Recitation 1: Uncertainity and Error Propagation

8.S50

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Basic Statistics

Average

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Variance

$$s_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \langle x \rangle)^2$$

"Standard Deviation"

$$s_x \text{ or } \sigma$$

Uncertainty

- Any representation of these three measurements are meaningless without uncertainty
- A physical measurement has two components: the best value and its uncertainty

Uncertainty or *error* is the estimate of how close our answer is to the true value.

Types of Uncertainty

Systematic	Statistical
Systematic Imperfection in • Method • Apparatus • Calibration • Environment • Model/Hypothesis	Random/Statistical Variation • Underlying physical process ◦ e.g. Poisson fluctuations • Instrument noise
Not independent from measurement to measurement	Independent from measurement to measurement • “averages out” when repeating the measurement
Limits accuracy	Limits Precision

Measurements of Uncertainty

- Standard deviation σ is often used as the estimate of uncertainty
- Standard Deviation of the Mean (Standard Error) $SE = \frac{\sigma}{\sqrt{N}}$ can also be reported for the uncertainty in the mean.
- For counting experiments with N observed events, the estimated uncertainity on the mean of the underlying Poisson distribution is $\sqrt{N}$

Error Propagation (Propagation of Uncertainty)

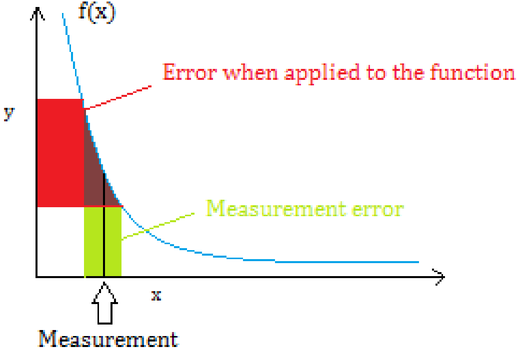
- Often, out experiment will measure one observable, but we will want to report a quantity derived from the observable.
- Also, often the reported final result will depend on multiple observables.
- Finally, the uncertainty on some observables may have contributions from multiple sources.
- We therefore need to understand how to propagate uncertainties on various components to the final result.

Error Propagation

- Suppose we measure two observables. $u \pm \sigma_u$ and $v \pm \sigma_v$
- Say we want to report the uncertainty of a measurement that is dependent on both $x = f(u, v)$
- Think of a Taylor expansion. (with σ_u, σ_v small and independent):

$$\sigma_x^2 = \sigma_u^2 (\frac{\partial x}{\partial u})^2 + \sigma_v^2 (\frac{\partial x}{\partial v})^2$$

Error Propagation



Error Propagation

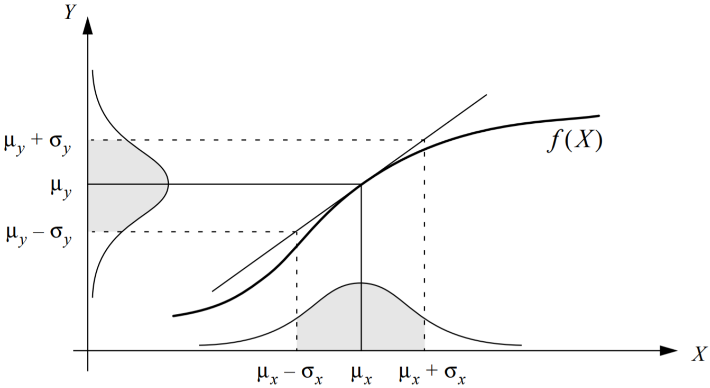


Figure 2: *One-dimensional case of a nonlinear error propagation problem*

Error Propagation

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- Say we want to report the uncertainty of a measurement that is dependent on both $x = f(u, v)$
- Think of a Taylor expansion. (with σ_u, σ_v small and independent):

$$\boxed{\sigma_x^2 = \sigma_u^2 (\frac{\partial x}{\partial u})^2 + \sigma_v^2 (\frac{\partial x}{\partial v})^2}$$

Propagation Rules - Sum and Difference

- Absolute uncertainties *add*

$$\underline{x = u \pm v}$$

$$\rightarrow \frac{\partial x}{\partial u} = \frac{\partial x}{\partial v} = 1$$

$$\sigma_x^2 = \sigma_u^2 + \sigma_v^2$$

$$\boxed{\sigma_x = \sqrt{\sigma_u^2 + \sigma_v^2}}$$

Propagation Rules - Products or Quotients

- Fractional uncertainties *add*

$$\underline{x = auv}$$

$$\rightarrow \frac{\partial x}{\partial u} = av, \frac{\partial x}{\partial v} = au$$

$$\frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2}$$

$$\boxed{\sigma_x = x\sqrt{(\frac{\sigma_u}{u})^2 + (\frac{\sigma_v}{v})^2}}$$

Propagation Rules - Powers

- Fractional uncertainties *multiply* by power

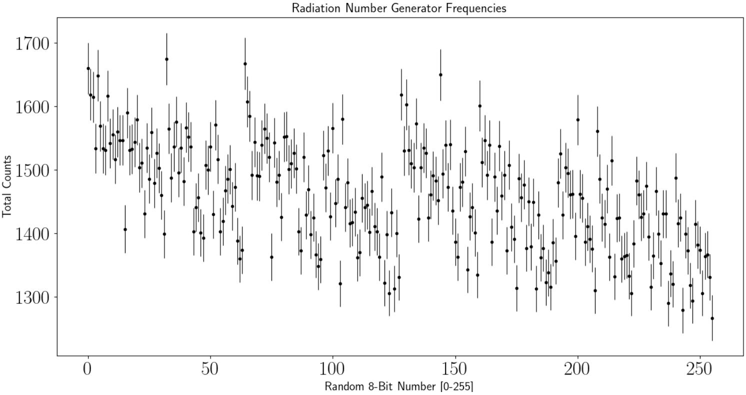
$$\underline{x = au^p}$$

$$\rightarrow \frac{\partial x}{\partial u} = apu^{p-1} = p\frac{x}{u}$$

$$\frac{\sigma_x}{x} = p\frac{\sigma_u}{u}$$

$$\boxed{\sigma_x = xp\frac{\sigma_u}{u}}$$

Representing Uncertainty



Representing Uncertainty

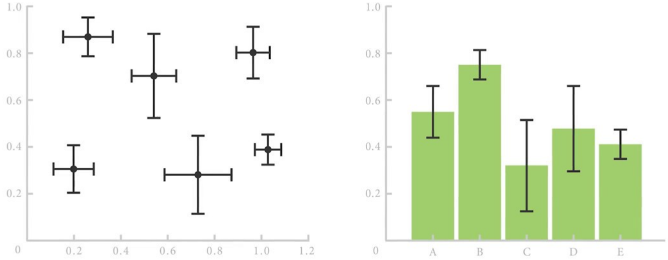
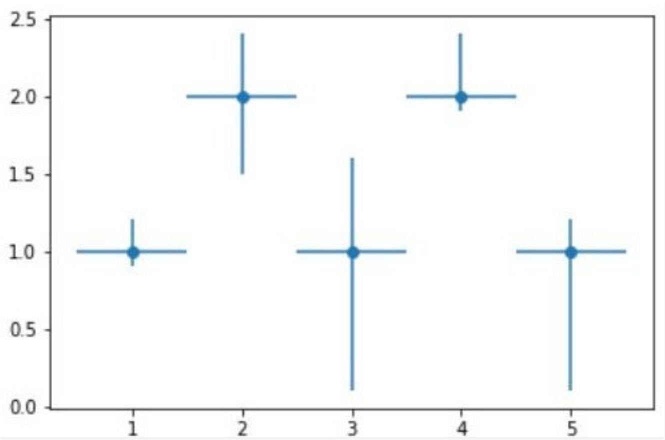


Figure: The Data Visualization Catalog

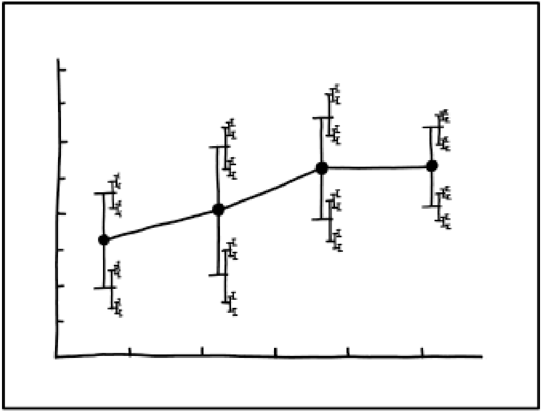
matplotlib.pyplot.errorbar

```
import matplotlib.pyplot as plt

x =[1, 2, 3, 4, 5]
y =[1, 2, 1, 2, 1]

y_errormin =[0.1, 0.5, 0.9,
             0.1, 0.9]
y_errormax =[0.2, 0.4, 0.6,
             0.4, 0.2]

x_error = 0.5
y_error =[y_errormin, y_errormax]

# ploting graph
# plt.plot(x, y)
plt.errorbar(x, y,
             yerr = y_error,
             xerr = x_error,
             fmt ='o')
```

pyplot errobars



Relevant XKCD



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.