

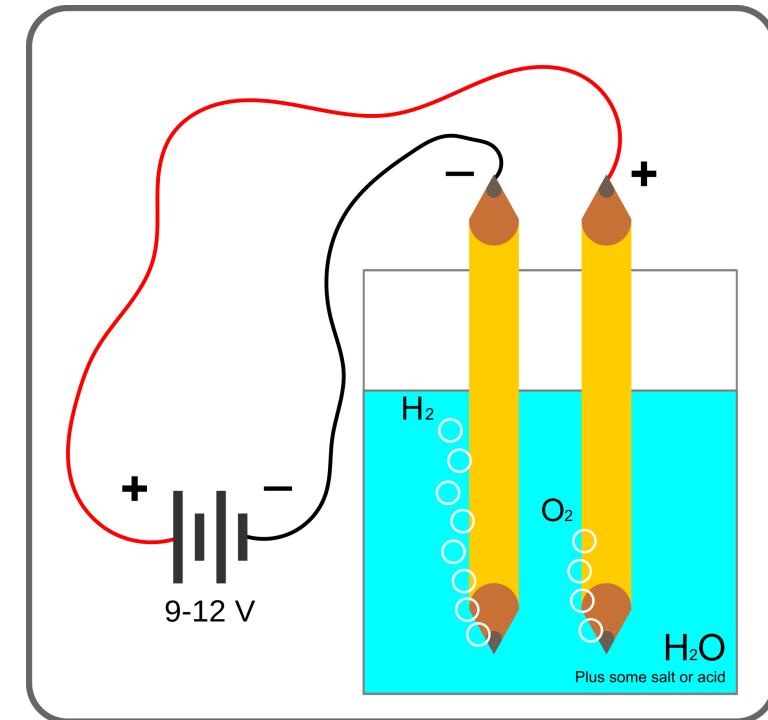
# Particle Flow and Calorimeter Clustering



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July 18, 2023

# Identifying particles – the early years

- Back in the 1800s, most scientists thought that atoms were the smallest physical unit
  - Some thought that larger atoms were built out of hydrogen though
- Using contemporary techniques in electrolysis, one could measure mass of the substance deposited on one electrode when a current of 1 ampere is passed for 1 second
  - This is called the “electrochemical equivalent”
  - Faraday’s law of electrolysis tells us that this is the mass/charge ( $m/q$ ) ratio
- This technique was used to measure the  $m/q$  ratio of the hydrogen ion
  - We now know that that’s actually the proton

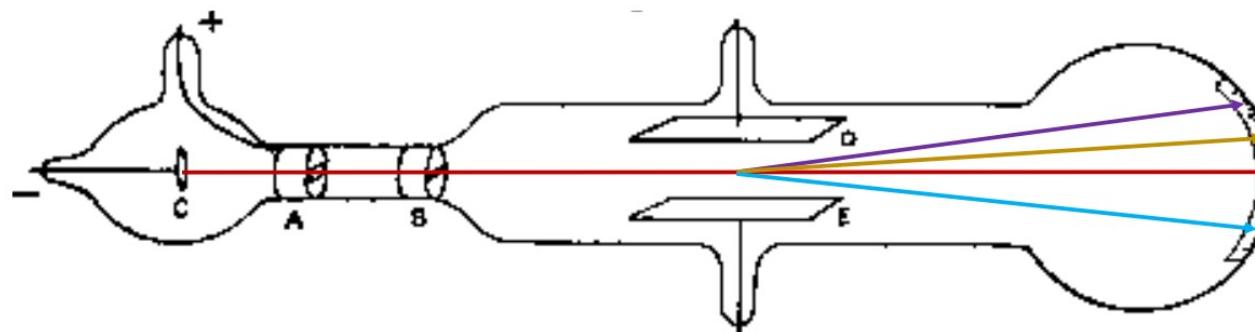


Simple electrolysis setup (in case it's been a while since you've taken chemistry)

# Identifying particles – the early years

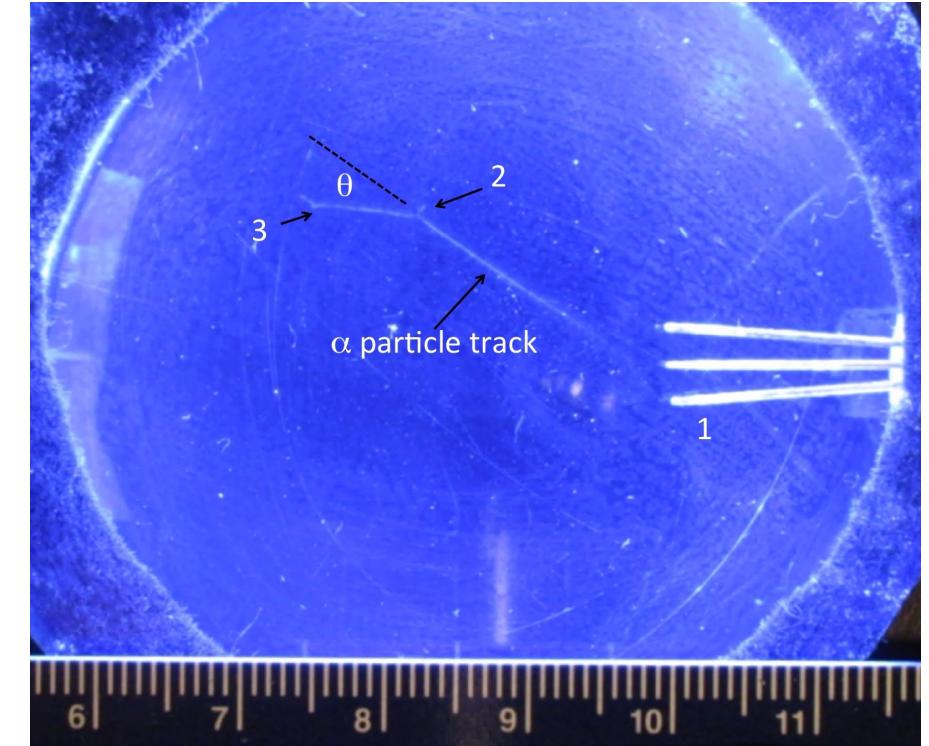
- The first massive particle to be isolated was the electron in 1897 by JJ Thomson
  - “Cathode Rays”: [\*Philosophical Magazine, 44, 293-316 \(1897\)\*](#)
- Measured mass/charge ratio of emitted cathode rays did not depend on material that the rays come from
  - Supports hypothesis of a single particle type that is present in all materials: **the electron**
  - Thomson noted that electron’s  $m/q$  is about 1/1700 of hydrogen ion  $m/q$
  - Charge (and thus mass) later measured in Millikan Oil Drop Experiment
- A much more interesting history and experiment than you might think!
  - Please see [these slides](#) for more detail

Red: no voltage diff between D and E  
Purple: D+ and E-, larger del. V  
Gold: D+ and E-, smaller del. V  
Blue: D- and E+



# Identifying particles – the early years

- Rutherford discovered alpha and beta rays in 1899
  - Becquerel showed that beta rays have the same m/q as the electron (they *are* electrons after all)
- In 1911, Wilson created the cloud chamber, which allowed for particle visualization!
- Also enables early “Particle ID”
  - You can analyze particle trajectories in magnetic fields and ionizing path density



Cloud chambers

Alpha particle emitted from 1 hits some gas molecule at 2, and is scattered before being absorbed a little after a second scattering at 3

# Particle ID and new particles

- Anderson's discovery of positron:
  1. He could tell that particle was moving up, since it lost momentum when traversing lead plate
  2. He could tell charge sign (positive) from bending direction in magnetic field
  3. He could tell that charge magnitude was less than 2x that of electron (based on visual comparison of trajectory to electron trajectories and energy loss in lead plate)
  4. He could tell that the particle had mass less than 20x that of electron based on curvature (energy loss in lead would have been too low if the particle mass was higher)
- Based on above mass argument, this wasn't a proton
- Anderson postulated that this was Dirac's anti-electron

C.D. Anderson, "The positive electron",  
Phys. Rev., **43**, 1933

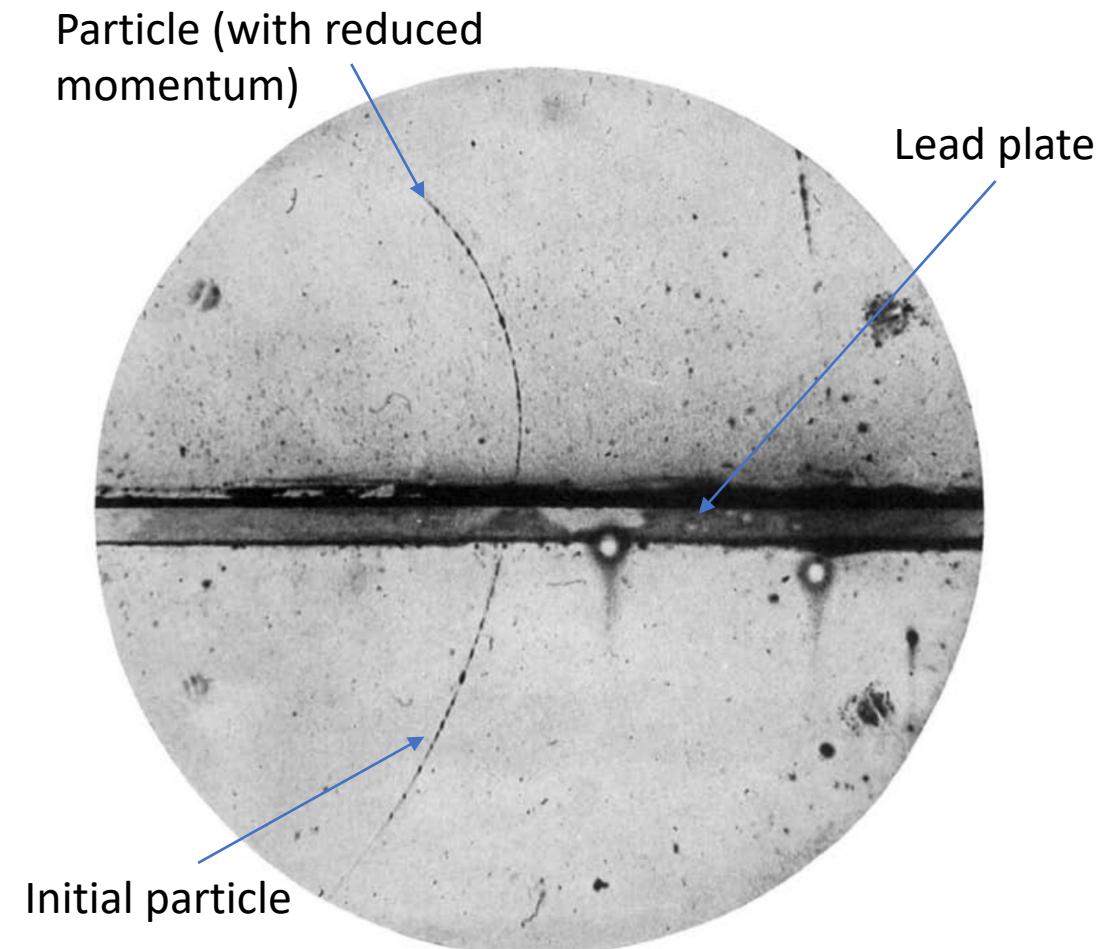


FIG. 1. A 63 million volt positron ( $H\rho = 2.1 \times 10^5$  gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ( $H\rho = 7.5 \times 10^4$  gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

Skipping a few decades...

# Learning new things *without* particle ID

- “Early” collider experiments allowed for lab-controlled observation of processes like  $e^+e^- \rightarrow \mu^+\mu^-$  or  $e^+e^- \rightarrow q\bar{q}$ 
  - Muons could be observed directly and tagged with similar considerations to Anderson – muons were highly penetrating charged particles (tagging based on particle passing through some dense absorber)
  - Conversely, quarks give rise to showers of particles
- Initial jets studies performed using event shape variables like sphericity or thrust
  - In QCD, as final state quark energy increases, you expect “jets” to become narrower
  - I.e. “sphericity” decreases with increasing  $e^+e^-$  center of mass energy

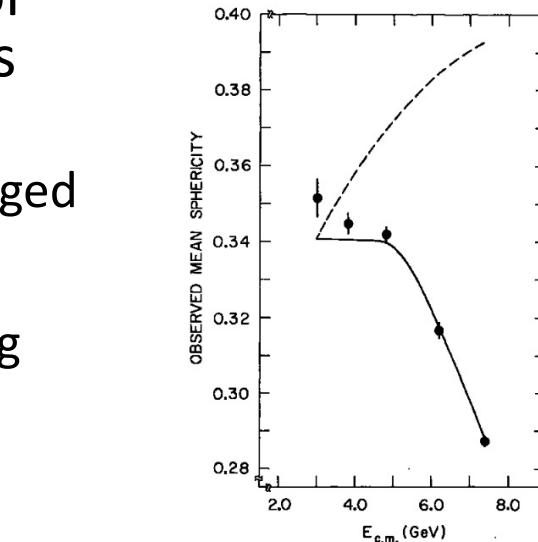


FIG. 1. Observed mean sphericity versus center-of-mass energy  $E_{c.m.}$  for data, jet model with  $\langle p_T \rangle = 315 \text{ MeV}/c$  (solid curve), and phase-space model (dashed curve).

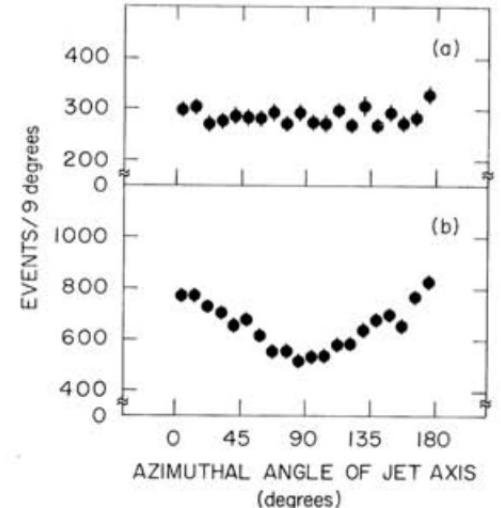


FIG. 3. Observed distributions of jet-axis azimuthal angles from the plane of the storage ring for jet axes with  $|\cos\theta| \leq 0.6$  for (a)  $E_{c.m.} = 6.2 \text{ GeV}$  and (b)  $E_{c.m.} = 7.4 \text{ GeV}$ .

Using jets, one could deduce that quarks had spin  $\frac{1}{2}$  (same angular distribution as dimuons)

# Learning new things without particle ID

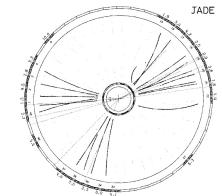


Figure 10.4. A three-jet event measured by the JADE Collaboration, viewed along the beam axis.  
[P. Soding and G. Wolf, Ann. Rev. Nucl. Part. Sci., 31, 231 (1981).]

- But what goes into a jet?
- In these early years (e.g. late '70s), a jet could basically be defined by charged particles alone
  - $e^+e^-$  initial state leads to very clean events – no underlying event or pileup, so any observed particle is part of the final state of interest
  - Isolated final state electrons, muons, and photons could be relatively easily identified in “sparse” events using tracker, ECAL, and absorber setup
  - Even “busy” events (with jets) were not busy by modern standards, and tracker technology was good enough to determine momenta with adequate accuracy
- Basic jets were all that was needed to discover the gluon!
  - Did not use “jet algorithms” as we think of them now, but defined multiple axes to minimize or maximize certain dot products with charged particle momentum
  - No calorimeters involved!

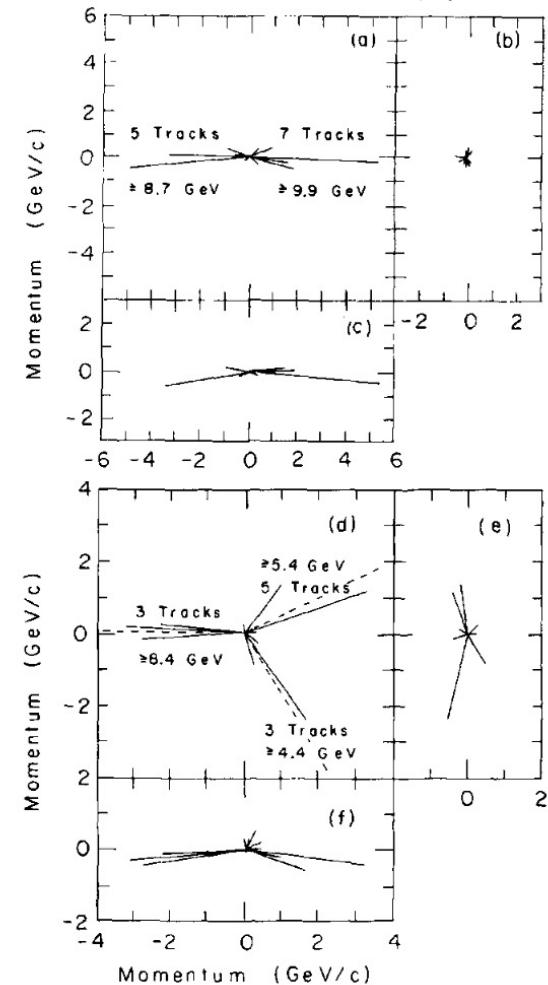


Fig. 6. Momentum space representation of a two-jet event (a)–(c) and a three-jet event (d)–(f) in each of three projections. (a), (d)  $\hat{n}_2-\hat{n}_3$  plane; (b), (e)  $\hat{n}_1-\hat{n}_2$  plane; (c), (f)  $\hat{n}_1-\hat{n}_3$  plane.

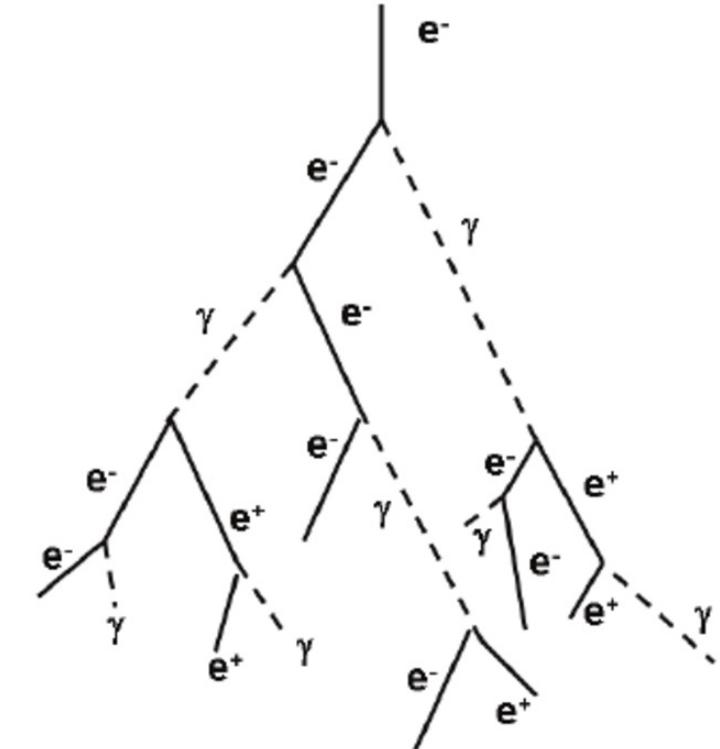
# Speaking of calorimeters

# What happens when a particle hits a calorimeter?

- For high energy electrons/positrons and photons:
  - Bremsstrahlung and  $e^+e^-$  pair production
- Shower is characterized by “Radiation length” ( $X_0$ )
  - Distance where electron loses all but  $1/e$  of its energy and  $7/9$  of mean free path of photon for pair production

$$X_0 = 716.4 \text{ g cm}^{-2} \frac{A}{Z(Z+1) \ln \frac{287}{\sqrt{Z}}}$$

- Electromagnetic showers have characteristic transverse dimension: “Moliere radius”
  - 90% of shower contained in cylinder with 1 moliere radius, and 95% within 2 moliere radii
  - $R_M = 0.0265 X_0 (Z + 1.2)$



Material	$X_0$ [cm]	Moliere rad [cm]
Fe	1.76	1.71
Pb	0.56	1.60
Si	9.37	4.94
$\text{PbWO}_4$	0.89	1.96

# What happens when a particle hits a calorimeter?

- For high energy hadrons:

- Hadronic shower
- (A bit more complicated than electromagnetic shower, but *contains* an electromagnetic component due to neutral pion creation and decay)

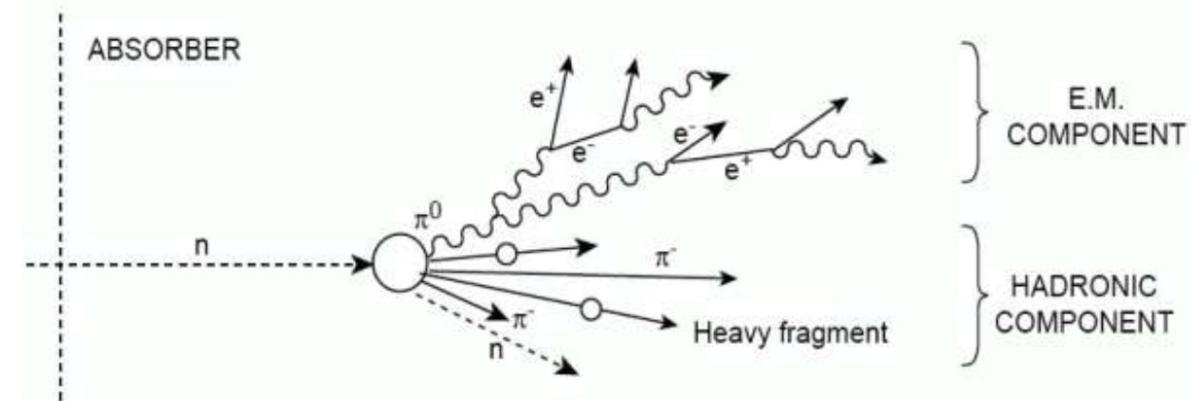
- Characterized by “nuclear interaction length” ( $\lambda_A$ )

$$\lambda_A = \frac{A}{[N\rho\sigma_{nA(\text{inelastic})}]} \quad \sigma_{nA(\text{inelastic})} \approx 41.2A^{0.711} \text{ [mb]}$$

Approximation:  $\lambda_A \approx 35 \frac{A^{1/3}}{\rho} \text{ cm}$

Depth:  $L_{95\%} \approx (6.2 + 0.8 \ln(E/100 \text{ GeV})) \lambda_A$

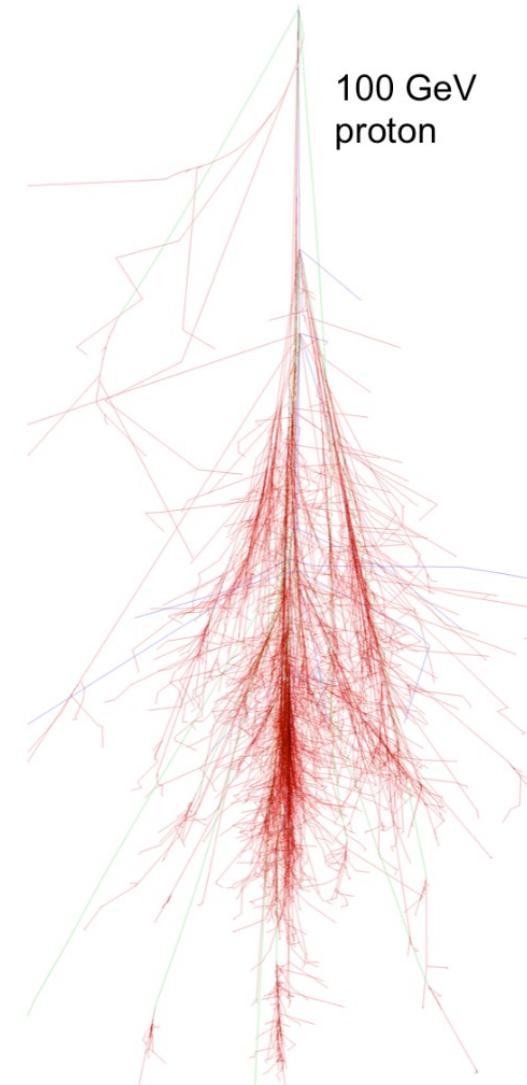
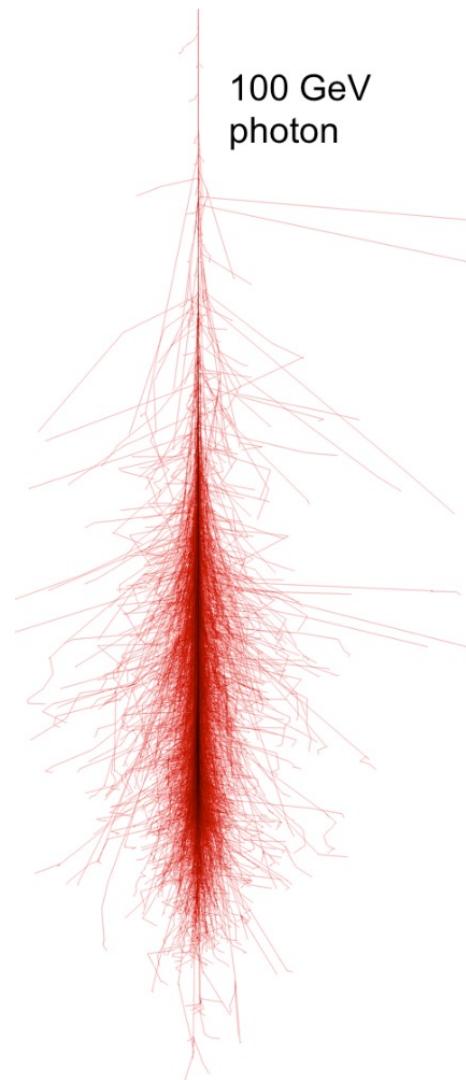
Lateral containment:  $d_{95\%} \approx \lambda_A$



Material	$\lambda$ [cm]
Fe	16.77
Pb	17.59
Si	46.52
PbWO <sub>4</sub>	20.27

- Physics of cascading shower generation and propagation in matter: principles of high-energy, ultra high-energy and compensating calorimetry: [Rept. Prog. Phys. 63 \(2000\) 505](#)
- Hadronic Calorimeter Shower Size: Challenges and Opportunities for Jet Substructure in the Superboosted Regime [\[1506.02656\]](#)

# What happens when a particle hits a calorimeter?

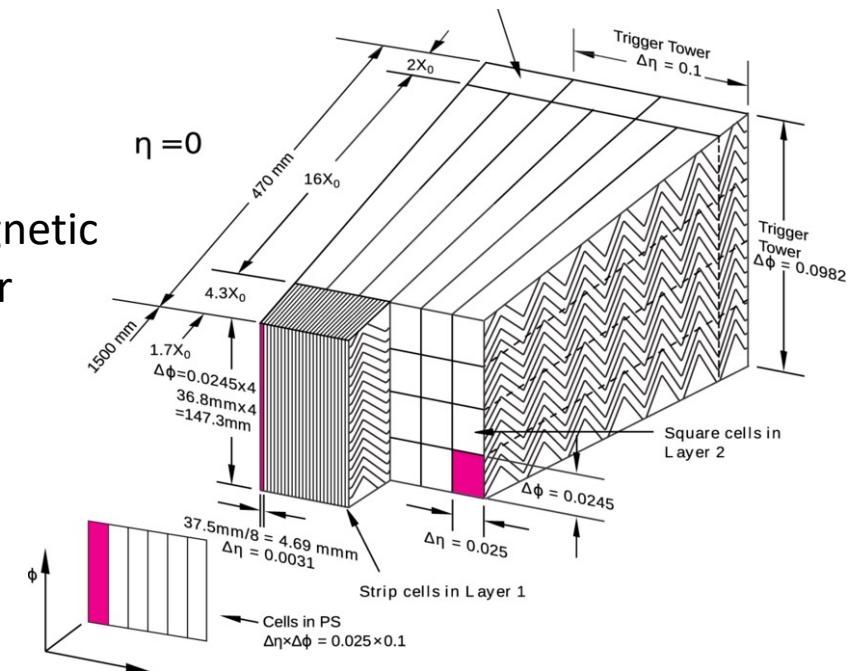


# Designing a calorimeter

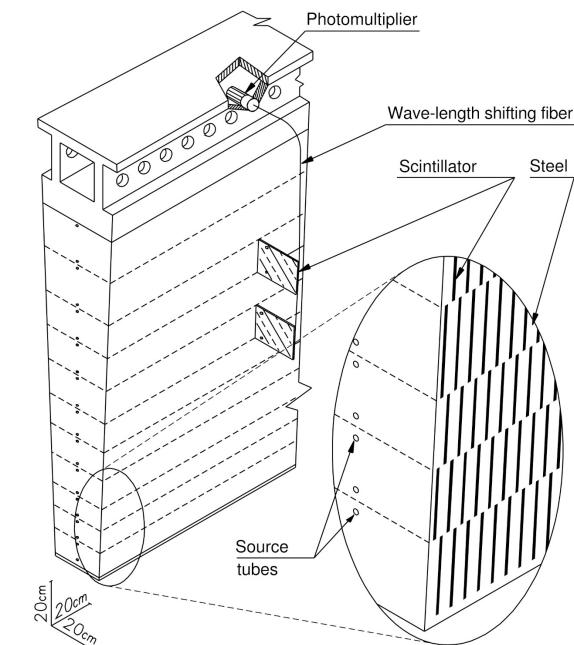
- When designing a calorimeter, you should keep in mind typical particle energy, multiplicity, and shower sizes
- Typically, calorimeter will be comprised of “cells” (often rectangles, but some use hexagons or triangles – just something that tessellates calorimeter face)
  - Many calorimeters have multiple layers (can be stacked into towers)
  - Cells are instrumented to read out deposited energy
  - Particle showers cover multiple cells. Shower width (and number of cells covered) depends on material – see previous two slides
- Two main types in use: sampling calorimeters or crystal calorimeters
  - At LHC:
    - CMS: crystal ECAL and sampling HCAL
    - ATLAS: sampling ECAL *and* HCAL

ATLAS

Electromagnetic  
Calorimeter



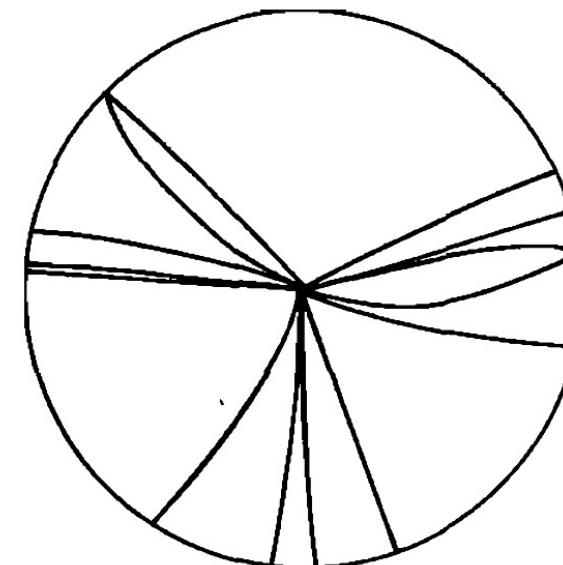
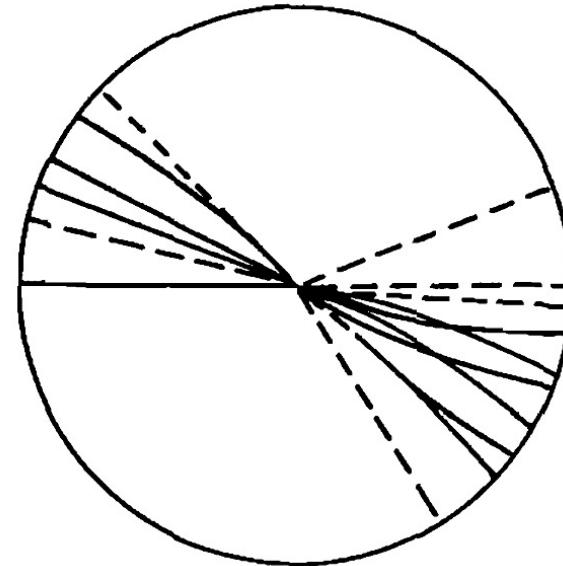
ATLAS  
Hadronic  
Calorimeter



# Calorimeter clustering (jet algorithms)

# What is a jet? (revisited)

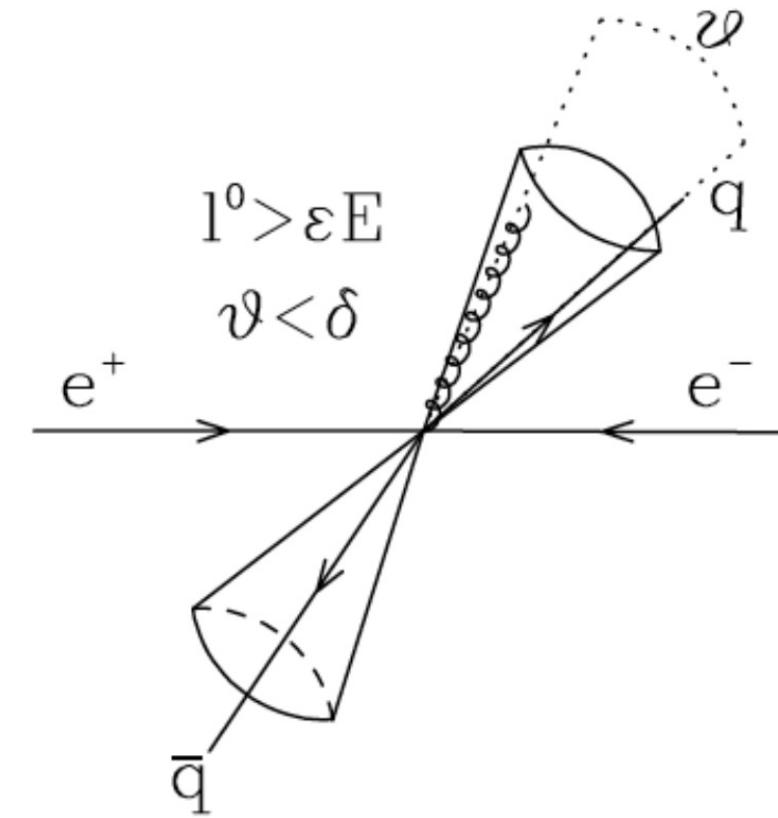
- In the 70's, at  $e^+e^-$  colliders, jets were defined somewhat "imprecisely"
  - Normally used only charged particles
  - Experimentalists would examine event planarity or check how "pencil like" event was
    - Very spherical events didn't really have jets (typically lower energy), "Pencil-like" events were two jets, planar events that weren't very spherical often had 3 jets
- No real notion of clustering here
- Also, early event shape variables were not easy to actually calculate



# What is a jet? (revisited)

[G. Sterman and S. Weinberg, “Jets from Quantum Chromodynamics”,](#)  
[Phys. Rev. Lett. 39, 1436 \(1977\)](#)

- In 1977, Sterman and Weinberg introduced a more precise jet definition by introducing a “cone”, allowing them to integrate over QCD singularities (soft and collinear radiation)
- As such, early jet algorithms were primarily cone-based
- In the 80’s experiments started deploying “sequential recombination algorithms”, which give an exact prescription for clustering particles or energy together



# The $k_t$ jet algorithm

For a much more detailed reference on jet algorithm and jets in general, please see “Towards Jetography” [[0906.1833](#)]

- Example prescription (this is actually the algorithm for  $e^+e^-$  events)\*

1. For each pair of particles  $i, j$  work out the distance

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{Q^2} \quad (3)$$

where  $Q$  is the total energy in the event,<sup>7</sup>  $E_i$  is the energy of particle  $i$  and  $\theta_{ij}$  the angle between particles  $i$  and  $j$ .

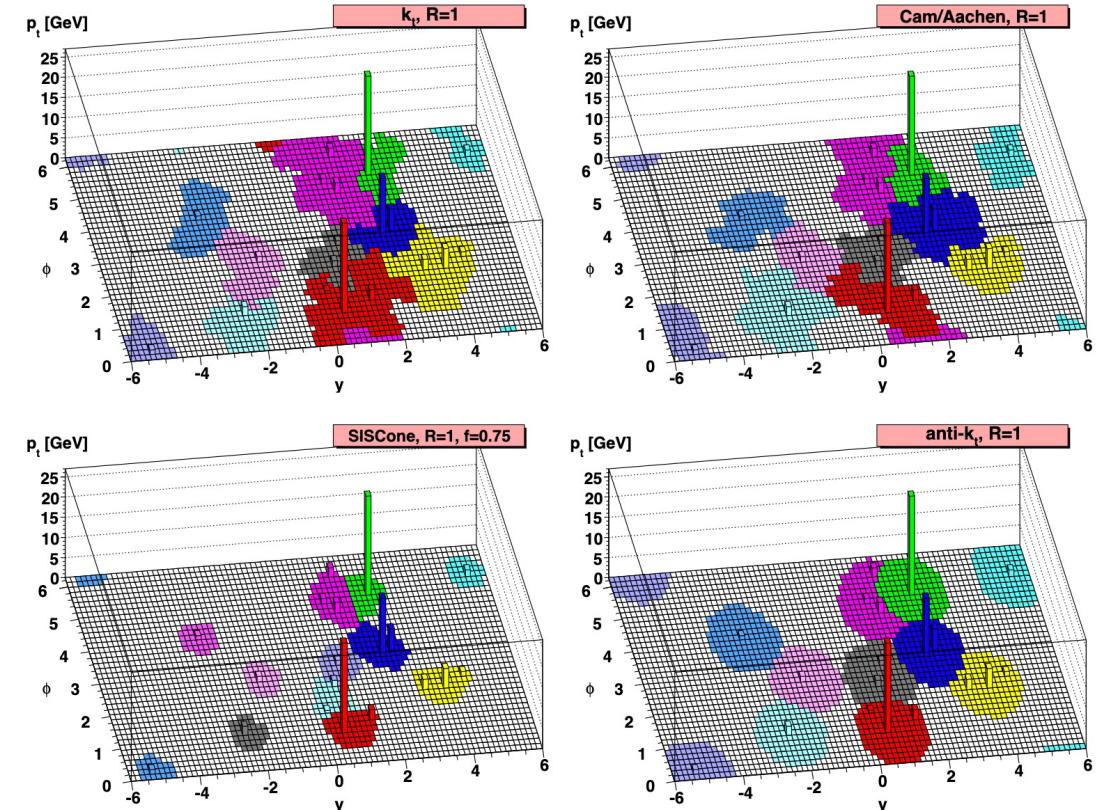
2. Find the minimum  $y_{\min}$  of all the  $y_{ij}$ .
3. If  $y_{\min}$  is below some *jet resolution threshold*  $y_{\text{cut}}$ , then recombine  $i$  and  $j$  into a single new particle (or “pseudojet”) and repeat from step 1.
4. Otherwise, declare all remaining particles to be jets and terminate the iteration.

\*Other algorithms often modify the distance metric introduced in (3)

# Jets in application

- Different jet algorithms result in different jets
- In early LHC days (and I think at the Tevatron), jet algorithms were run on calorimeter cells only, not “particles”\*
- There is often an association between a jet and an underlying parton (quark or gluon) that initiated the jet
  - This is a somewhat tenuous connection, and it's not always true
  - For example, if a boosted W decays hadronically, you can end up with a single jet corresponding to the whole W (two quarks... kind of)
- Jets, especially calorimeter-only jets are “contaminated” by particles from pileup interactions and the underlying event of hard-scatter pp collisions
  - Calorimeter-only applications often have some pileup-subtraction scheme that uses tracking

\*Reconstructing “particles” can be very difficult in busy events, and actually making “particles” is one of the key points in these slides



# Particle flow

# Reconstructing particles

- Jet algorithms gave us a systematic process for “clustering” calorimeter energy (i.e. associating calorimeter cells together)
  - These “clusters” more or less corresponded to parton showers, which take place *before* the calorimeter and contain multiple free particles
- To reconstruct individual particles, we need to associate calorimeter cells together that correspond to the electromagnetic and hadronic showers *of* those particles *in* the calorimeter
  - If we can do that, then we can combine that information with information from the tracker to create “particles”

# Particle flow

“Particle-flow reconstruction and global event description with the CMS detector”, [[1706.04965](#)]

- According to CMS, using particle flow to reconstruct all particles in an event improves reconstruction of jets and taus, MET resolution, and electron and muon identification

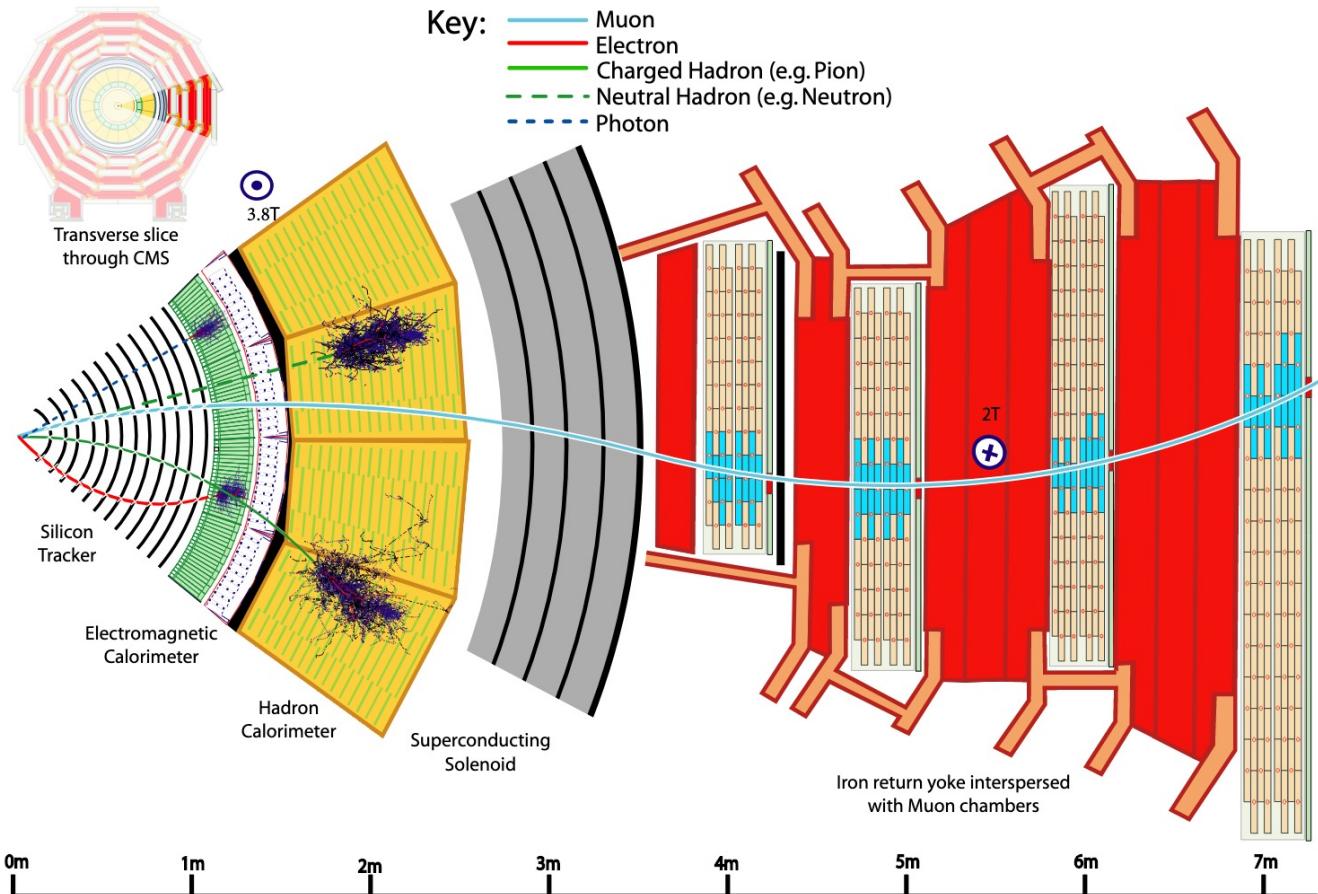
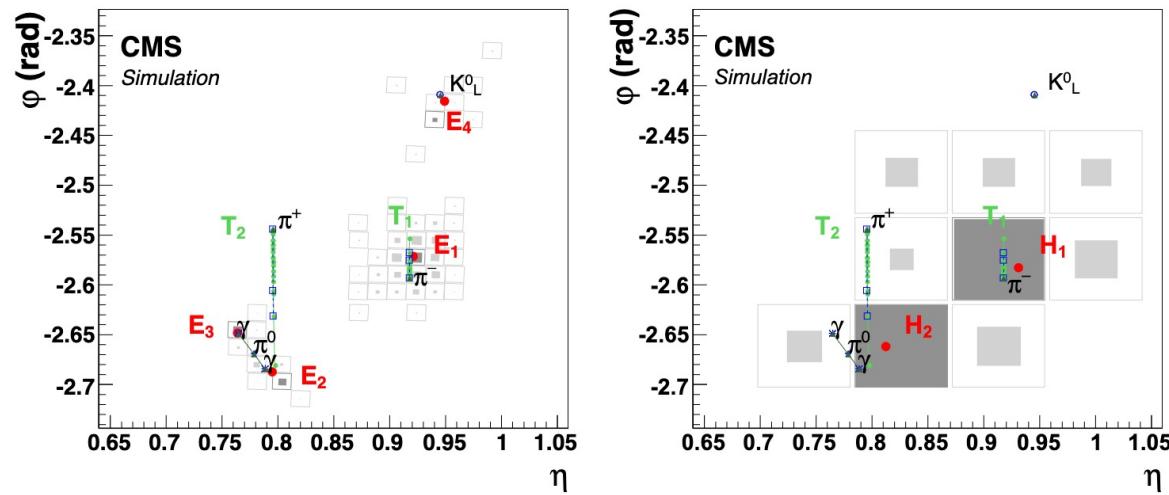
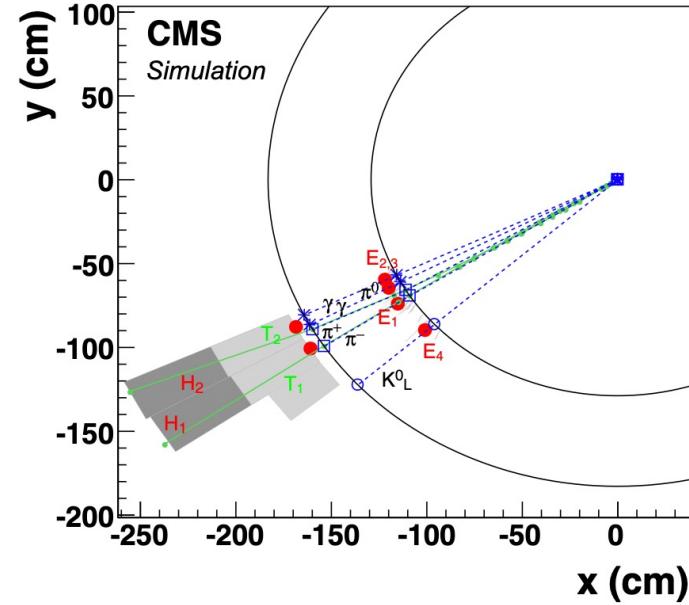


Figure 1: A sketch of the specific particle interactions in a transverse slice of the CMS detector, from the beam interaction region to the muon detector. The muon and the charged pion are positively charged, and the electron is negatively charged.

# Particle flow

“Particle-flow reconstruction and global event description with the CMS detector”, [[1706.04965](#)]

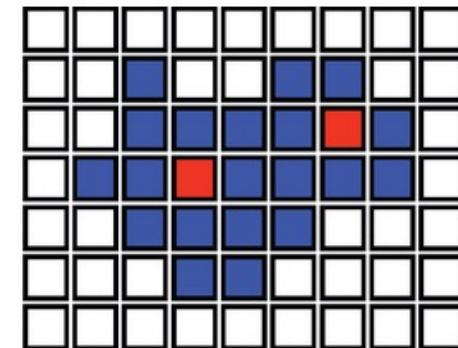
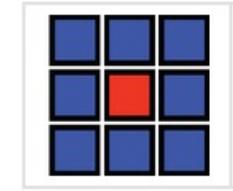
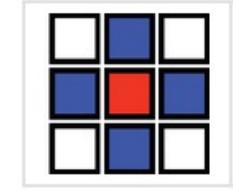


# Making calorimeter clusters

- We want to make calorimeter clusters for a few reasons:
  - Reconstruct neutral particles (photons from ECAL clusters, neutral hadrons from HCAL clusters)
  - Separate neutral hadrons from charged hadron energy deposits
  - Recombine bremsstrahlung photons into electrons
  - Above a certain energy, calorimeter measurement is more precise than tracker measurement (for charged particles)

# CMS clustering algorithm

1. Identify seed cells (with energy above a certain threshold and not next to a higher energy cell)
  - Idea: showers typically deposit most energy in 1 or 2 cells or towers; these cells/towers are the seeds
2. “Topological clusters” are grown outward from seeds through neighboring cells (typically either 4 full neighboring cells or 8 cells that share at least one corner with seed)
  - Topological clusters can contain multiple seeds

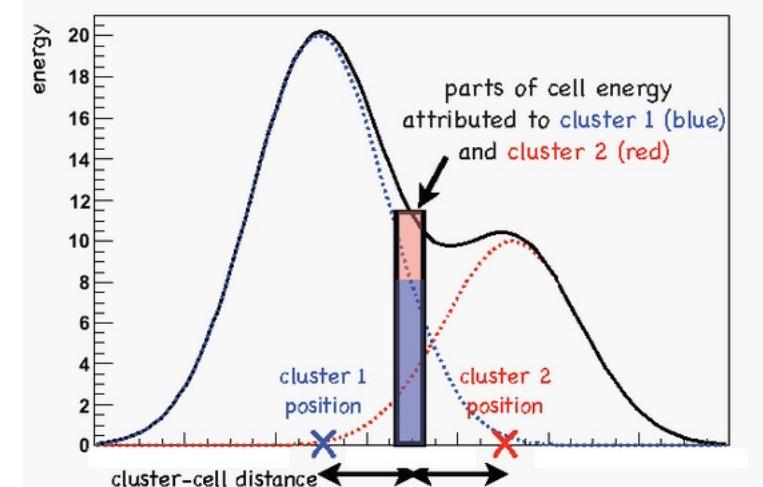


# CMS clustering algorithm

3. If a topological cluster contains  $N > 1$  seeds then it is split into  $N$  clusters
  - Showers are taken to have 2D Gaussian shape, and topological clusters are split based on a Gaussian-mixture model
  - Cells can be shared by multiple clusters
  - The fraction of a cell's,  $j$ 's, energy corresponding to a particular cluster,  $i$ , is

$$f_{ji} = \frac{A_i e^{-(\vec{c}_j - \vec{\mu}_i)^2 / (2\sigma^2)}}{\sum_{k=1}^N A_k e^{-(\vec{c}_j - \vec{\mu}_k)^2 / (2\sigma^2)}}$$

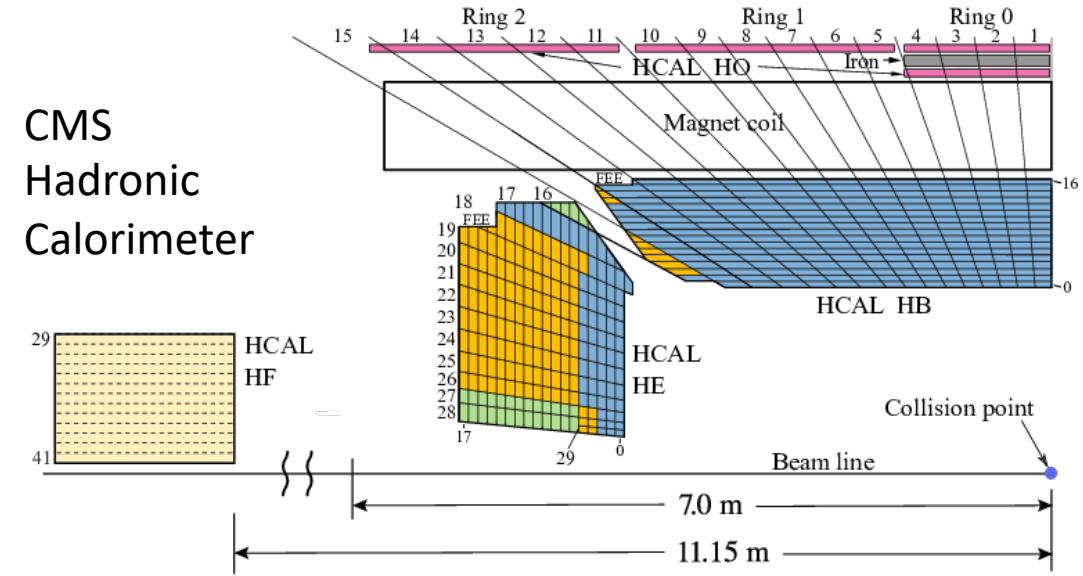
$$A_i = \sum_{j=1}^M f_{ji} E_j, \vec{\mu}_i = \sum_{j=1}^M f_{ji} E_j \vec{c}_j$$



# CMS clustering algorithm

4. CMS HCAL has multiple layers (number depends on where in calorimeter you are). Cells from different single layers are combined into multi-depth clusters based on an eta-phi matching scheme

Some parameters used in CMS calorimeter clustering algorithm  
(For using a similar algorithm in a different calorimeter, these values should be adjusted based on e.g. calorimeter material and expected particle energies)



	ECAL		HCAL		Preshower
	barrel	endcaps	barrel	endcaps	
Cell $E$ threshold (MeV)	80	300	800	800	0.06
Seed # closest cells	8	8	4	4	8
Seed $E$ threshold (MeV)	230	600	800	1100	0.12
Seed $E_T$ threshold (MeV)	0	150	0	0	0
Gaussian width (cm)	1.5	1.5	10.0	10.0	0.2

# Some subtleties

- In dense environments, particle showers in the calorimeter can overlap significantly leading to merged clusters (meaning fewer clusters than the number of particles), distorted energy measurements
  - This can be worsened by pileup, and especially in e.g. forward regions at LHC, where there is high particle multiplicity
  - As a note, high energy jets are often high-density
- Some later splitting of clusters can be performed by using tracking information (e.g. if a cluster's energy is significantly higher than a track pointing into it, it can be split, and the leftover is considered to come from a neutral particle)
- In regions that aren't covered by a tracker, you can't really do particle flow
  - You can end up with "hadrons" and "EM" "particles" though
- In realistic calorimeters (and when there's pileup), sometimes cell energy thresholds are applied, where any cell with energy below the threshold is ignored
  - Real calorimeters are somewhat noisy!

# Calibration

- A cluster calibration is normally performed after a clustering algorithm is established
  - Cluster energies are often biased (either low, due to systematically leaving out energy on the fringes of showers or hit energy thresholds, or high, due to e.g. pileup)

Primary effect is on response,  
though calibration also improves  
resolution at low energies

CMS cluster energies biased low  
by default before calibration

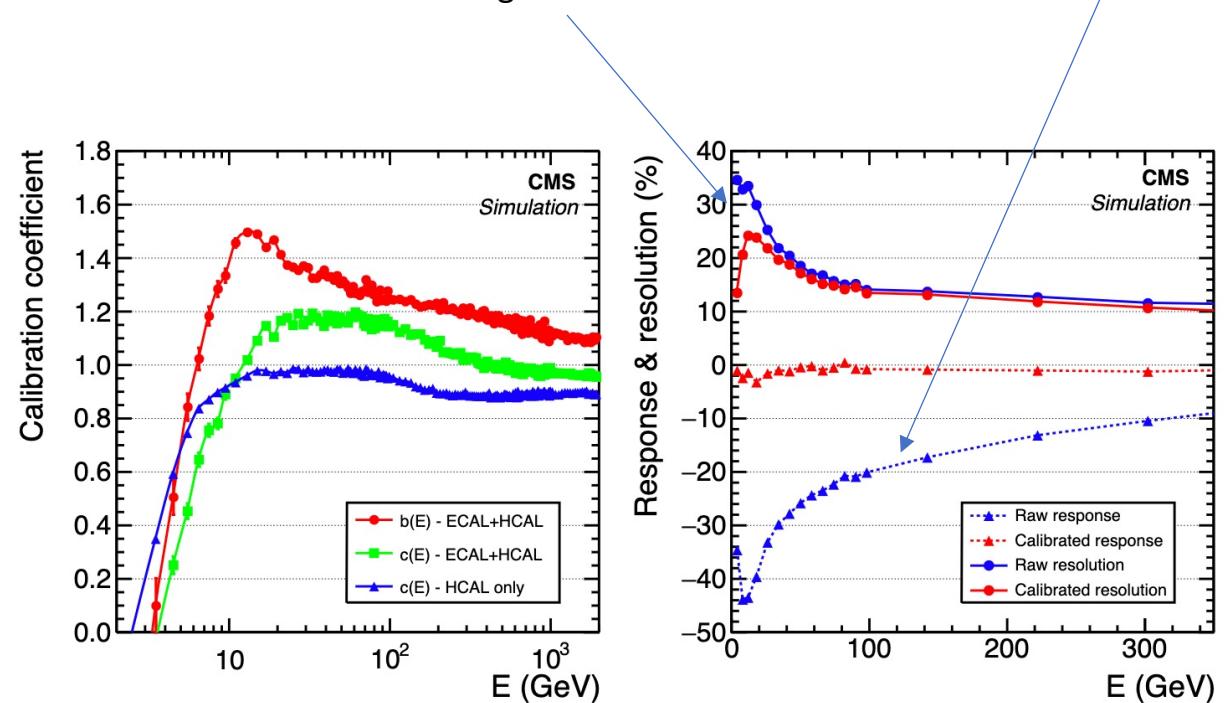


Figure 8: Left: Calibration coefficients obtained from single hadrons in the barrel as a function of their true energy  $E$ , for hadrons depositing energy only in the HCAL (blue triangles), and for hadrons depositing energy in both the ECAL and HCAL, for the ECAL (red circles) and for the HCAL (green squares) clusters. Right: Relative raw (blue) and calibrated (red) energy response (dashed curves and triangles) and resolution (full curves and circles) for single hadrons in the barrel, as a function of their true energy  $E$ . Here the raw (calibrated) response and resolution are obtained by a Gaussian fit to the distribution of the relative difference between the raw (calibrated) calorimetric energy and the true hadron energy.

# Performance metrics

- As hinted at on the last slide, cluster **energy response and resolution** are two main metrics considered
- One should also consider the number of clusters vs the number of particles
  - there is often a discrepancy
    - Typically it's more important to get the higher-energy particles right, especially when eventually using particles in jets
    - This discrepancy can be folded into response/resolution considerations by e.g. only considering particles/clusters above a certain threshold
- One thing that I haven't mentioned at all is algorithm timing performance
  - In offline processing, timing isn't normally a very strong consideration, but one could consider running clustering offline in a trigger if it's fast enough
  - In any case, a faster algorithm is normally preferable to a slower one, assuming equivalent performance
    - Note: ML and coprocessors (like GPUs) can be useful here

# Backup

# Traditional approach at hadron colliders

“Particle-flow reconstruction and global event description with the CMS detector”, [[1706.04965](#)]

- *Jets* consist of hadrons and photons, the energy of which can be inclusively measured by the calorimeters without any attempt to separate individual jet particles. Jet reconstruction can therefore be performed without any contribution from the tracker and the muon detectors. The same argument applies to the *missing transverse momentum*<sup>1</sup> ( $p_T^{\text{miss}}$ ) reconstruction.
- The reconstruction of *isolated photons and electrons* primarily concerns the ECAL.
- The *tagging* of jets originating from hadronic  $\tau$  decays and from b quark hadronization is based on the properties of the pertaining charged particle tracks, and thus mostly involves the tracker.
- The identification of *muons* is principally based on the information from the muon detectors.

# Traditional approach at hadron colliders

“Particle-flow reconstruction and global event description with the CMS detector”, [[1706.04965](#)]

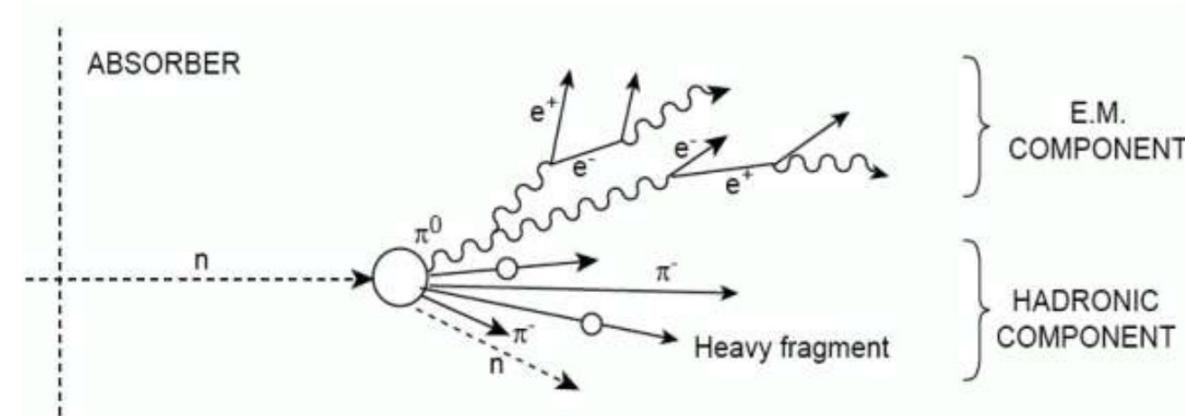
The PF concept was developed and used for the first time by the ALEPH experiment at LEP [2] and is now driving the design of detectors for possible future  $e^+e^-$  colliders, ILC and CLIC [3, 4], FCC-ee [5], and CEPC [6]. Attempts to repeat the experience at hadron colliders had not met with success so far. A key ingredient in this approach is the fine spatial granularity of the detector layers. Coarse-grained detectors may cause the signals from different particles to merge, especially within jets, thereby reducing the particle identification and reconstruction capabilities. Even in that case, however, the tracker resolution can be partially exploited by locally subtracting from the calorimeter energy either the energy expected from charged hadrons or the energy measured within a specific angle from the charged hadron trajectories. Such *energy-flow* algorithms [7–14] are used in general to improve the determination of selected hadronic jets or hadronic tau decays. If, on the other hand, the subdetectors are sufficiently segmented to provide good separation between individual particles, as shown for CMS in Fig. 2, a global event description becomes possible, in which all particles are identified. From the list of identified particles, optimally reconstructed from a combined fit of all pertaining measurements, the physics objects can be determined with superior efficiencies and resolutions.

# Hadron showers are hard

[Exploring the structure of hadronic showers and the hadronic energy reconstruction with highly granular calorimeters](#)  
(presentation from CALICE collaboration at ICHEP 2020)

- **Hadronic showers are quite complex**

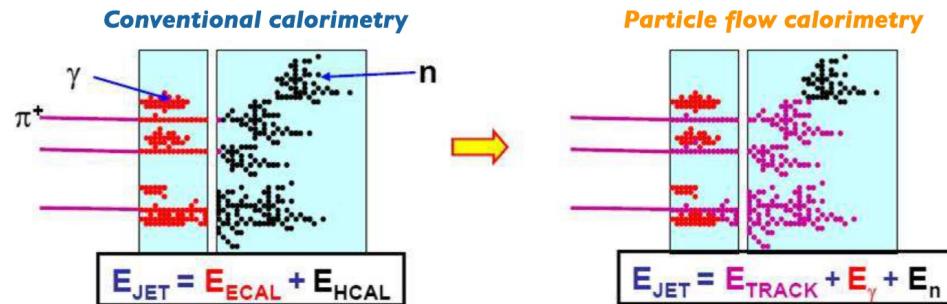
- Initiated by hard collision of incident hadron with a nucleus
- Narrow core of electromagnetic cascades by photons from  $\pi^0/\eta^0$
- Surrounding halo dominated by charged hadrons
- Large event-by-event fluctuation of electromagnetic and hadronic components ratio
- Invisible energies as nuclear binding energy, nuclear recoil, late component  
→ **limited hadronic energy resolution**



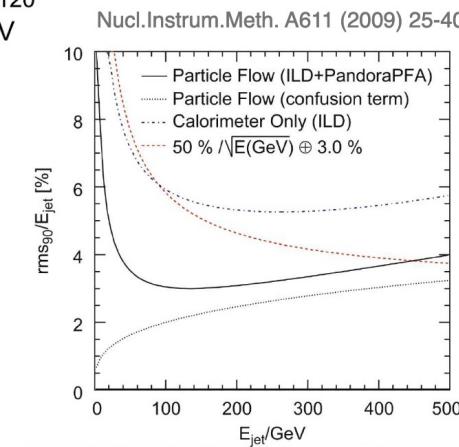
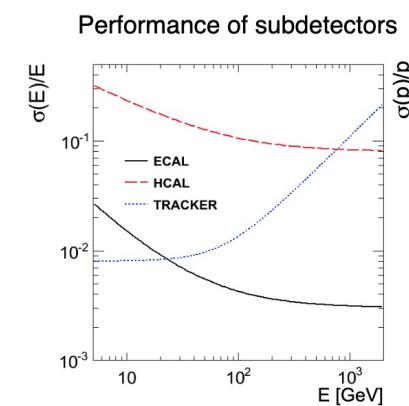
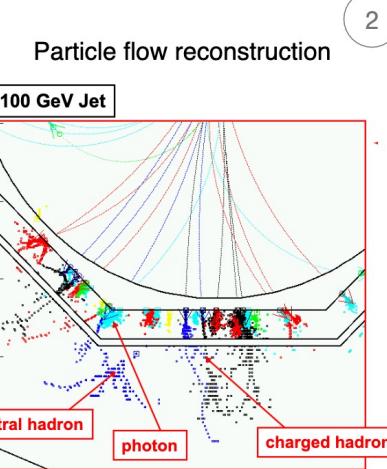
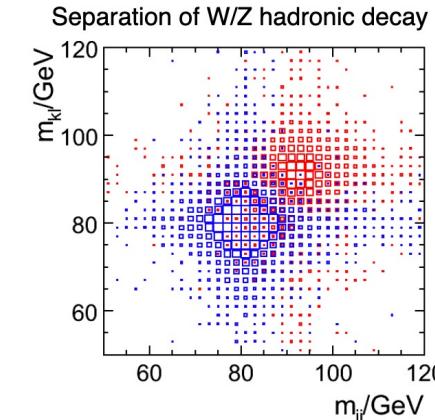
# Some nice particle flow plots

[Exploring the structure of hadronic showers and the hadronic energy reconstruction with highly granular calorimeters](#) (presentation from CALICE collaboration at ICHEP 2020)

- Physics with jets at future e<sup>+</sup>e<sup>-</sup> collider experiments requires unprecedented jet energy resolution of 3-4%
- Particle Flow Algorithm (PFA)
  - Measurements with best suited detectors depending on particle type
    - Charged particles → tracker
    - Photon → ECAL
    - Neutral hadrons → HCAL
  - PFA requires reconstruction of all visible particles in a jet with
    - Highly granular calorimeters
    - High precision tracker



## Particle Flow Calorimetry for Best Jet Energy Reconstruction



3-4% jet energy resolutions!

# Shower shape studies

[Exploring the structure of hadronic showers and the hadronic energy reconstruction with highly granular calorimeters](#) (presentation from CALICE collaboration at ICHEP 2020)

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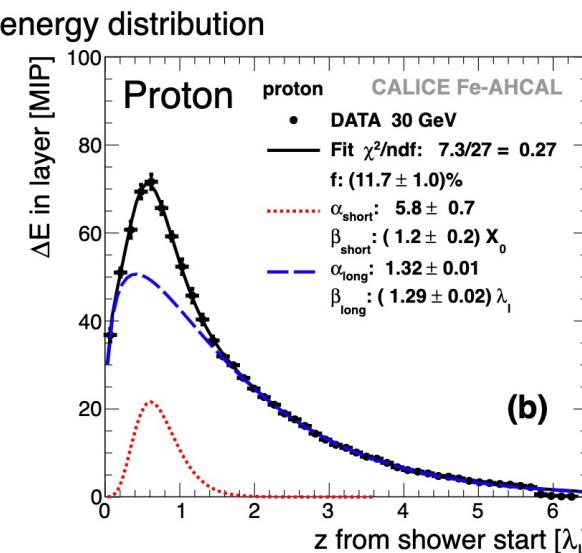
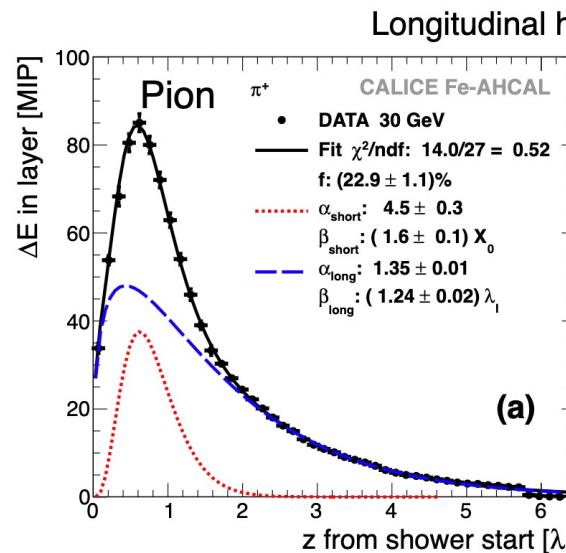
## Hadronic Shower Studies AHCAL

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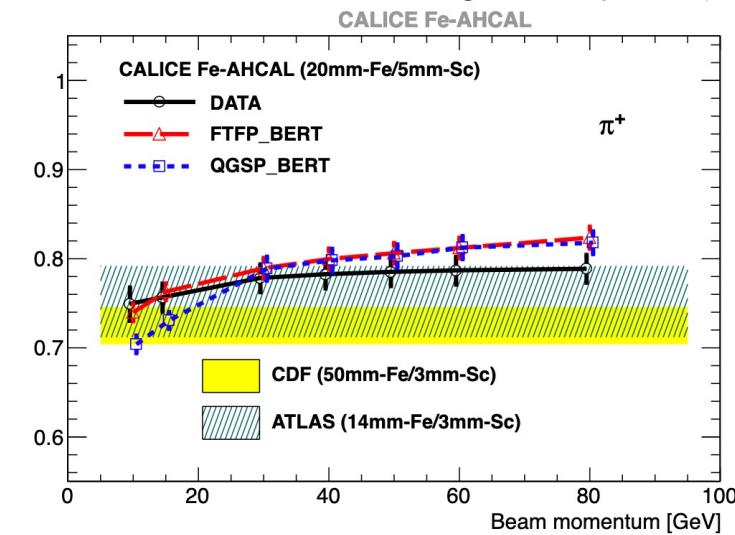
### • Longitudinal shower profile measured by AHCAL

- Test beam data: positive pion and proton 10-80GeV@CERN and FNAL
- Decompose shower components
  - Short component: electromagnetic component
  - Long component: hadronic component
- Extract ratio of hadronic to electromagnetic response (h/e)

$$\Delta E(z) = A \cdot \left\{ \frac{f}{\Gamma(\alpha_{\text{short}})} \cdot \left( \frac{z}{\beta_{\text{short}}} \right)^{\alpha_{\text{short}}-1} \cdot \frac{e^{-z/\beta_{\text{short}}}}{\beta_{\text{short}}} + \frac{1-f}{\Gamma(\alpha_{\text{long}})} \cdot \left( \frac{z}{\beta_{\text{long}}} \right)^{\alpha_{\text{long}}-1} \cdot \frac{e^{-z/\beta_{\text{long}}}}{\beta_{\text{long}}} \right\}$$



### Ratio of hadronic to electromagnetic response (h/e)



# Shower shape studies

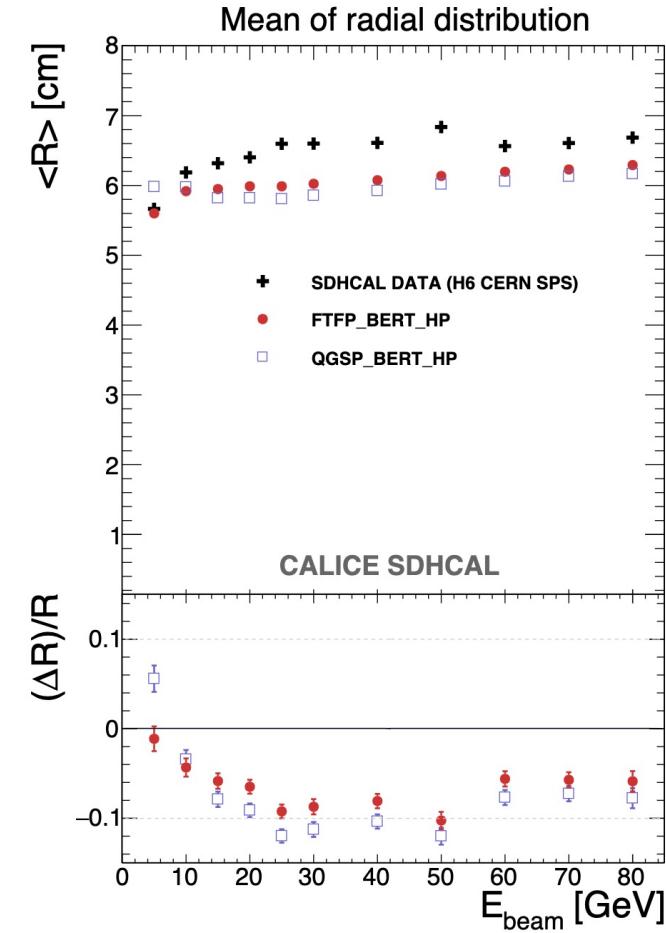
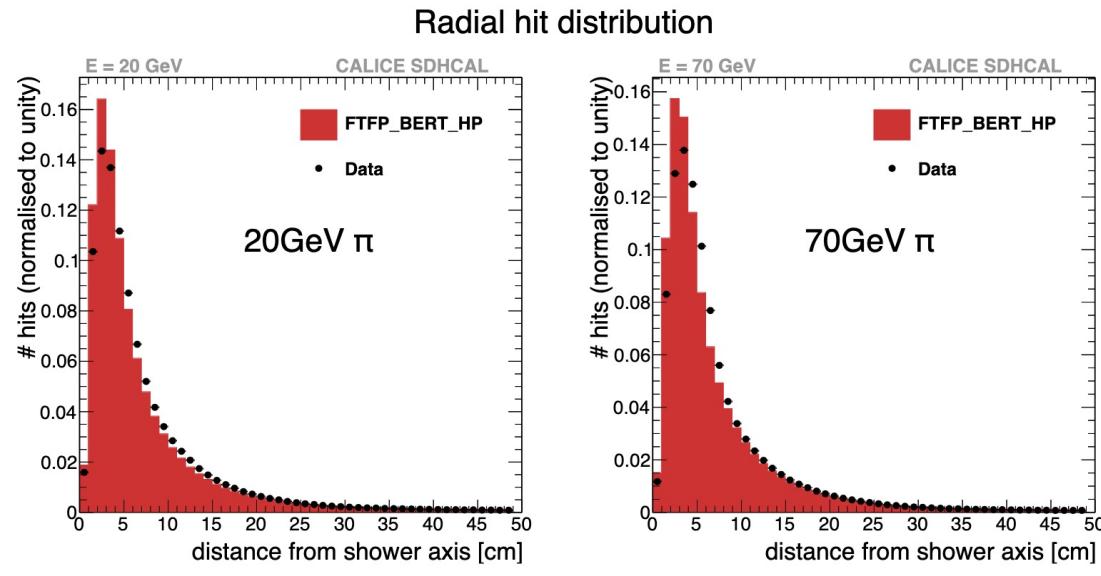
[Exploring the structure of hadronic showers and the hadronic energy reconstruction with highly granular calorimeters](#) (presentation from CALICE collaboration at ICHEP 2020)

## Hadronic Shower Studies SDHCAL

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- Finer transverse segmentation of SDHCAL is useful for radial profile study
  - Test beam: 5-80GeV pions @CERN SPS
  - Simulation: GEANT4 ver9.6 with High Precision (HP) package
- Radial profile is narrower in simulation



# Future calorimeters

- High granularity HCALs
  - [CMS High Granularity HCAL \(HGCAL\)](#)
- Calorimeters with timing
  - [Precision Timing in HGCAL](#)

