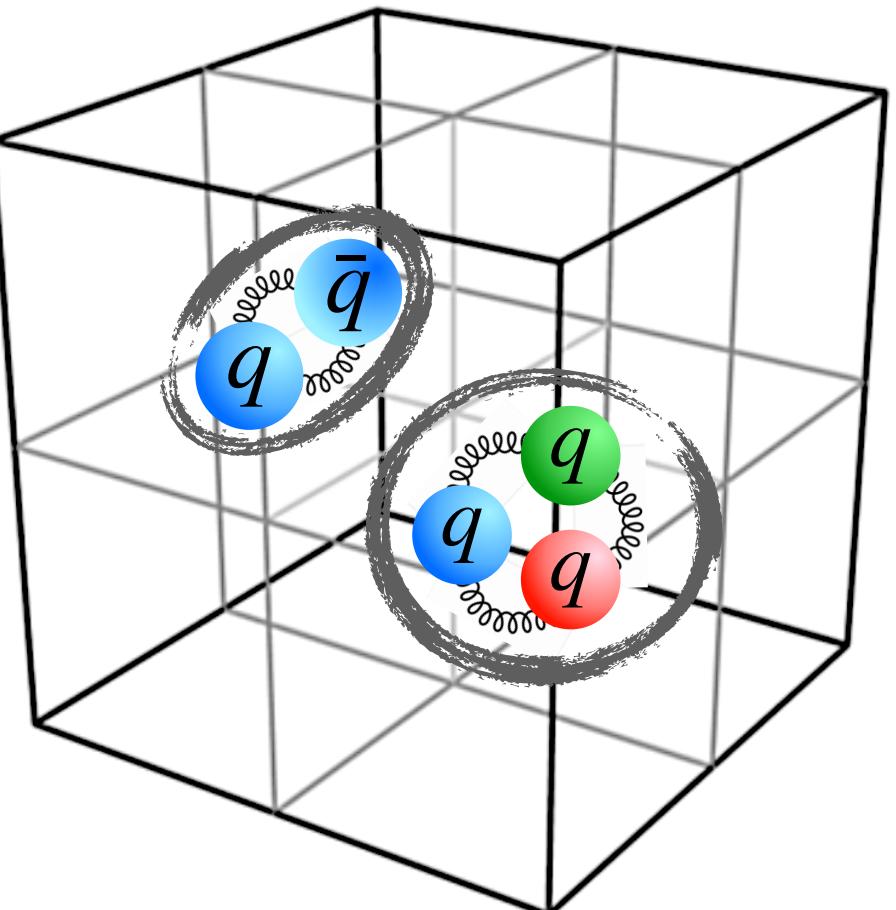
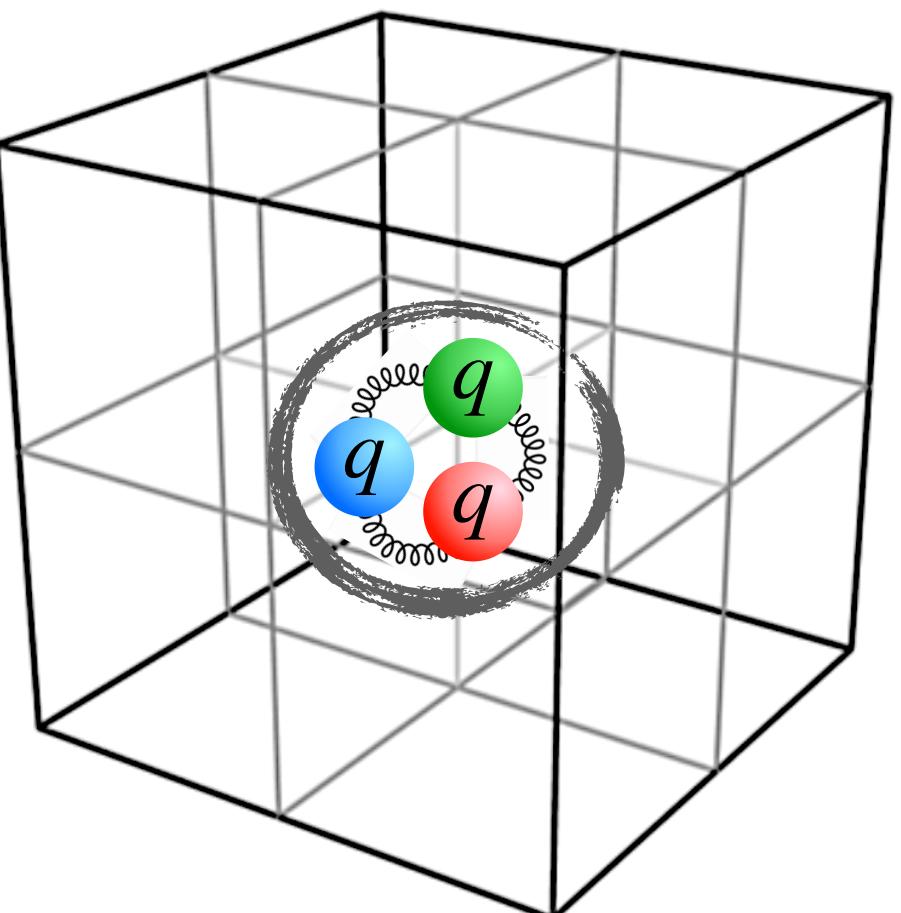
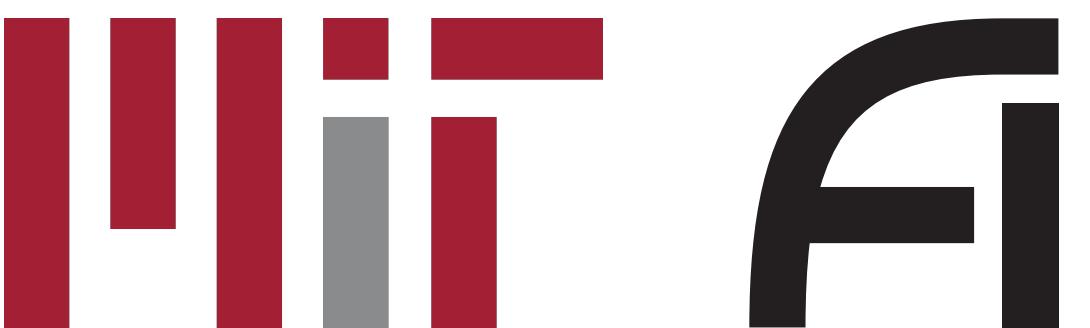


An Introduction to Lattice QCD

Fernando Romero-López
MIT

fernando@mit.edu

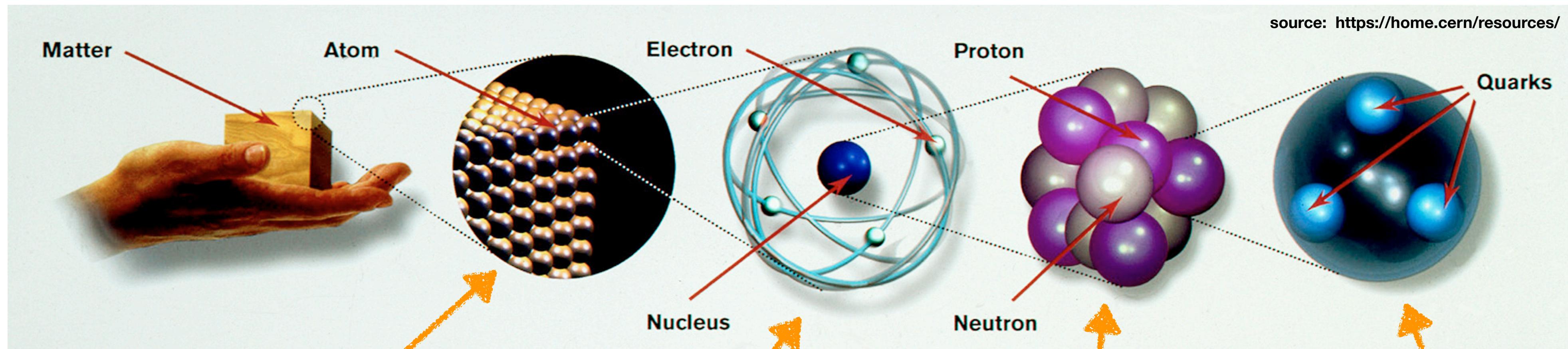


Outline

1. The strong interaction
2. Lattice Field Theory
3. Sampling in LFT
4. The QCD spectrum
5. Lattice QCD for SM tests

The strong interaction in the standard model

From atoms to quarks



Atoms bound together by
van der Waals forces
(electromagnetic)

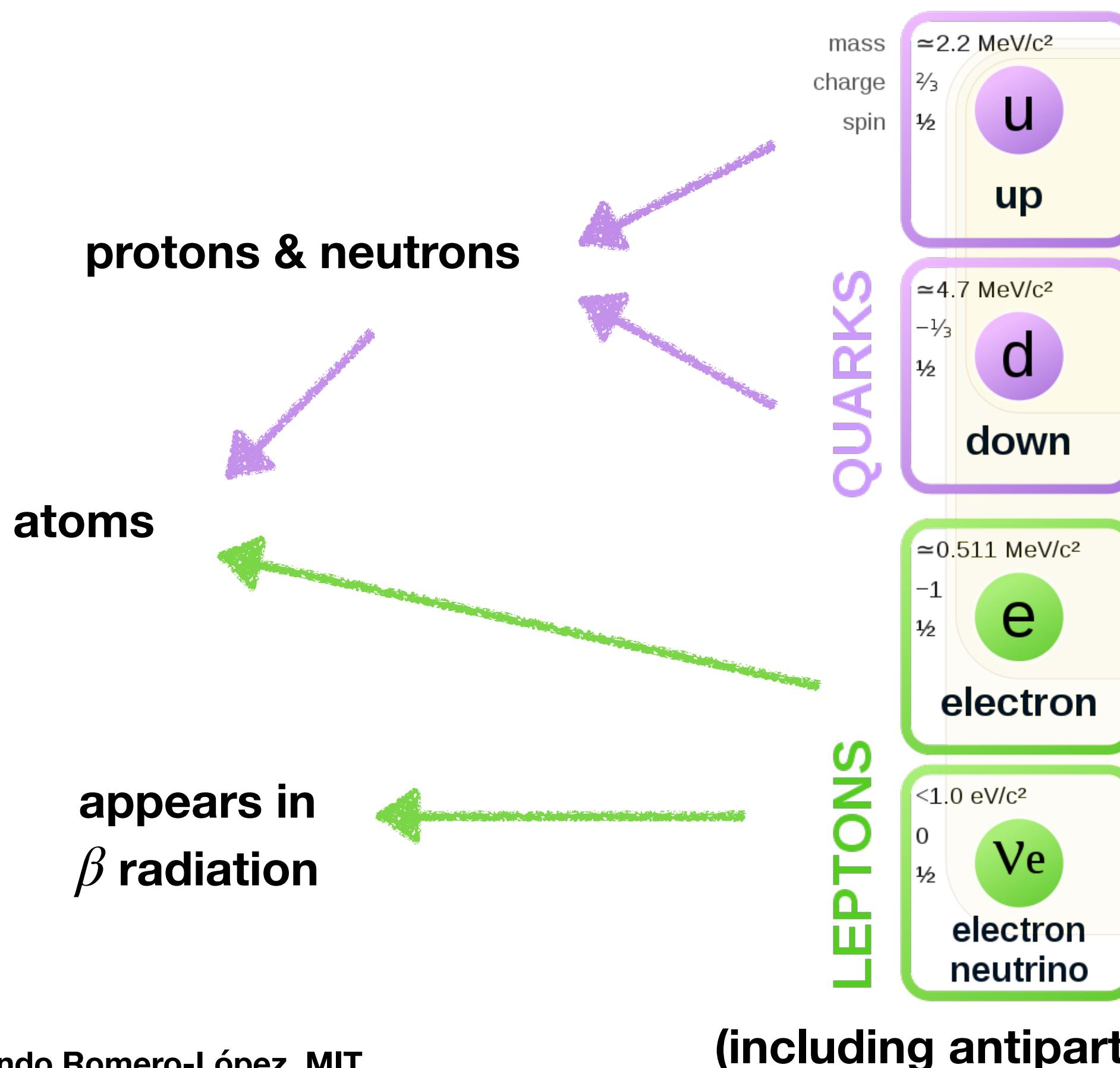
Electron-nucleus attraction
by Coulomb forces
(electromagnetic)

protons and neutrons
bound by the strong force

quarks confined in protons
by the strong force

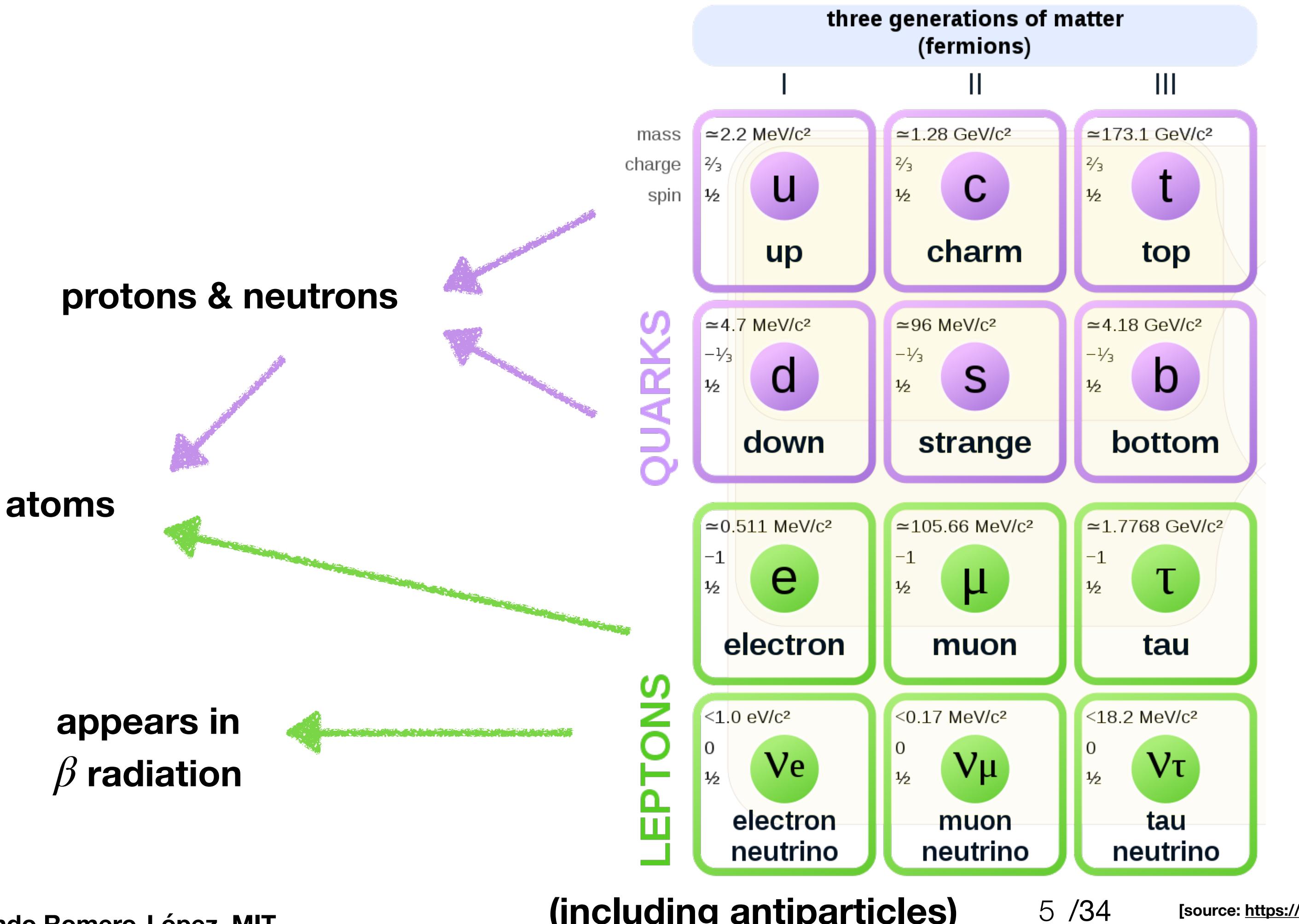
Standard Model of Particle Physics

very successful description of the subatomic world



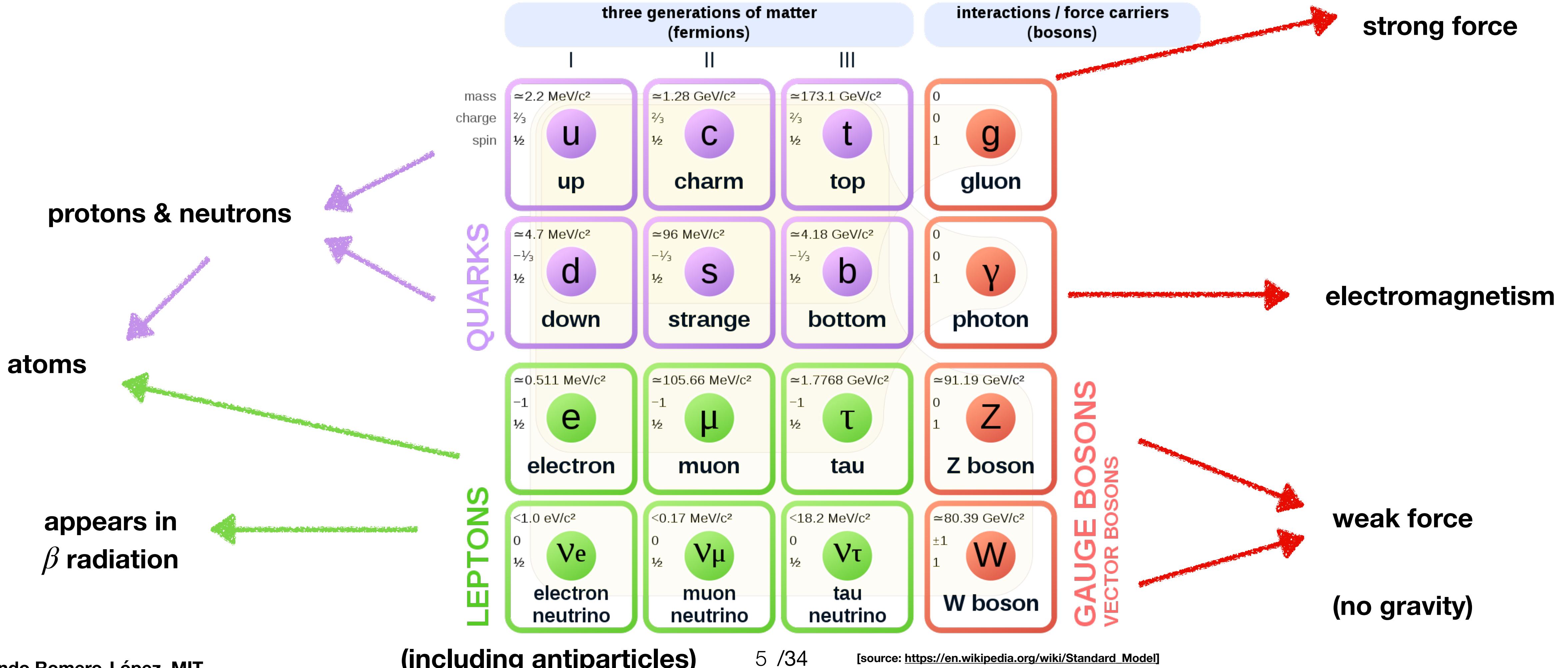
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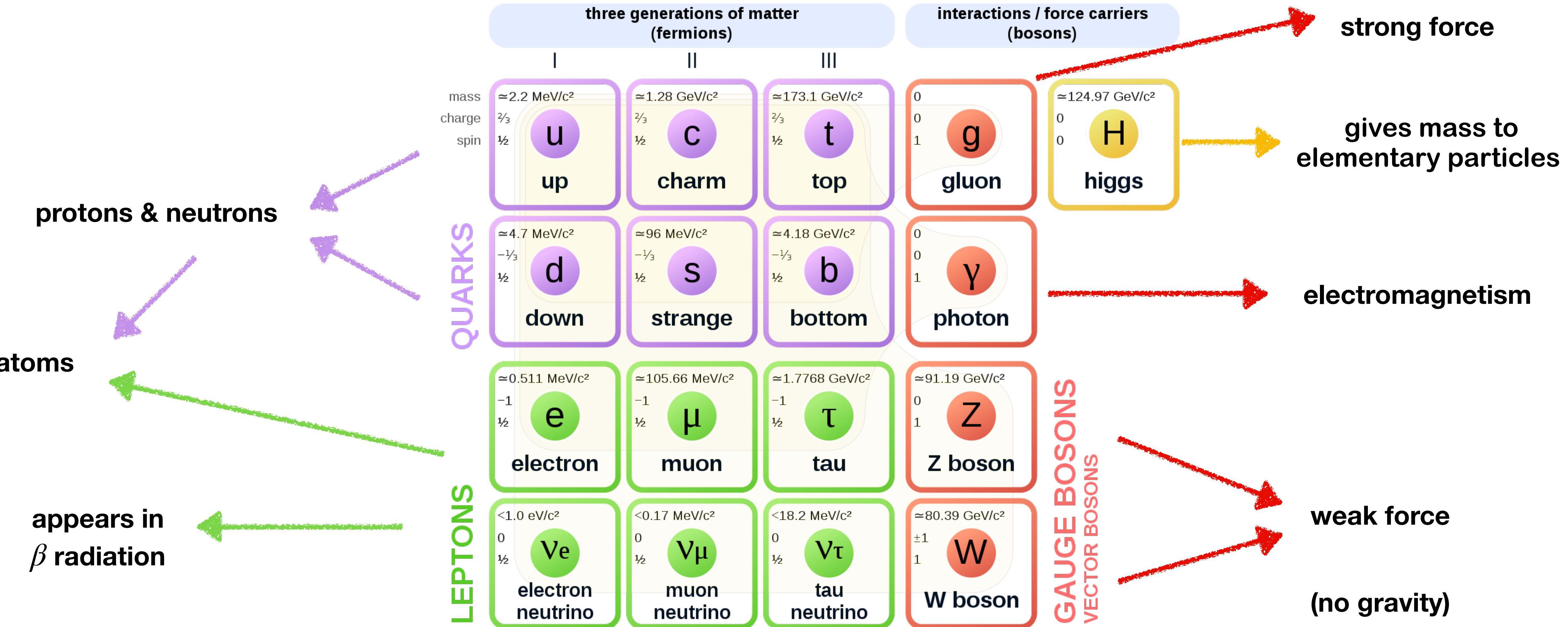
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Standard Model of Particle Physics

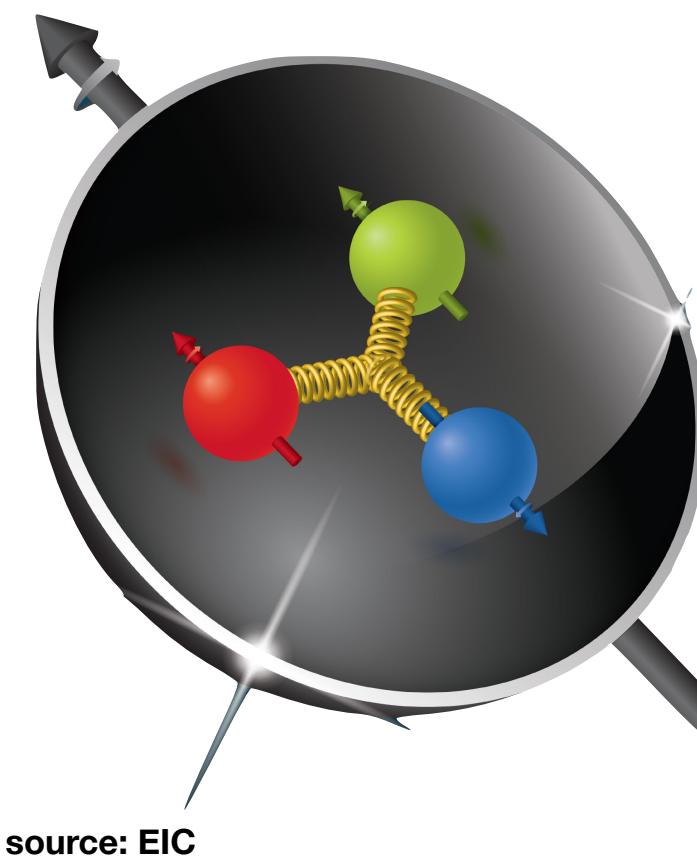
very successful description of the subatomic world



A successful theory

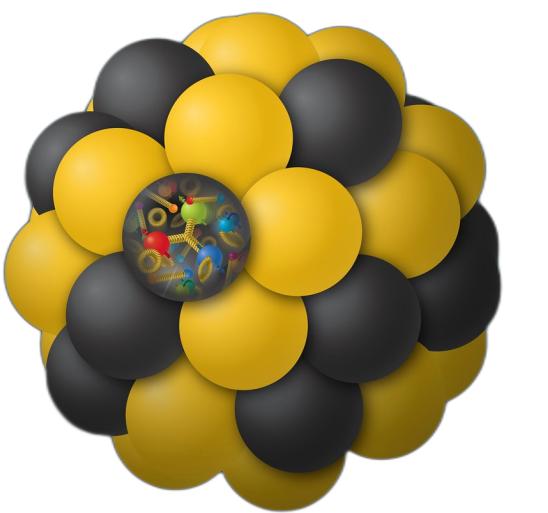
periodic table of elements

proton



Standard Model

nuclei



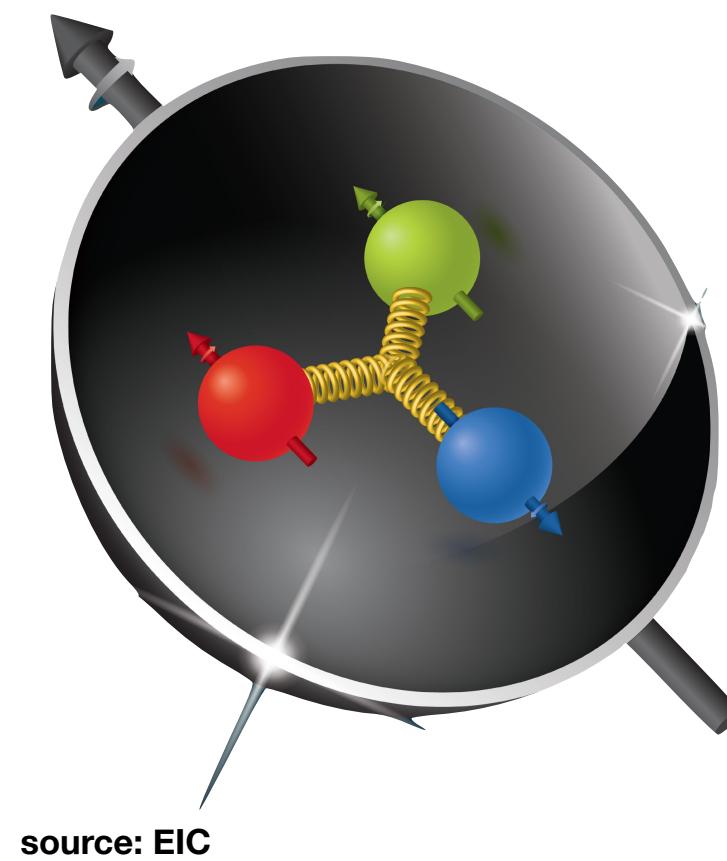
Standard Model?

PubChem	
Atomic Number	Symbol
1 H	Hydrogen Nonmetal
3 Li	Lithium Alkali Metal
4 Be	Beryllium Alkaline Earth Metal
11 Na	Sodium Alkali Metal
12 Mg	Magnesium Alkaline Earth Metal
19 K	Potassium Alkali Metal
20 Ca	Calcium Alkaline Earth Metal
21 Sc	Scandium Transition Metal
22 Ti	Titanium Transition Metal
23 V	Vanadium Transition Metal
24 Cr	Chromium Transition Metal
25 Mn	Manganese Transition Metal
26 Fe	Iron Transition Metal
27 Co	Cobalt Transition Metal
28 Ni	Nickel Transition Metal
29 Cu	Copper Transition Metal
30 Zn	Zinc Transition Metal
31 Ga	Gallium Post-Transition Metal
32 Ge	Germanium Metalloid
33 As	Arsenic Metalloid
34 Se	Selenium Nonmetal
35 Br	Bromine Halogen
36 Kr	Krypton Noble Gas
5 B	Boron Metalloid
6 C	Carbon Nonmetal
7 N	Nitrogen Nonmetal
8 O	Oxygen Nonmetal
9 F	Fluorine Halogen
10 Ne	Helium Noble Gas
13 Al	Aluminum Post-Transition Metal
14 Si	Silicon Metalloid
15 P	Phosphorus Nonmetal
16 S	Sulfur Nonmetal
17 Cl	Chlorine Halogen
18 Ar	Argon Noble Gas
20 Ca	Calcium Alkaline Earth Metal
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36 Kr	Krypton Noble Gas
37 Rb	Rubidium Alkali Metal
38 Sr	Sodium Alkaline Earth Metal
39 Y	Yttrium Transition Metal
40 Zr	Zirconium Transition Metal
41 Nb	Niobium Transition Metal
42 Mo	Molybdenum Transition Metal
43 Tc	Technetium Transition Metal
44 Ru	Ruthenium Transition Metal
45 Rh	Iridium Transition Metal
46 Cd	Rhodium Transition Metal
47 Ag	Palladium Transition Metal
48 Pd	Silver Transition Metal
49 In	Cadmium Transition Metal
50 Sn	Indium Transition Metal
51 Sb	Tin Antimony Metalloid
52 Te	Antimony Metalloid
53 I	Tellurium Metalloid
54 Xe	Iodine Halogen
55 Cs	Cesium Alkali Metal
56 Ba	Barium Alkaline Earth Metal
57 La	Lanthanum Lanthanide
58 Ce	Cerium Lanthanide
59 Pr	Praseodymium Lanthanide
60 Nd	Neodymium Lanthanide
61 Pm	Promethium Lanthanide
62 Sm	Samarium Lanthanide
63 Eu	Europium Lanthanide
64 Gd	Gadolinium Lanthanide
65 Tb	Terbium Lanthanide
66 Dy	Dysprosium Lanthanide
67 Ho	Holmium Lanthanide
68 Er	Erbium Lanthanide
69 Tm	Thulium Lanthanide
70 Yb	Ytterbium Lanthanide
71 Lu	Lutetium Lanthanide
89 Ac	Actinium Actinide
90 Th	Thorium Actinide
91 Pa	Protactinium Actinide
92 U	Uranium Actinide
93 Np	Neptunium Actinide
94 Pu	Plutonium Actinide
95 Am	Americium Actinide
96 Cm	Curium Actinide
97 Bk	Berkelium Actinide
98 Cf	Californium Actinide
99 Es	Einsteinium Actinide
100 Fm	Fermium Actinide
101 Md	Mendelevium Actinide
102 No	Nobelium Actinide
103 Lr	Lawrencium Actinide

A successful theory

periodic table of elements

proton

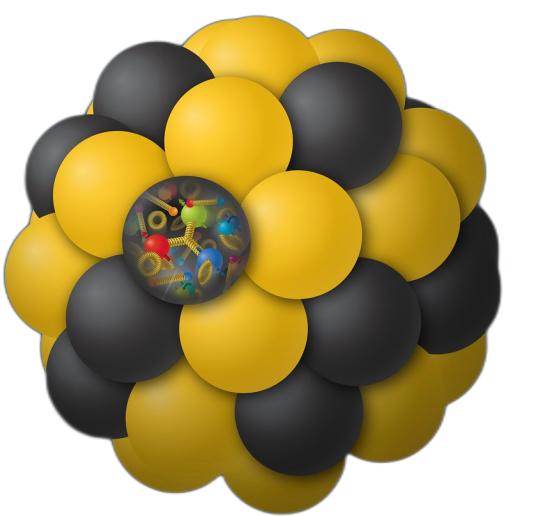


source: EIC

Standard Model



nuclei



Standard Model?



PubChem	
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27 Nb	28 Ne
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29 Tc	30 Ne
30 Ru	31 Ne
31 Cd	32 Ne
32 Ag	33 Ne
33 Ge	34 Ne
34 As	35 Br
35 Se	36 Kr
36 Br	37 Ne
37 Rb	38 Ne
38 Sr	39 Ne
39 Y	40 Ne
40 Zr	41 Ne
41 Nb	42 Ne
42 Mo	43 Ne
43 Tc	44 Ne
44 Ru	45 Ne
45 Rh	46 Ne
46 Pd	47 Ne
47 Ag	48 Ne
48 Cd	49 Ne
49 In	50 Sn
50 Sb	51 Te
51 Te	52 I
52 I	53 Xe
53 Xe	54 Ne
54 Cs	55 Ne
55 Ba	56 Ne
56 Cs	57 Hf
57 Ba	58 Ta
58 Ra	59 W
59 Ra	60 Re
60 Ra	61 Os
61 Ra	62 Ir
62 Ra	63 Pt
63 Ra	64 Au
64 Ra	65 Hg
65 Ra	66 Tl
66 Ra	67 Pb
67 Ra	68 Bi
68 Ra	69 Po
69 Ra	70 At
70 Ra	71 Rn
71 Lu	72 La
72 Lu	73 Ce
73 Lu	74 Pr
74 Lu	75 Nd
75 Lu	76 Pm
76 Lu	77 Sm
77 Lu	78 Eu
78 Lu	79 Gd
79 Lu	80 Tb
80 Lu	81 Dy
81 Lu	82 Ho
82 Lu	83 Er
83 Lu	84 Tm
84 Lu	85 Yb
85 Lu	86 Lu
86 Lu	87 Ac
87 Lu	88 Th
88 Lu	89 Pa
89 Lu	90 U
90 Lu	91 Np
91 Lu	92 Am
92 Lu	93 Pu
93 Lu	94 Cm
94 Lu	95 Bk
95 Lu	96 Cf
96 Lu	97 Es
97 Lu	98 Fm
98 Lu	99 Md
99 Lu	100 No
100 Lu	101 Lr
101 Lu	102 No
102 Lu	103 Lr

However, it is incomplete!

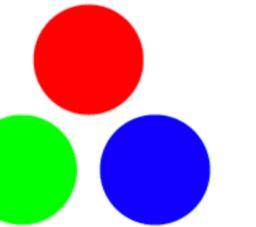
- Dark matter and dark energy
- Matter-antimatter asymmetry
- Neutrino masses
- Gravity

...

Confinement ≠ hadrons

Our understanding of the SM is limited by the complexity of the strong force

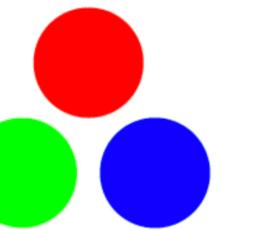
- Quarks and gluons carry the strong charge: the so-called “color”



Confinement ≠ hadrons

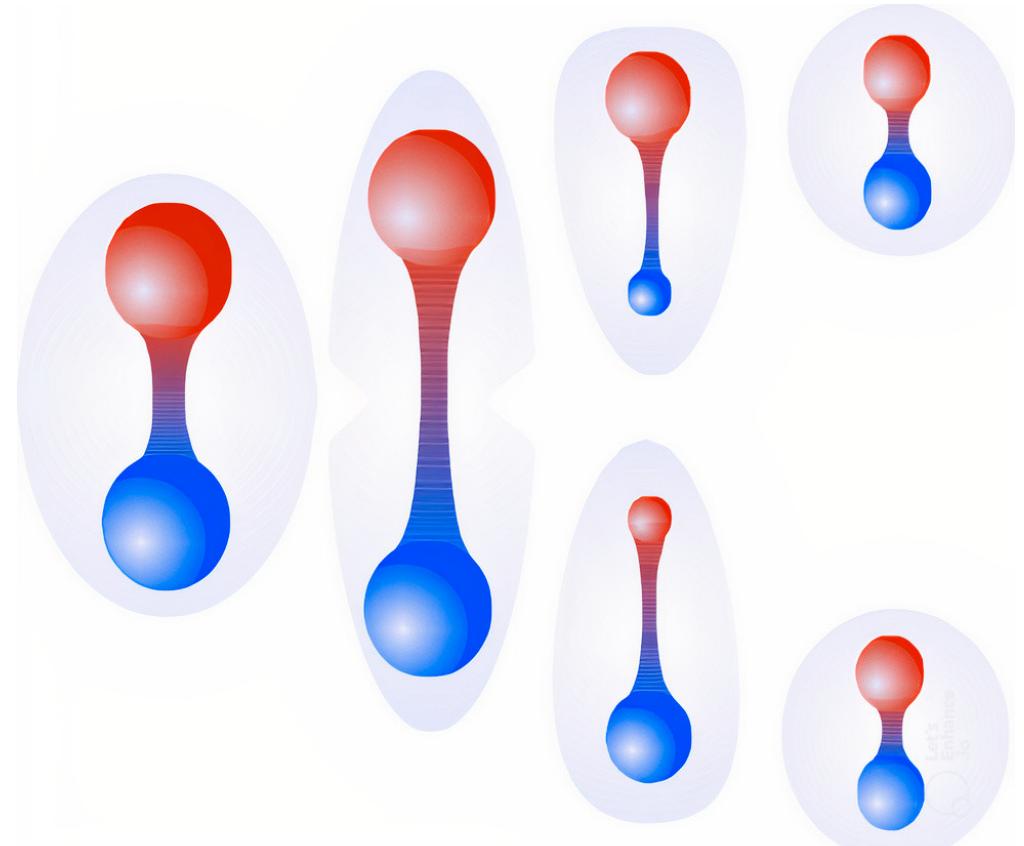
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Confinement

Quarks and gluons can only be found within colorless composite states. These are called “hadrons”.

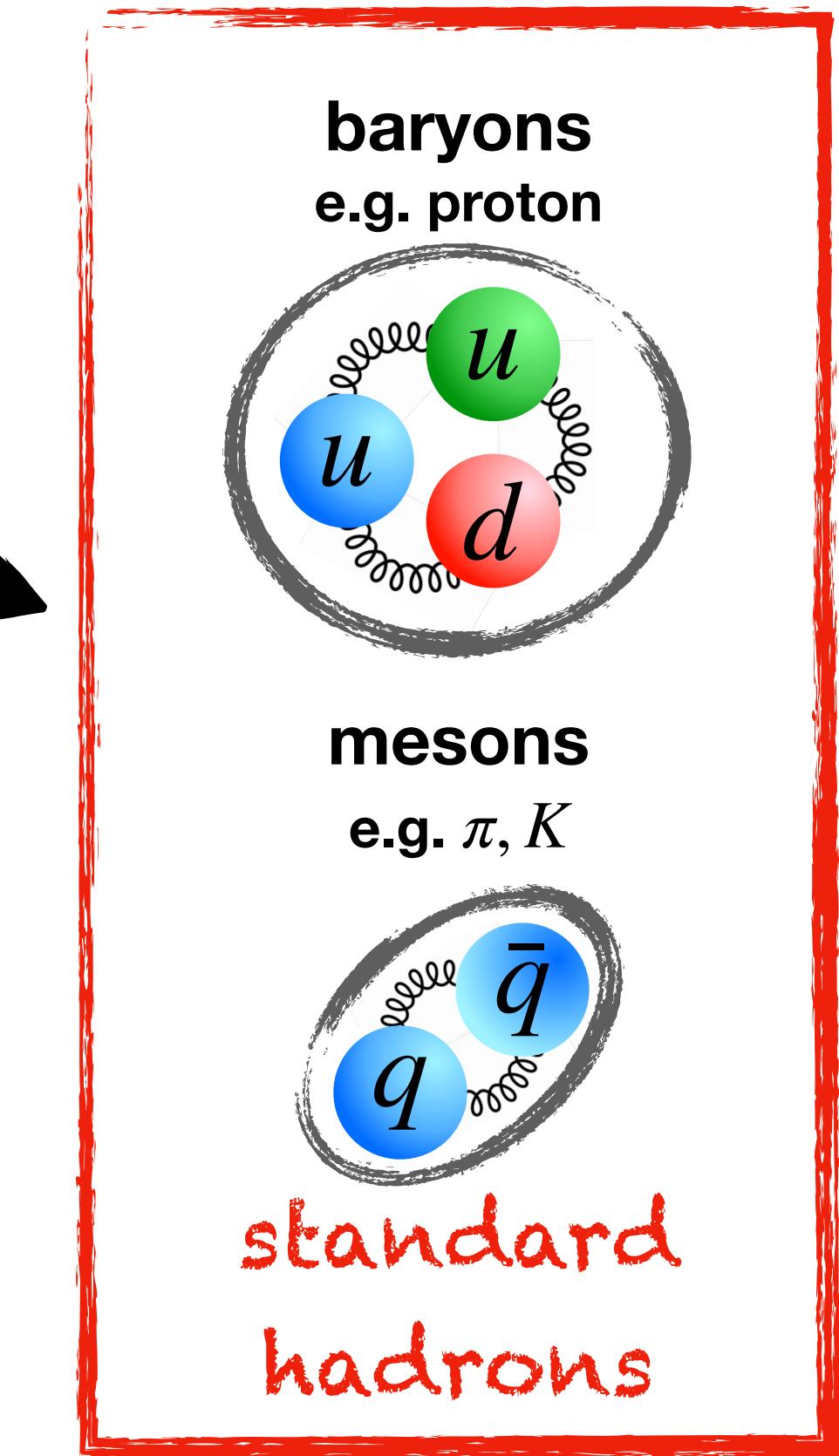
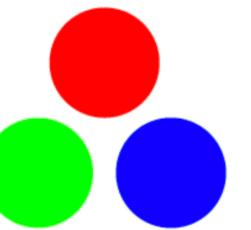


source: IFIC

Confinement $\not\equiv$ hadrons

Our understanding of the SM is limited by the complexity of the strong force

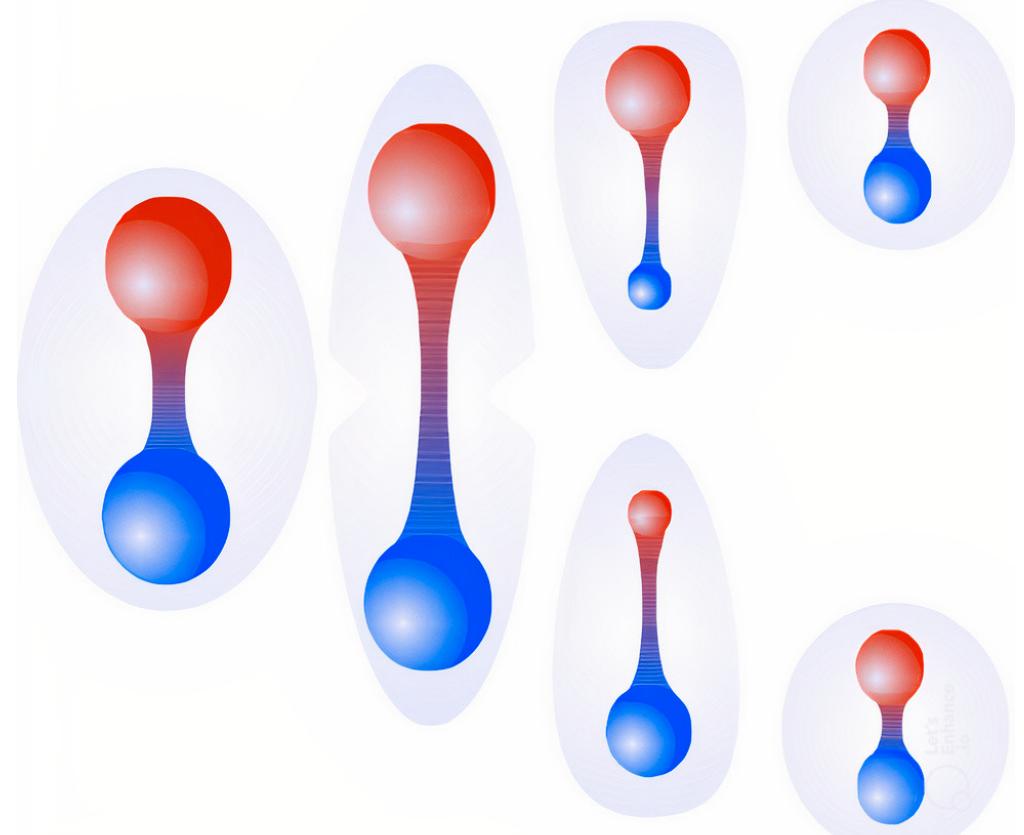
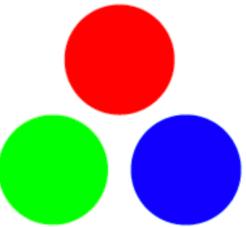
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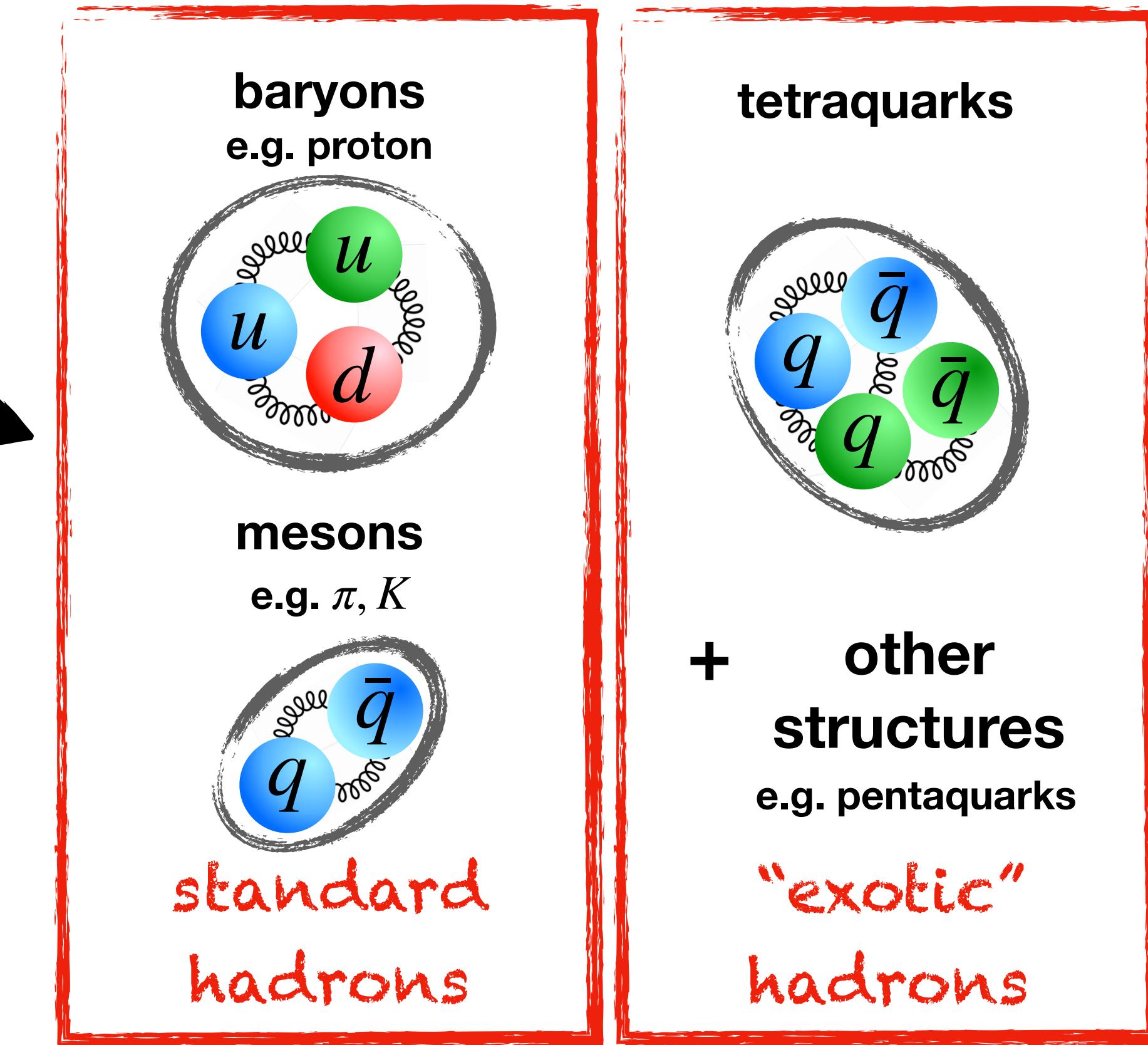
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source: IFIC



Quantum Chromodynamics

Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex.

Frank Wilczek

[<https://physicstoday.scitation.org/doi/10.1063/1.1310117>]

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$$\mathcal{L}_{QCD} = \bar{q} \left(D_\mu \gamma^\mu + m_q \right) q + \frac{1}{4g_s^2} G_{\mu\nu}^a G_a^{\mu\nu}$$

Quantum Chromodynamics

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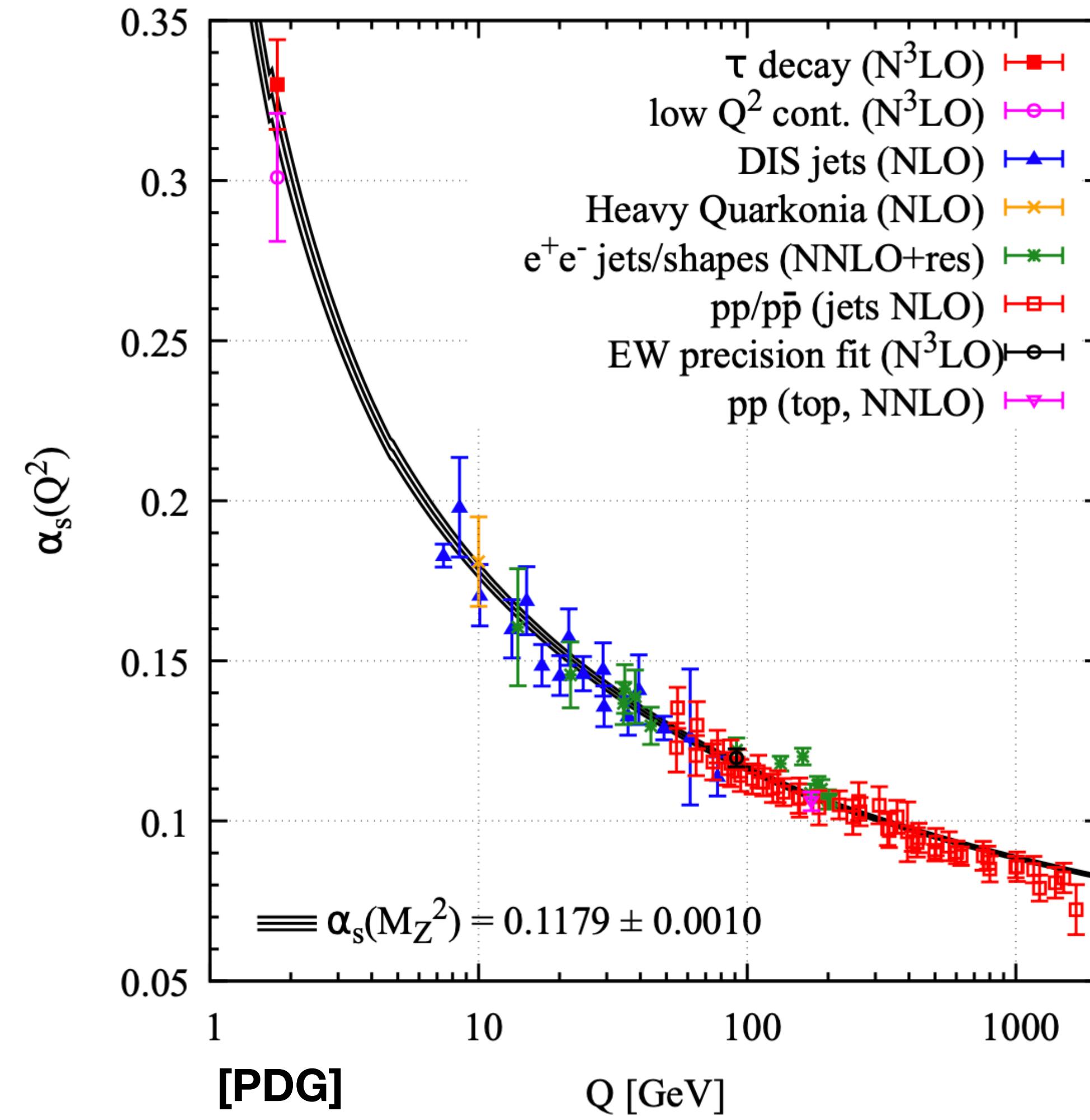
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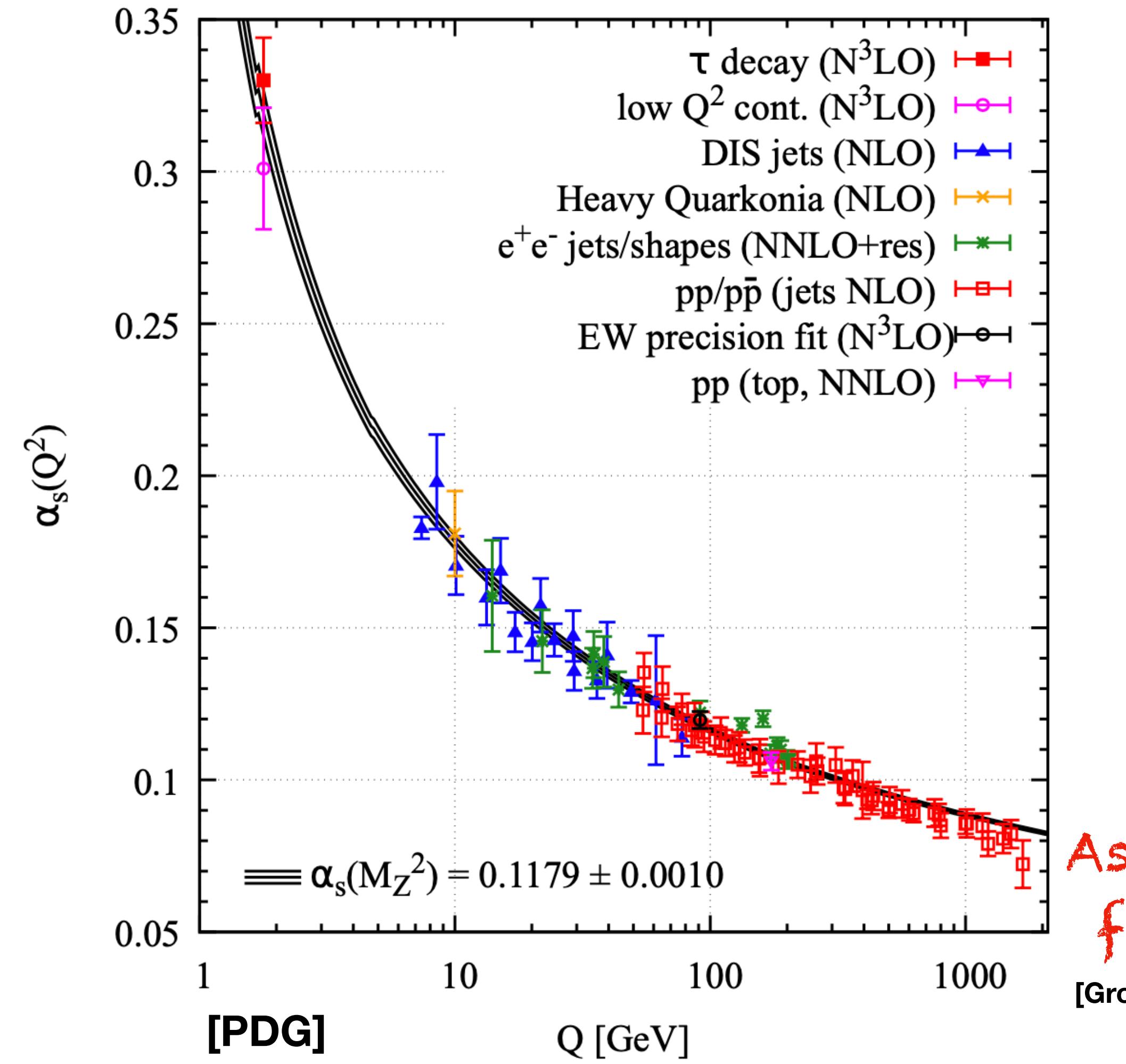
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The QCD coupling



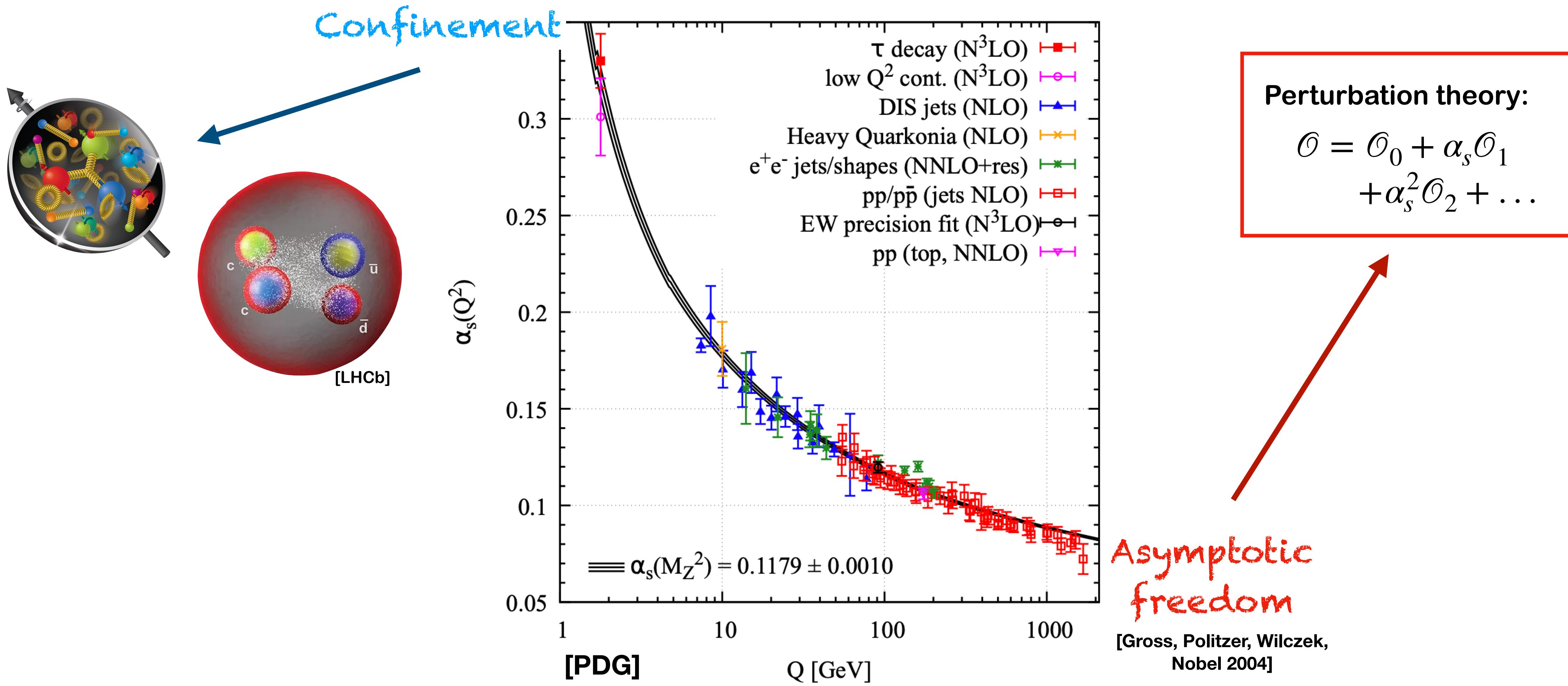
The QCD coupling



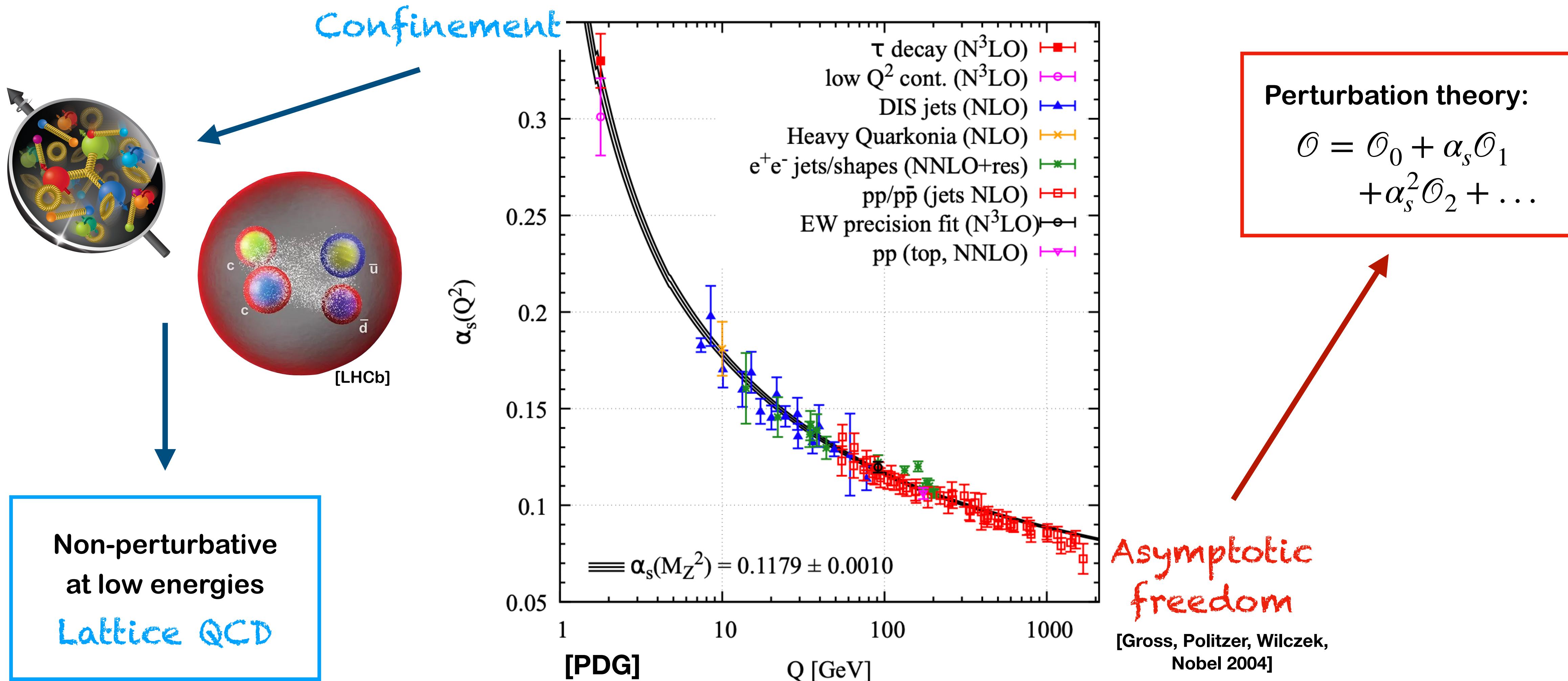
Perturbation theory:

$$\mathcal{O} = \mathcal{O}_0 + \alpha_s \mathcal{O}_1 + \alpha_s^2 \mathcal{O}_2 + \dots$$

The QCD coupling



The QCD coupling



Lattice Field Theory

Path Integral Formalism (I)

- QFTs can be formulated using the Feynman path integral formalism

$$S[\phi] = \int d^4x \mathcal{L}(\phi)$$

↑
Action ↑
 Lagrangian ↑
 Quantum
 Fields

$$\mathcal{Z} = \int D\phi e^{iS[\phi]}$$

↑
Partition
function

resembles
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 Lagrangian Quantum
 Fields

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partition function
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statistical mechanics

- Compute n-point correlation functions:

$$\langle \phi(x_1) \dots \phi(x_n) \rangle = \frac{\int D\phi e^{iS[\phi]} \phi(x_1) \dots \phi(x_n)}{\mathcal{Z}}$$

contain information
about interactions
n-particle interactions

Path Integral Formalism (II)

- Partition function almost looks like the normalization of some probability distribution

$$\mathcal{Z} = \int D\phi e^{-iS(\phi)}$$

- ▶ But complex numbers
- ▶ Infinite number variables

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- Perform Wick rotation to Euclidean spacetime:

$$x^0 \rightarrow -ix_E^0, \quad \int d^4x \rightarrow i \int d^4x_E \quad \partial_\mu \partial^\mu \rightarrow -\partial_\mu \partial_\mu$$

- Path integral in Euclidean or imaginary time: statistical meaning

$$\mathcal{Z} = \int D\phi e^{-S_E(\phi)} \quad , \text{ where } \quad S_E(\phi) = \int d^4x_E \mathcal{L}_E(\phi) \quad \text{Euclidean action}$$

↑
Boltzmann factor

Lattice Field Theory

- LFT is the first-principles treatment of the generic QFT

- Path integral in Euclidean or imaginary time: statistical meaning

$$Z = \int D\phi e^{-S_E(\phi)} \quad , \text{ where} \quad S_E(\phi) = \int d^4x_E \mathcal{L}_E(\phi)$$

Euclidean action

“Boltzmann factor”

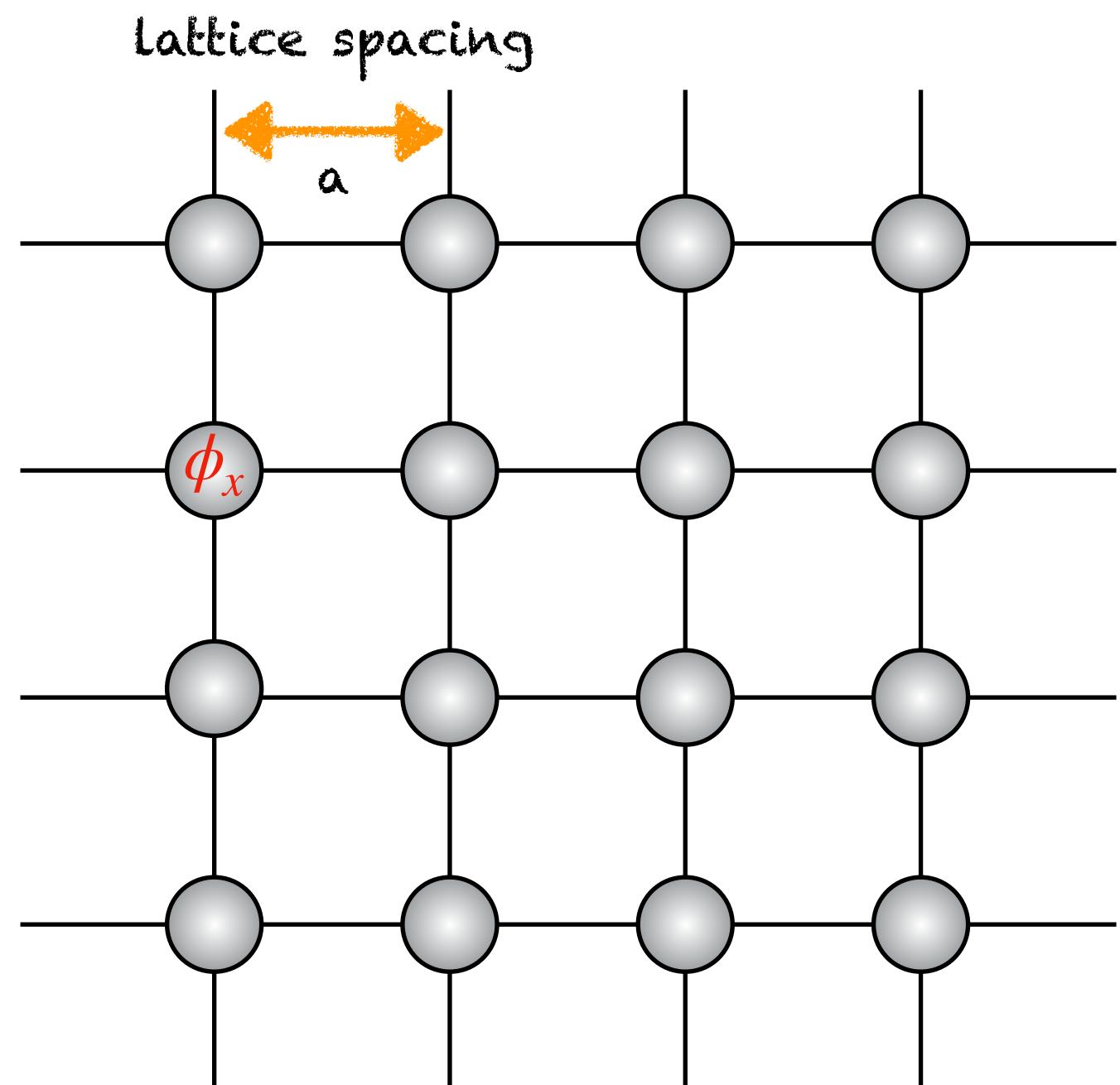
- Discretize quantum fields (real scalars):

Continuum:

$$S_E = \int d^4x \left[\frac{1}{2} \partial_\mu \phi(x)^2 + \frac{m^2}{2} \phi(x)^2 + \lambda \phi(x)^4 \right]$$

Lattice:

$$S_E = a^4 \sum_x \frac{1}{2a^2} (\phi_{x+\mu} - \phi_x)^2 + \frac{m^2}{2} \phi_x^2 + \lambda \phi_x^4$$

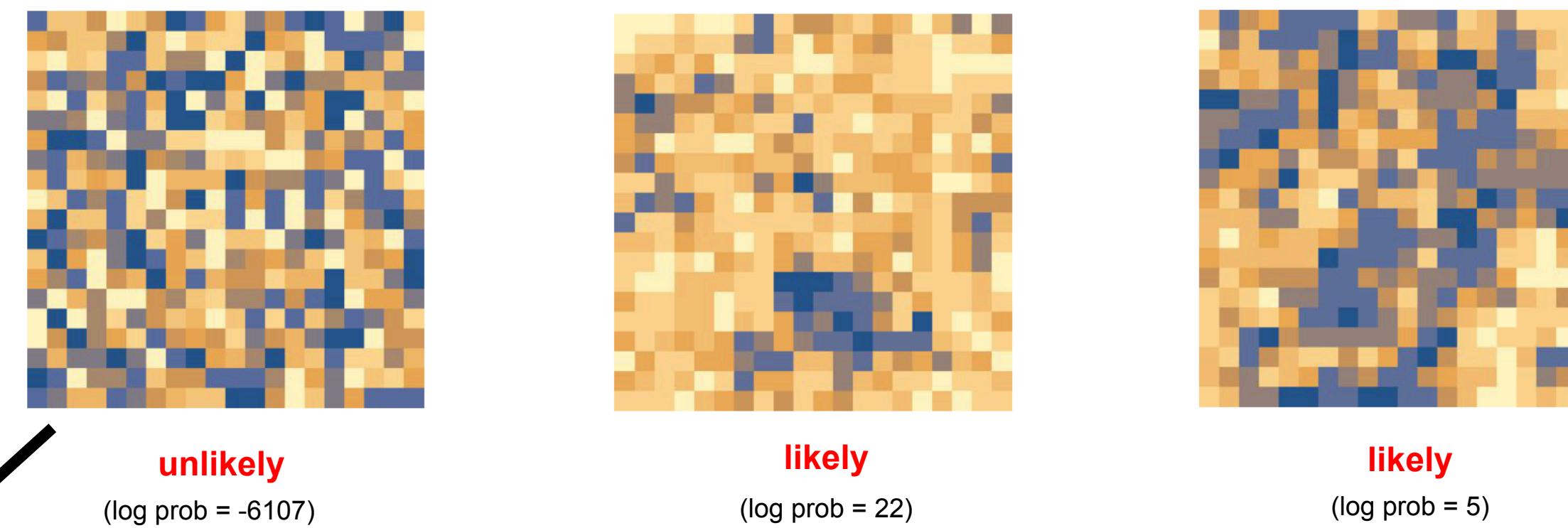


Test case: scalar theory

- A **real scalar field** per lattice site in a 2D lattice.

$$\phi(x) \in (-\infty, +\infty)$$

A real number in each point of space time in the lattice



no structure, i.e.
looks like random noise

Field configurations with probability:

$$P[\phi(x)] \sim e^{-S[\phi(x)]}$$

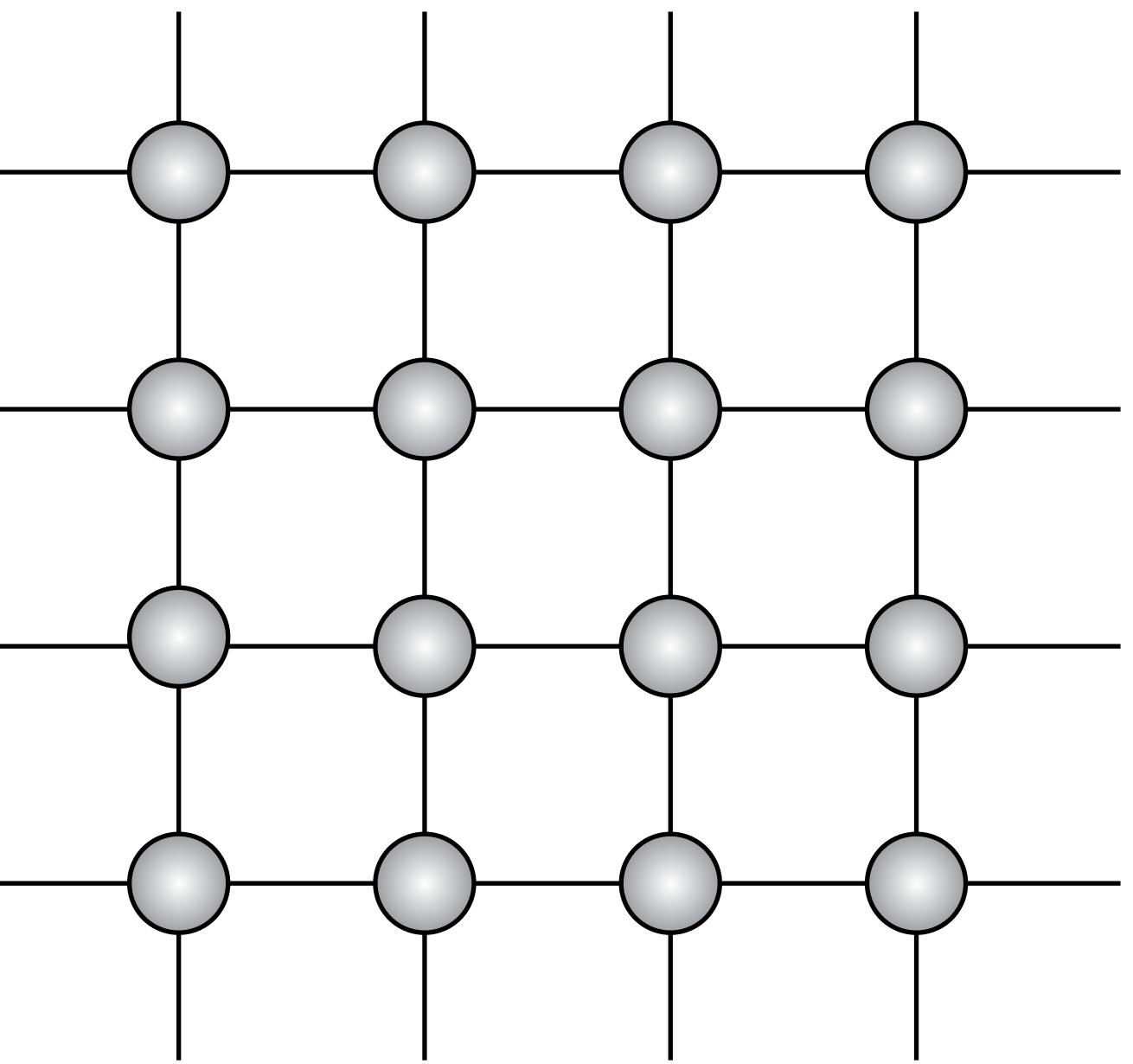
- Lattice action:

$$S = a^4 \sum_x \frac{1}{2a^2} (\phi_{x+\mu} - \phi_x)^2 + \frac{m^2}{2} \phi_x^2 + \lambda \phi_x^4$$

long-range correlations

Lattice QCD Basics

- Lattice QCD is the state-of-the-art treatment of the strong interaction at hadronic energies

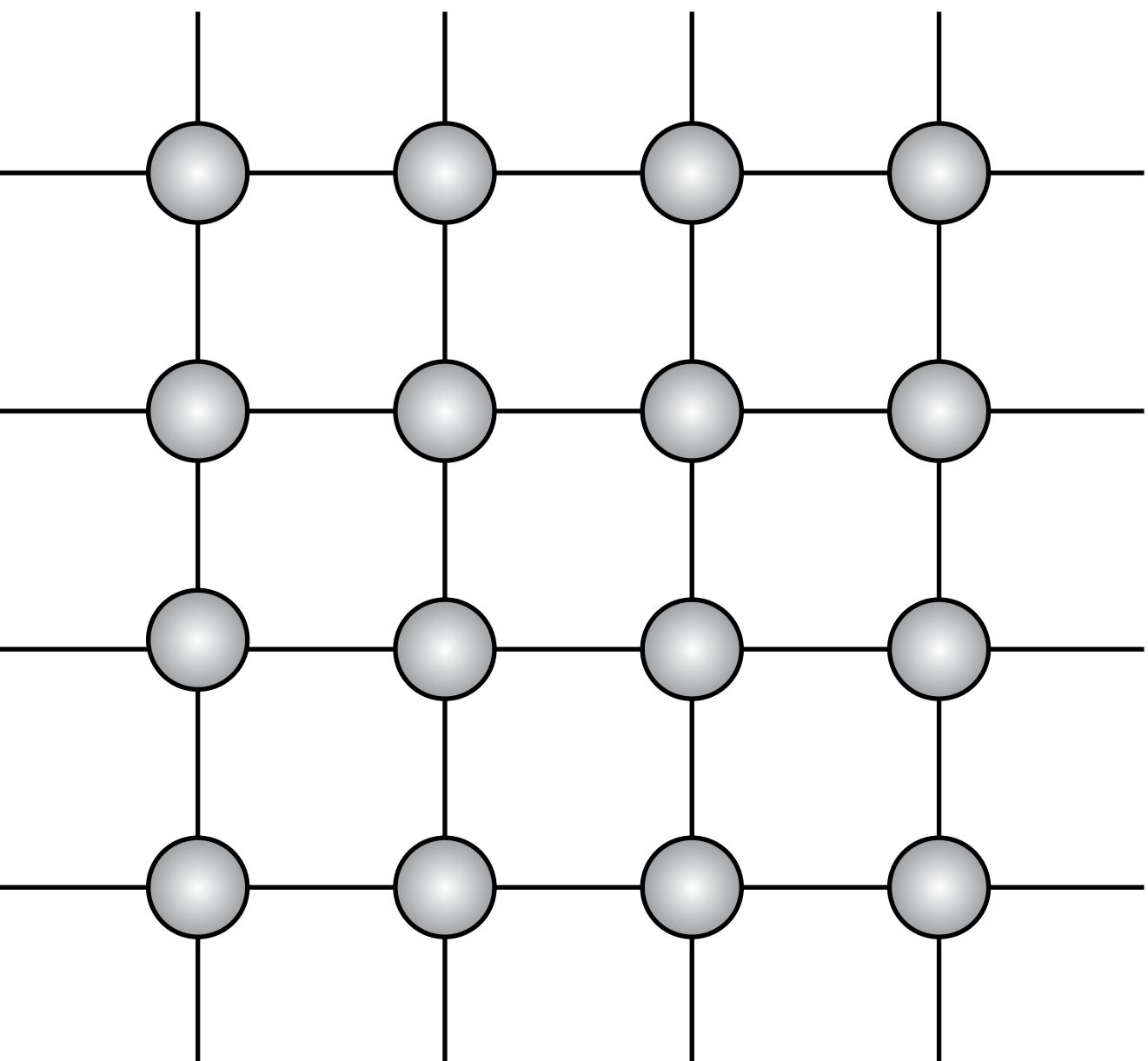


Lattice QCD Basics

- Lattice QCD is the state-of-the-art treatment of the strong interaction at hadronic energies
- Euclidean space-time: action has statistical meaning

$$Z = \int D\psi D\bar{\psi} DA e^{-S_E(\psi, \bar{\psi}, A_\mu)}$$

→ “Boltzmann factor”



Lattice QCD Basics

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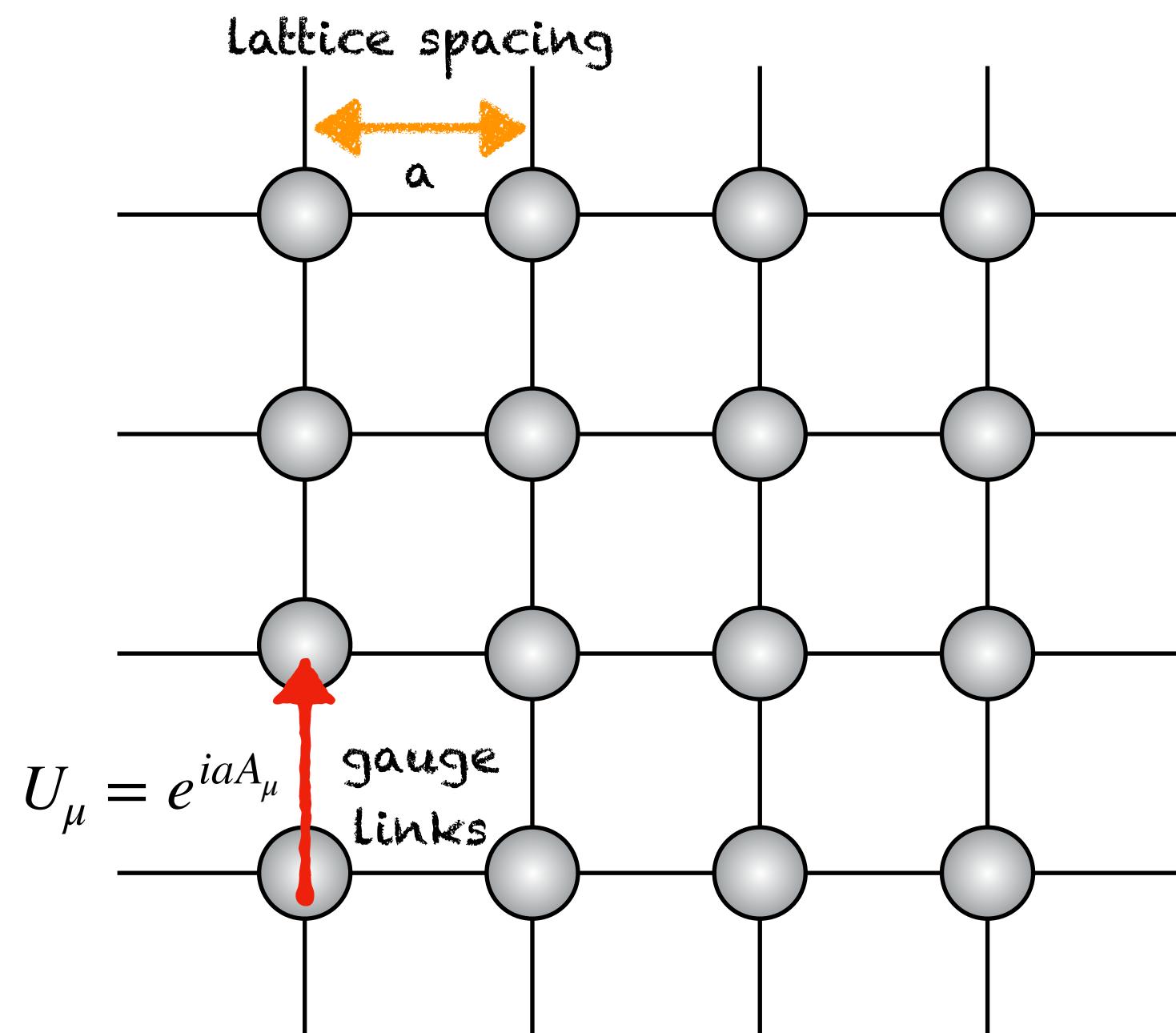
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“Boltzmann factor”

- Discretize gauge fields and fermion fields:

→ Under control but technical

→ Need continuum limit



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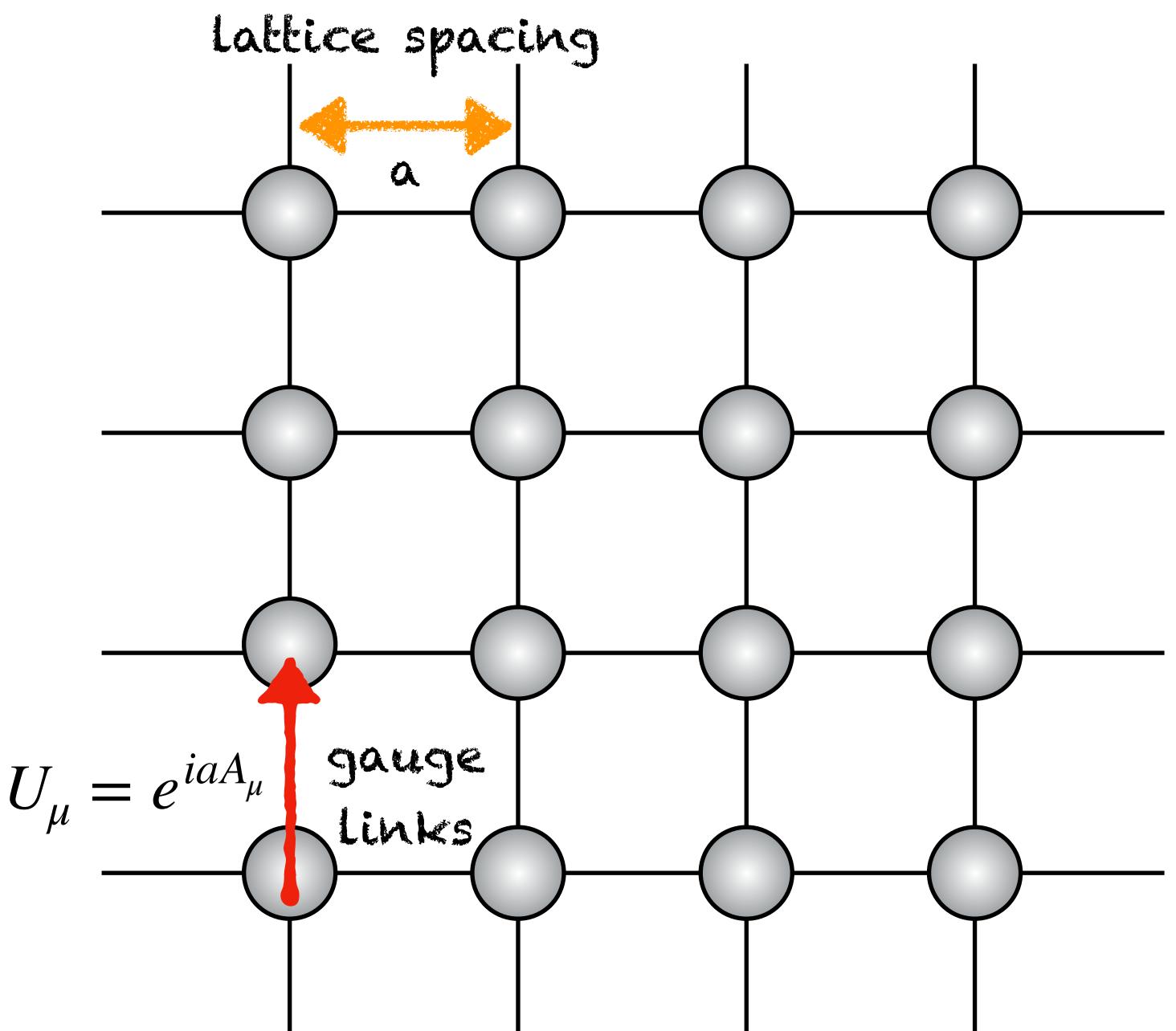
- Discretize gauge fields and fermion fields:

→ Under control but technical

→ Need continuum limit

- Compute observables as expectation values

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D\psi D\bar{\psi} DA \mathcal{O} e^{-S_E(\psi, \bar{\psi}, A_\mu)}$$



An integral over many variables:
 $2 \times 3^2 \times 4 \times L^4 \simeq 10^{10}$

Sampling in Lattice Field Theory

Markov Chain Monte Carlo

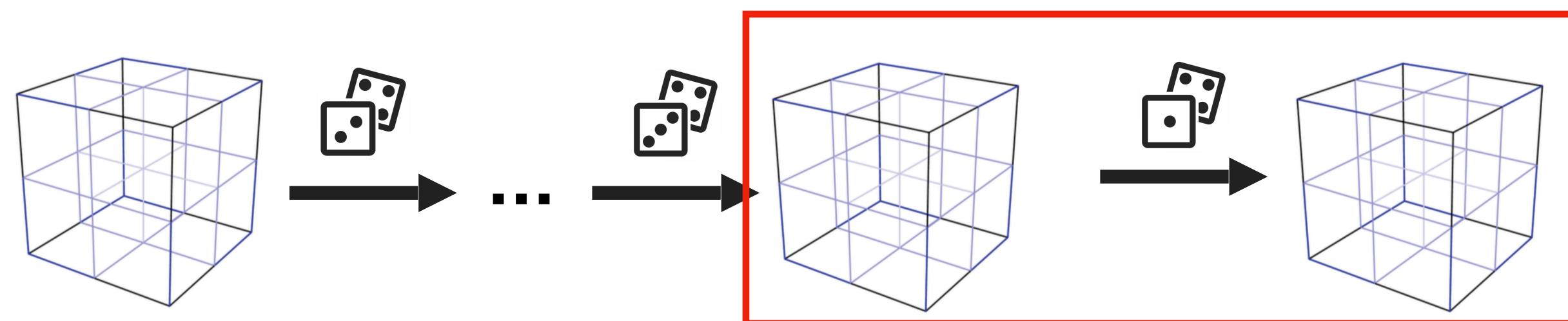
Lattice QCD is sampling problem over a very large number of variables

Markov Chain Monte Carlo

Lattice QCD is sampling problem over a very large number of variables

STEP 1: Generate field configurations:

$$\{U_i\} \sim p(U) = \frac{e^{-S_E(U)}}{\mathcal{Z}}$$



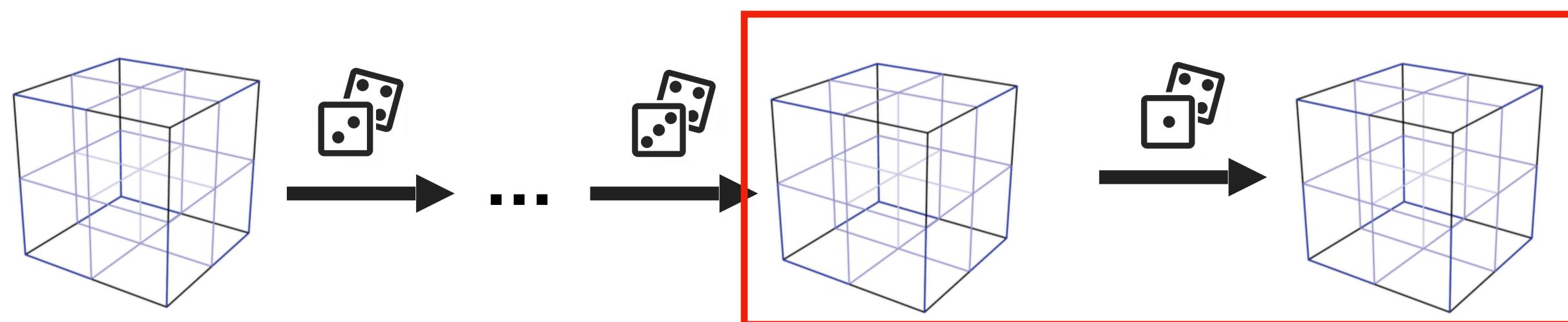
“autocorrelation” \equiv Computational cost

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STEP 2: Compute observables:



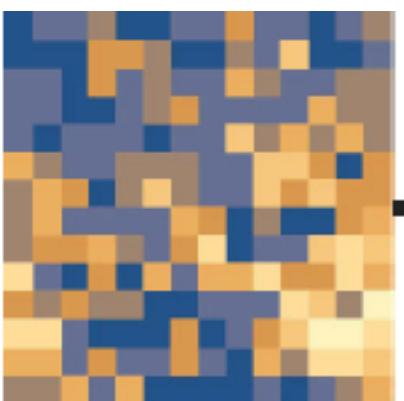
For instance, the proton mass!

$$\langle \mathcal{O} \rangle \simeq \sum_{i=0}^{N_{conf}} O(U_i)$$

Standard algorithms

- The simplest sampling algorithm is the Metropolis-Hastings algorithm

initial
config

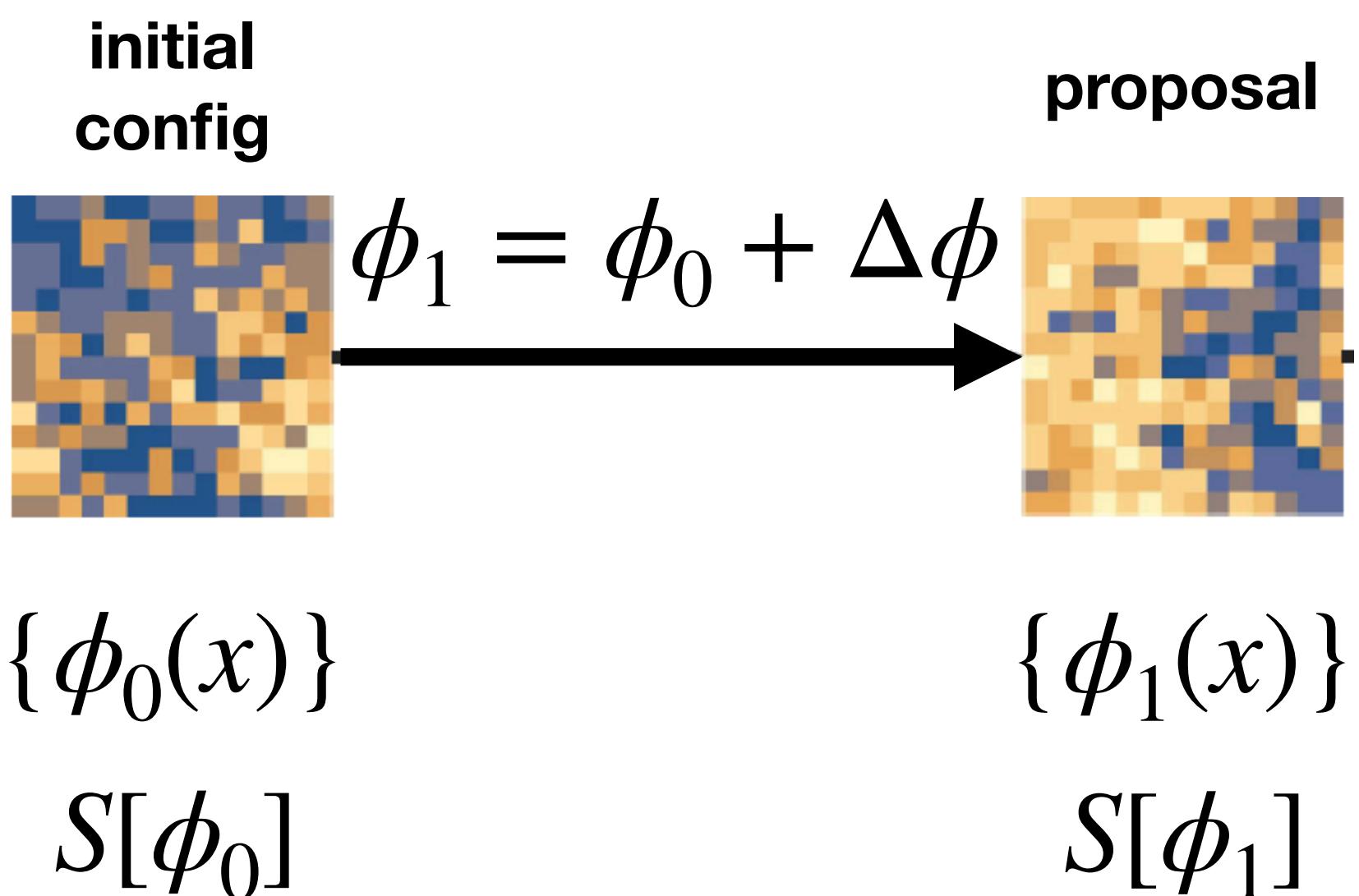


$$\{\phi_0(x)\}$$

$$S[\phi_0]$$

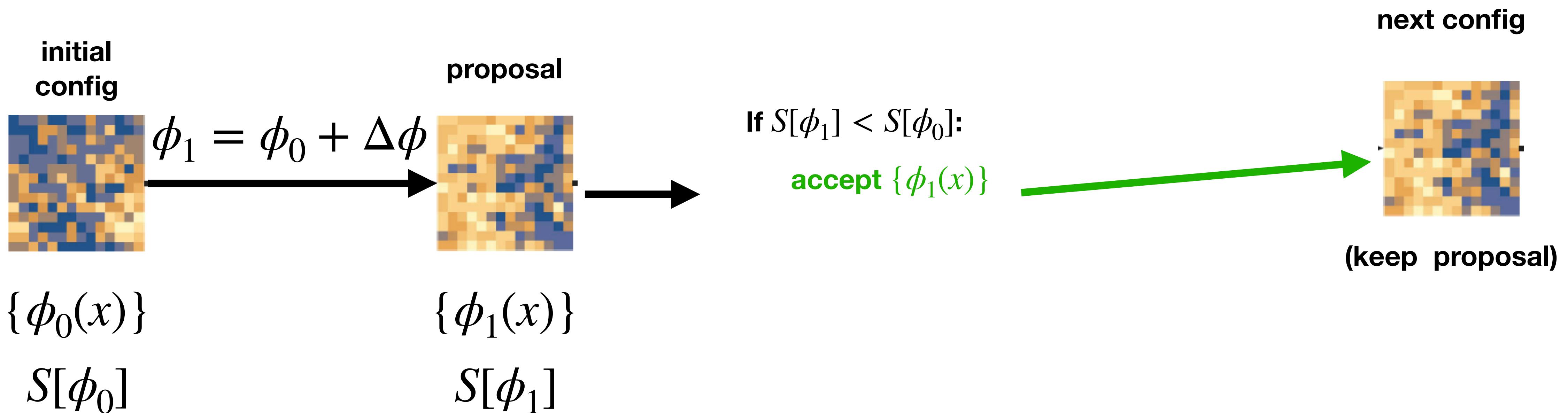
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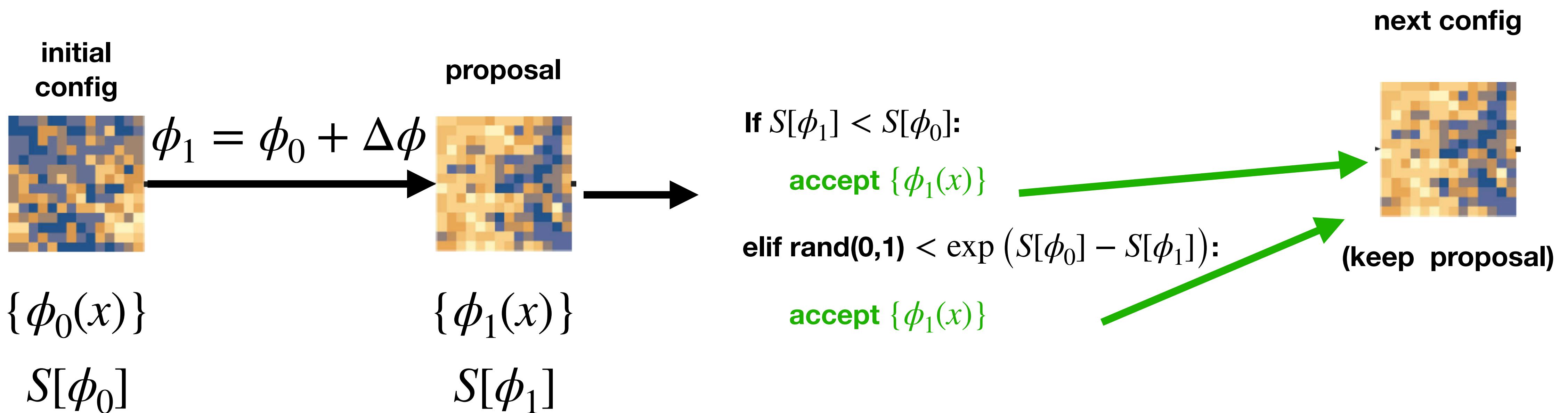
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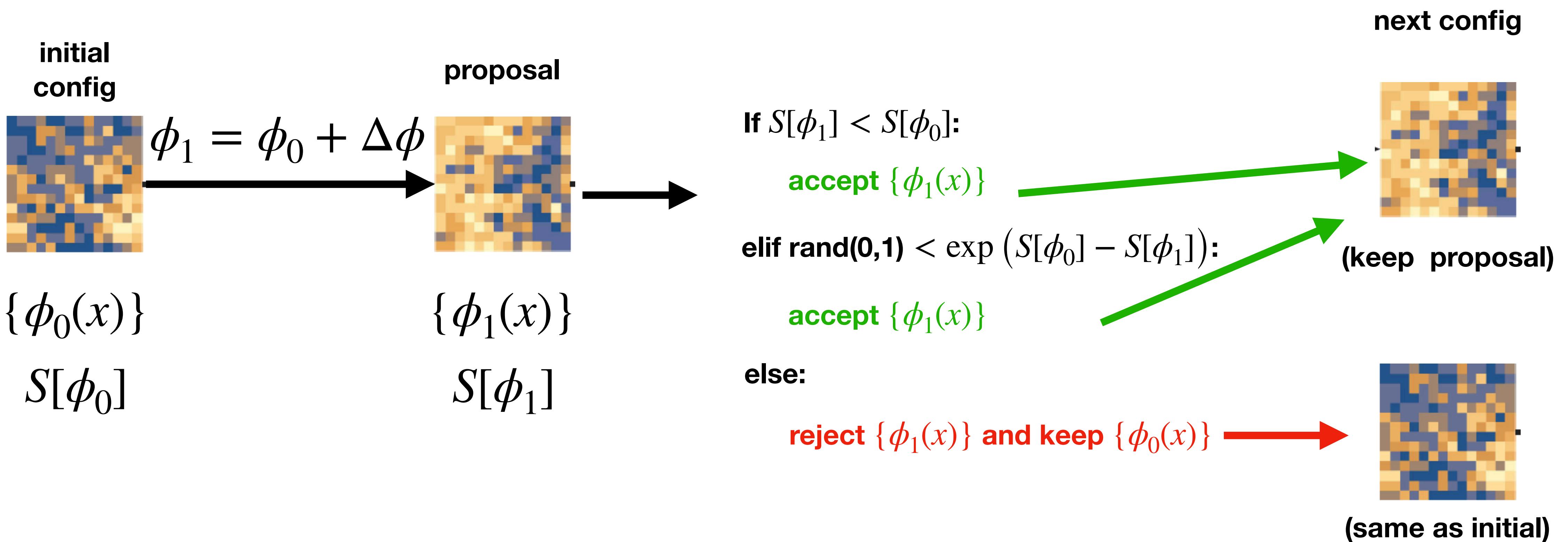
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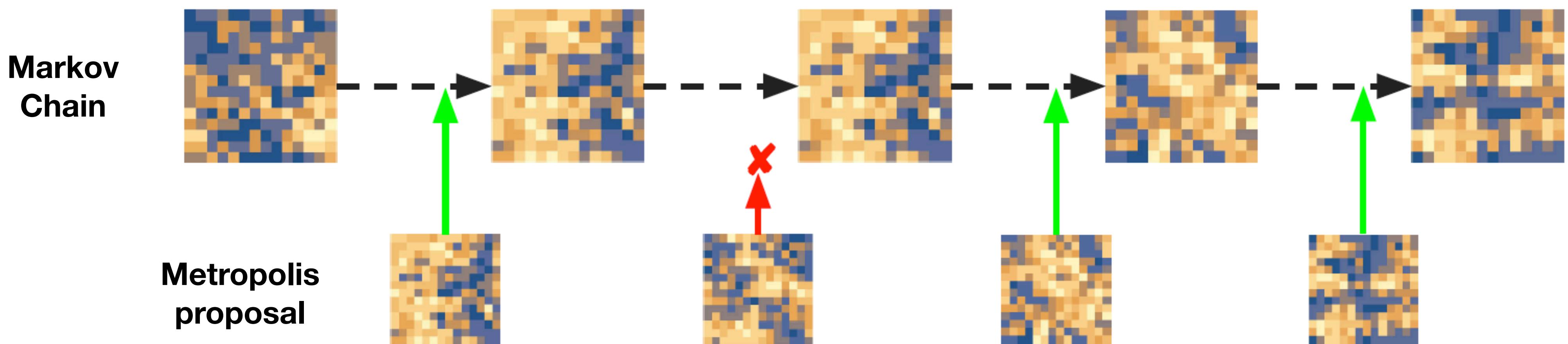
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Markov Chain Monte Carlo

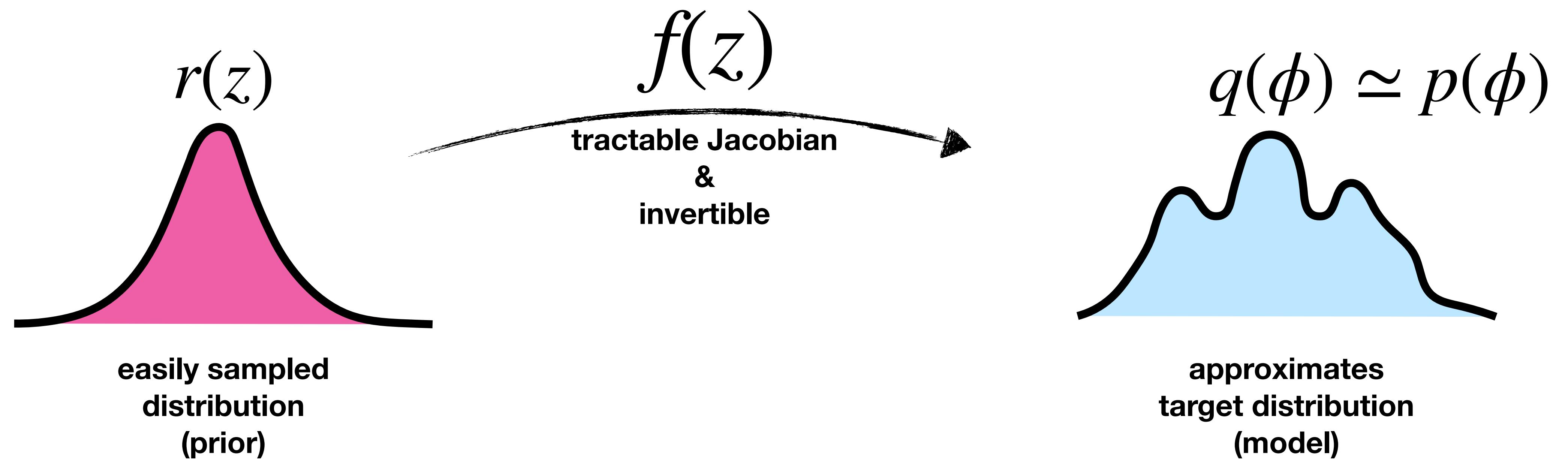
- “Ensembles” of configurations are generated by building a Markov Chain



- State-of-the-art calculations use the Hybrid Monte Carlo (HMC) algorithm
 - ▶ Based on Hamilton equations of motion

Generative flow models

[Rezende, Mohamed, 1505.05770]



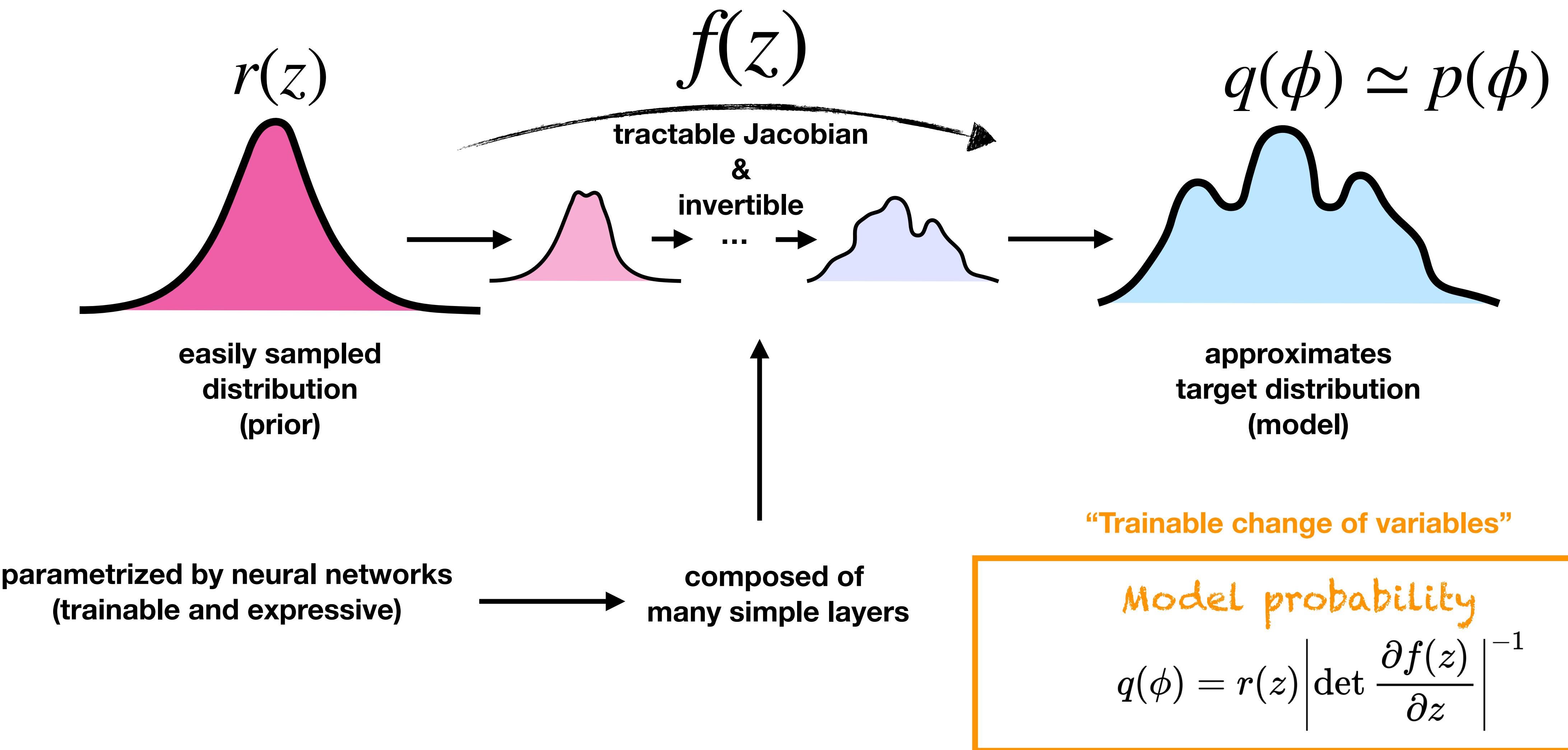
“Trainable change of variables”

Model probability

$$q(\phi) = r(z) \left| \det \frac{\partial f(z)}{\partial z} \right|^{-1}$$

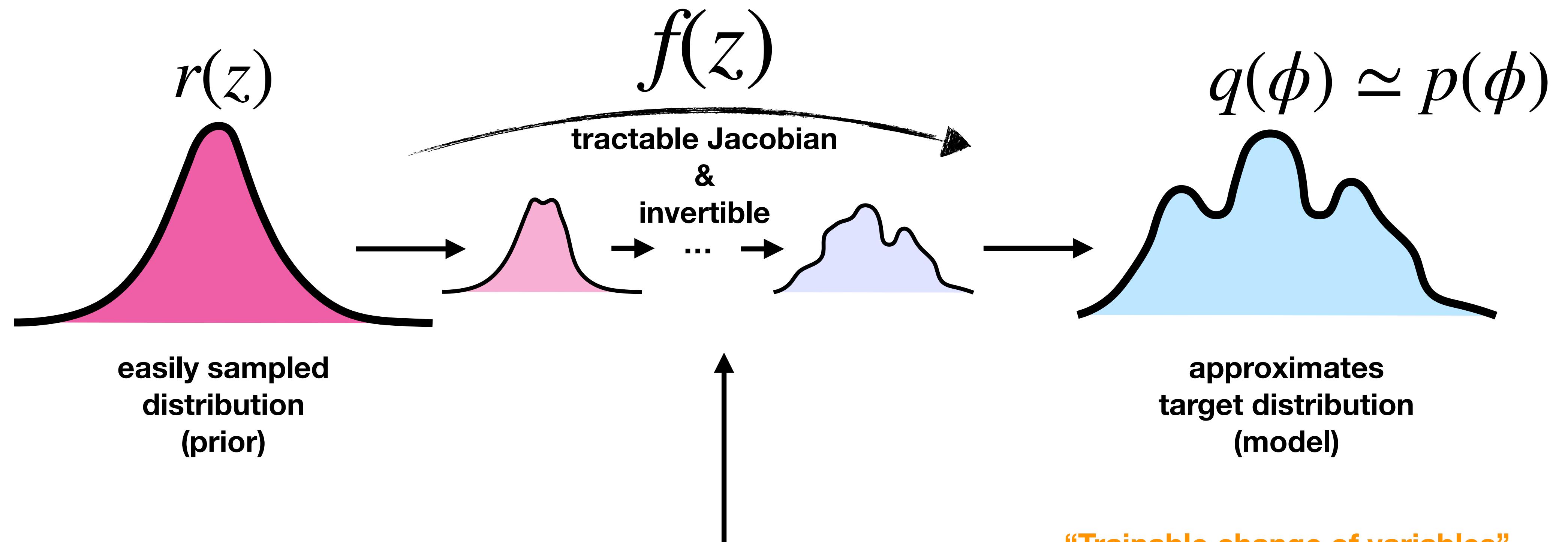
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[Rezende, Mohamed, 1505.05770]



Generative flow models

[Rezende, Mohamed, 1505.05770]



parametrized by neural networks
(trainable and expressive)

! Trained models are not perfect.

But exact sampling can be recovered via Markov Chain

composed of
many simple layers

“Trainable change of variables”

Model probability

$$q(\phi) = r(z) \left| \det \frac{\partial f(z)}{\partial z} \right|^{-1}$$

Application: Schwinger Model

- QED in 1+1 dimesions: a toy model for QCD
 - ▶ Confinement, topology, symmetry breaking
 - ▶ Standard algorithms affected by topology freezing

MCMC with
traditional algorithms

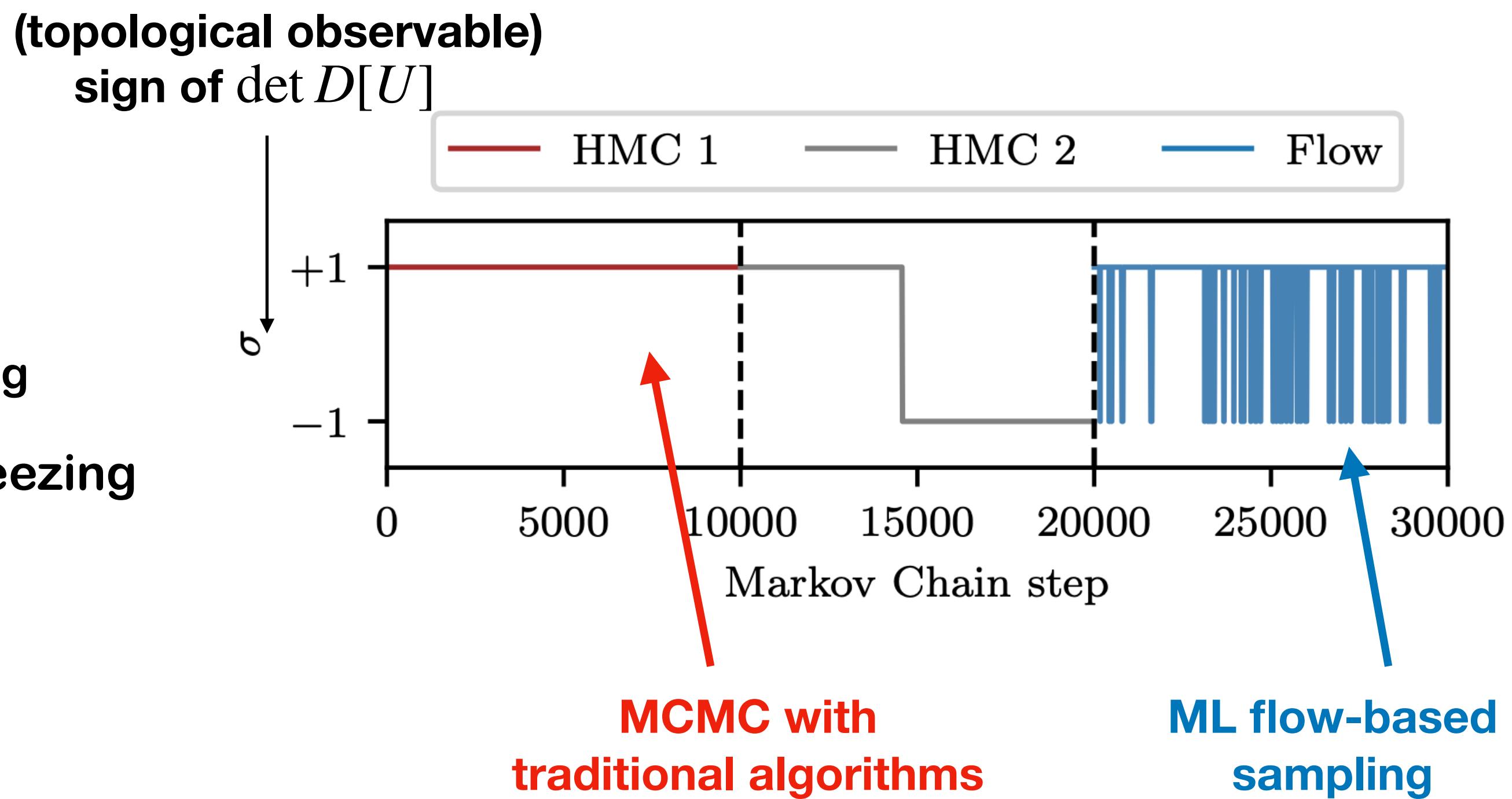
ML flow-based
sampling

[Albergo, Boyda, Cranmer, Hackett, Kanwar, Racanière, Rezende, FRL, Shanahan, Urban 2202.11712]

Application: Schwinger Model

QED in 1+1 dimesions: a toy model for QCD

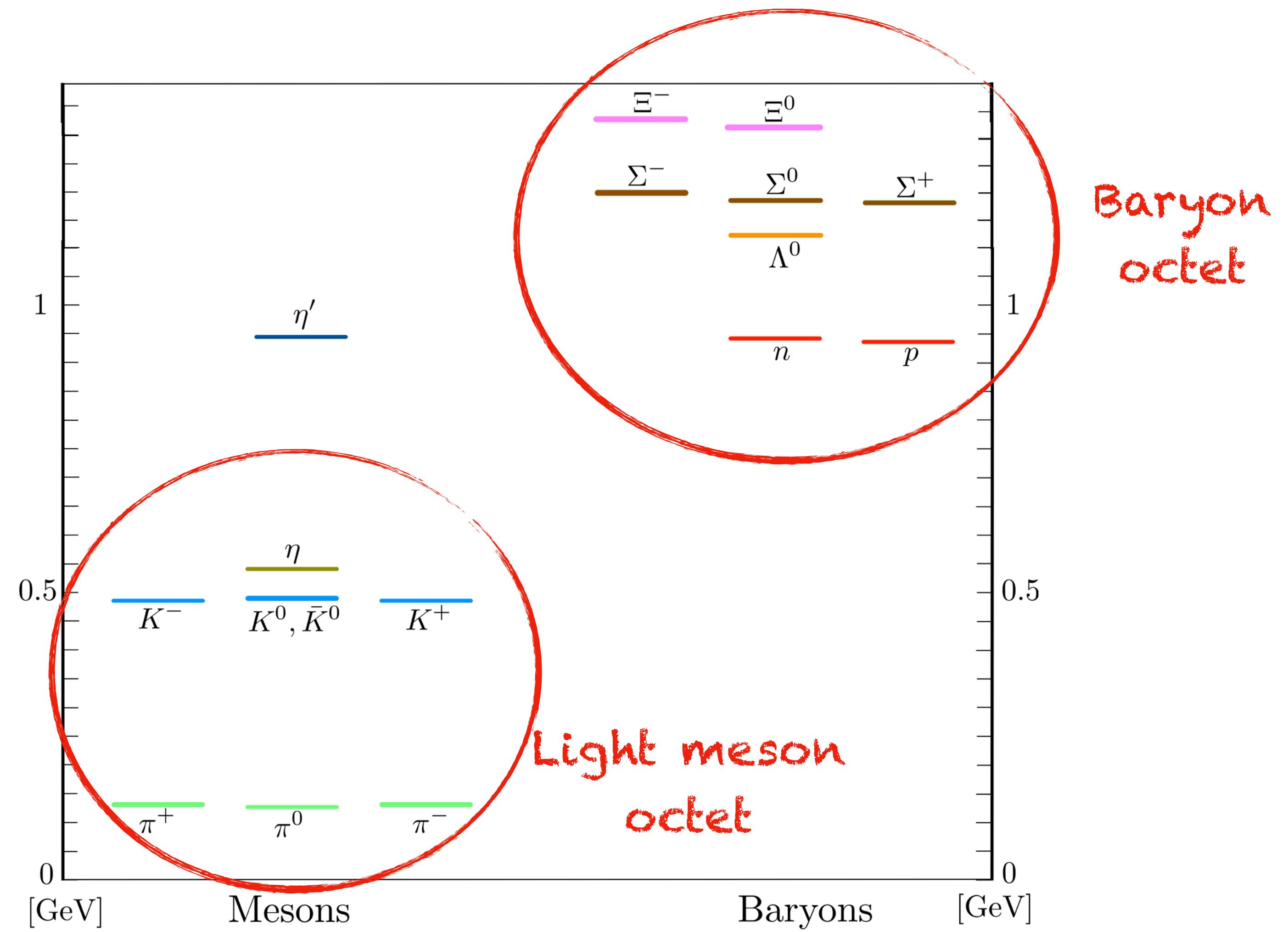
- ▶ Confinement, topology, symmetry breaking
- ▶ Standard algorithms affected by topology freezing
- ✓ Flow-based sampling mitigates topology freezing



[Albergo, Boyda, Cranmer, Hackett, Kanwar, Racanière, Rezende, FRL, Shanahan, Urban 2202.11712]

The QCD spectrum from Lattice QCD

The QCD spectrum



Lattice QCD

- We measure **energy levels** and **matrix elements**: "Spectral decomposition"

$$\begin{aligned} C(t) &= \langle \mathcal{O}^\dagger(t) \mathcal{O}(0) \rangle = \sum_n \langle 0 | \mathcal{O}^\dagger(t) | n \rangle \langle n | \mathcal{O}(0) | 0 \rangle \\ &= \sum_n \langle 0 | e^{Ht} \mathcal{O}^\dagger(0) e^{-Ht} | n \rangle \langle n | \mathcal{O}(0) | 0 \rangle \\ &= \sum_n \left| \langle 0 | \mathcal{O}(0) | n \rangle \right|^2 e^{-E_n t} \end{aligned}$$

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Ground state

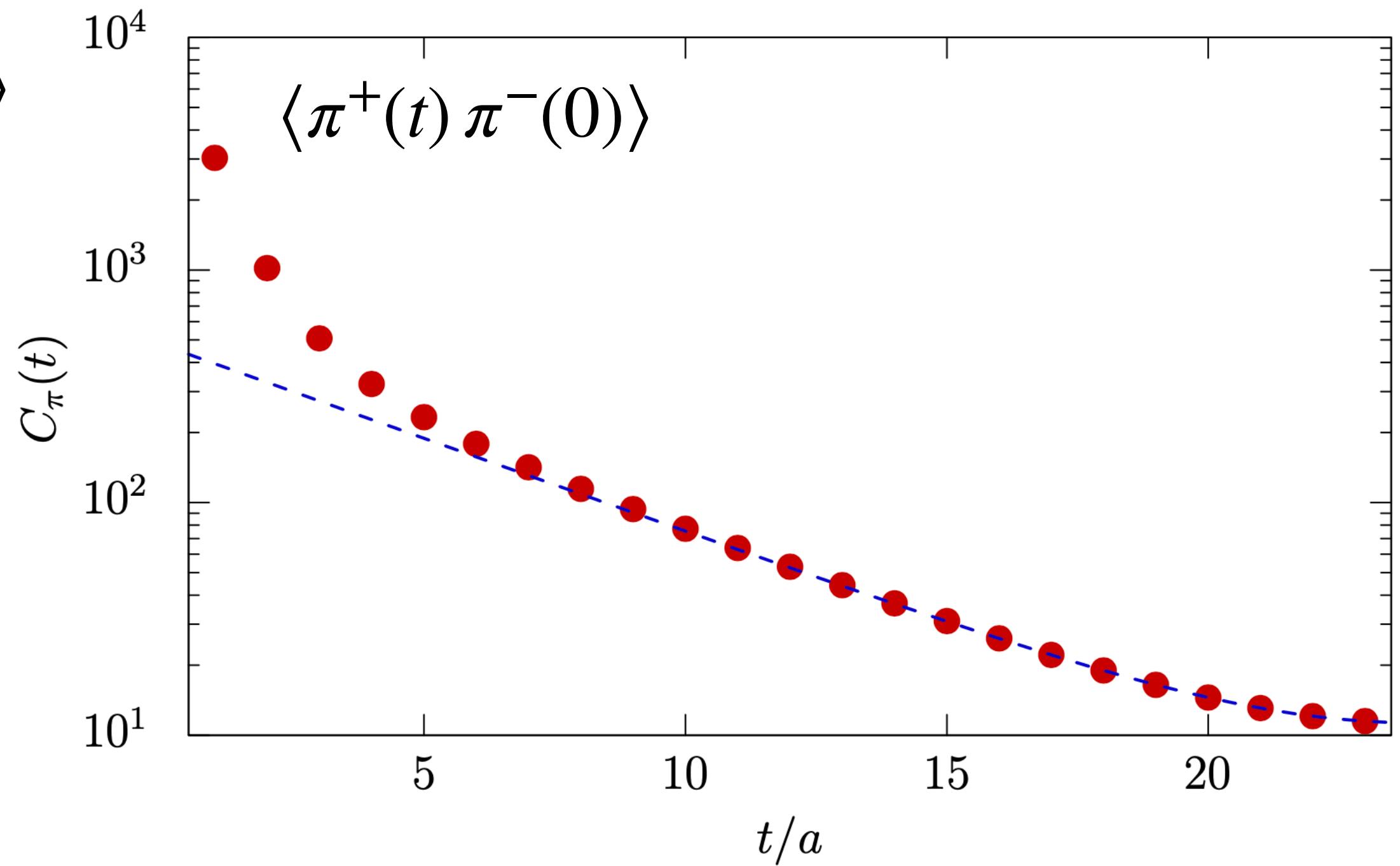
$$\lim_{t \rightarrow \infty} C(t) = A_0 e^{-E_0 t}$$

Lattice QCD

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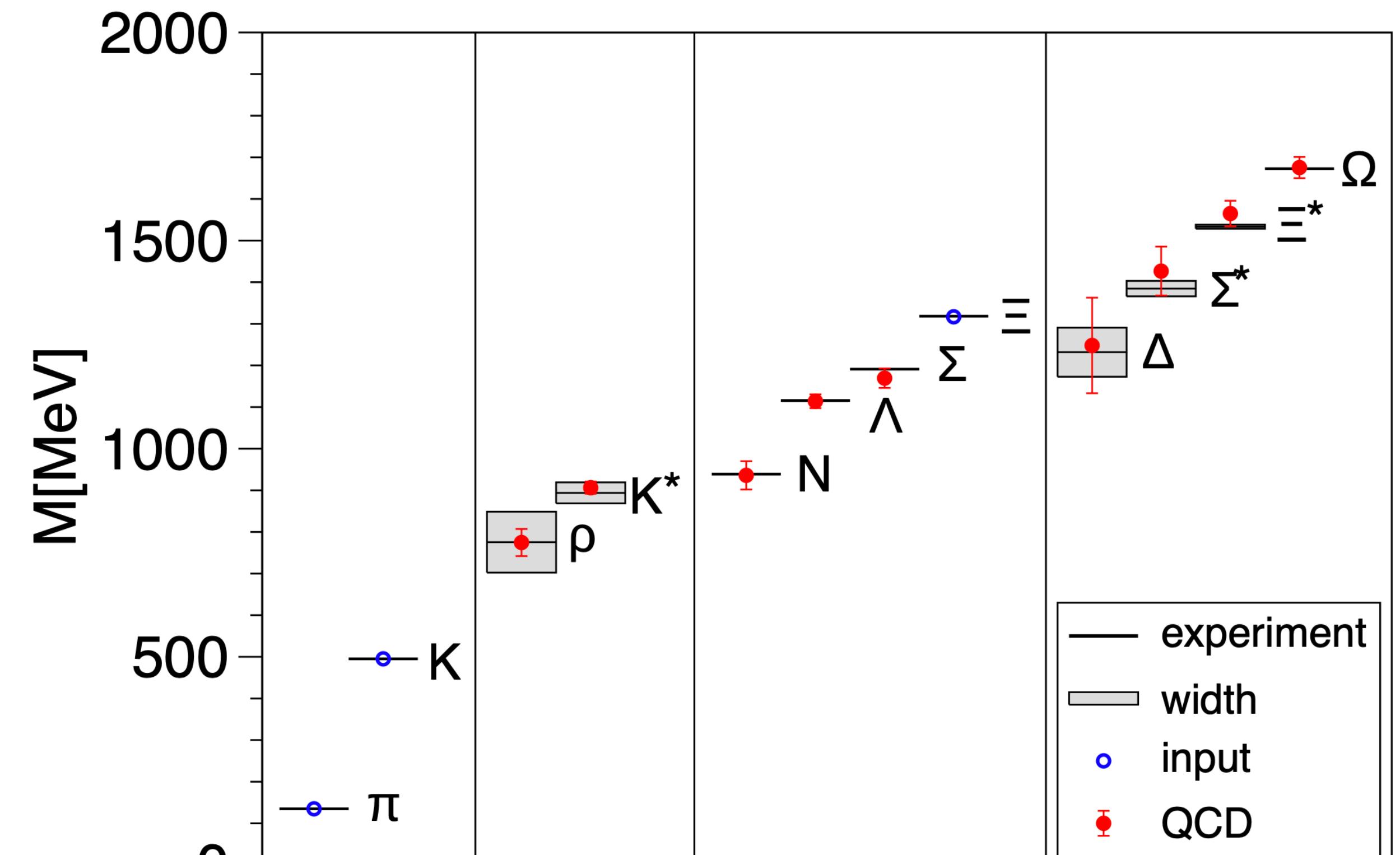
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Ground state

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The QCD spectrum (I)

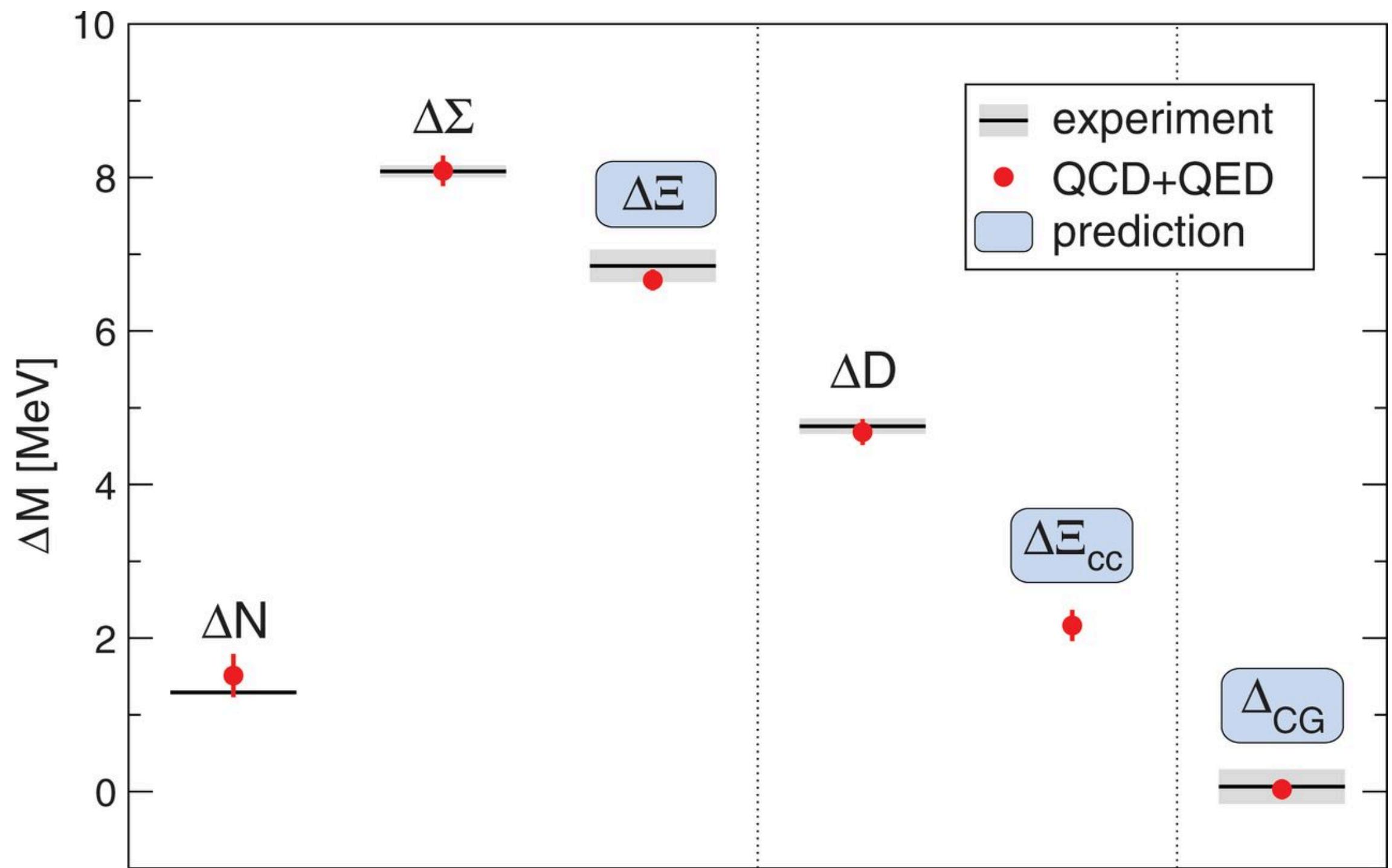
- Consider isospin-symmetric QCD ($m_u = m_d$)
- Neglect QED effects in hadrons
- Need a few inputs to fix quark masses
- Reproduce the lowest-lying hadrons!



[BMW collaboration, 2008]

The QCD spectrum (II)

- More recently: add QED and $m_u \neq m_d$
- Can reproduce neutron/proton mass difference
- More precise than some experimental results

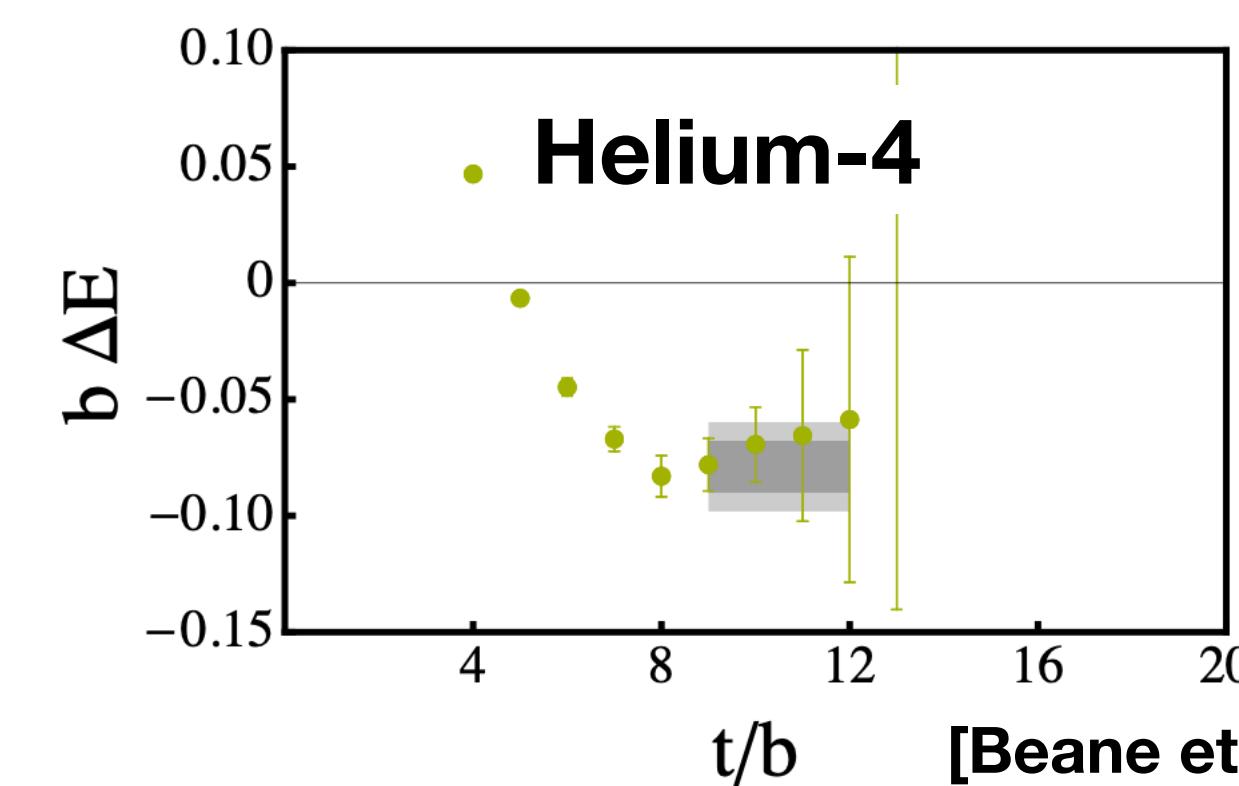
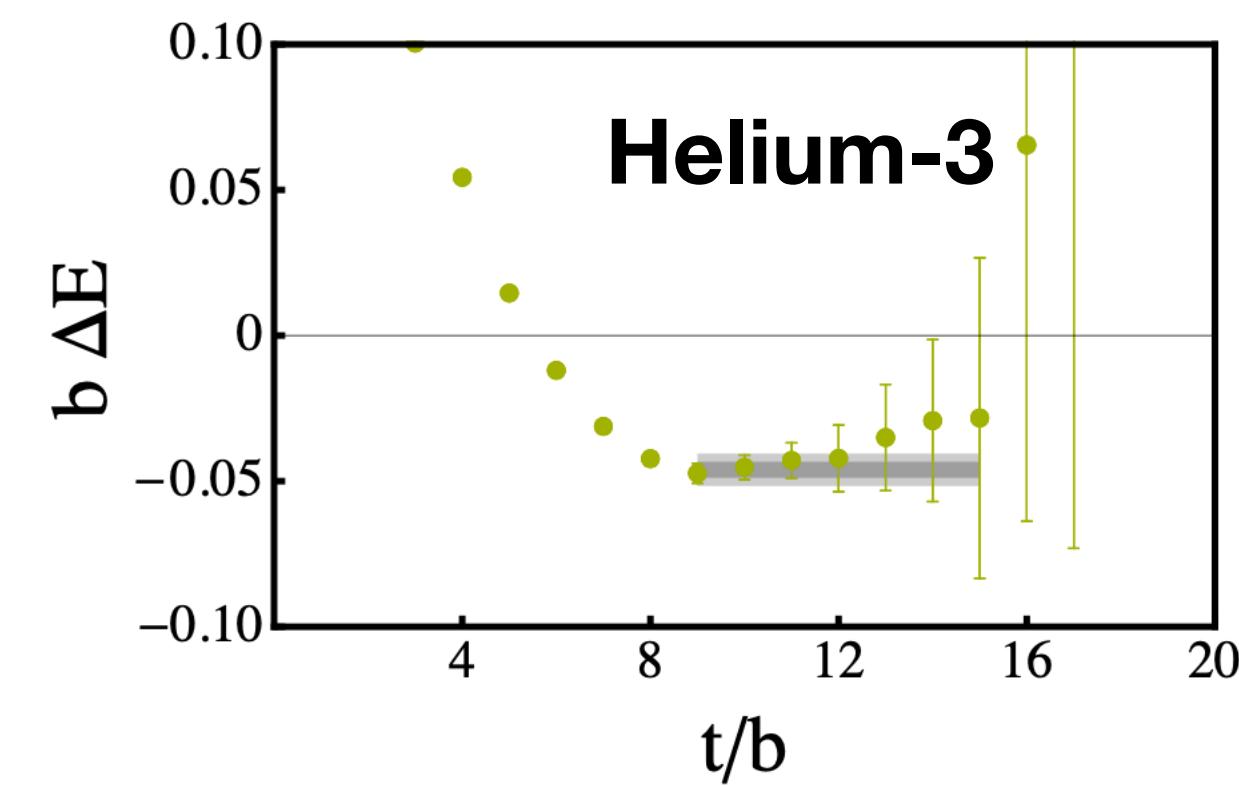
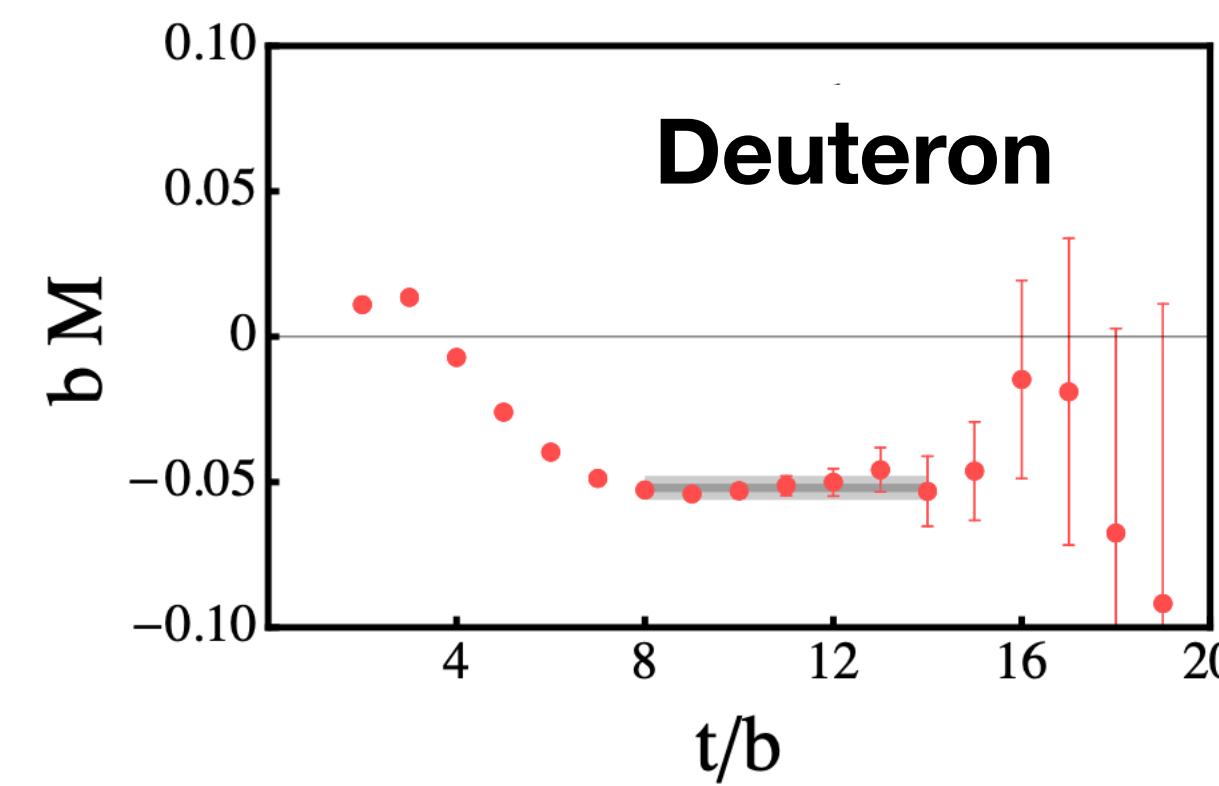


[BMW collaboration (2015)]

[\[https://www.science.org/doi/full/10.1126/science.1257050\]](https://www.science.org/doi/full/10.1126/science.1257050)

Nuclear Physics from LQCD

- Enormous progress in computing properties of nuclei
- Signal-to-noise problem grows rapidly with the baryon number



[Beane et al (NPLQCD), 2012]

- However, computational cost of Wick contractions grows (naively) factorially
- # contractions $\sim (3A)!$

Lattice QCD for precision SM tests

Example: muon g-2

- Muon magnetic moment marks the interaction of a muon with a magnetic field

$$H_I \propto g_\mu \vec{B} \cdot \vec{S}$$

Interacting Hamiltonian

Magnetic moment

external magnetic field

muon spin

“anomalous magnetic moment”

$$a_\mu = \frac{1}{2} (g_\mu - 2)$$

Example: muon g-2

- Muon magnetic moment marks the interaction of a muon with a magnetic field

Magnetic moment

“anomalous magnetic moment”

muon spin

Interacting Hamiltonian

$H_I \propto g_\mu \vec{B} \cdot \vec{S}$

$a_\mu = \frac{1}{2} (g_\mu - 2)$

► Dirac limit

$a_\mu = 0$

► First QFT correction [J. Schwinger (1948)]

$a_\mu = \frac{\alpha_{em}}{2\pi} \simeq 0.0011614$

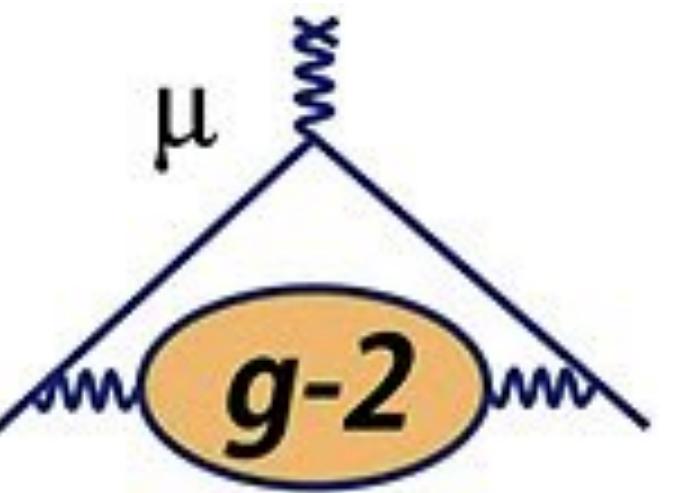
► Most precise number in nature

$a_\mu(\text{exp}) = 116592061(41) \cdot 10^{-11}$

$a_\mu(\text{theory}) = 116591810(43) \cdot 10^{-11}$

Example: muon g-2

Contribution	Section	Equation	Value $\times 10^{11}$
Experiment (E821)		Eq. (8.13)	116 592 089(63)
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)
HVP (e^+e^- , LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)

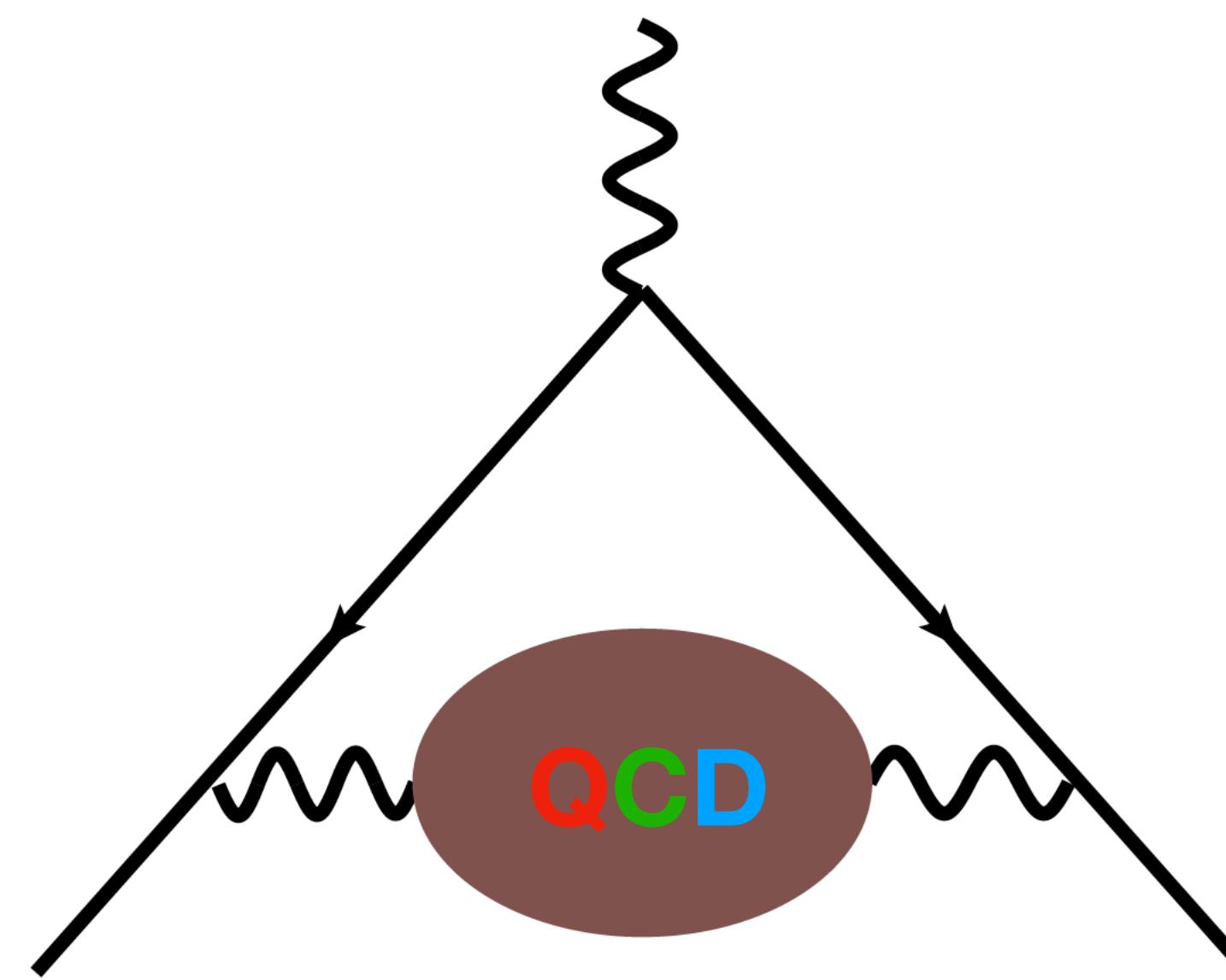


[<https://muon-g-2.fnal.gov>]

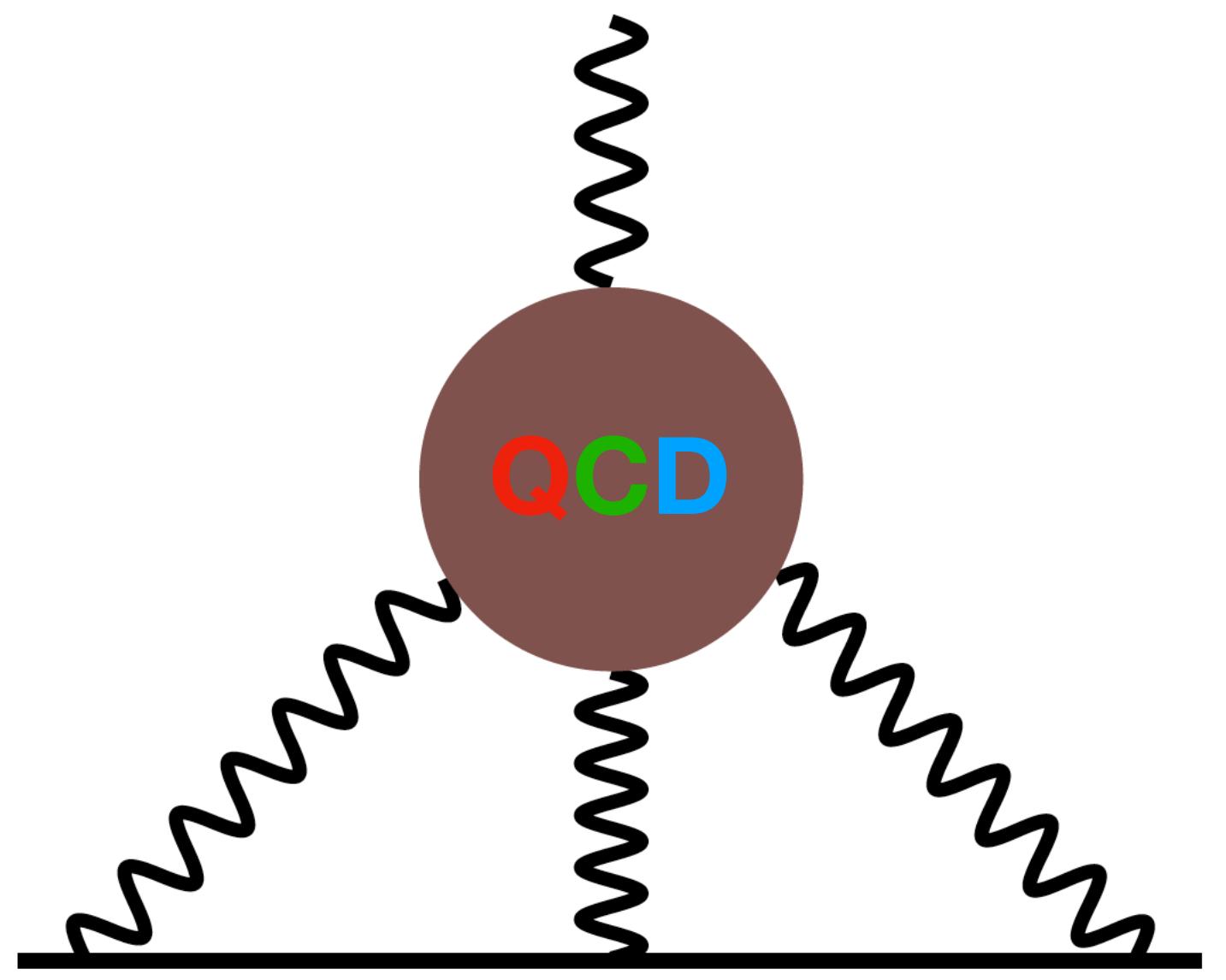
[<https://arxiv.org/pdf/2006.04822.pdf>]

Example: muon $g-2$

- Lattice QCD can contribute in the computation of diagrams involving hadrons



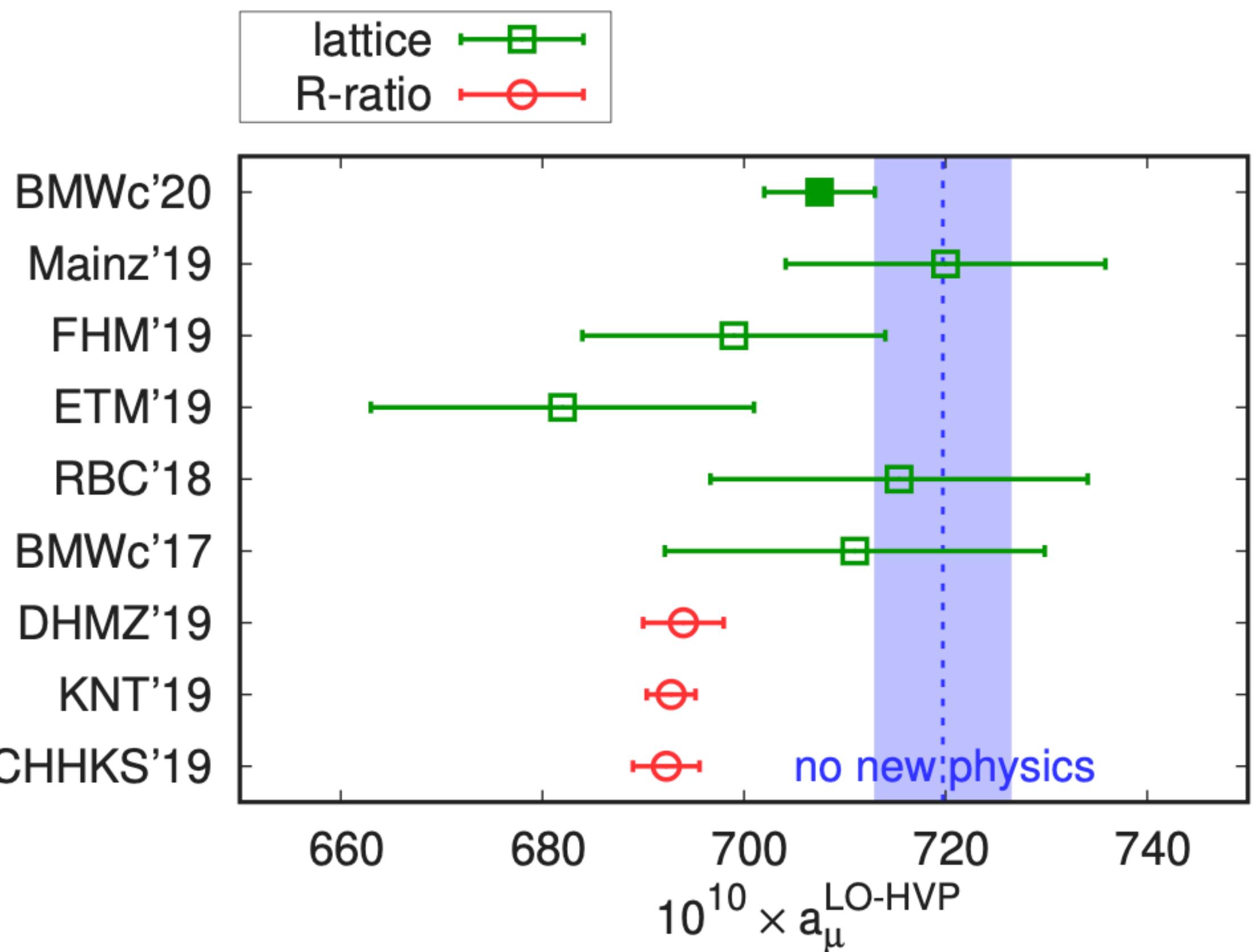
Hadronic Vacuum
Polarization (HVP)



Hadronic Light-by-light
scattering (HLbL)

Lattice QCD results for $g-2$ HVP

- Competitive precision from Lattice QCD
- Discrepancy between LQCD and data driven approach
- Essential calculation for the search of BSM Physics



Summary

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- Quantum Chromodynamics is the theory of the strong interaction
- Lattice QCD is the first-principle treatment of the strong interaction at hadronic energies
- Lattice Field Theory is a sampling problem over a very large number of variables
- Markov-Chain Monte Carlo approaches can be used for efficient sampling (even including ML!)
- Great success in computing important quantities for hadronic, nuclear and particle physics.
 - ▶ But many challenges remain: nuclear physics, SM tests...

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Thanks!

Backup slides

The QCD spectrum

QCD Lagrangian

$$(m_u, m_d, m_s \ll \Lambda_{\text{QCD}})$$

$$SU(3)_L \otimes SU(3)_R$$

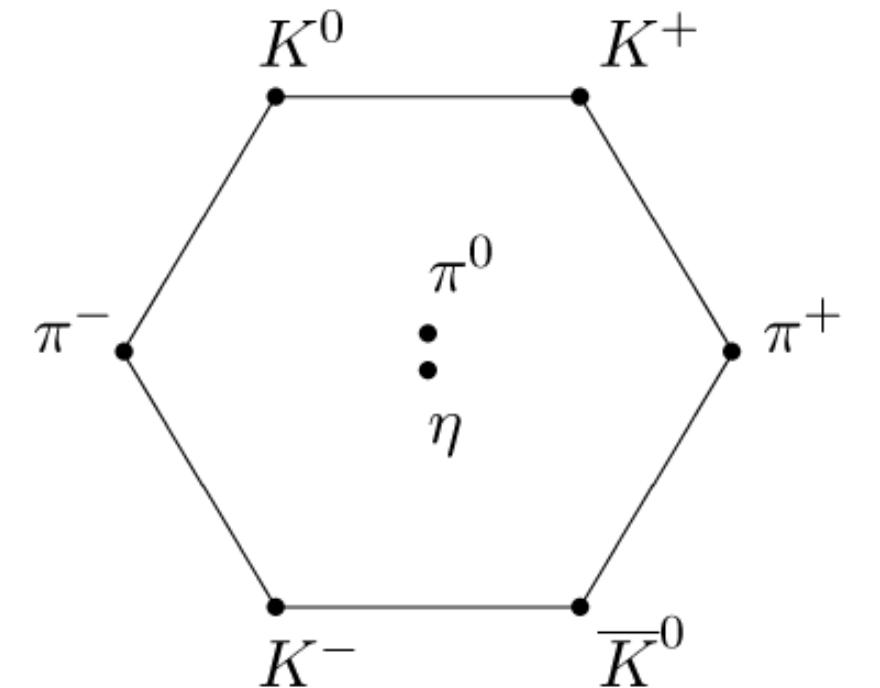
$$\langle \bar{q}q \rangle \neq 0$$

Spontaneous chiral symmetry breaking

$$SU(3)_V$$



octet of
pseudo-Goldstone bosons



The QCD spectrum

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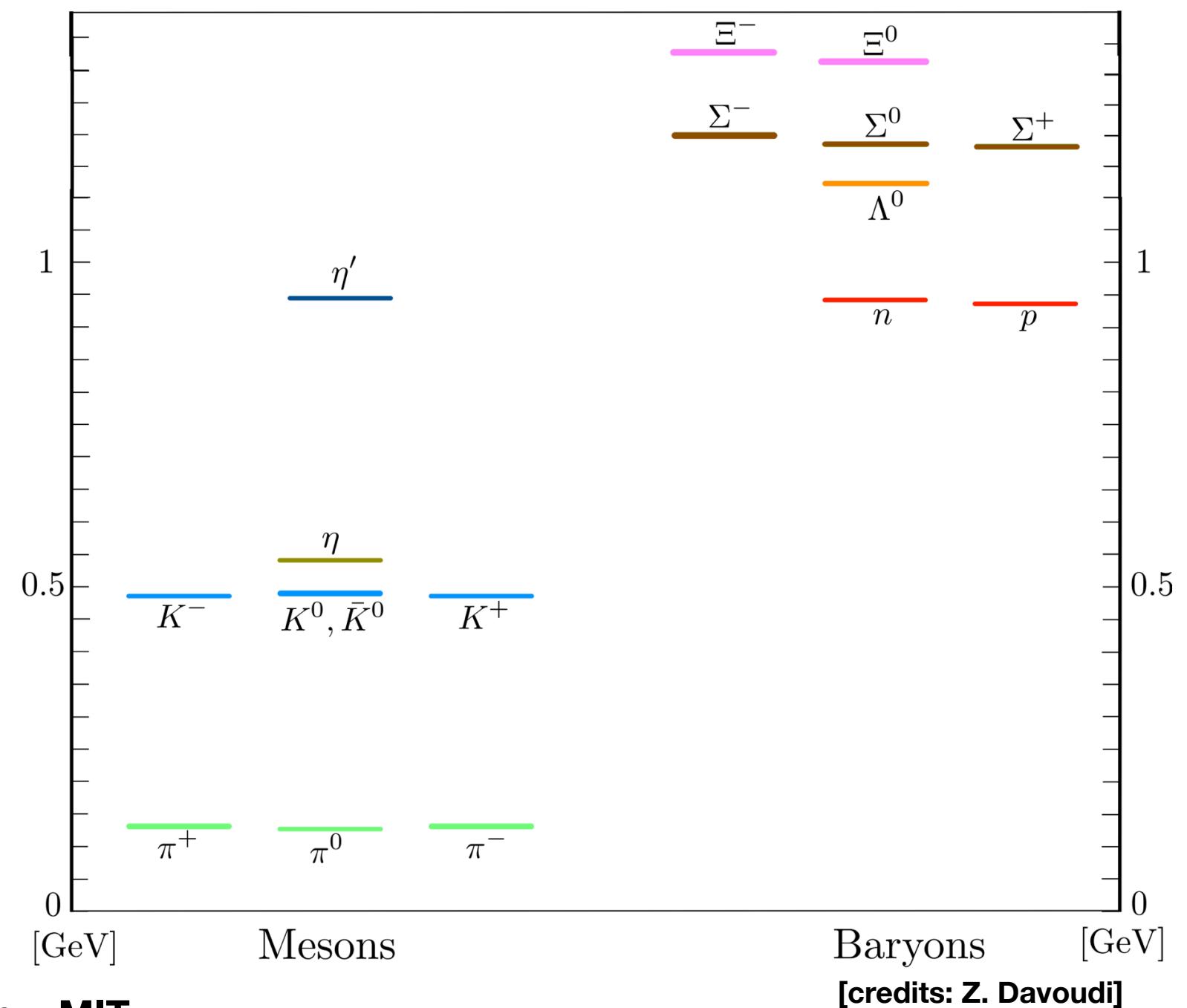
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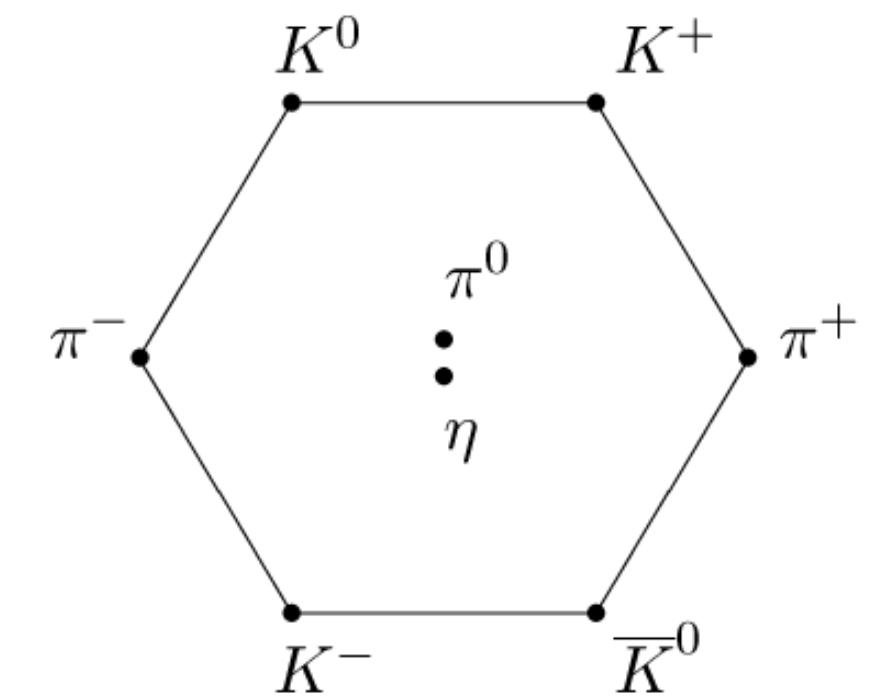
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