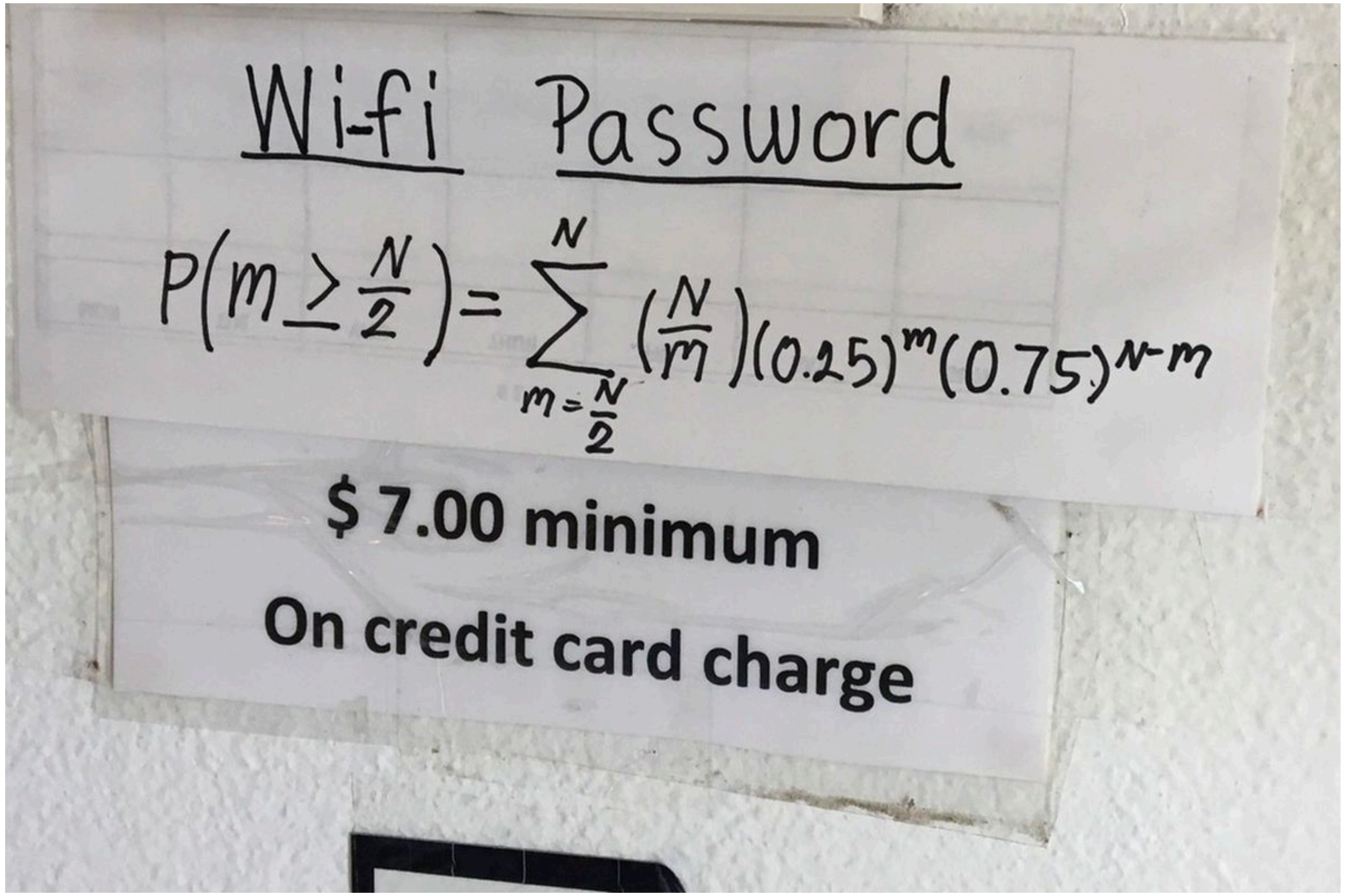
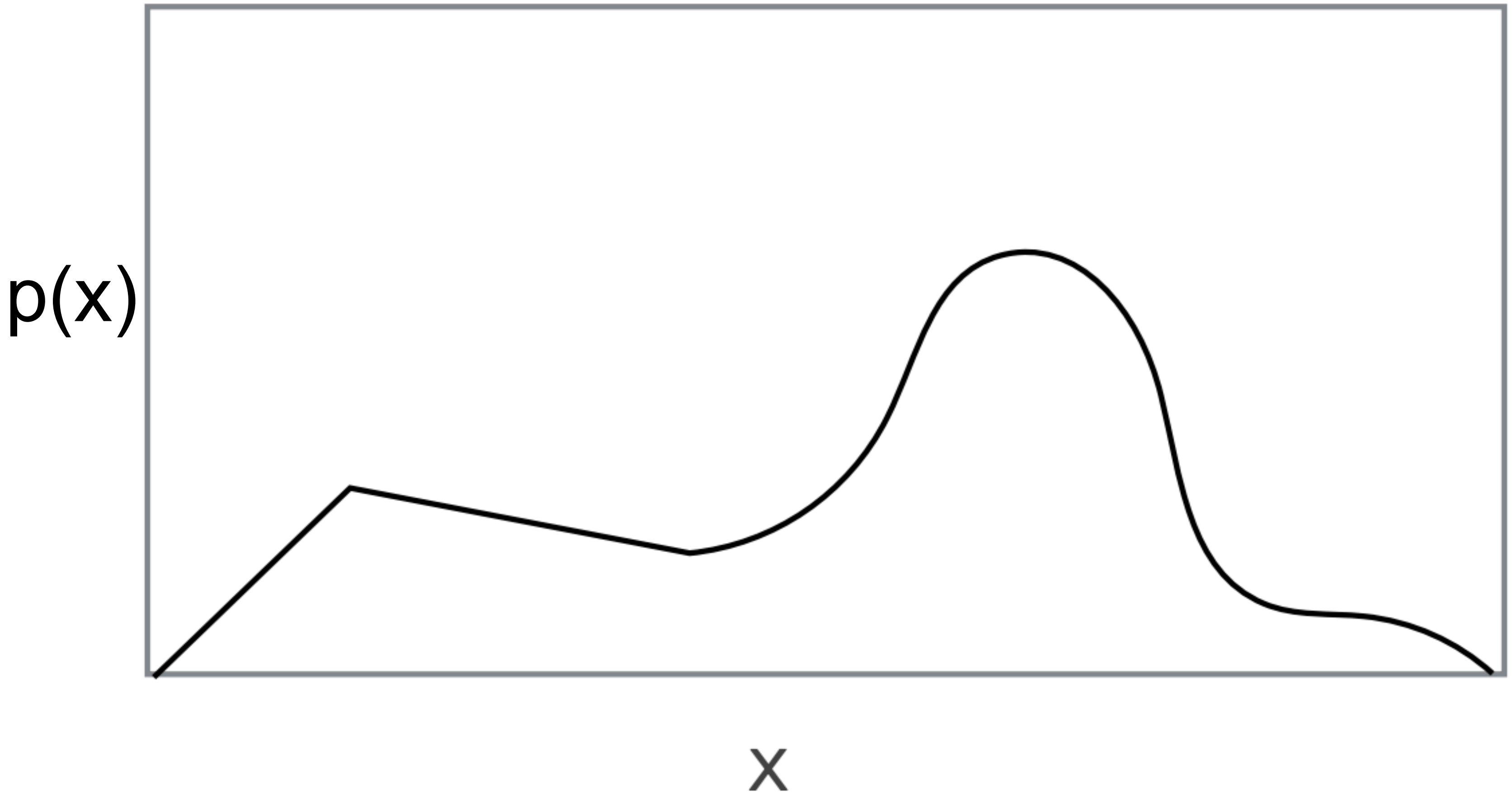


Lecture 2 : Distributions & Unc

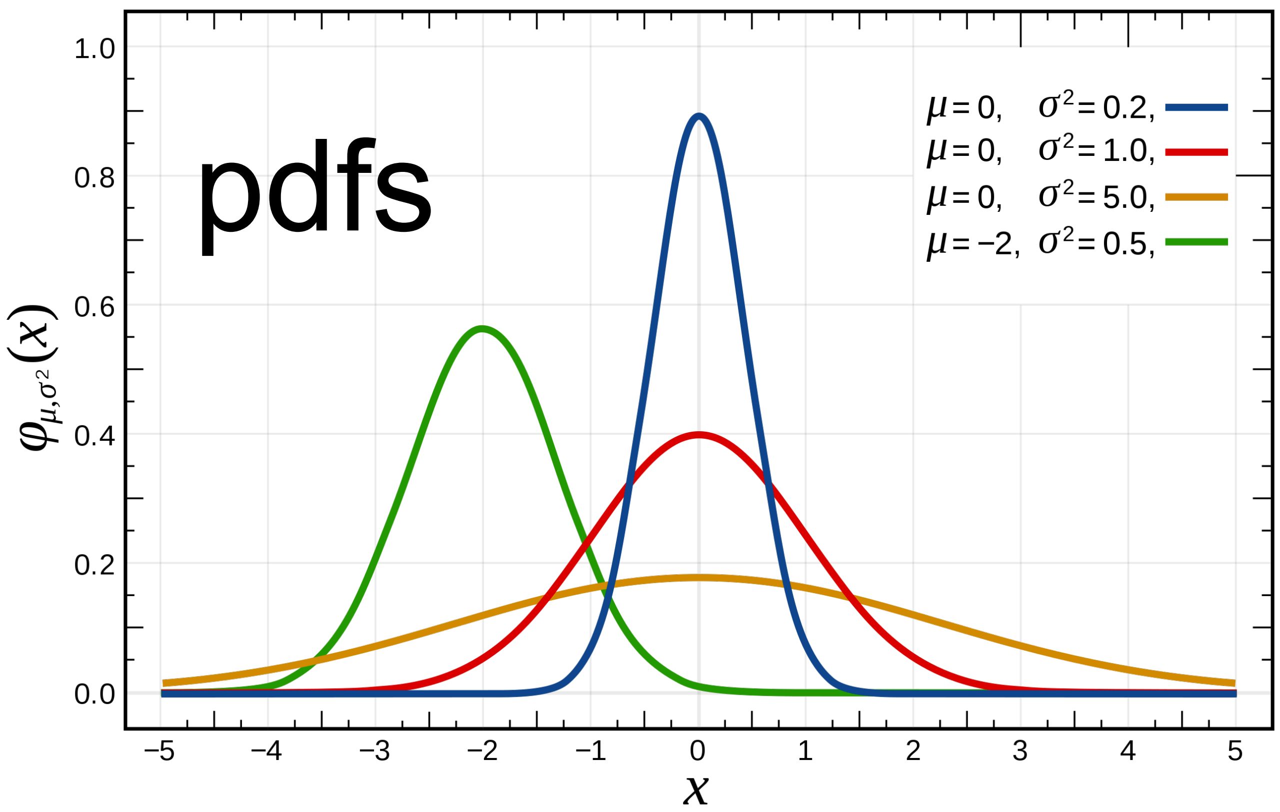
PDFs

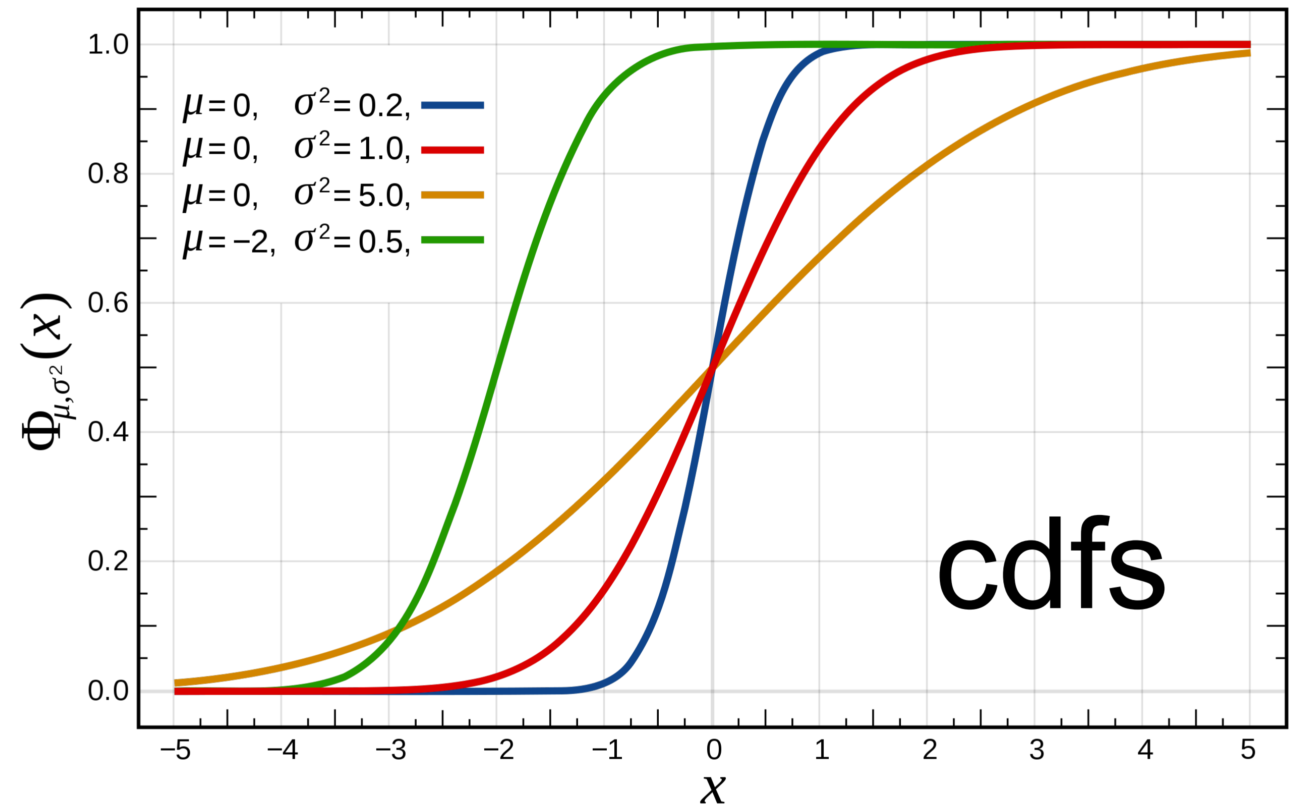


- Probability distribution(density) function *p(x)* sometimes *f(x)*
  - Probability of being between *x* and *x+dx*
  - $P(x \in [x, x + dx]) = p(x)dx$
  - $P(x \in [a, b]) = \int_a^b p(x)dx$

Probability can be disjoint

CDFs





- Cumulative distribution(density) functions or sometime CDFs
- $\mathrm{cdf}(\mathrm{p}(\mathrm{x}), \mathrm{a}) = \int_{-\infty}^{\mathrm{a}} \mathrm{p}(\mathrm{x})\mathrm{dx}$

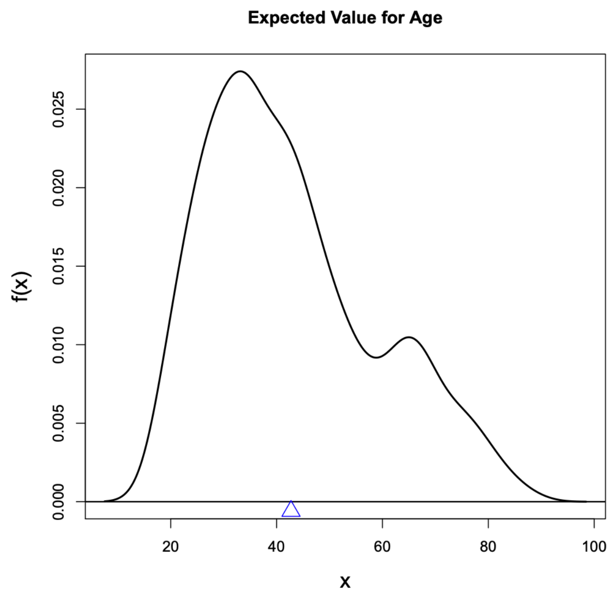
Expectation

The expected value of a random variable X is denoted by $E[X]$ and is a measure of central tendency of X. Roughly speaking, an expected value is like a weighted average (weighted by probability of occurrence).

The expected value of a discrete random variable X is defined as

$$E[X] = \sum_{\text{all } x} x \cdot f_X(x).$$

The expected value of a continuous random variable X is defined as



$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x) dx.$$

Expectation Balance Point of a distribution

Expectation

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\bar{x} = \sum_{\text{all } x_i} x_i \cdot f(x_i), \text{ where } f(x_i) = \frac{1}{N}$$

$$E[b] = b$$

$$E[aX] = aE[X]$$

$$E[aX + b] = aE[X] + b$$

$$E\left[\sum_{i=1}^{k} X_i\right] = E[X_1] + \cdots + E[X_k]$$

Variance

The expected value of a function $g()$ of the random variable X, written $g(X)$, is denoted by $E[g(X)]$ and is a measure of central tendency of $g(X)$.

The variance is a special case of this, and the variance of a random variable X (a measure of its dispersion) is given by

$$V[X] = E[(X - E[X])^2]$$

It is the expectation of the squared distances from the mean.

Variance is a measure of the width of our distribution

Variance

For a discrete random variable X

$$V[X] = \sum_{\text{all } x} (x - E[X])^2 f_X(x)$$

For a continuous random variable X

$$V[X] = \int_{-\infty}^{\infty} (x - E[X])^2 f_X(x) dx$$

Suppose a and b are constants and X is a random variable. Then

$$V[b] = 0$$

$$V[aX] = a^2 V[X]$$

$$V[aX + b] = a^2 V[X] + 0$$

Variance

Suppose we have k independent random variables $X_1, \ldots, X_k$. If $V[X_i]$ exists for all $i = 1, \ldots, k$, then

$$V\left[\sum_{i=1}^{k} X_i\right] = V[X_1] + \cdots + V[X_k]$$

Standard Deviation is defined as $\sigma = \sqrt{V[X_i \ldots.]}$

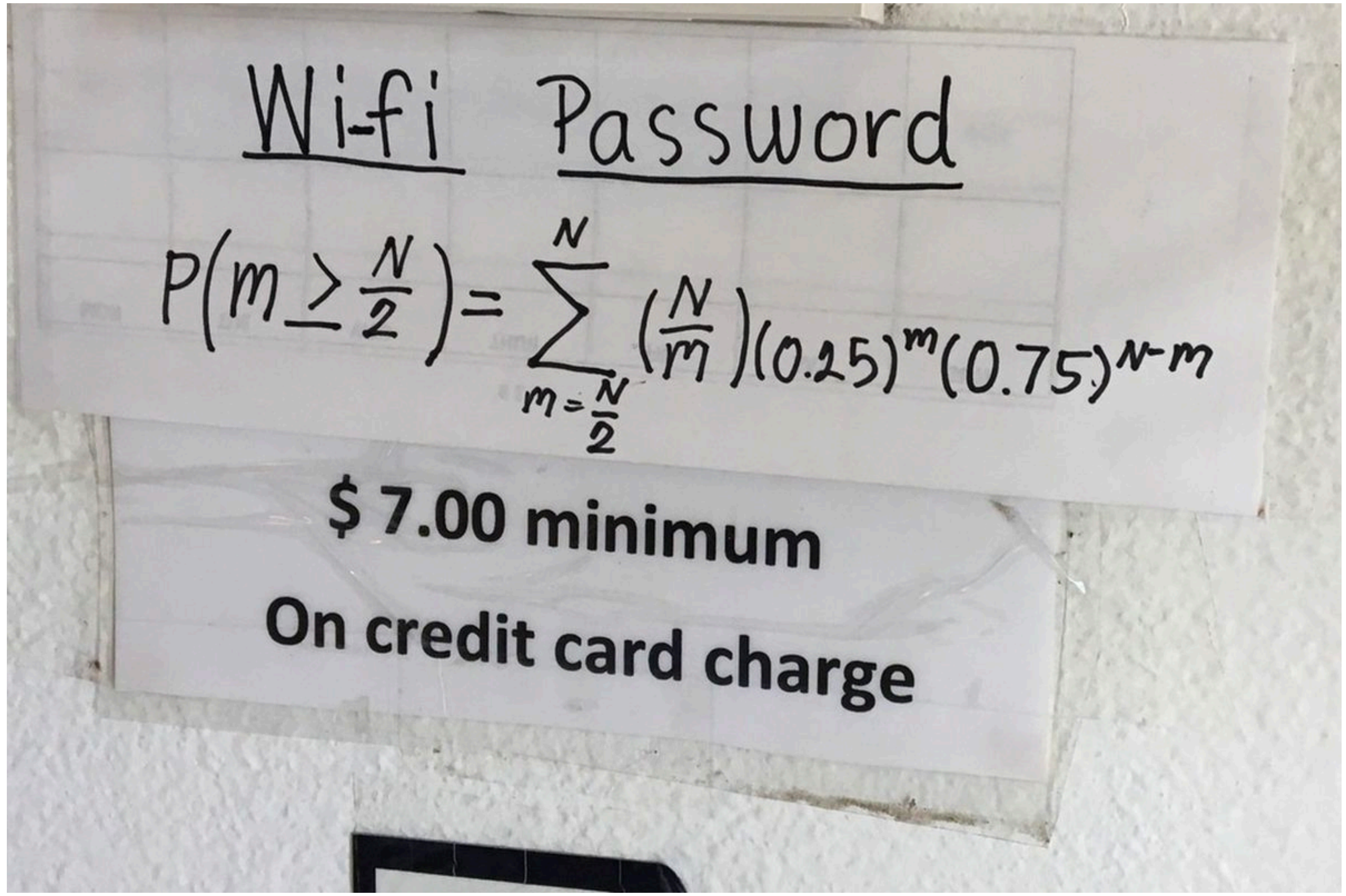
It is a measure of the width of a distribution

Label standard deviation to imply that we have chosen this for our uncertainty

Standard deviation is what we often use for **uncertainty**

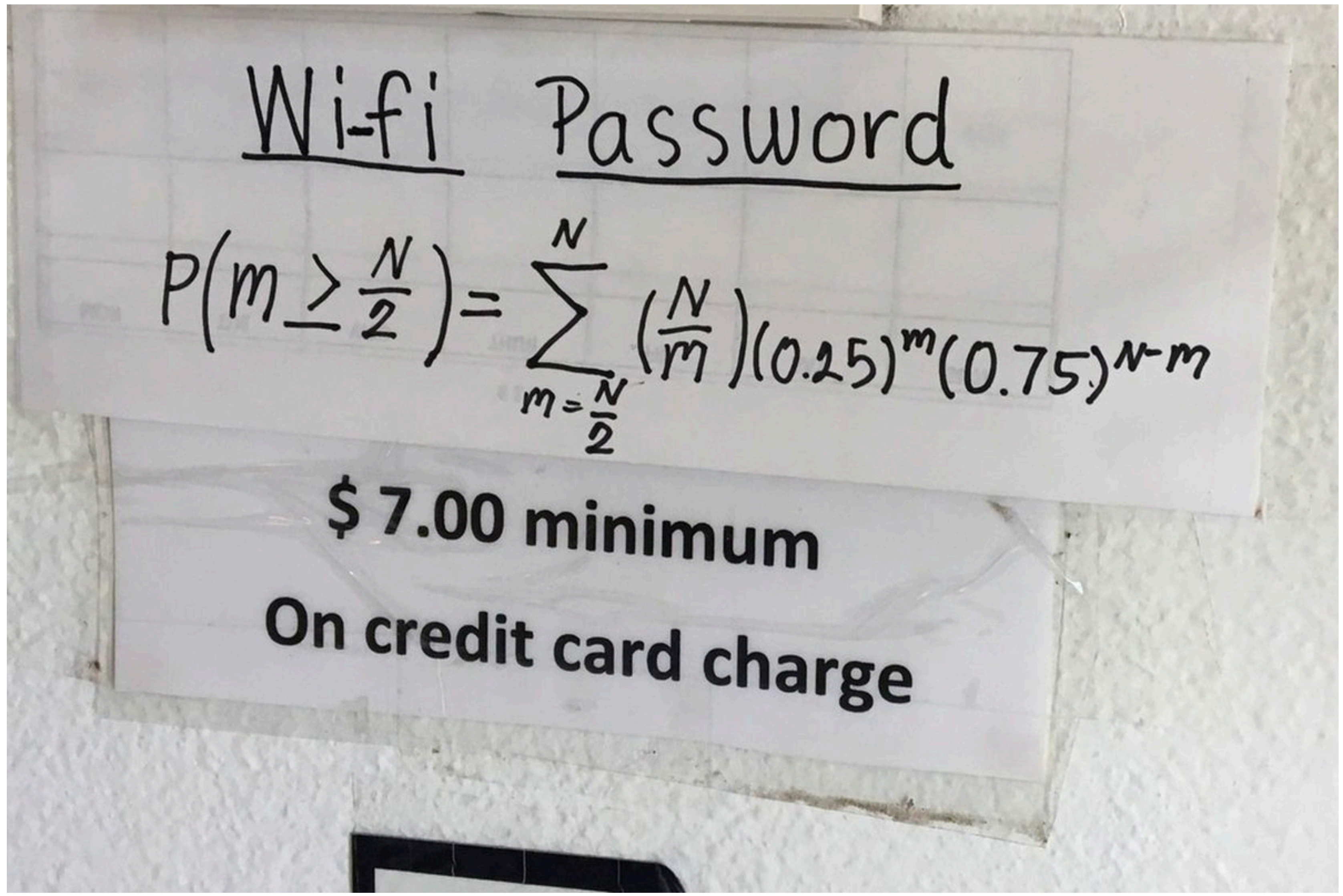
$$p(x' = x1 + x2)$$

Whats the password?



Hint: We are at a coffee shop?

Whats the password?



Hint: We are at a coffee shop?
Binomial(Buy No Meal)

Binomial Distribution

Given a coin with a probability *p* for heads what is the probability of *k* heads in *n* tosses

$$\boxed{\binom{n}{m}}\ \boxed{(1-p)^{n-m}}\ \boxed{p^m}$$

Number of possible heads combinations

Probaility of *n-k* tails

Probaility of *k* heads

Binomial is the foundation of all statistics

$$E[f(m)] = np \qquad V[f(m)] = np(1-p)$$

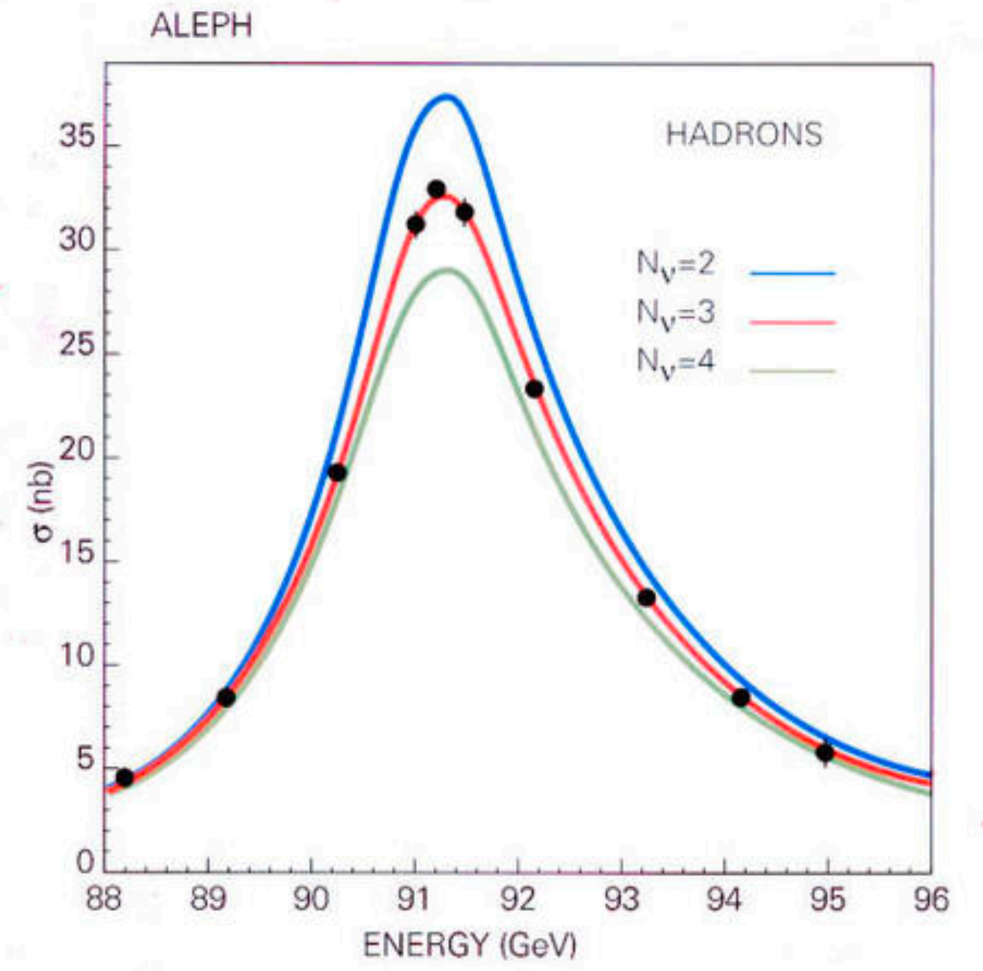
- Function summary

Error Propagation

- An important element of variance is that it propagates
  - Take f(x)
  - What is the variance of f(x) given x
  - Take: $f(x+\sigma) \approx f(x) + \sigma \frac{df}{dx}$
  - $\mathrm{VAR}[\mathrm{f(x)}] = \sigma^2 \left(\frac{\mathrm{df}}{\mathrm{dx}}\right)^2$

Dealing with uncertainties

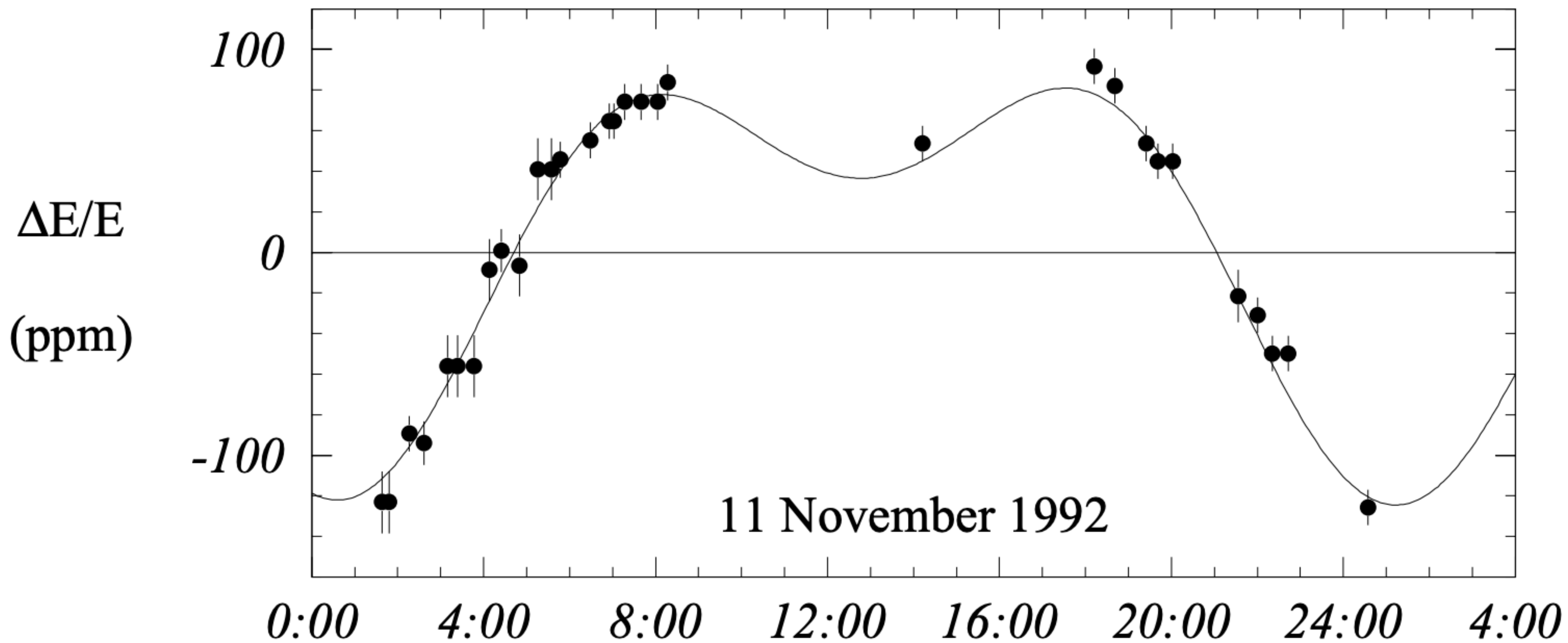
LEP: Large Elctron-Positron Collider



Collider was used to measure rates of collisions (and all their properties) Precisely

Func with Unceratinties

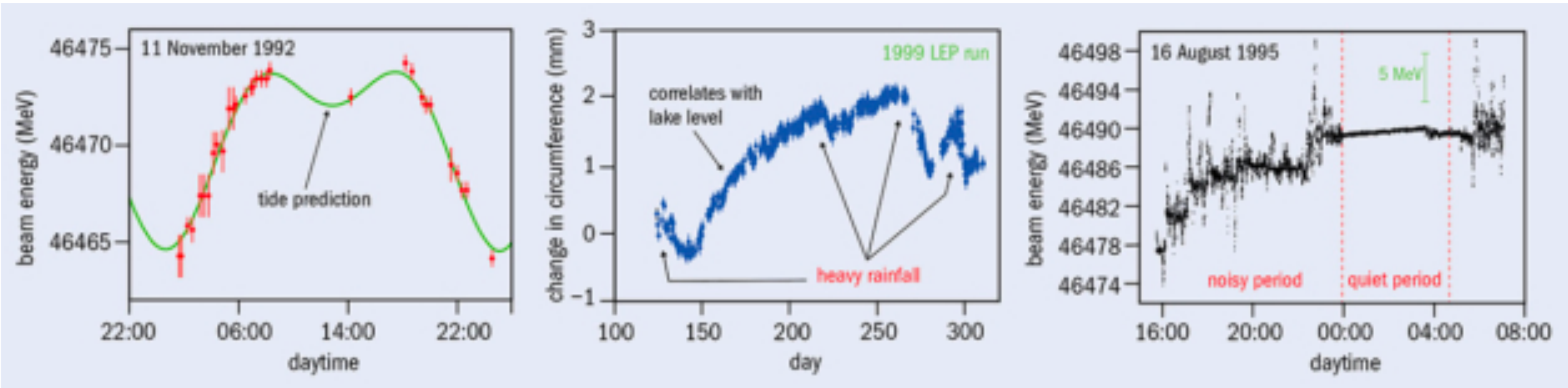
How precise is this?



Any guesses whats causing this?

Paper of this plot

Lots of Uncertainties

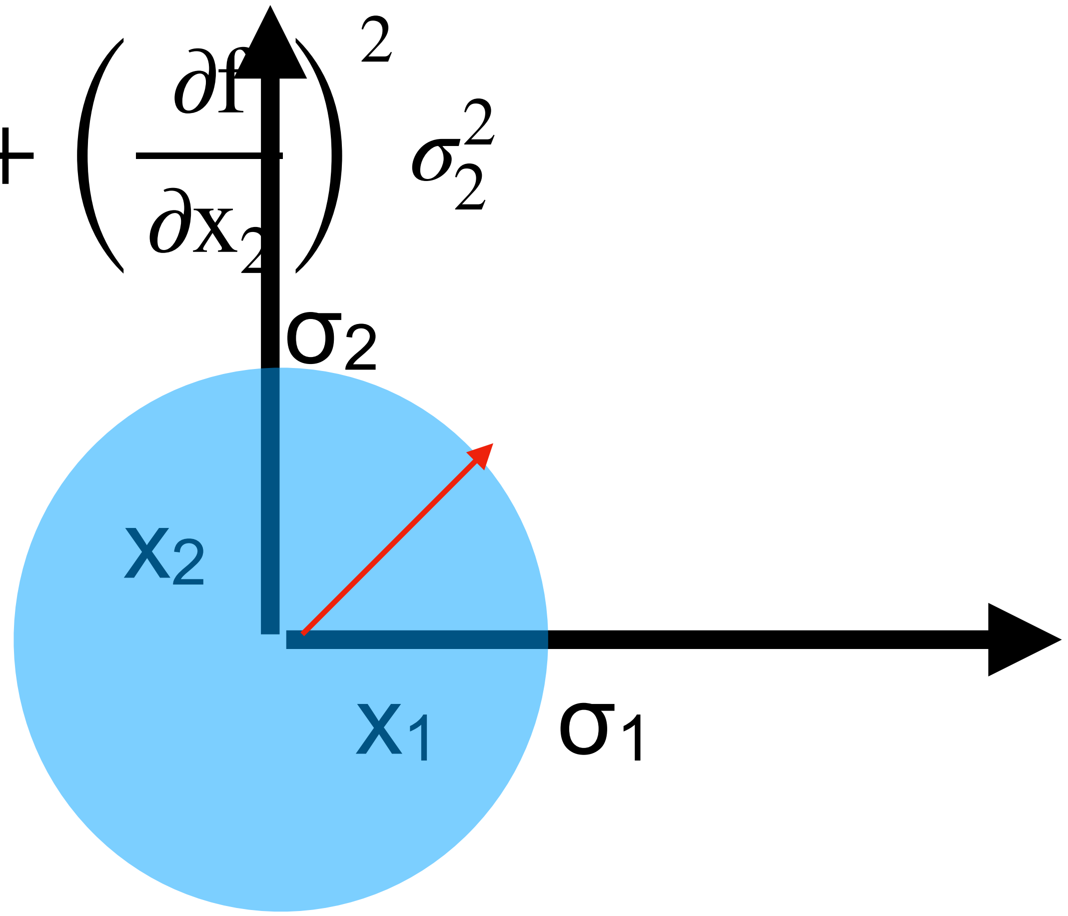


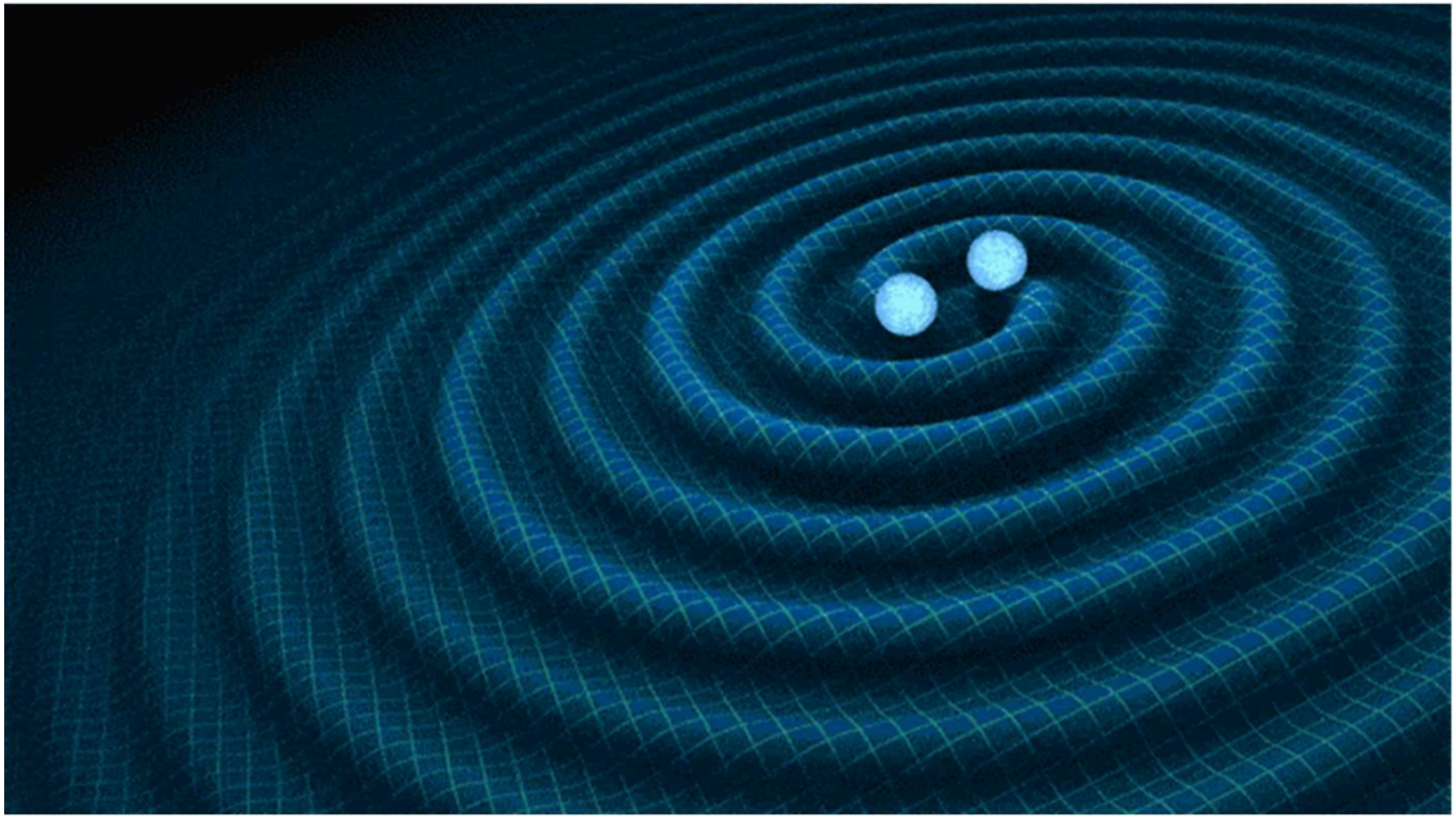
- The collider would change parameters due to :
    - The water level in the nearby lake
    - The TGV train schedule
    - The orbits of the moon

Questions?

What if you have two unc?

- Now consider another function:
- $f(x_1,x_2)$ where x_1 and x_2 are independent variables
- In this case we treat the total uncertainty as the sum
- Since they are independent we can visualize as a circle
- Take: $\mathrm{VAR}[f(x_1, x_2)] = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_2^2$

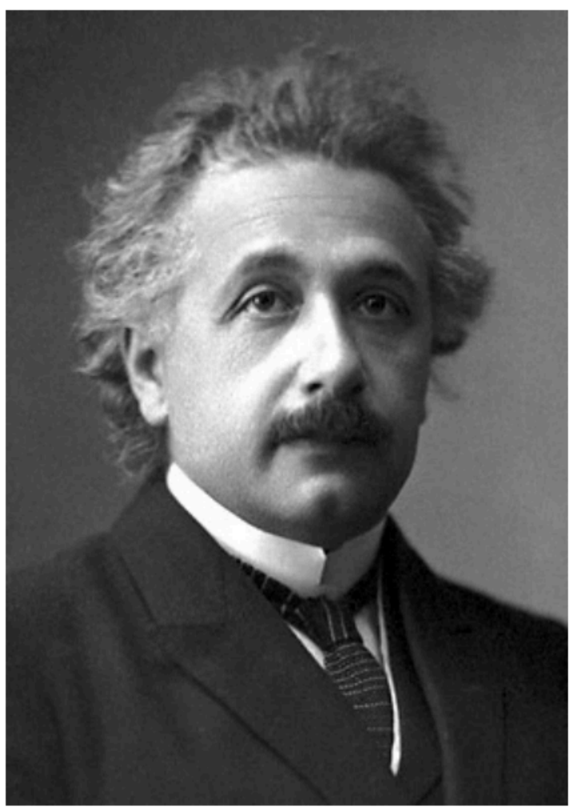




Searching For Gravitational Waves

Gravity

There is a limiting speed in Nature, the speed of light (1905)



geometry of spacetime

stress-energy tensor

$$\boxed{G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}}$$

Gravity: manifestation of spacetime curvature (1915)

Slides

- See this link for animations:
- https://www.dropbox.com/s/44ew35qlz89xono/PCH_Lecture2_LigoAnims_8S50.pptx?dl=0