Karlsruher Institut für Technologie

Migdal effect in helium

Neutrino floor and calculation of transition probabilities

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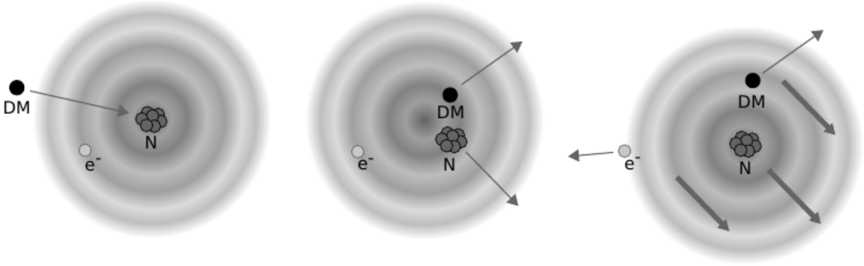
Migdal effect in direct dark matter detection

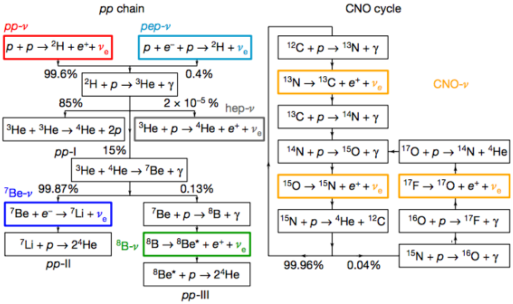
- NR: atom recoils as a unit
- Reality:
  - High recoil velocity
  - Electrons only follow the atom after some time
  - QM yields ionization (and excitation) probability
- Migdal effect: ionization of an atom via nuclear recoil

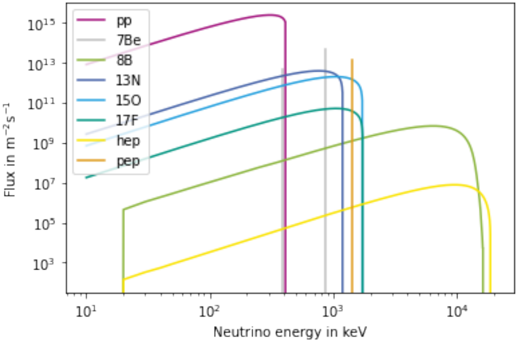


Dolan, Kahlhoefer & McCabe

Neutrino floor - solar neutrinos

- nuclear reaction chains in sun produce neutrinos

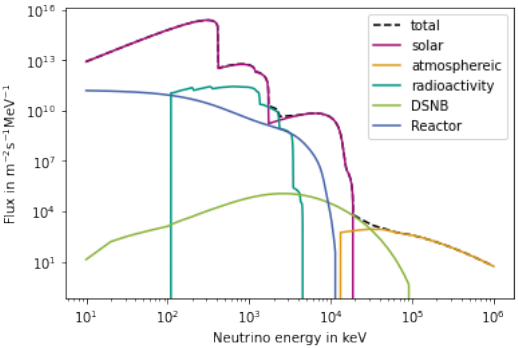




source: Borexino Collaboration

Neutrino floor - other neutrino sources

- atmospheric neutrinos: cosmic rays initiate particle showers
- geoneutrinos: radioactive decays in the earth crust
- diffuse SN background: SNs produce neutrinos
- man-made neutrinos: mostly reactors



Neutrino floor - cross section

- cross section for nuclear recoils

$$\frac{\mathrm{d}\sigma_{\mathrm{NR}}}{\mathrm{d}E_R} = \frac{G_F^2}{4\pi} Q_w^2 m_N \left(1 - \frac{m_N E_R}{2E_\nu^2}\right) F^2\left(E_R\right)$$

$$Q_w = N - \left(1 - 4\sin^2\theta_w\right) Z$$

- ionisation from factor

$$\left|Z_{\mathrm{ion}}\left(E_e\right)\right|^2 = \frac{1}{2\pi} \sum_{n,l} \int \mathrm{d}E_e \frac{\mathrm{d}}{\mathrm{d}E_e} p\left(nl \rightarrow E_e\right)$$

- cross section for the Migdal effect

$$\frac{\mathrm{d}^2\sigma_{\mathrm{Mig}}}{\mathrm{d}E_R \mathrm{d}E_e} = \frac{G_F^2}{4\pi} Q_w^2 m_N \left(1 - \frac{m_N E_R}{2E_\nu^2}\right) F^2\left(E_R\right) \frac{\mathrm{d}|Z_{\mathrm{ion}}(E_e)|^2}{\mathrm{d}E_e}$$

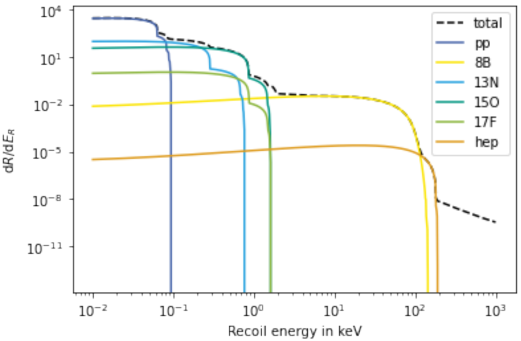
Neutrino floor - differential recoil rate

- calculation:

$$\frac{\mathrm{d}R_{\mathrm{NR}}}{\mathrm{d}E_R} = N_T \int_{E_\nu^{\min}}^{E_\nu^{\max}} \frac{\mathrm{d}\Phi}{\mathrm{d}E_\nu} \frac{\mathrm{d}\sigma_{\mathrm{NR}}}{\mathrm{d}E_R} \mathrm{d}E_\nu$$

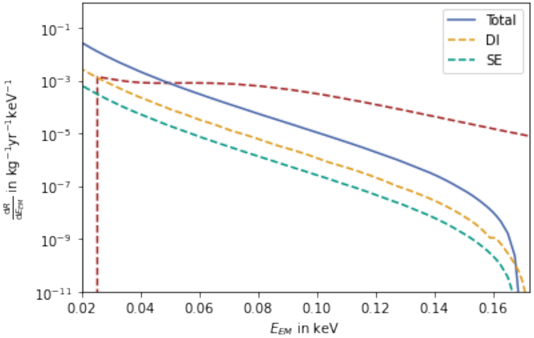
$$\frac{\mathrm{d}R_{\mathrm{Mig}}}{\mathrm{d}E_R} = N_T \int_{E_R^{\min}} \int_{E_\nu^{\min}}^{E_\nu^{\max}} \frac{\mathrm{d}\Phi}{\mathrm{d}E_\nu} \frac{\mathrm{d}^2\sigma_{\mathrm{Mig}}}{\mathrm{d}E_R \mathrm{d}E_{\mathrm{EM}}} \mathrm{d}E_\nu \mathrm{d}E_{\mathrm{EM}}$$

- ^{8}B neutrinos and hep neutrinos have similar recoil spectrum to dark matter

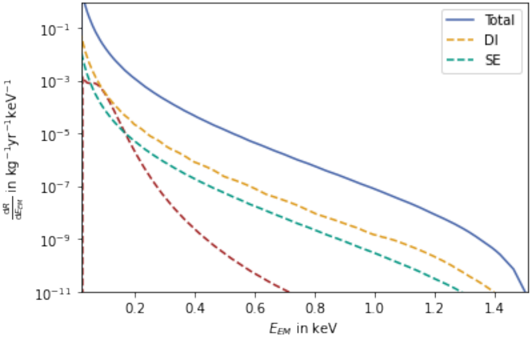


Neutrino floor - differential recoil rate for Migdal

- for DM mass of 50 MeV

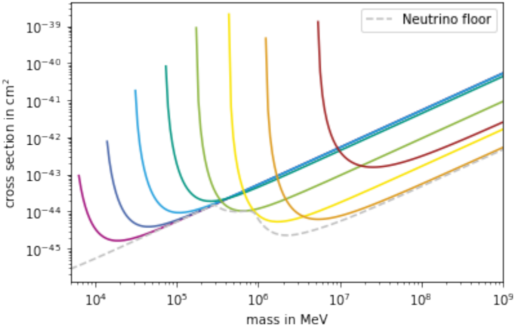


- for DM mass of 500 MeV

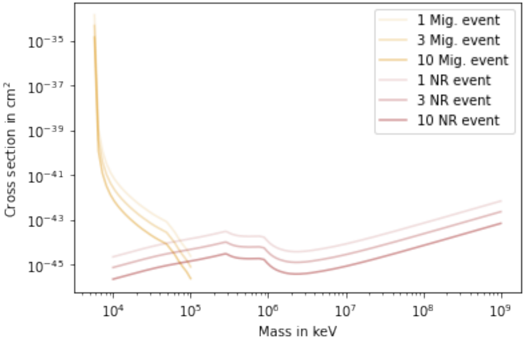


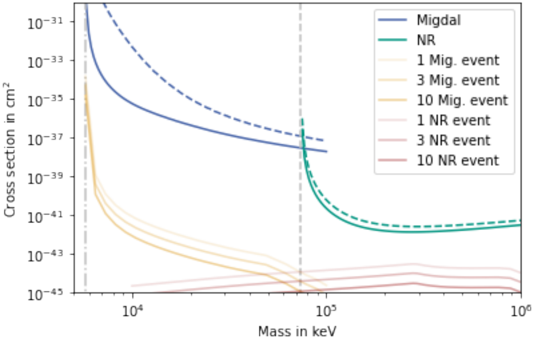
Neutrino floor - construction (for NR)

- generate a set of exclusion limits for different energy thresholds
- the exposure is set to obtain one neutrino event
- take the lowest cross section for each mass



Neutrino floor reults





Probability calculation - single electron matrix element I

- going into the nucleus rest system imideatly after the recoil by applying

$$\exp\left(im_{\mathrm{e}}\vec{v}\vec{r}\right)$$

- single electron transition matrix

$$M_{n_i,\kappa_i,m_i}^{n_f,\kappa_f,m_f} = \langle n_f,\kappa_f,m_f|\exp\left(im_{\mathrm{e}}\vec{v}\vec{r}\right)|n_i,\kappa_i,m_i\rangle$$

- Dirac spinor

$$\langle\vec{r}|n,\kappa,m\rangle = \frac{1}{r}\begin{pmatrix} P_{n,\kappa}(r)\chi_{\kappa,m}(\theta,\varphi) \\ iQ_{n,\kappa}(r)\chi_{-\kappa,m}(\theta,\varphi)\end{pmatrix}$$

  satisfies

$$\langle\vec{r}|H|n,\kappa,m\rangle = \langle\vec{r}|\vec{\alpha}\vec{p}+\beta m_{\mathrm{e}}+V(\vec{r})|n,\kappa,m\rangle = E_n\langle\vec{r}|n,\kappa,m\rangle$$

Probability calculation - single electron matrix element II

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- spherical tensor expansion

$$
\begin{aligned}
M_{n_i,\kappa_i,m_i}^{n_f,\kappa_f,m_f} =& \sqrt{4\pi} \sum_{L,M} i^L \sqrt{2L+1}\, Y_L^M(\hat{v})\, d_M^L(j_f, m_f; j_i, m_i) \\
&\cdot \int_0^\infty \mathrm{d}r\, j_L(m_e v r) \left[P_{n_i,\kappa_i}(r) P_{n_f,\kappa_f}(r) + Q_{n_i,\kappa_i}(r) Q_{n_f,\kappa_f}(r) \right]
\end{aligned}
$$

- note: energy and velocity dependence only in the radial integral
- addition of angular momentum in the angular coefficients:

$$
\begin{aligned}
d_M^L(j_f, m_f; j_i, m_i) =& (-1)^{2j_f - m_f + 1/2} \sqrt{2j_i+1}\sqrt{2j_f+1}\,\Pi^e(\kappa_i, \kappa_f; L) \\
&\cdot \begin{pmatrix} j_f & L & j_i \\ -m_f & M & m_i \end{pmatrix} \begin{pmatrix} j_f & L & j_i \\ 1/2 & 0 & -1/2 \end{pmatrix}
\end{aligned}
$$

Probability caluclaion - atomic state functions and probability

- atomic state functions in Slater-determinant basis

$$|\Phi_i\rangle = |\phi_{i,1}\phi_{i,2}\rangle$$
$$|\Phi_f\rangle = |\phi_{f,1}\phi_{f,2}\rangle$$

- single transition matrix elements

$$\left(M^{\gamma'\gamma}\right)_{\beta\alpha} = \langle\phi_{f,(\gamma')_\beta}|\,\exp\left(im_e\vec{v}\vec{r}\right)|\phi_{i,a(\gamma)_\alpha}\rangle$$

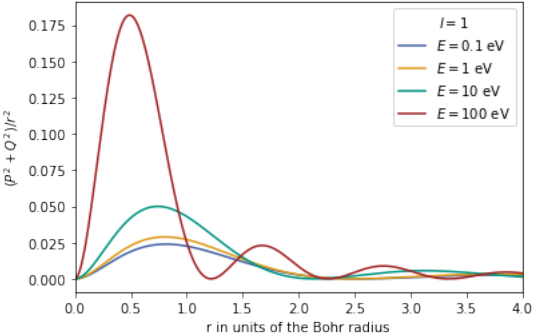
- transition probability

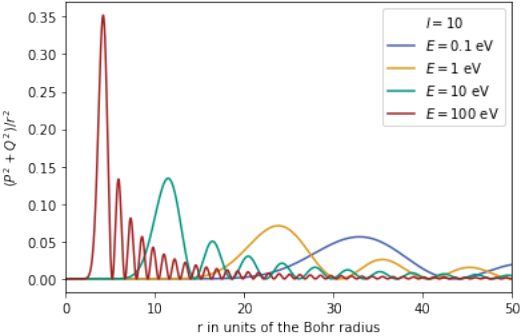
$$p_v\left(|\Phi_i\rangle \rightarrow |\Phi_f\rangle\right) = |\det M|^2$$

- disclaimer: this is a simplified version of the calculation and only applies to ground-to-ground state scattering

Probability caluclaion - continuous wavefunctions

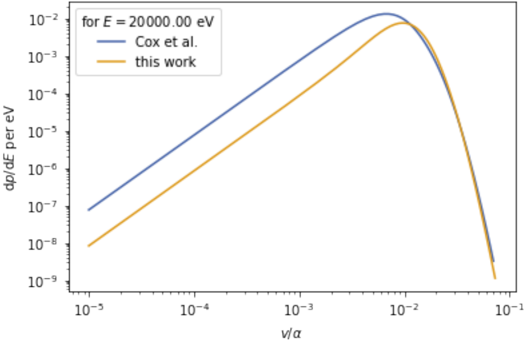
- ground states calculated with GRASP
- continuum states calculated with RATIP

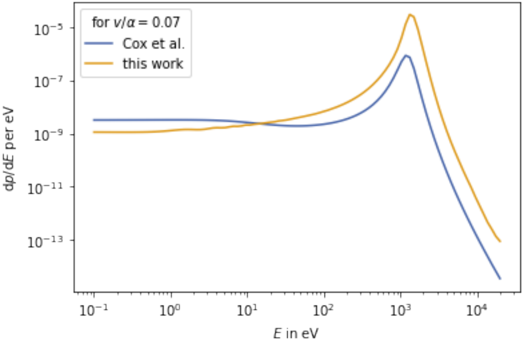




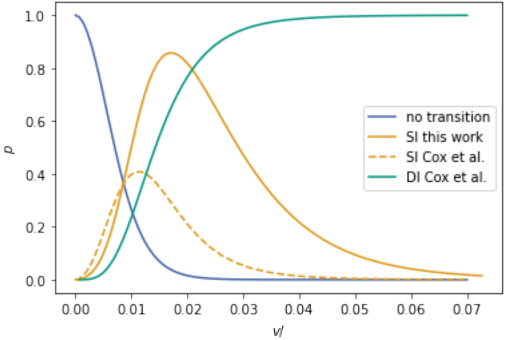
Probability caluclaion - results

- ground-to ground state agree
- SI :

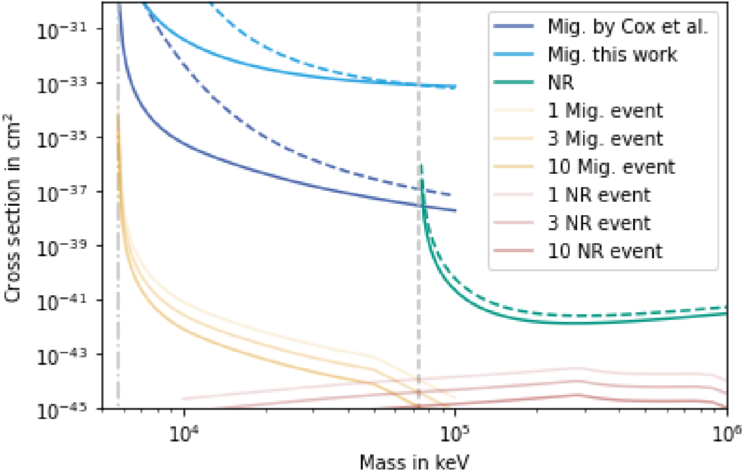




Probability caluclaion - results



Conclusion



Mig. by Cox et al.
Mig. this work
NR
1 Mig. event
3 Mig. event
10 Mig. event
1 NR event
3 NR event
10 NR event
Cross section in cm^2
Mass in keV
10^{-31}
10^{-33}
10^{-35}
10^{-37}
10^{-39}
10^{-41}
10^{-43}
10^{-45}
10^4
10^5
10^6

Literature

- Migdal effect
  - Ibe et al.: Migdal Effect in Dark Matter Direct Detection Experiments.
  - Dolan, Kahlhoefer, McCabe: Directly detecting sub-GeV dark matter with electrons from nuclear scattering.
  - Cox et al.: Precise predictions and new insights for atomic ionisation from the Migdal effect.
- Halo model
  - Barger et al.: Electromagnetic properties of dark matter: dipole moments and charge form factor.
  - Fitzpatrick, Zurek: Dark Moments and the DAMA-CoGeNT Puzzle.
- Form factor
  - Lewin, Smith: Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil.
- Neutrino floor
  - O'Hare: Fog on the horizon: a new definition of the neutrino floor for direct dark matter searches.
  - Herrera: A neutrino floor for the Migdal effect.
  - Billard et al.: Implication of neutrino backgrounds on the reach of next generation dark matter direct detection experiments