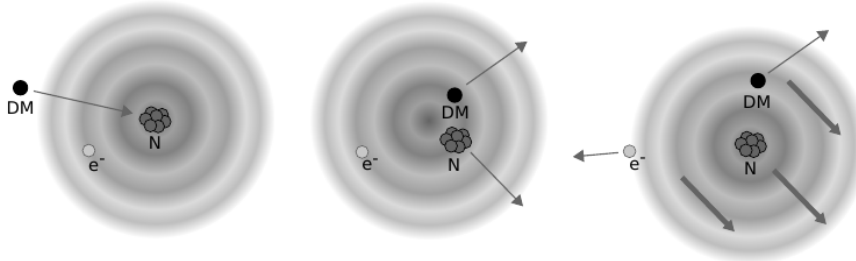


Migdal effect in helium

Rahel Gabriel | 21.02.24

Migdal effect in direct dark matter detection

- NR: atom recoils as a unit
- Reality:
 - High recoil velocity
 - Electrons only follow the atom after some time
 - QM yields ionization (and excitation) probability
- Migdal effect: ionization of an atom via nuclear recoil



Kinematics

- Energy conservation:

$$\frac{\vec{p}\vec{q}}{m_\chi} = \frac{pq}{m_\chi} \cos \theta = \frac{q^2}{2\mu_{\chi N}} + E_{EM}$$

- Minimal velocity:

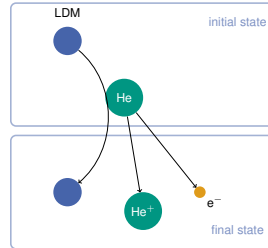
$$v_{min} = \frac{q}{2\mu_{\chi N}} + \frac{E_{EM}}{q}$$

$$= \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}} + \frac{E_{EM}}{\sqrt{2m_N E_R}}$$

- Condition for dark matter detection:

$$v_{min} < v_{max} = v_{esc} + v_E$$

$$E_i = \frac{p^2}{2m_\chi} = \frac{1}{2}m_\chi v^2$$



$$E_f = \frac{(\vec{p} - \vec{q})^2}{2m_\chi} + \frac{q^2}{2m_N} + E_{EM}$$

Migdal effect: motivation

$$v_{max} = \frac{q}{2\mu_{\chi N}} + \frac{E_{EM}}{q}$$

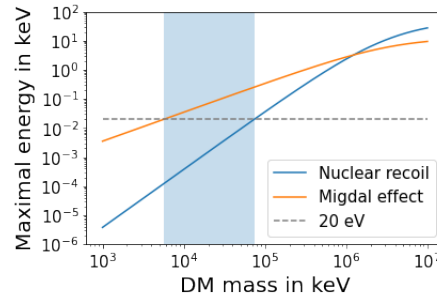
$$= \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}} + \frac{E_{EM}}{\sqrt{2m_N E_R}}$$

Nuclear recoil:

$$E_R^{max} = \frac{2\mu_{\chi N}^2 v_{max}^2}{m_N}$$

Migdal effect:

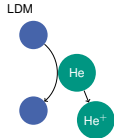
$$E_{EM}^{max} = \frac{1}{2}\mu_{\chi N} v_{max}^2$$



E_{th}	m_{NR}^{min}	m_{Mig}^{min}
20 eV	74 MeV	5.7 MeV
10 eV	52 MeV	2.9 MeV

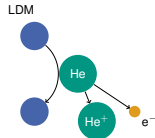
Dark matter direct detection: nuclear recoils

Nuclear recoil



$$R = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_{E_R^{\min}}^{E_R^{\max}} |F_N|^2 g(v_{\min}) dE_R$$

Migdal effect



$$\frac{dR}{dE_{EM}} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}) dE_R$$

Differential event rate per deposited EM energy

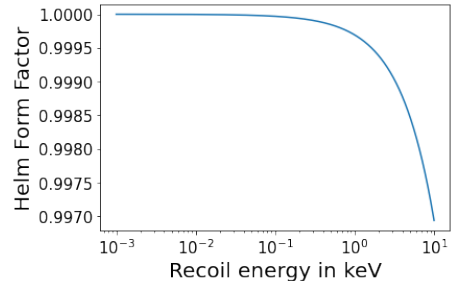
$$\frac{dR}{dE_{EM}} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}(E_R, E_{EM})) dE_R$$

Dark matter physics and target atom

- This term is a prefactor which depends on the physics of dark matter and the target atom

Helm form factor

- This term contains the properties of interactions with an atom
- Helm form factor: analytical expression from [Lewin & Smith](#)

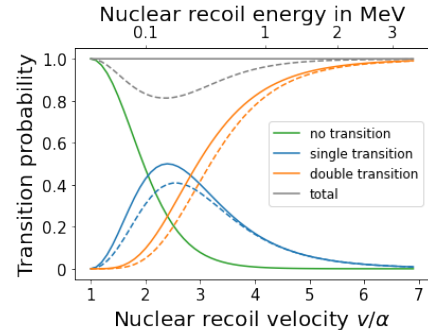


Differential event rate per deposited EM energy

$$\frac{dR}{dE_{EM}} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}(E_R, E_{EM})) dE_R$$

Migdal effect

- This term reduces the rate compared to the NR case and contains all information regarding the Migdal effect
- 3 cases:
 - Single ionization (SI)
 - Single ionization and excitation (SE)
 - Double ionization (DI)
 - integration over energy of the 2nd electron
- Data from [Cox et al.](#)
- Derivation in [lbe et al.](#)



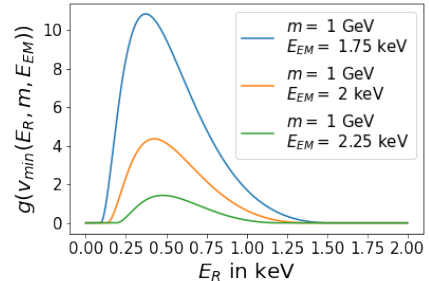
Differential event rate per deposited EM energy

$$\frac{dR}{dE_{EM}} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}(E_R, E_{EM})) dE_R$$

Halo velocity distribution

- This term contains all of the purely kinematic constraints
- Placement in the sum (E_{EM} depends on the binding energy of the Migdal electron)
- Standard halo model ([Barger et al.](#)) with:
 - $v_{\text{obs}} = 250 \text{ km s}^{-1}$
 - $v_0 = 238 \text{ km s}^{-1}$
 - $v_{\text{esc}} = 544 \text{ km s}^{-1}$

$$g(v_{\min}) = \int_{v_{\min}}^{v_{\text{esc}}} \frac{f(\vec{v}_\chi + \vec{v}_{\text{obs}})}{|\vec{v}_\chi|} d^3 \vec{v}_\chi$$



Result

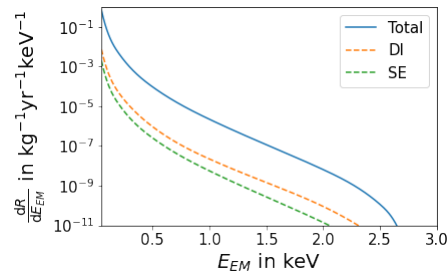
$$\frac{dR}{dE_{EM}} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}(E_R, E_{EM})) dE_R$$

Assumptions

- $\rho_\chi = 0.3 \text{ GeV cm}^{-3}$
- $\sigma_n = 1 \cdot 10^{-40} \text{ cm}^2$
- $m_\chi = 1 \text{ GeV}$

Observations:

- SI dominates
- DI and SE suppressed by more than one order of magnitude
- Different assumptions affect scaling and shape

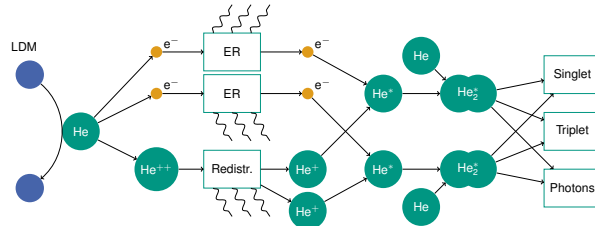
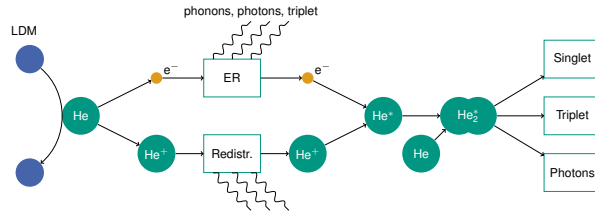


Migdal effect: signals in helium

- ER for every ionized electron
- Excimer for every recombination
- Current assumption: nucleus at rest after recoil

Processes:

- SI: excimer + ER
- SE: SI + energy from deexcitation (problem: energy is not quantified in the data)
- DI: double SI (looks most likely like one SI)



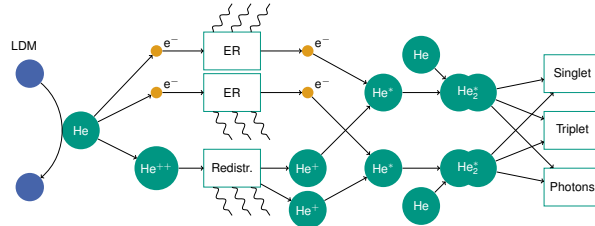
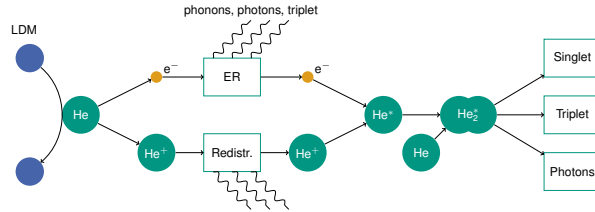
Migdal effect: signals in helium

Signal:

- Triplet: no sensitivity (due to large timescale)
- Singlet: decays \rightarrow photons
- ER: produces triplet, singlet, quasiparticles and photons

Note

One excimer will always be produced. Lowering the threshold further than the binding energy will not result in access to smaller DM masses.



Reconsideration: motivation

Threshold:

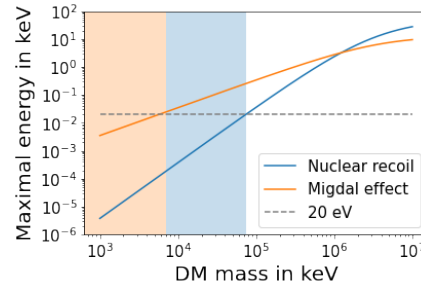
$$E_{EM} = E_e + E_b \geq E_{th}$$

$$E_e^{\min} = E_{th} - E_b \geq 0$$

This gives the minimal mass:

$$m_{Mig}^{\min} \geq 5.7 \text{ MeV}$$

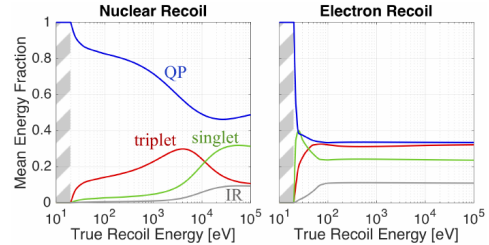
E_R^{\min}	m_{NR}^{\min}	m_{Mig}^{\min}
20 eV	74 MeV	5.7 MeV
10 eV	52 MeV	2.9 MeV



Exclusion plot

Assumptions:

- NR and ER energy partition according to [Hertel et al.](#)
- No triplet detection
- Singlet:triplet ratio is 1:1
- Photons are always detected
- Gaussian energy resolution with $\sigma = 5$ eV (not used in the following plot)



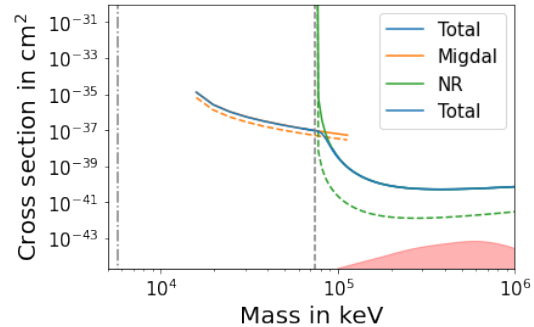
[Hertel et al.](#)

$$s_m^{\text{Mig}}(E_{EM}, E_R) = \left(\sum_{i \in S_{ER}} p_{i,\text{det}} p_i(E_e) + \frac{1}{2} p_{\text{singlet},\text{det}} \right)$$

$$s_m^{\text{NR}}(E_{EM}, E_R) = \sum_{i \in S_{NR}} p_{i,\text{det}} p_i(E_e)$$

Exclusion plot

- Exposure of 1 kg yr
- $E_{th} = 20$ eV
- Dashed lines: cross section with perfect detector
- Neutrino floor [O'Hare](#)



Conclusion

- Migdal effect enhances the accessible parameter space depending on the energy threshold
- The energy threshold can improve the detection up to the binding energy
- Improvements on the signal model?
 - Form factor
 - Ionization probabilities (wavefunction, electron energies in SE)
 - Signal channels
 - Detector resolution
- The Migdal effect can be included in the neutrino floor

Literature

- Migdal effect
 - Ibe et al.: Migdal Effect in Dark Matter Direct Detection Experiments.
 - Dolan, Kahlhoefer, McCabe: Directly detecting sub-GeV dark matter with electrons from nuclear scattering.
 - Cox et al.: Precise predictions and new insights for atomic ionisation from the Migdal effect.
- Halo model
 - Barger et al.: Electromagnetic properties of dark matter: dipole moments and charge form factor.
 - Fitzpatrick, Zurek: Dark Moments and the DAMA-CoGeNT Puzzle.
- Form factor
 - Lewin, Smith: Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil.
- Energy partition
 - Hertel et al.: A Path to the Direct Detection of sub-GeV Dark Matter Using Calorimetric Readout of a Superfluid ^4He Target.
- Neutrino floor
 - O'Hare: Fog on the horizon: a new definition of the neutrino floor for direct dark matter searches.
 - Herrera: A neutrino floor for the Migdal effect.

Exclusion plot: short mention of statistics

Statistical Assumptions:

- No events observed
- No background events observed

Poisson distribution gives the condition:

$$0.1 < p(N_{obs} = 0) = \exp(-N_{exp}) = \frac{N_{exp}^{N_{obs}}}{N_{obs}!} \exp(-N_{exp})$$
$$N_{exp} \gtrsim 2.3$$

Treatment of energies

Nuclear recoil

$$\frac{dR}{dE_R} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} |F_N|^2 g(v_{\min}(E_R, E_{EM}))$$

$$R = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_{E_{\text{th}}}^{E_R^{\max}} |F_N|^2 g(v_{\min}(E_R, E_{EM})) s_m^{\text{NR}}(E_R) dE_R$$

Migdal effect

$$\frac{dR}{dE_{EM}} = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}(E_R, E_{EM})) dE_R$$

$$R = \frac{\rho_\chi A^2 \sigma_n}{2m_\chi \mu_{\chi N}^2} \int_{E_{\text{th}}}^{E_{EM}^{\max}} \int_0^{E_R^{\max}} |F_N|^2 \sum_{n\kappa} \frac{dp_\nu(n\kappa \rightarrow E_e)}{dE_e} g(v_{\min}(E_R, E_{EM})) s_m^{\text{Mig}}(E_{EM}) dE_R dE_{EM}$$