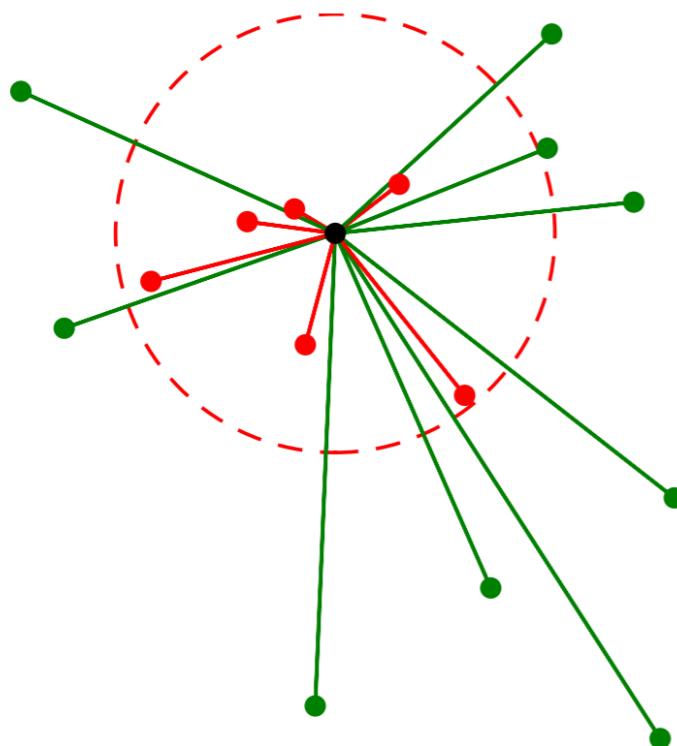
Henri Poincare →

He proved no solution existed!



Going to N-body

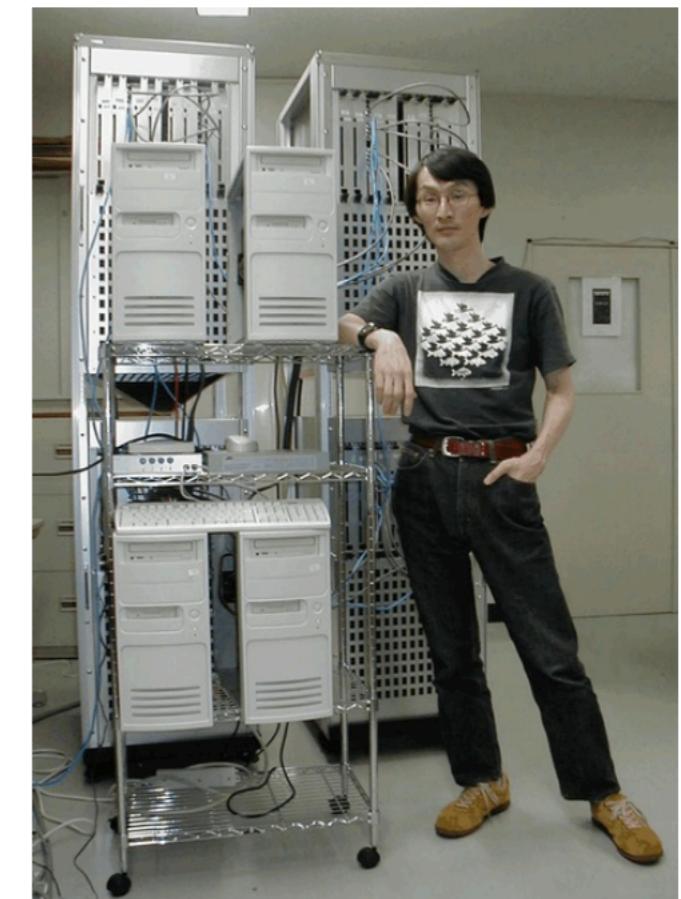
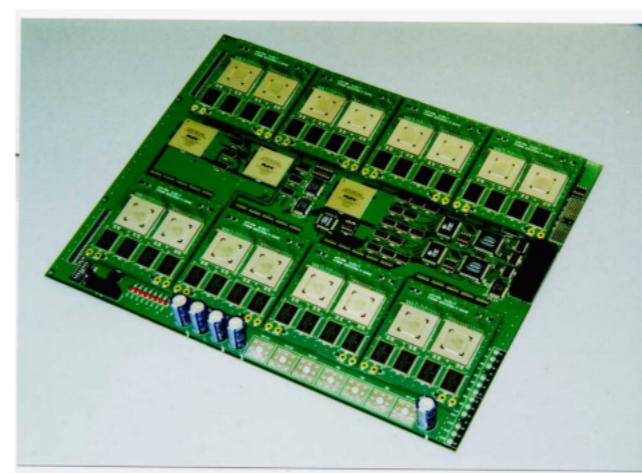
- The challenge of solving this numerically for n-body
 - This scales with the number of bodies N^2
 - Requires the computations of all pairwise distances



- For $N=1000$ (1 Million/step computations)
- For $N=10^6$ Something ridiculous

Historical Solutions

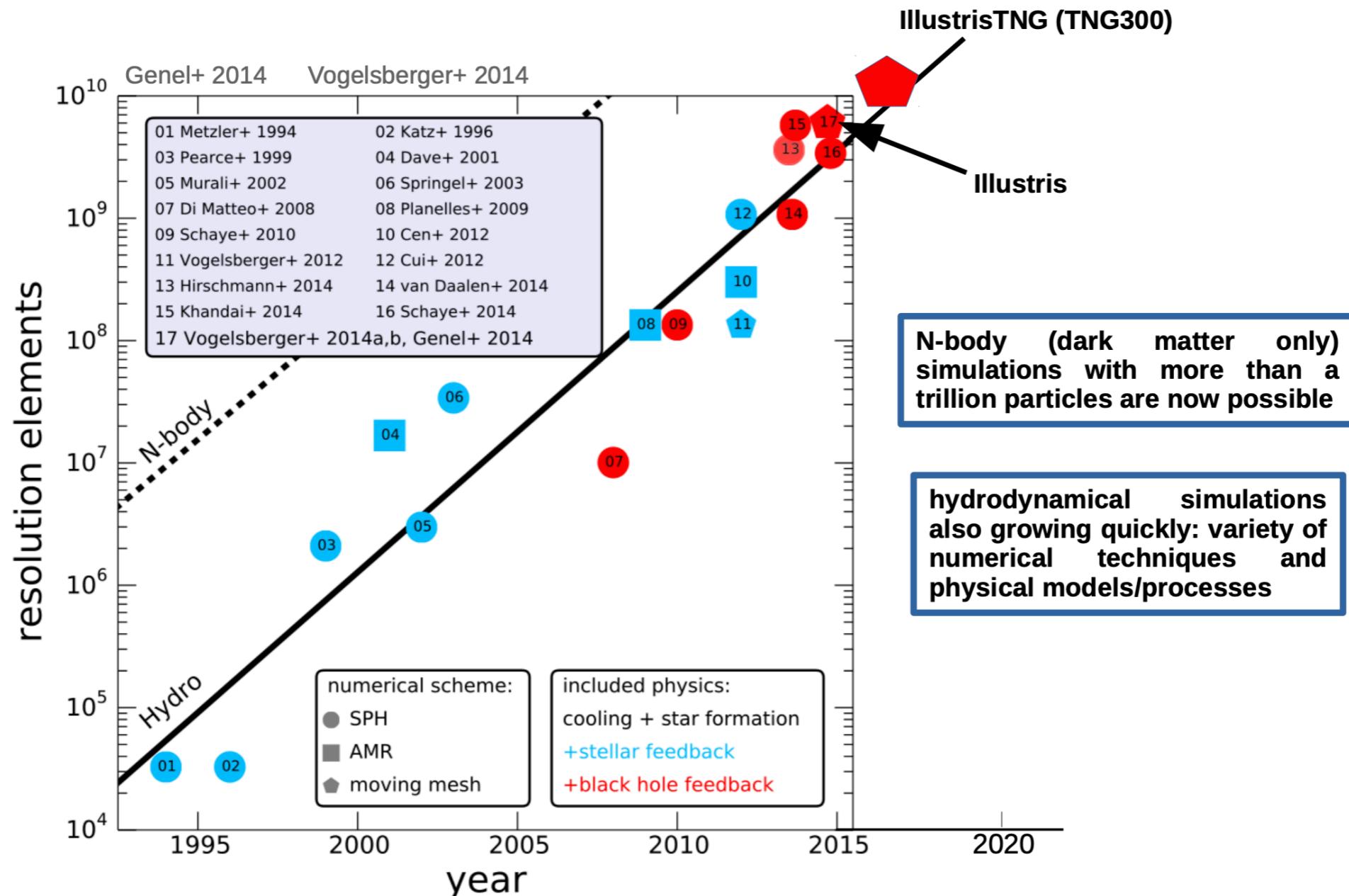
- One approach has been to build dedicated computing hardware
 - Dedicated hardware that can do large-scale parallel computation
 - Focused specifically on n-body simulation
 - GRAPE boards (GRAvity PipelinE)
- Now done with GPUs



Jun Makino with GRAPE-6

Scaling of n-body

The Evolution of Large-Scale Simulations

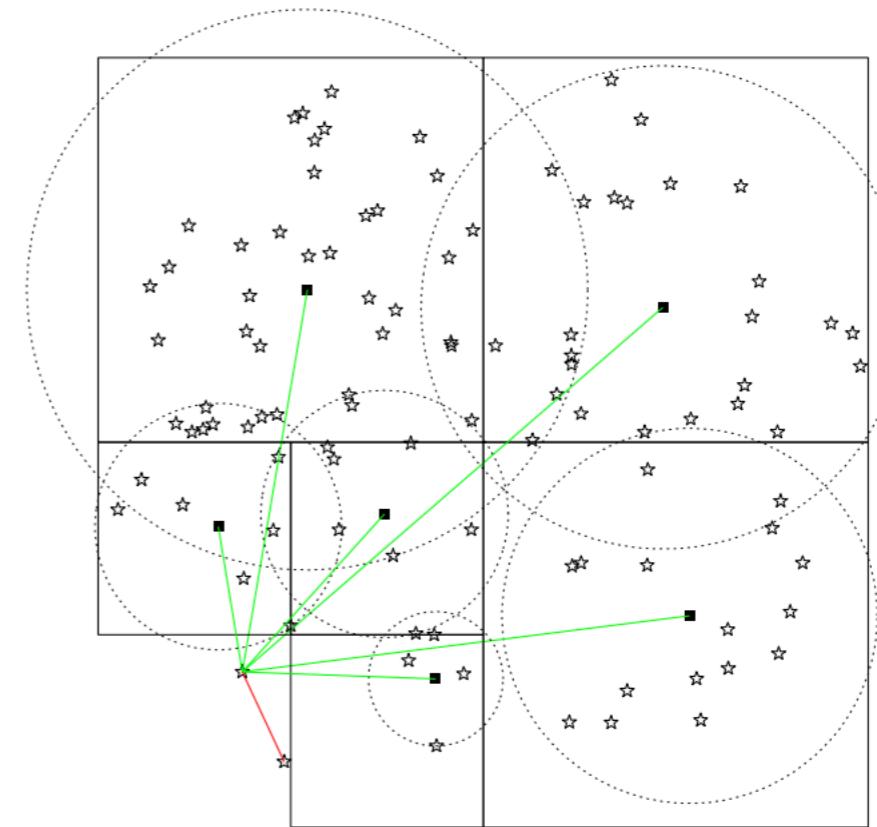
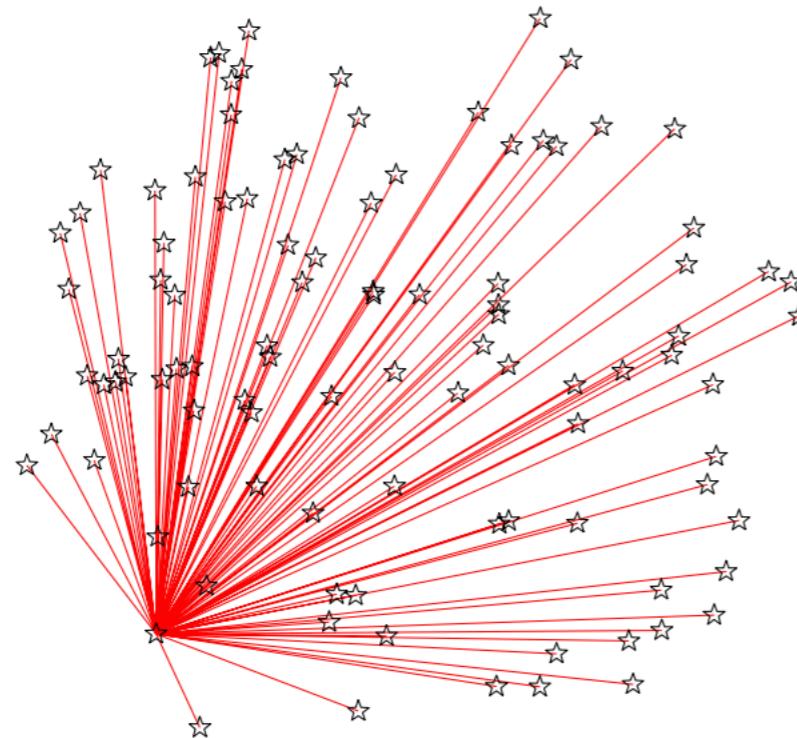


- M. Vogelsberger(https://indico.cern.ch/event/736594/contributions/3184103/attachments/1738225/2812076/talk_vogelsberger.pdf)

How²²

do you deal with N-body?

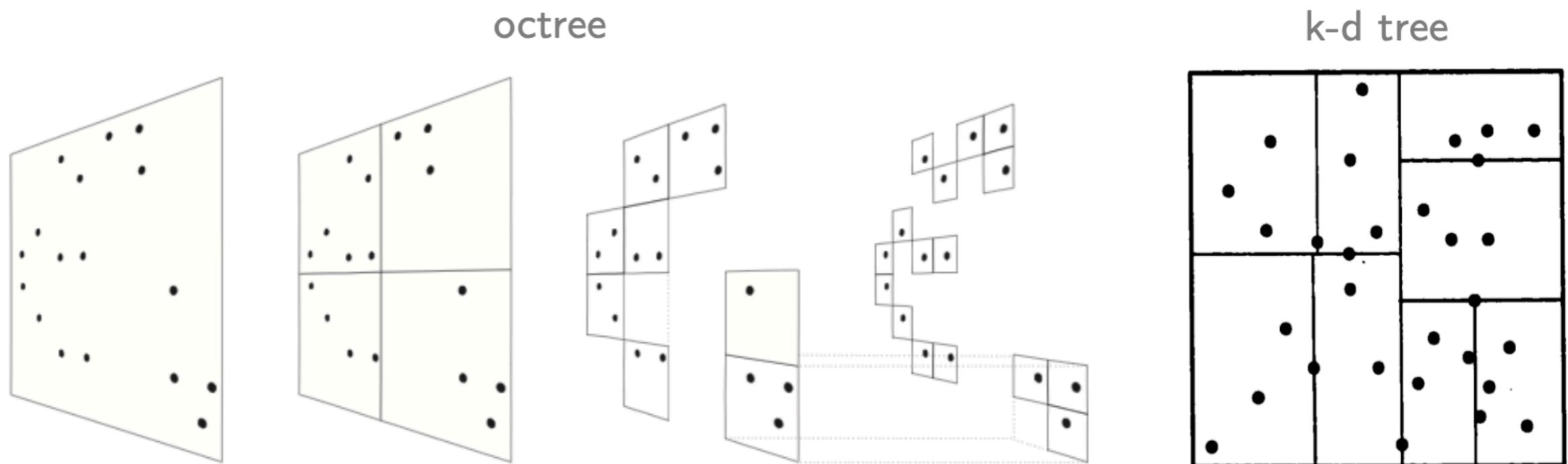
- Barnes-Hut Algorithm re-rank stars into a tree structure
 - Structure is a grid over the whole space



[from Dehnen & Read 2011]

Tree Construction

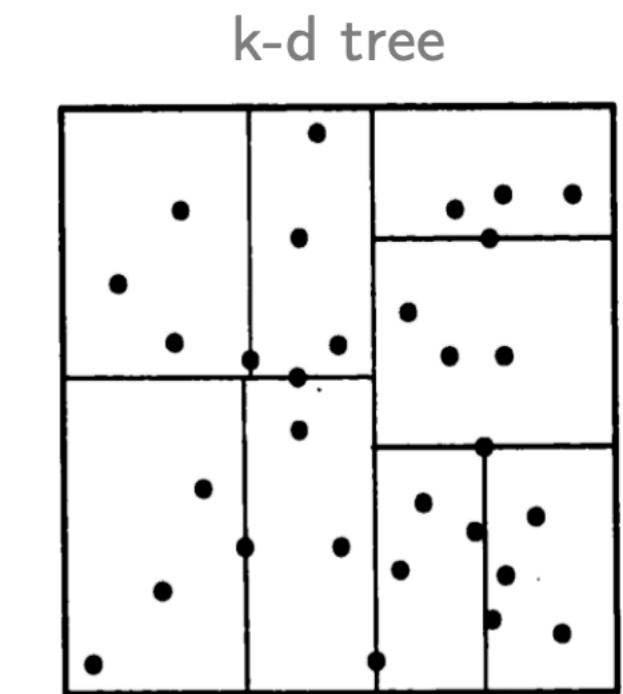
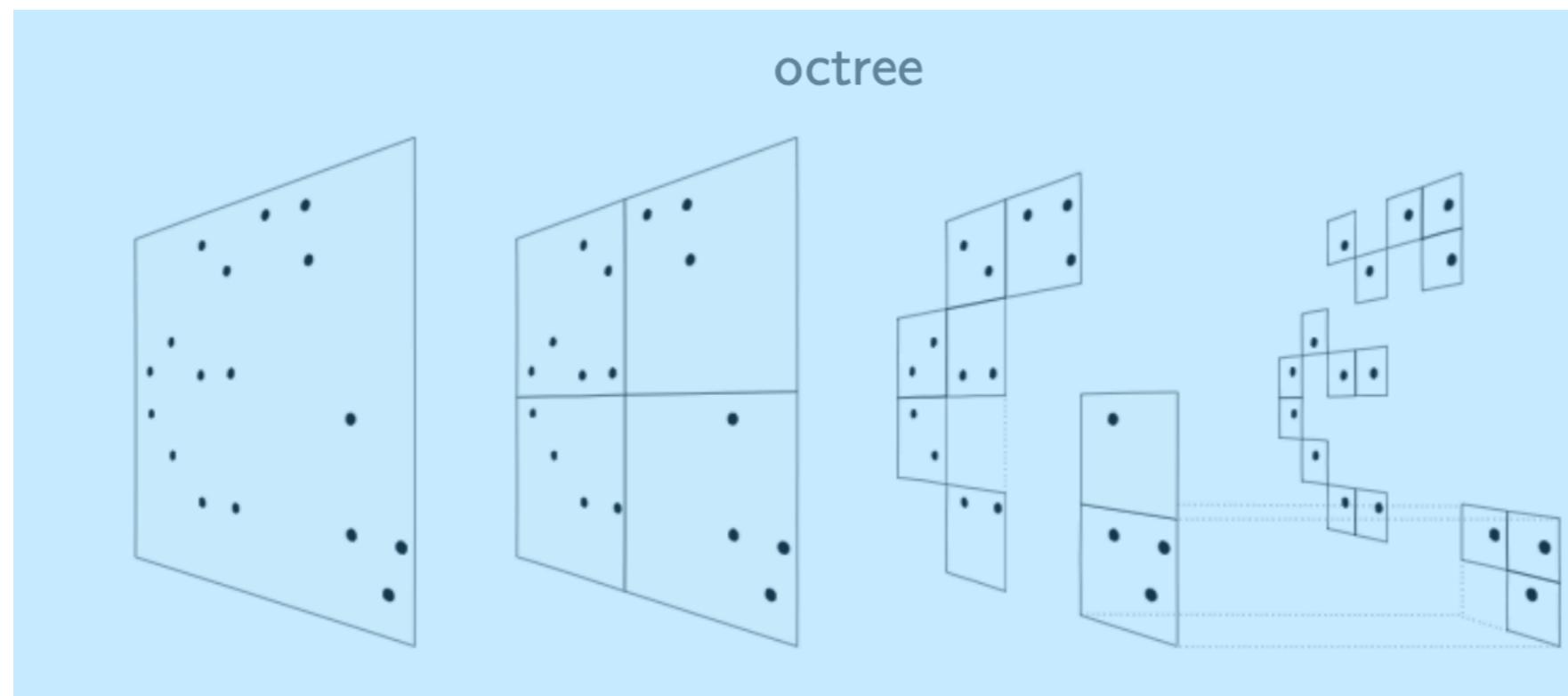
- QuadTree/OctTree
 - Split each square(cube) into 4(8) sub regions
- KD Tree
 - Use the data to draw equal numbered regions in space



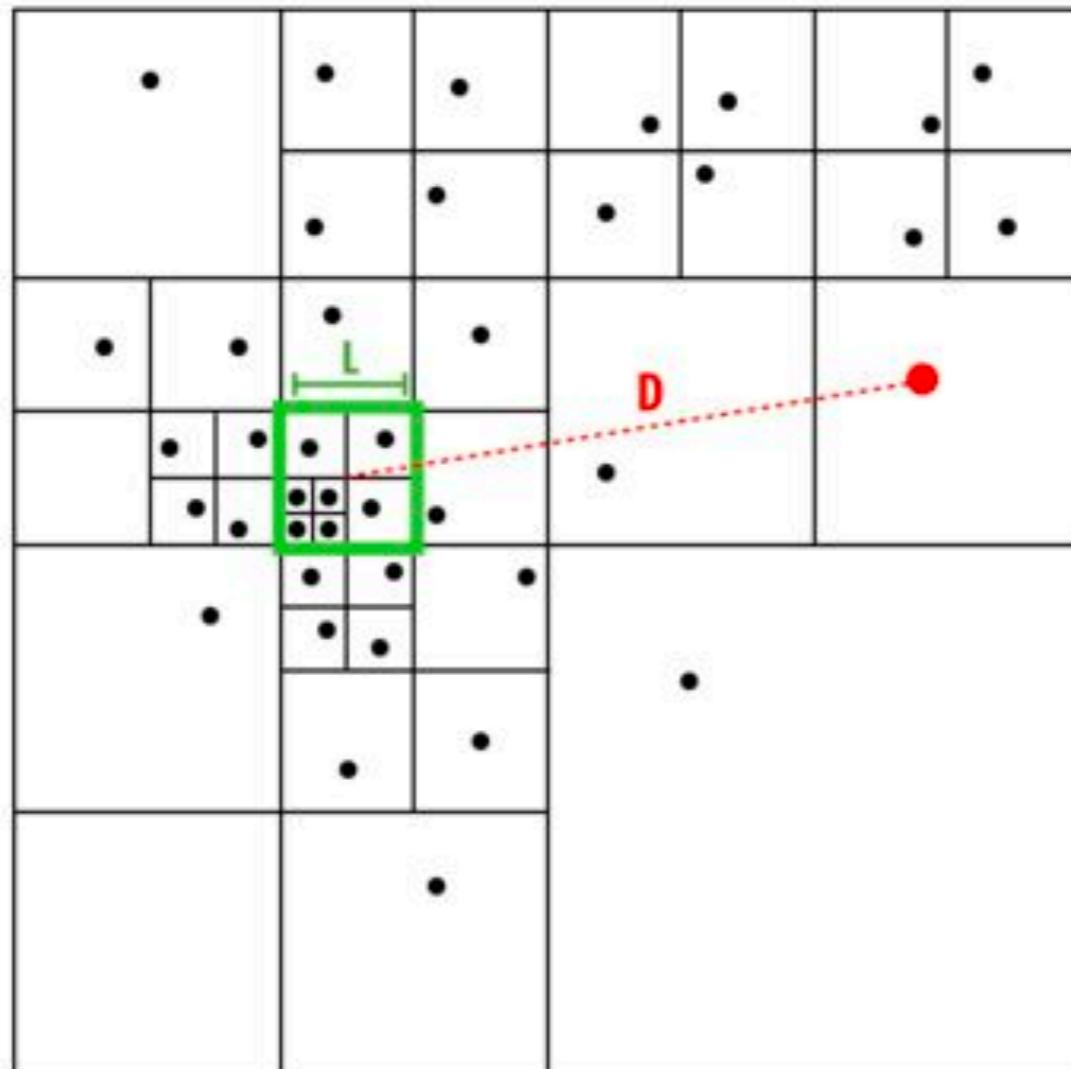
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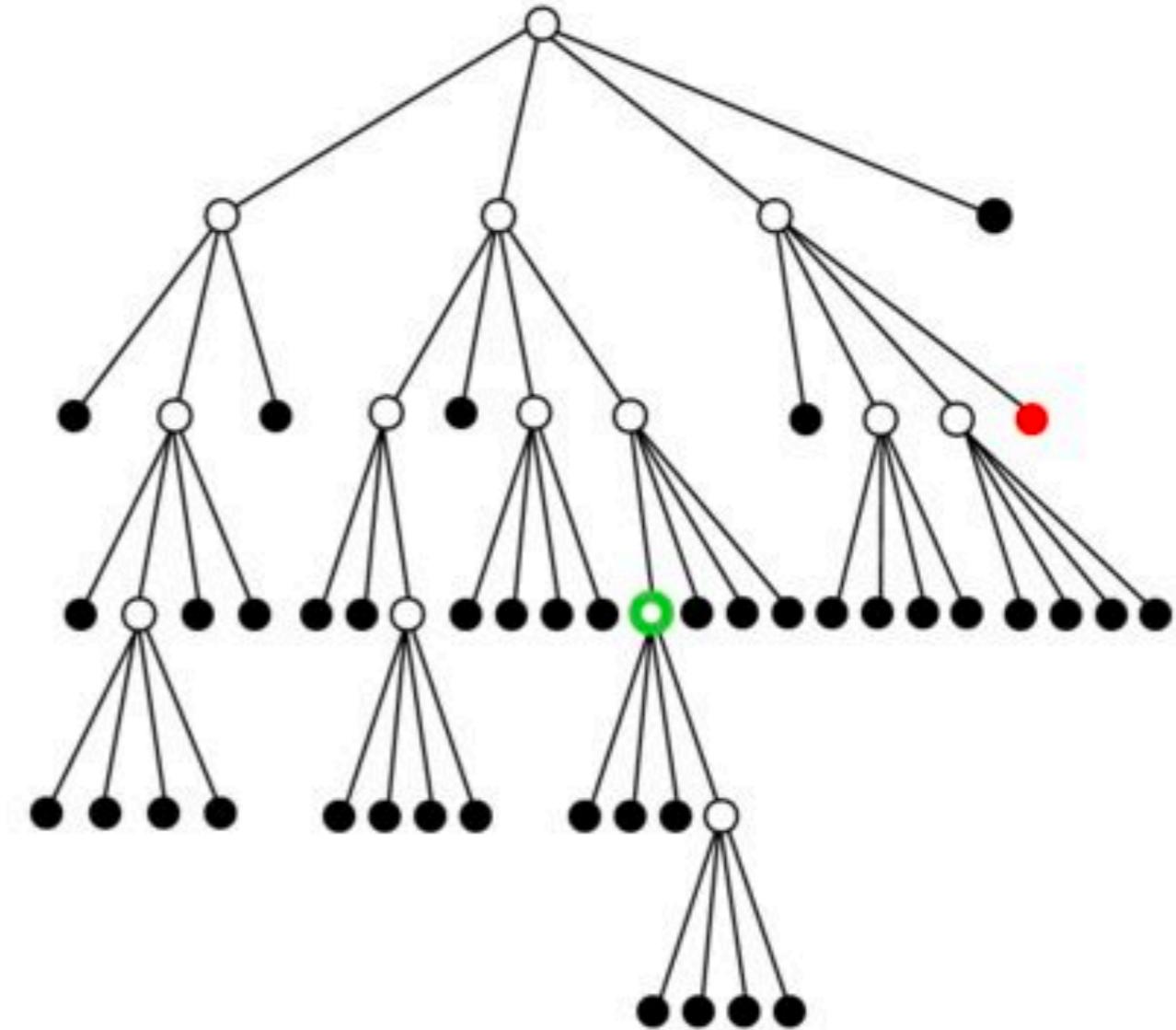
Just requires that we know the bounds of the space



Visualizing Tree



Spatial Domain



Quad-Tree Representation

Note that for the big grids, this equates to Gauss' law style approach
Treat each square as a star w/total mass at mass weighted center

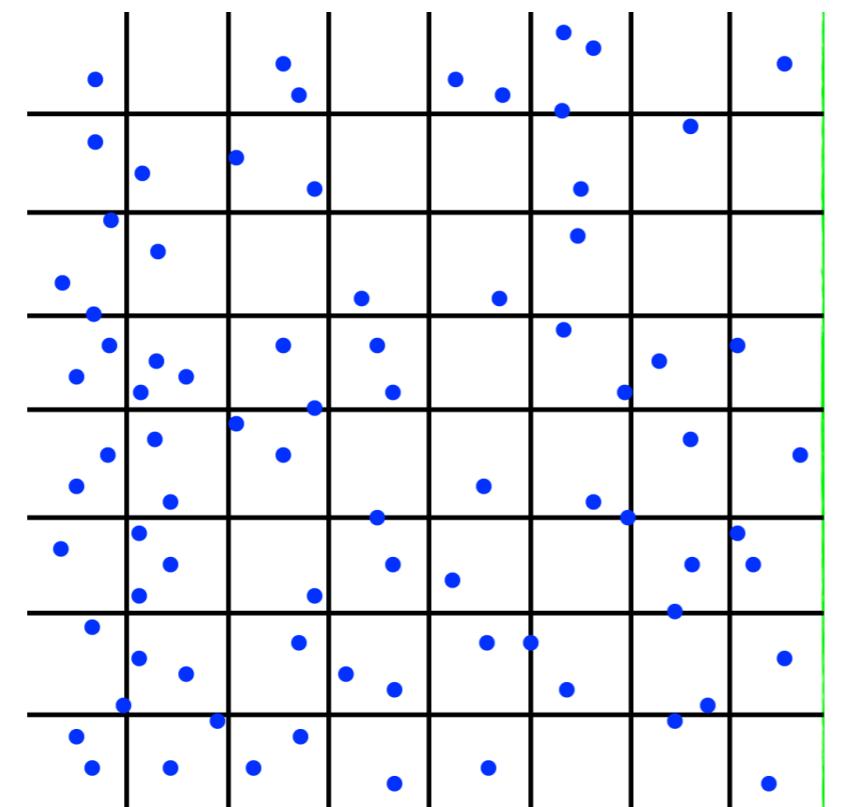
Barnes-Hut Algorithm

- We can follow a step by step construction of this :
 - 1. Construct tree structure with bounds
 - 2. Loop over stars and fill tree structure
 - 3. Loop over stars and compute distance
 - Full n-body computation for nearby trees only
 - 4. Step forward everything
- The above process is $N \log(N)$ in computational time

Larger Scale Concept

- For the really large scale n-body dynamics
 - To get global motion
 - Grid up the whole space and Fourier transform it
 - Evolve the system in Fourier space

FFT(



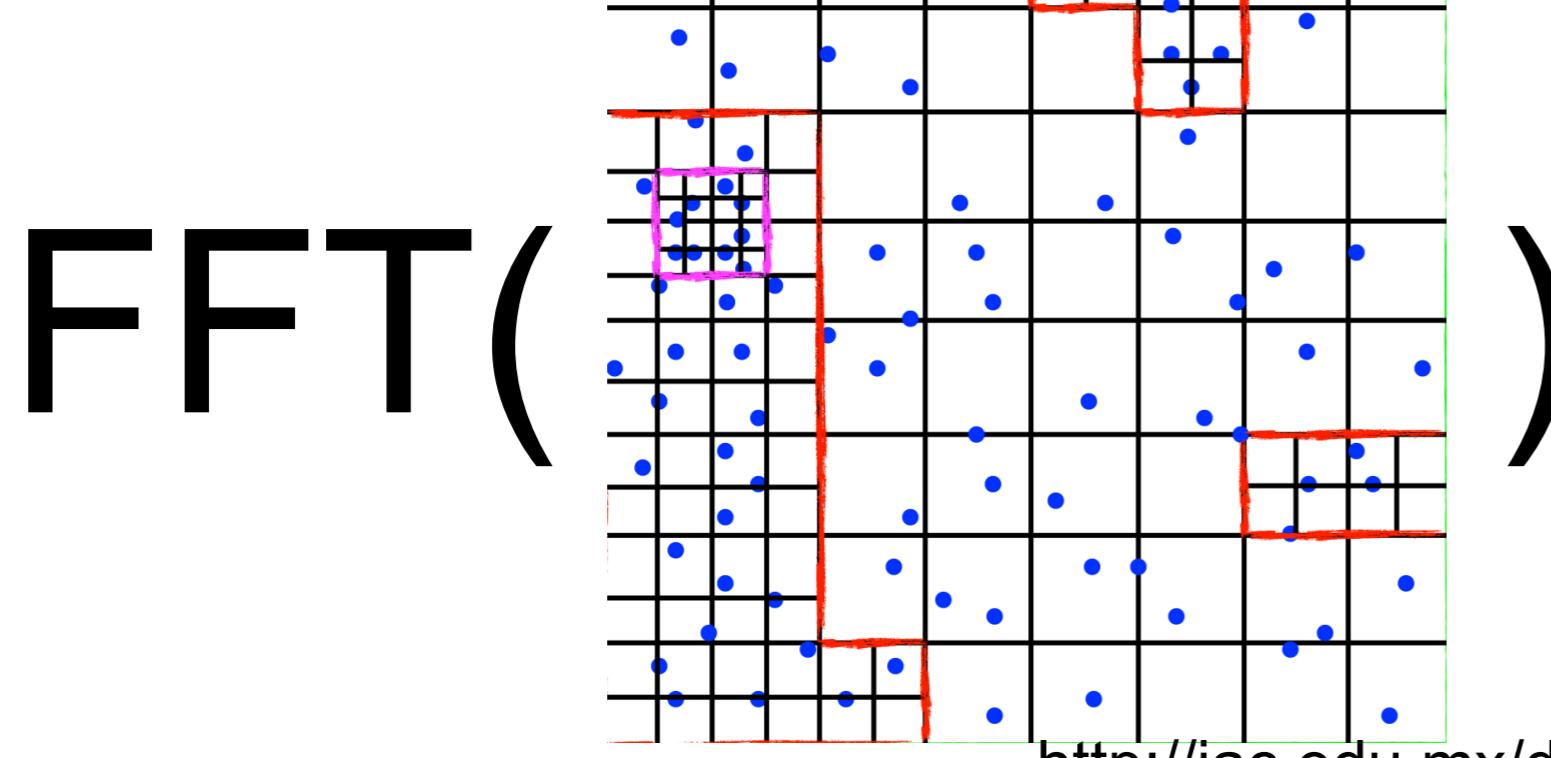
)

Solve for the Gravational
Potential in Fourier space

Step Potential

Larger Scale Concept

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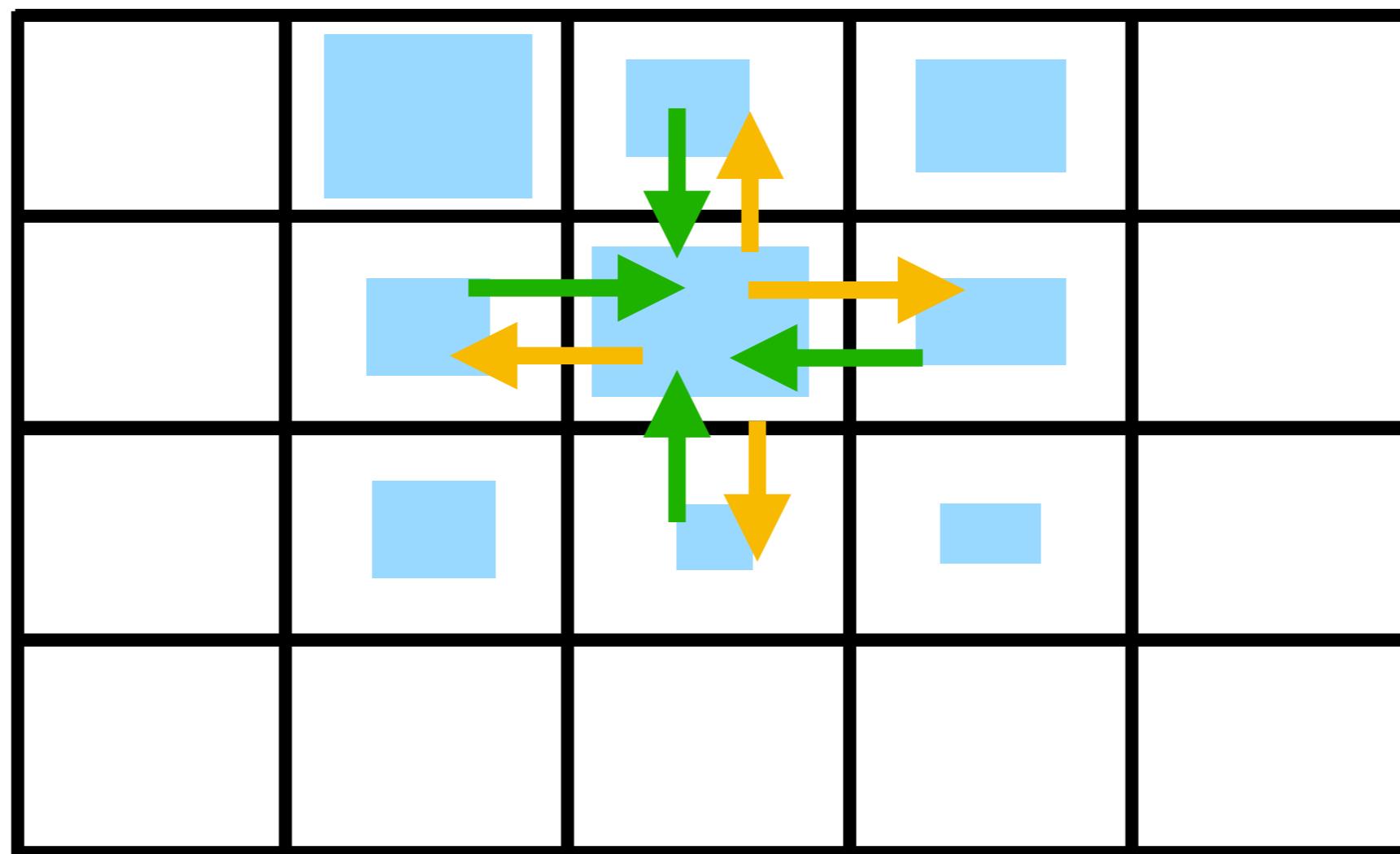


Solve for the Gravational Potential in Fourier space

Step Potential

Hydrodynamics

- Hydrodynamics we want to consider motion along a grid
- Each grid has a density
 - We consider motion in and out of specific grid



For Galactic Matter

- Unlike Dark matter, standard model interacts with each other
- Need to consider particle interactions in addition to gravity
 - These are governed by fluid flow adding particle interactions
 - When simulating you do n-body dynamics
 - However you need to save density information of body
 - When bodies collide need to solve fluid flow dynamics

Berger's Equation

- We will consider a few simple fluid equations:

- $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$ (Burger's equation)

- U is velocity and ν is the viscosity

Fluid Equations

- Inviscid Fluid flow is governed by the Euler Equations
 - Aka Navier-stokes with zero viscosity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

CONTINUITY

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \otimes u) + \nabla p = -\rho \nabla \phi$$

MOMENTUM

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot [\rho u (e + p/\rho)] = -\rho u \cdot \nabla \phi$$

ENERGY

mass
density

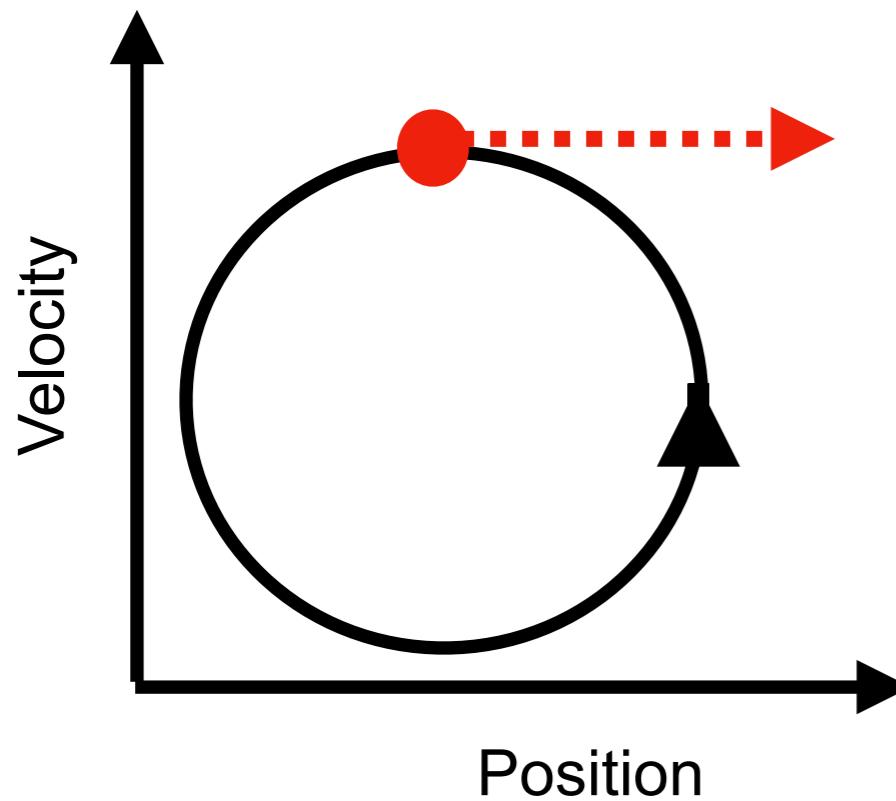
specific
energy

fluid
velocity

thermal
pressure

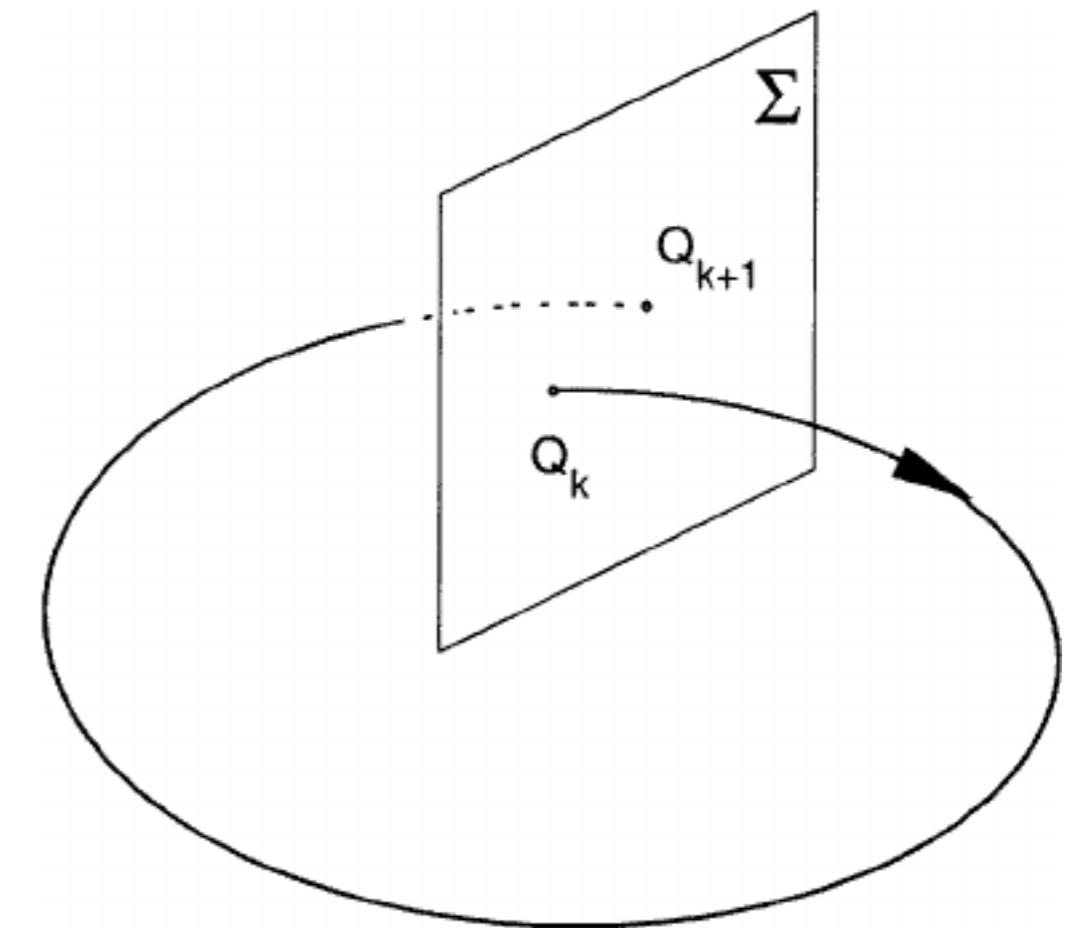
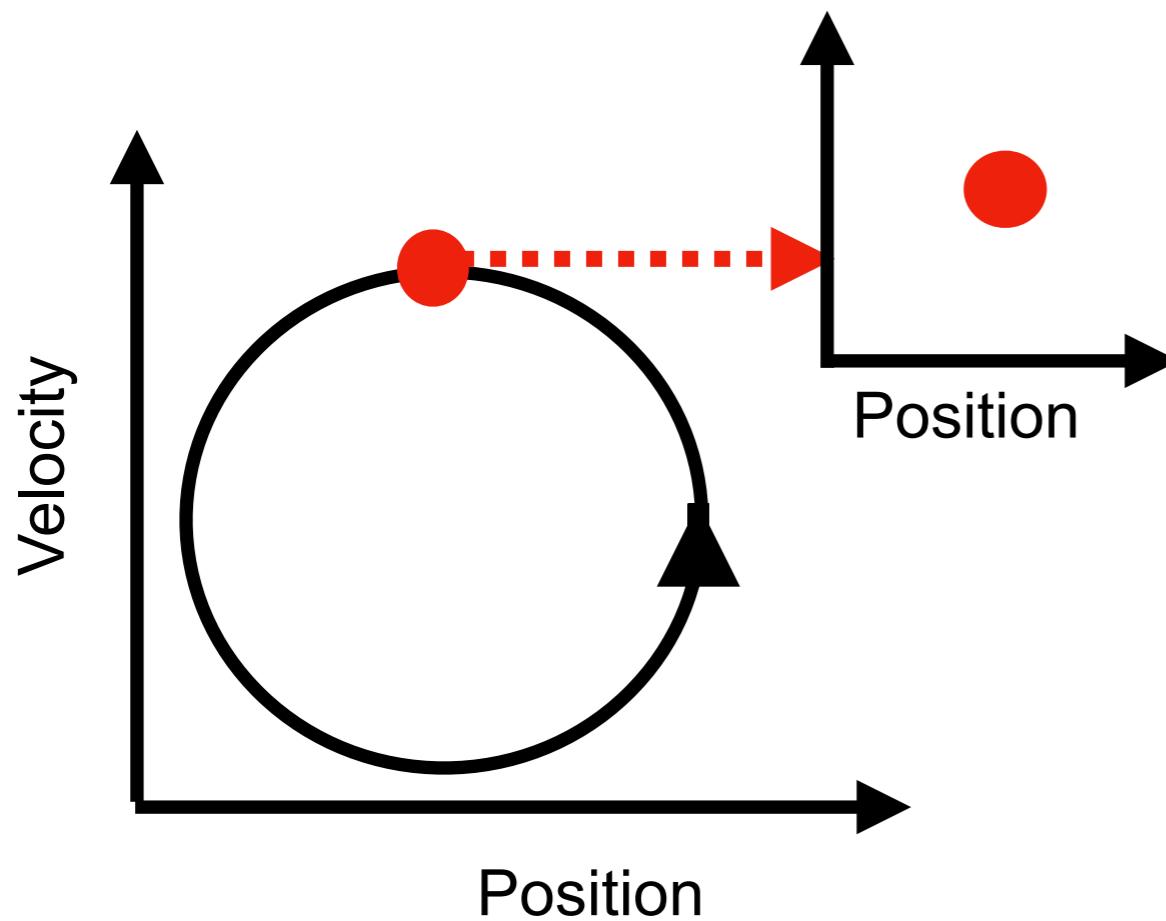
acceleration

Poincare Map



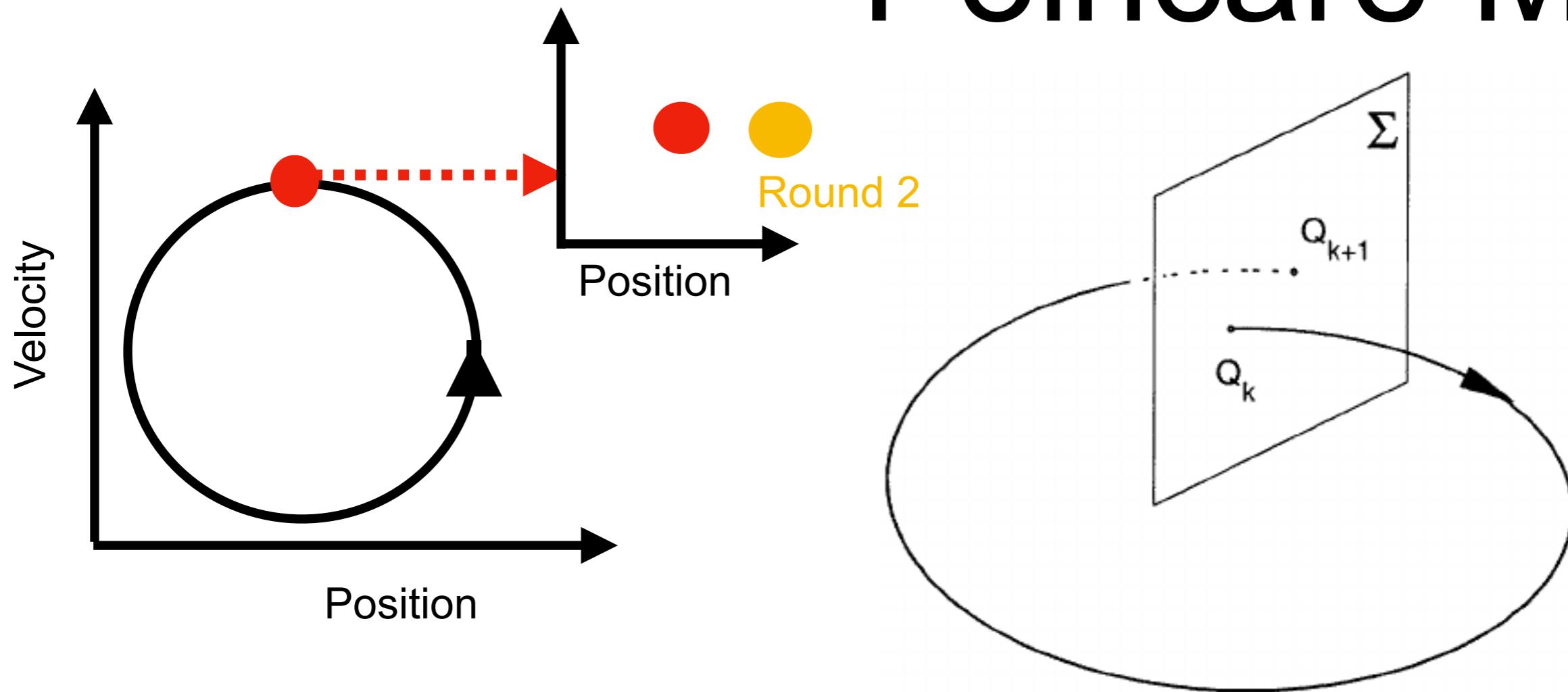
- Looking at the evolution for a fixed velocity or position point that a trajectory oscillates through defines a poincare map

Poincare Map



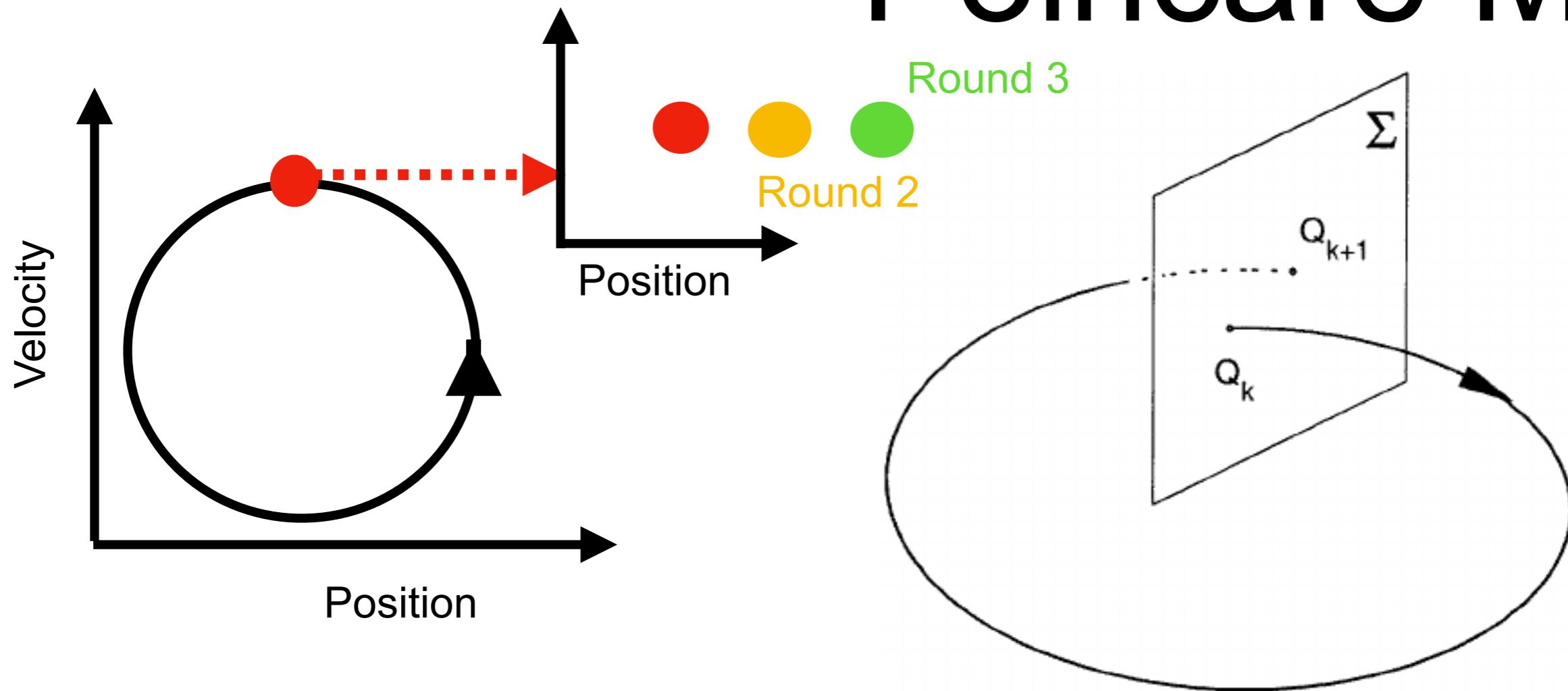
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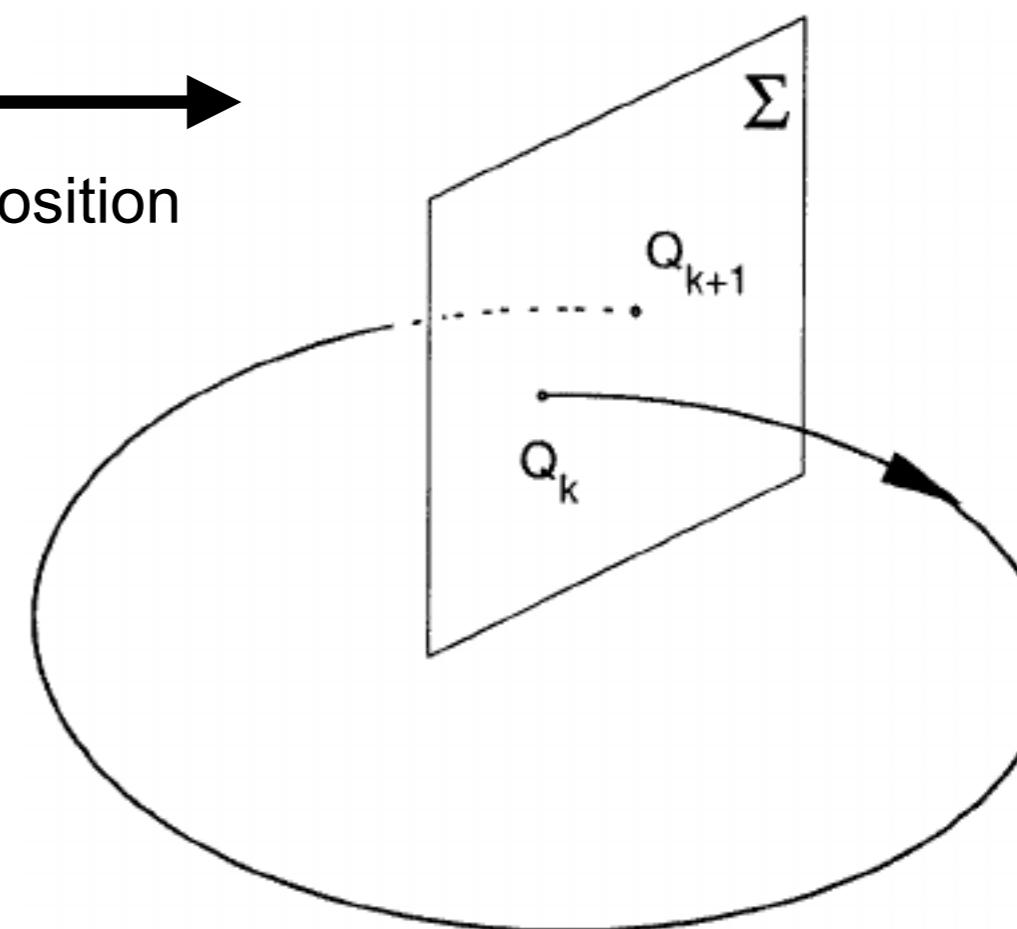
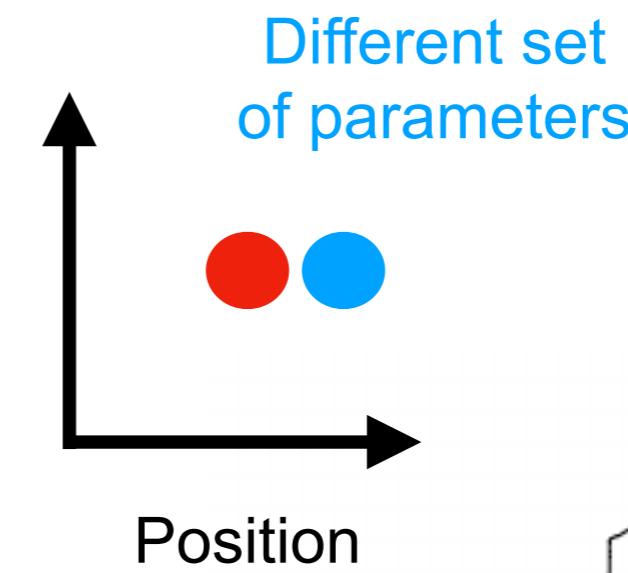
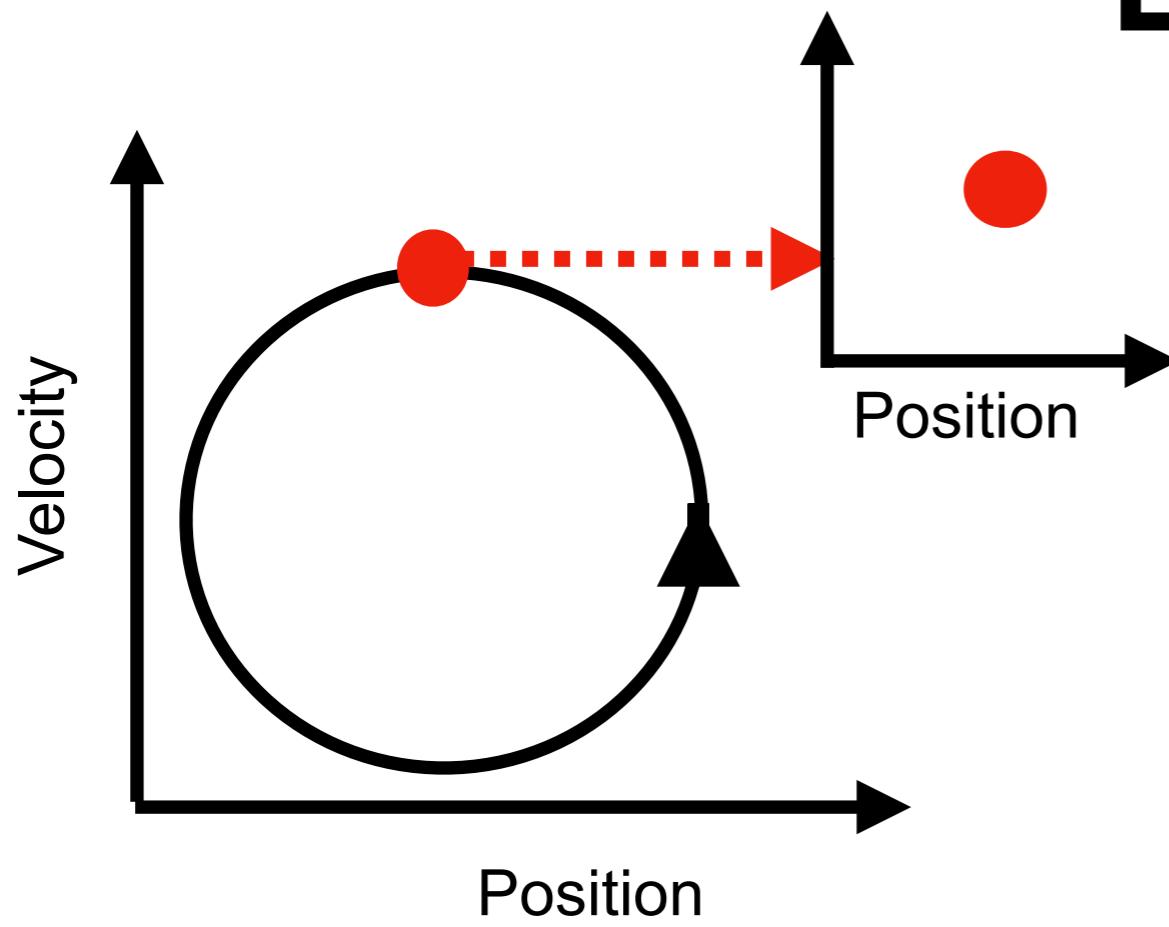
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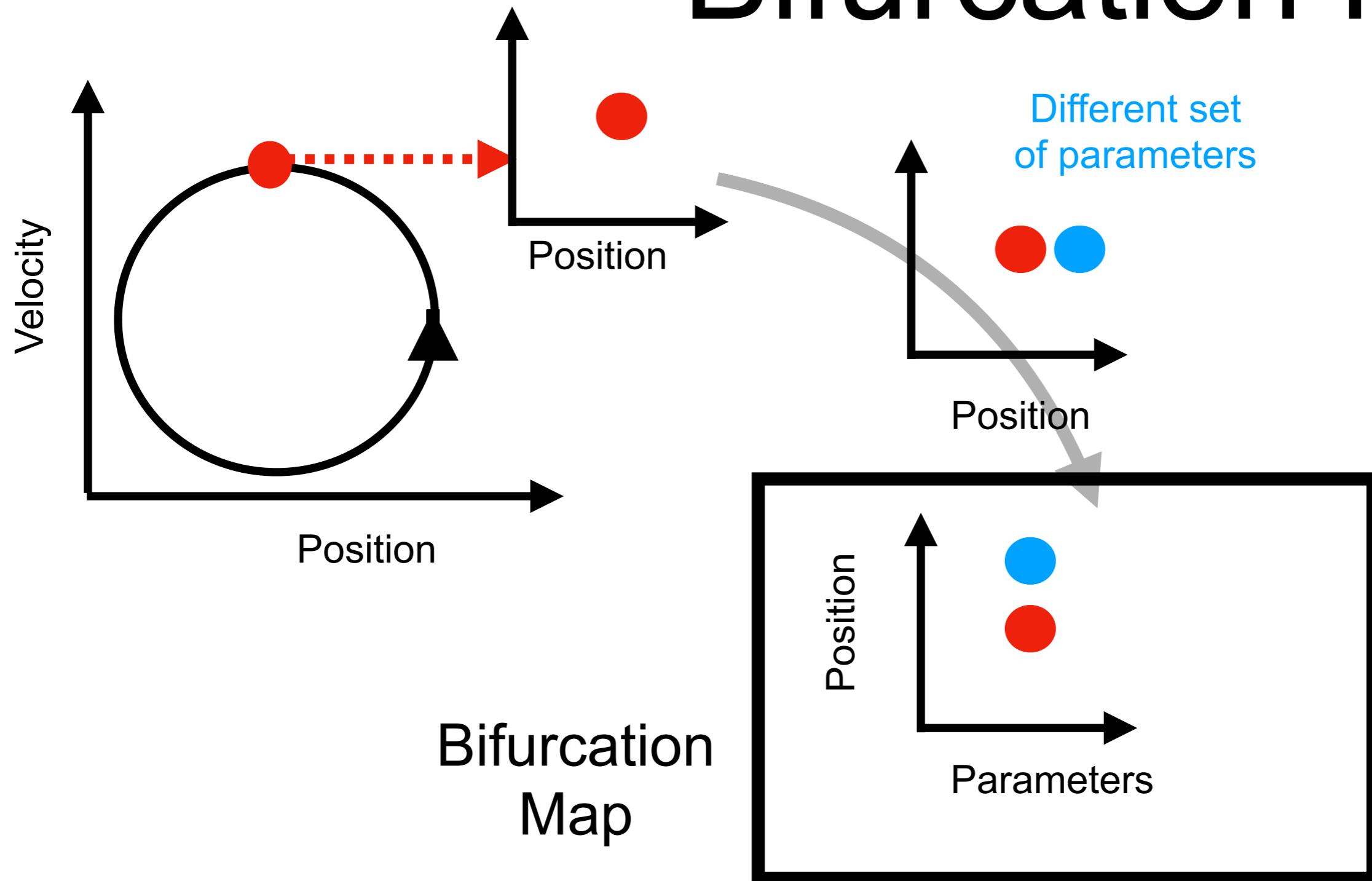
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Bifurcation Map



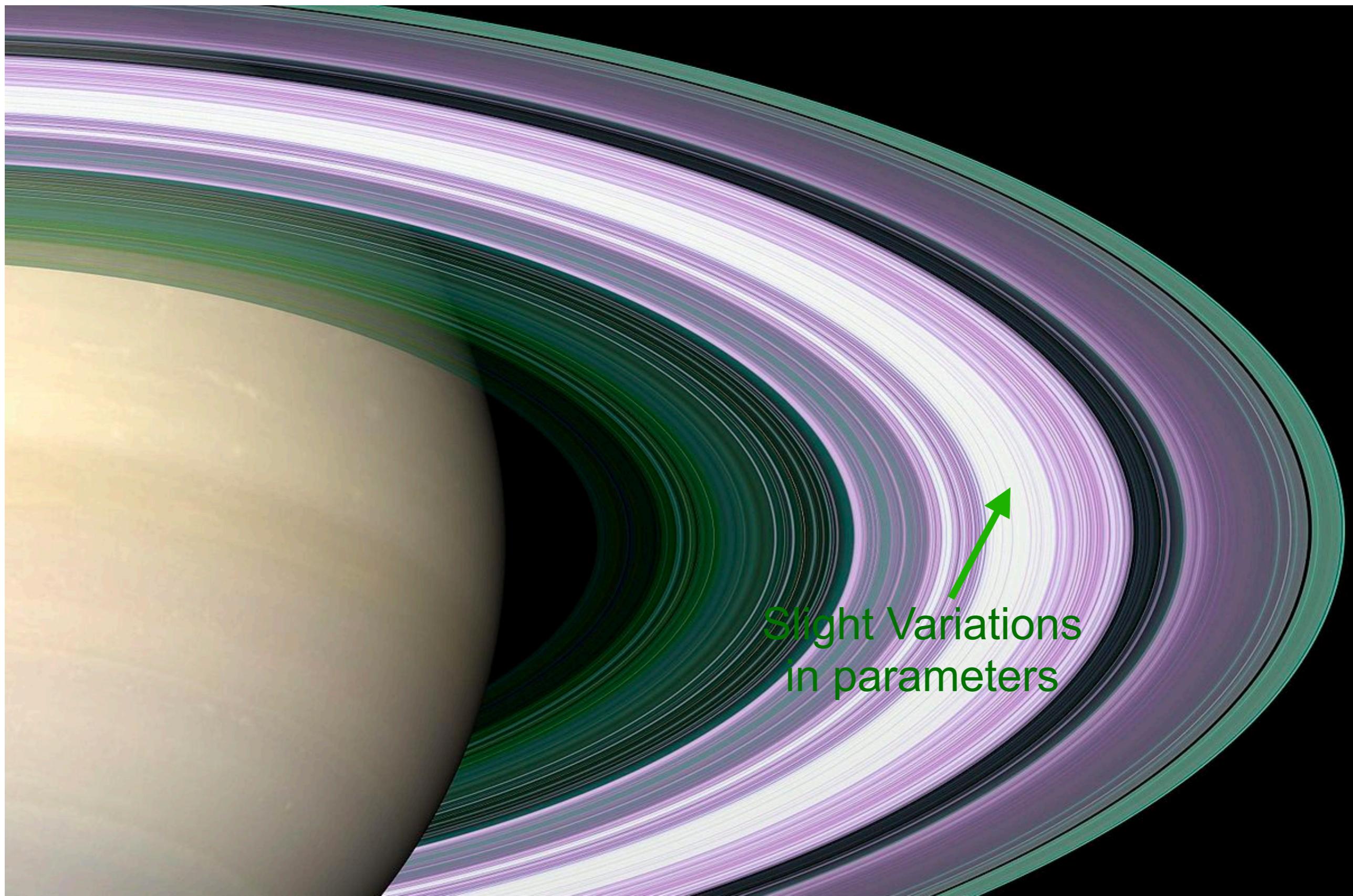
- We can look at behavior over parameters

Bifurcation Map

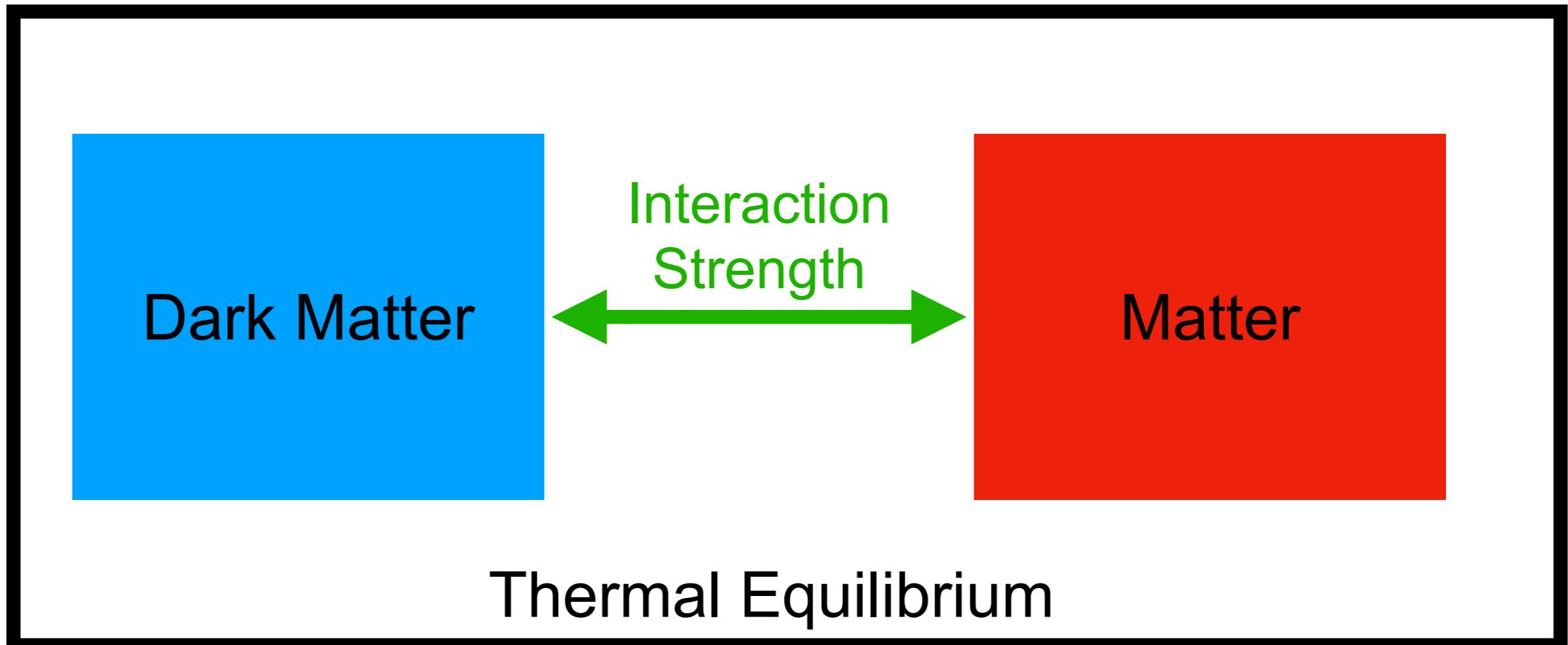


- We can look at behavior over parameters

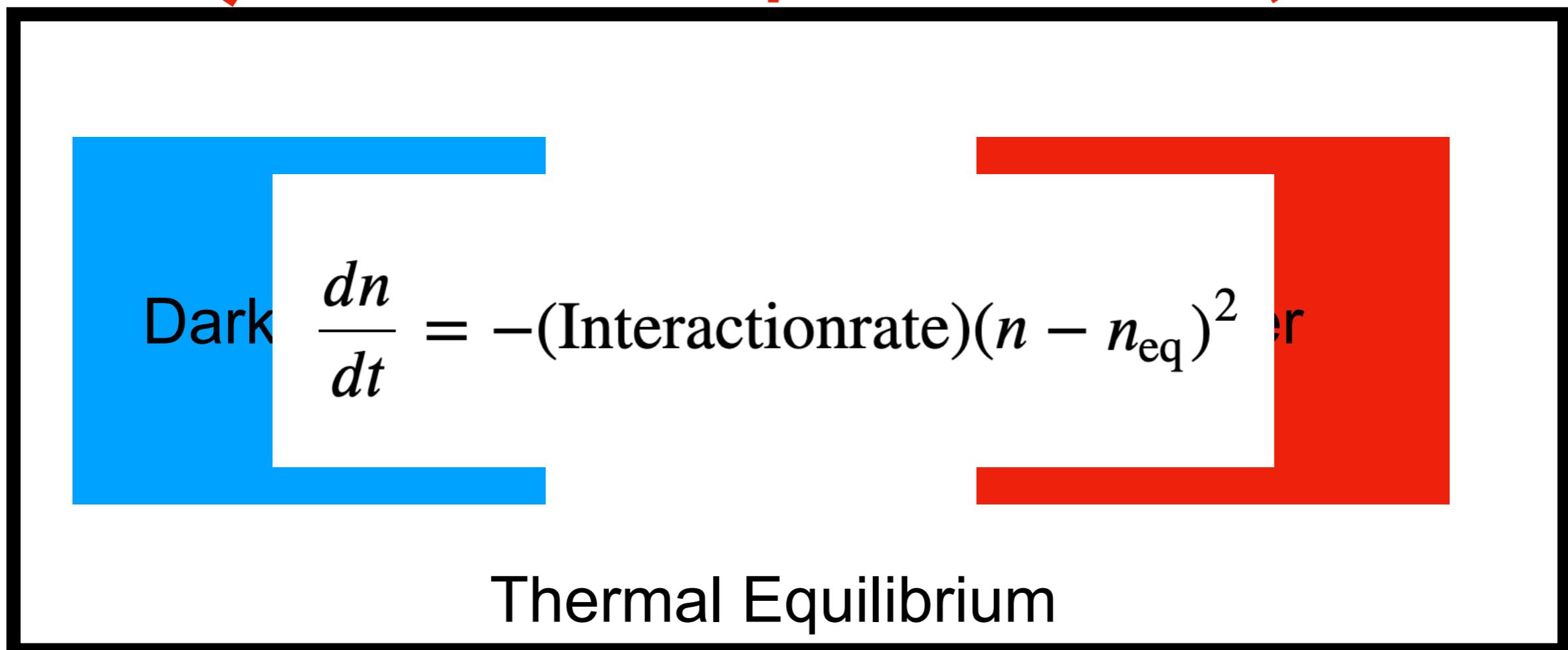
Saturn's Rings



Dark Matter



Dark Matter



University is Expanding