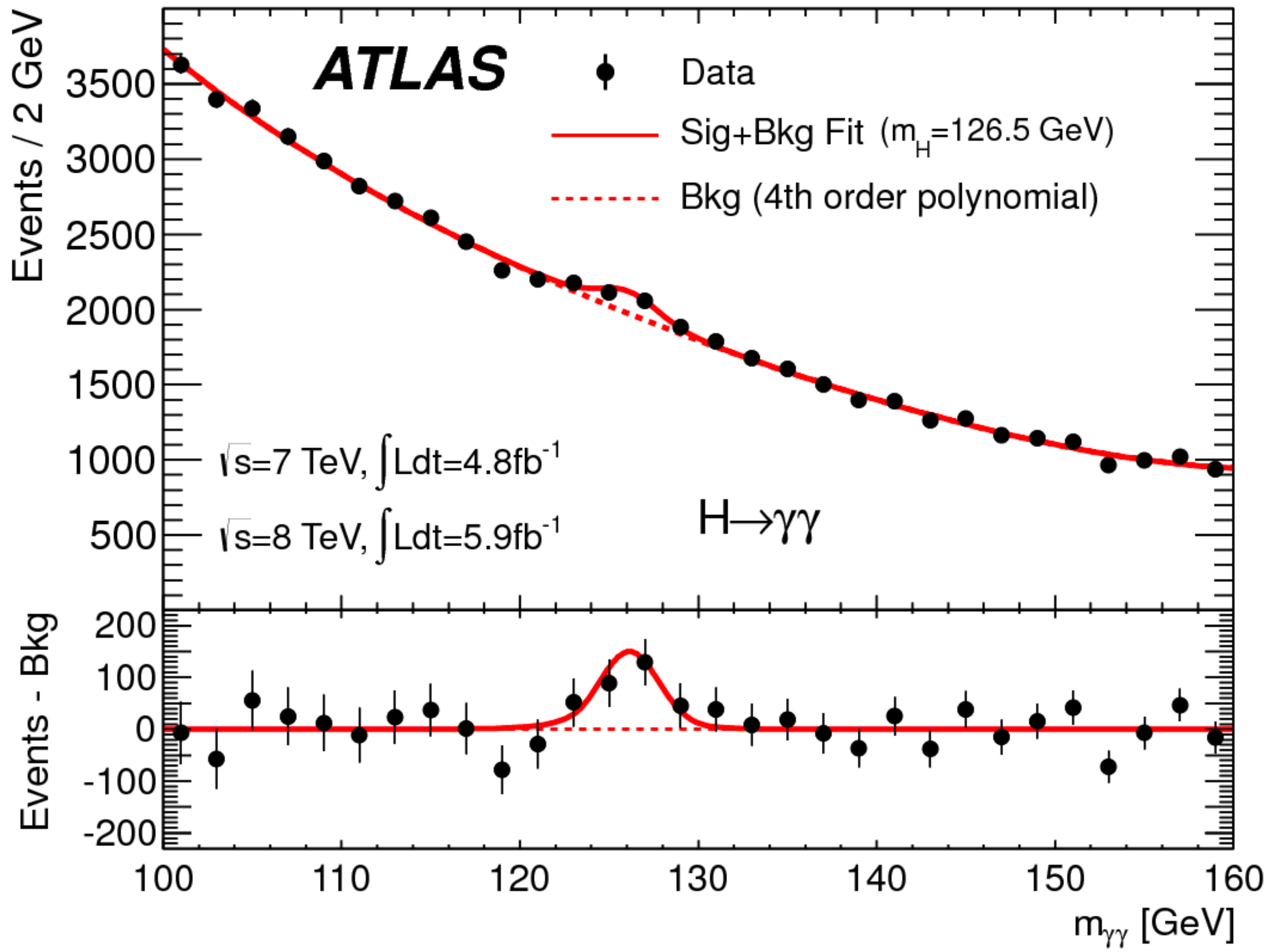


Lecture 10

Higgs Boson Discovery
Towards Deep Learning

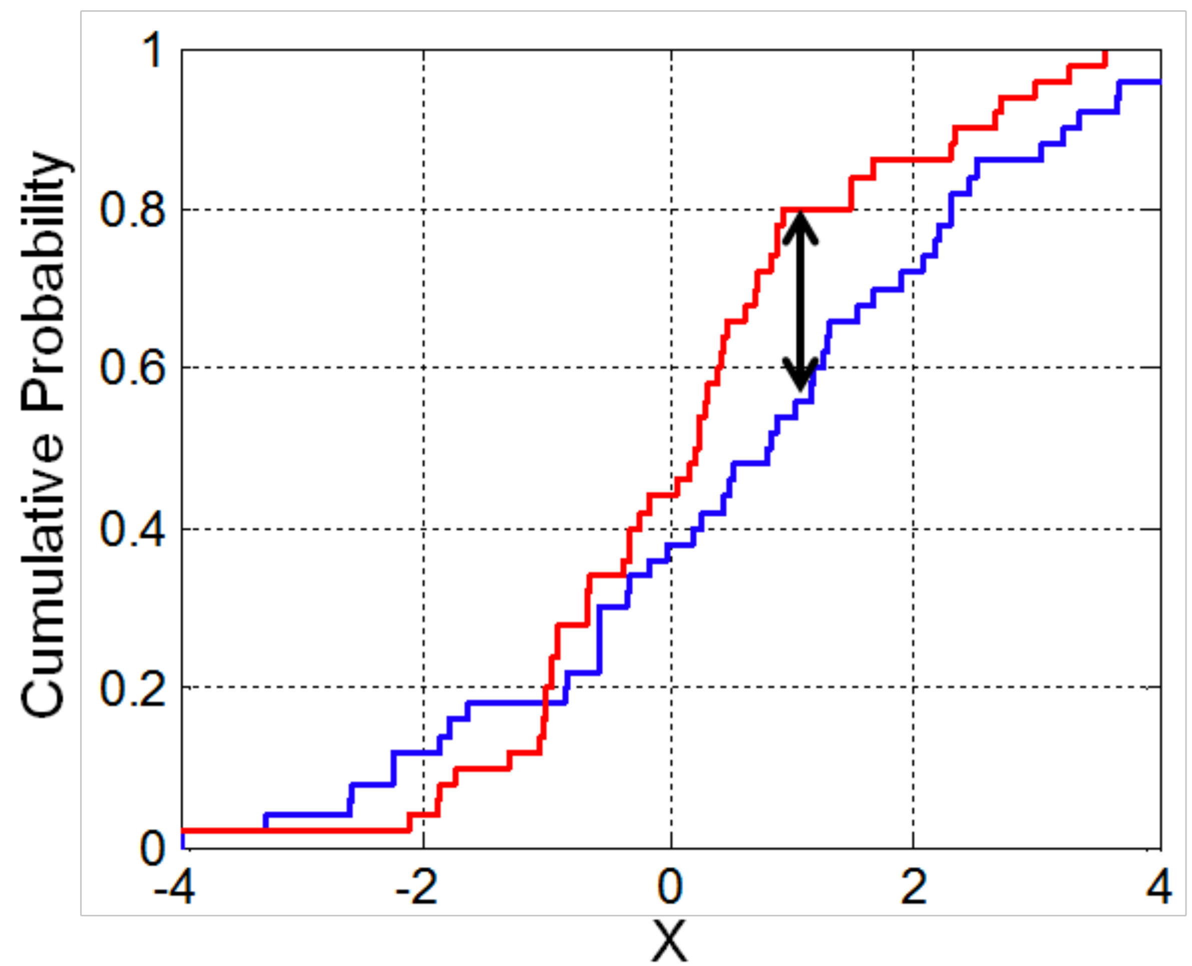
KS-test

- Komolgorov-Smirnov Test
  - Estimate the maximum difference between observed and predicted cumulative distribution functions and compare with expectations.
  - An unbinned way to check histograms are equal

$$\widehat{F}_n(t) = \frac{\text{number of elements in the sample} \leq t}{n} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{X_i \leq t},$$

$$D_{n,m} = \sup_x |F_{1,n}(x) - F_{2,m}(x)|,$$

Supremum Function (ie max difference)



Likelihood Ratio Test

- Perhaps the most important test is the likelihood ratio test
- This is what we will focus for the rest of these slides
  - Likelihood ratio is the standard in hypothesis testing
  - This approach is very similar to a a chi2 test

$\mathscr{L}(\theta_i)\forall\theta_i \in \Theta_0 \text{where } \Theta_0 \text{ refers to null} - \text{hypothesis}$

$\mathscr{L}(\theta_i)\forall\theta_i \in \Theta \text{where } \Theta \text{ refers to alternative} - \text{hypothesis}$

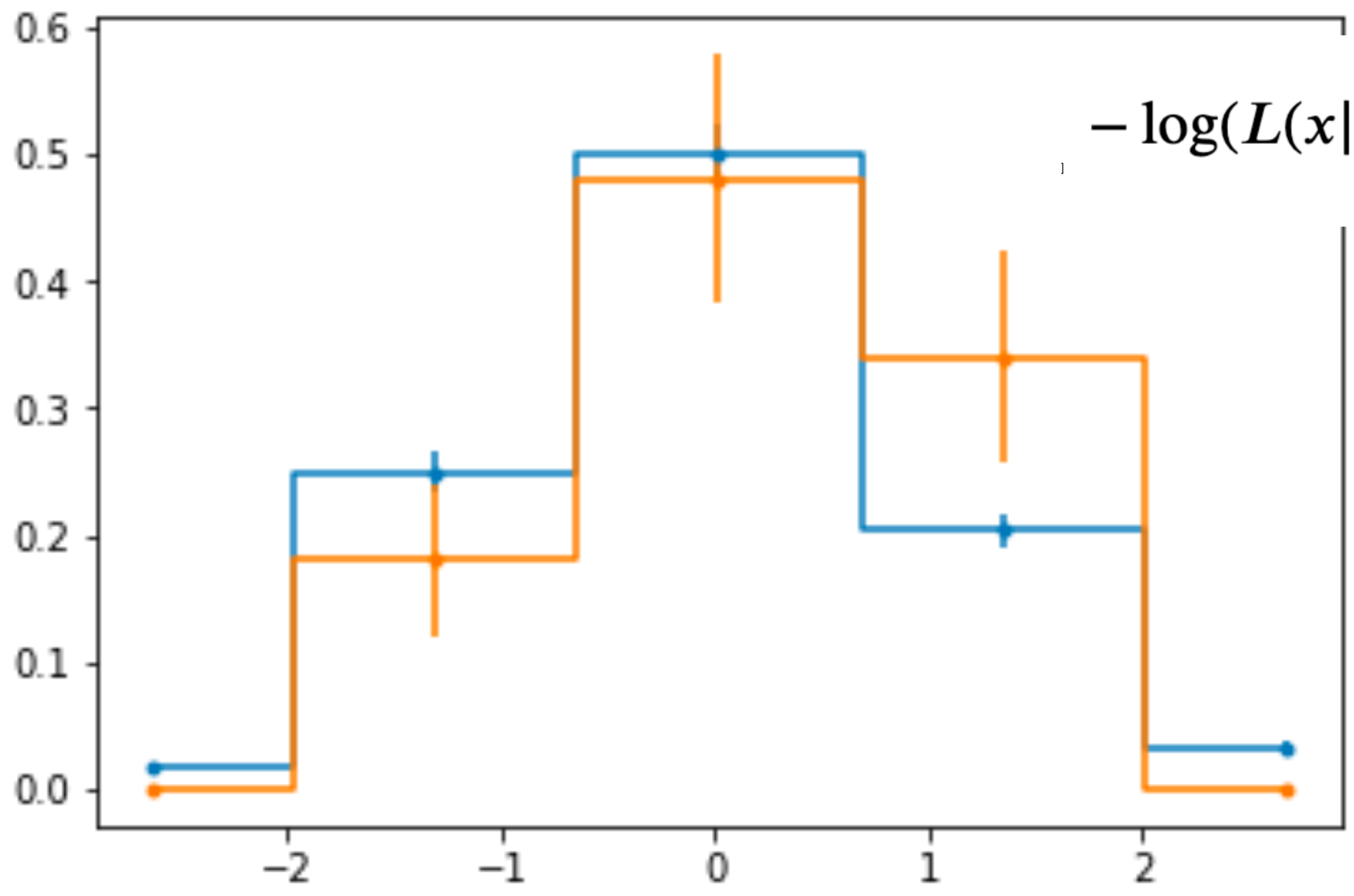
$\sup_{\theta\in\Theta_0}\mathscr{L}(\theta)$ is the maximum likelihood over all θ

$$\boxed{\lambda_{\mathrm{LR}} = -2\ln\left[\frac{\sup_{\theta\in\Theta_0}\mathcal{L}(\theta)}{\sup_{\theta\in\Theta}\mathcal{L}(\theta)}\right]}$$

Neyman-Pearson Lemma states the likelihood ratio converges to the optimal test statistic

Likelihood Test: Concept

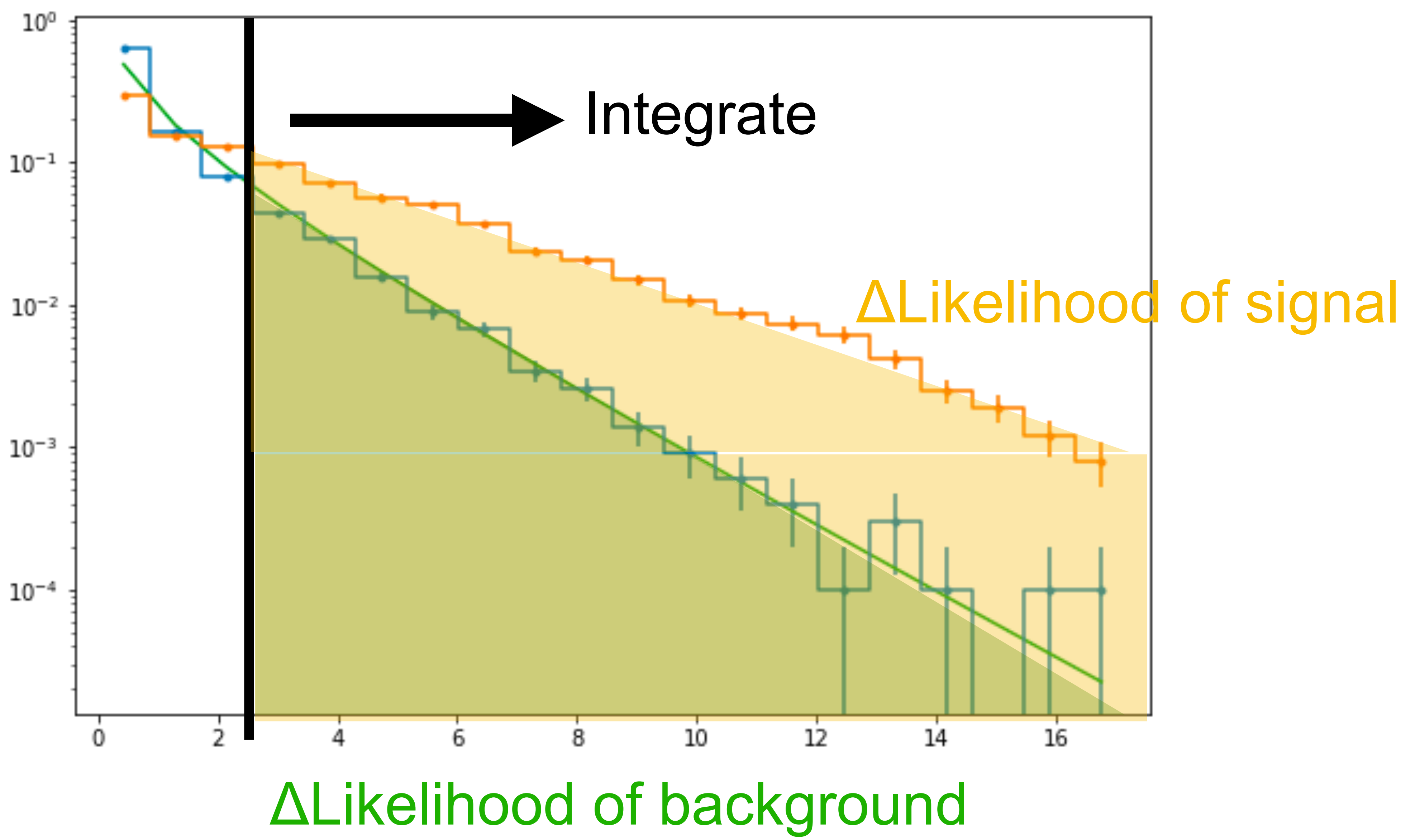
- We would like to see if these to histograms are consistent
  - Hypothesis: Orange is a guassian with mean 0 (blue)
  - True: Orange is sampled from a gaussian with mean 0.2



$$-\log(L(x|\lambda)) = \sum_{i-1}^{N} -\frac{1}{2}\log(2\pi\sigma^2) + \frac{(x_i-\mu)^2}{2\sigma^2}$$

Likelihood Test: Gaussian

- We can see how these on average a deviation



Frequentist: t-test

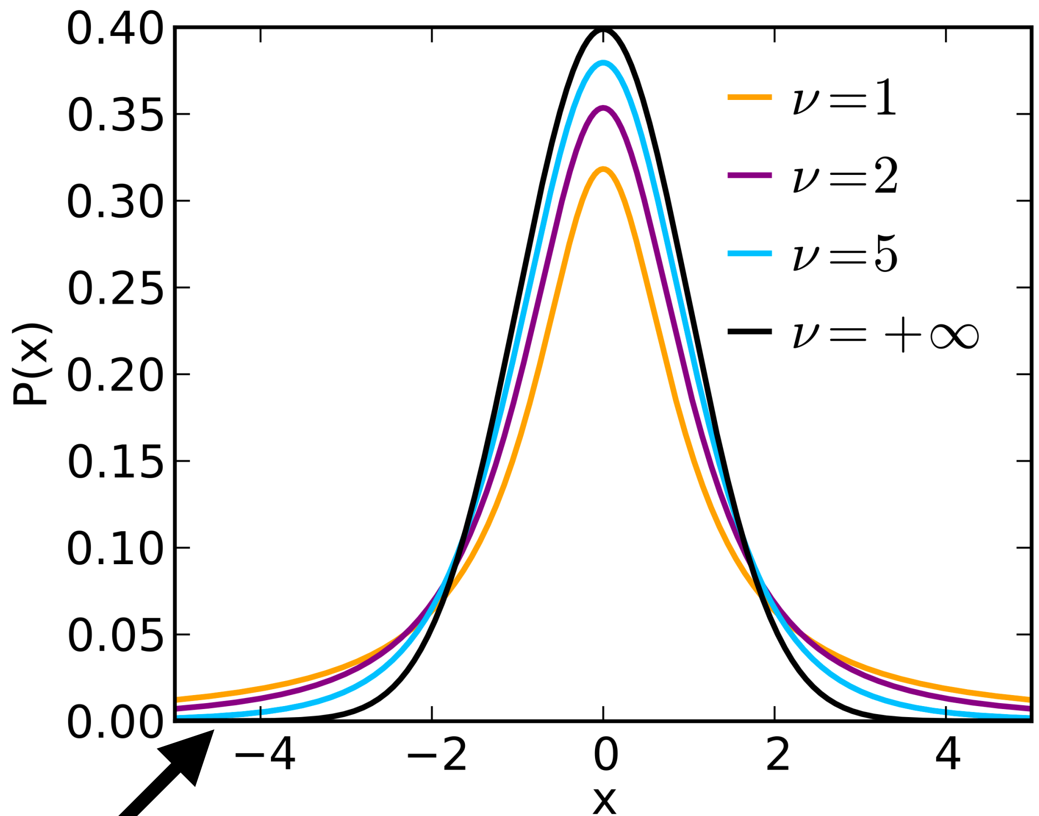
- Perform a t-test:
  - If samples $X_1, X_2, \ldots X_N$ from the same underlying distrbtion
  - Underlying distribution is Gaussian with mean μ and variance σ^2
  - If the above is true then we nave a measured mean+variance:
  - given by $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ and $S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$
  - And the distribution $\frac{\bar{X} - \mu}{S/\sqrt{N}}$ is given by a t-distribution

Frequentist :t-test

- Often called the “students” t-test (https://www.youtube.com/watch?v=U9Wr7VEPGXA)

- Invented by brewmaster to test flavor variations in Guinness

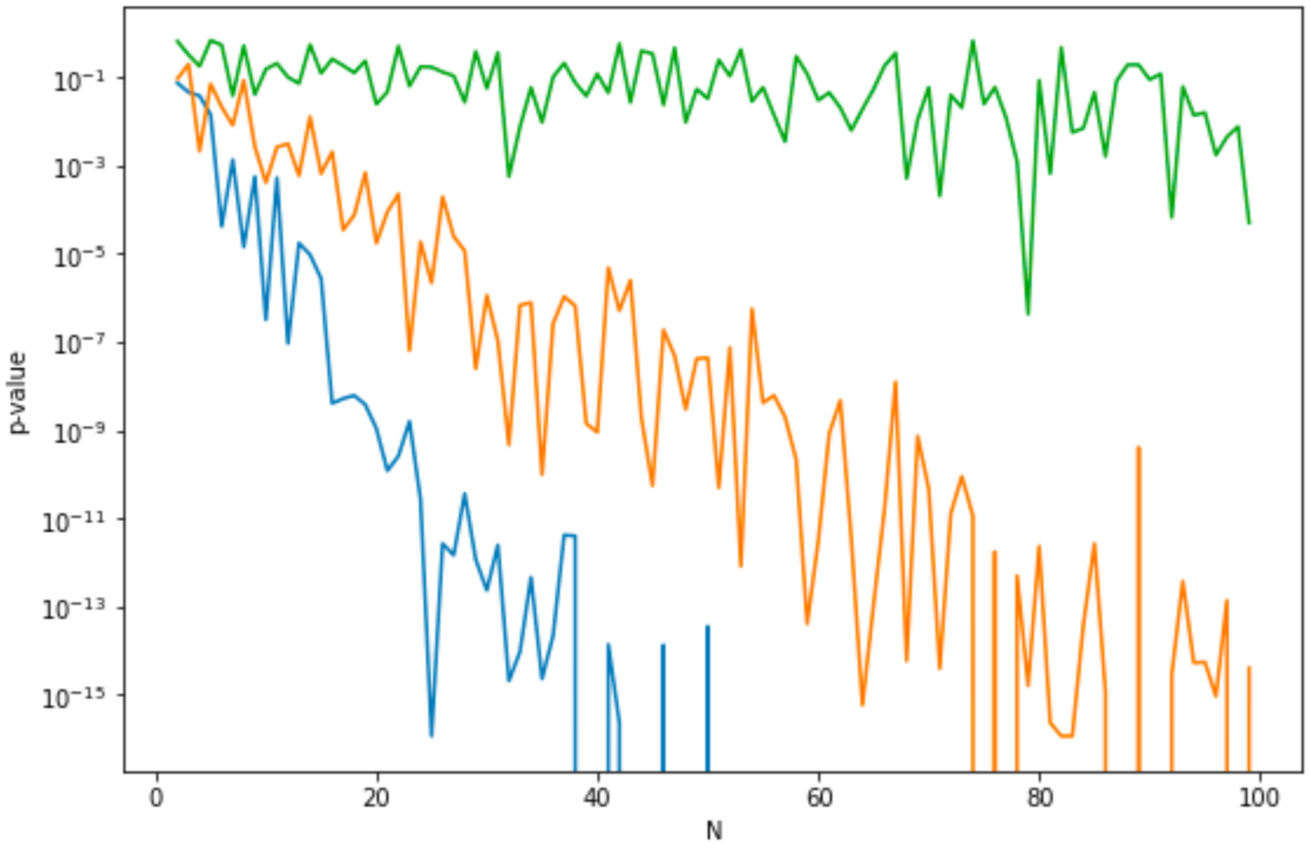
$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \qquad \nu = N - 1$$

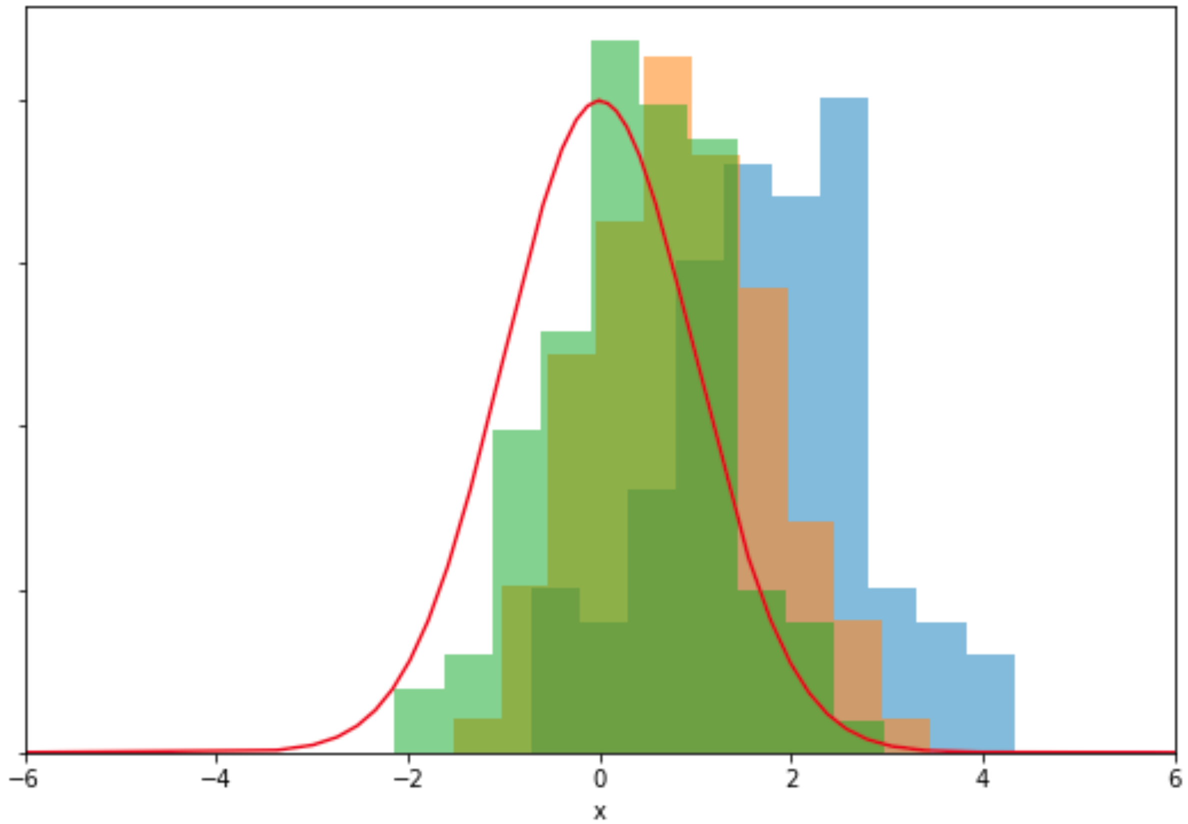


Normal Distribution

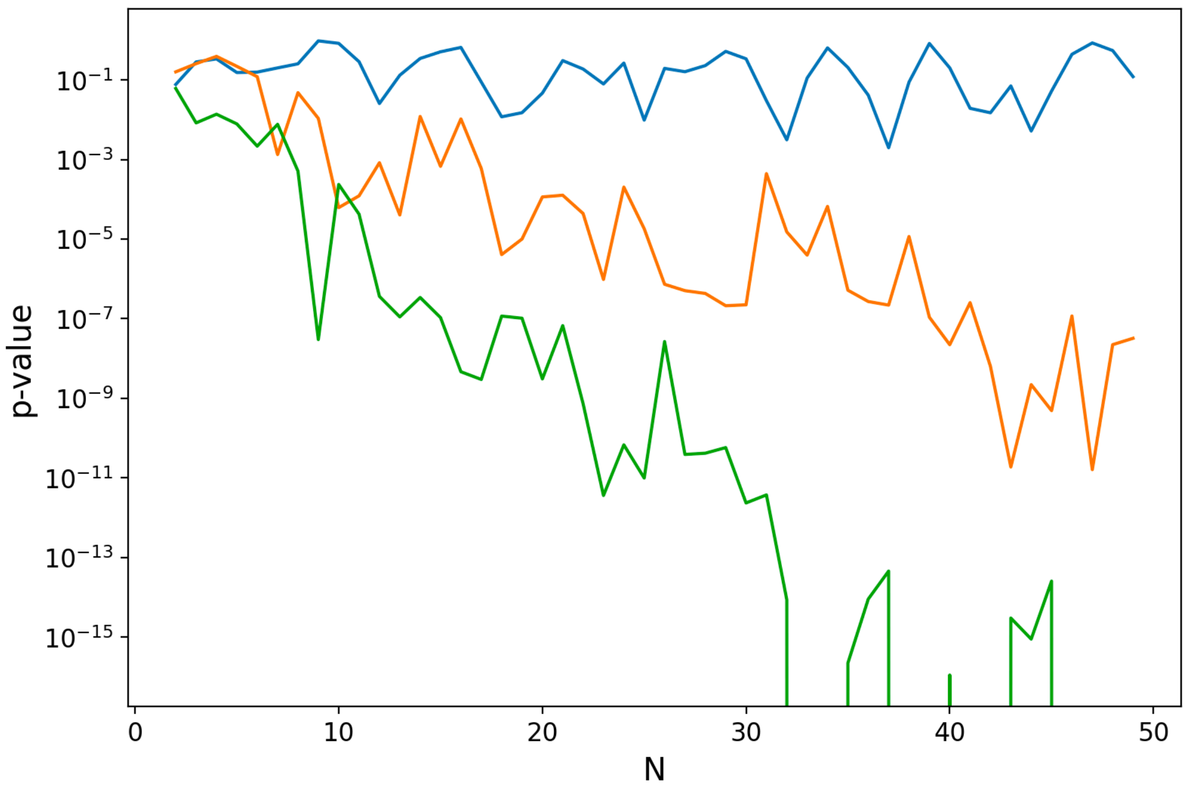
t-test Consistency

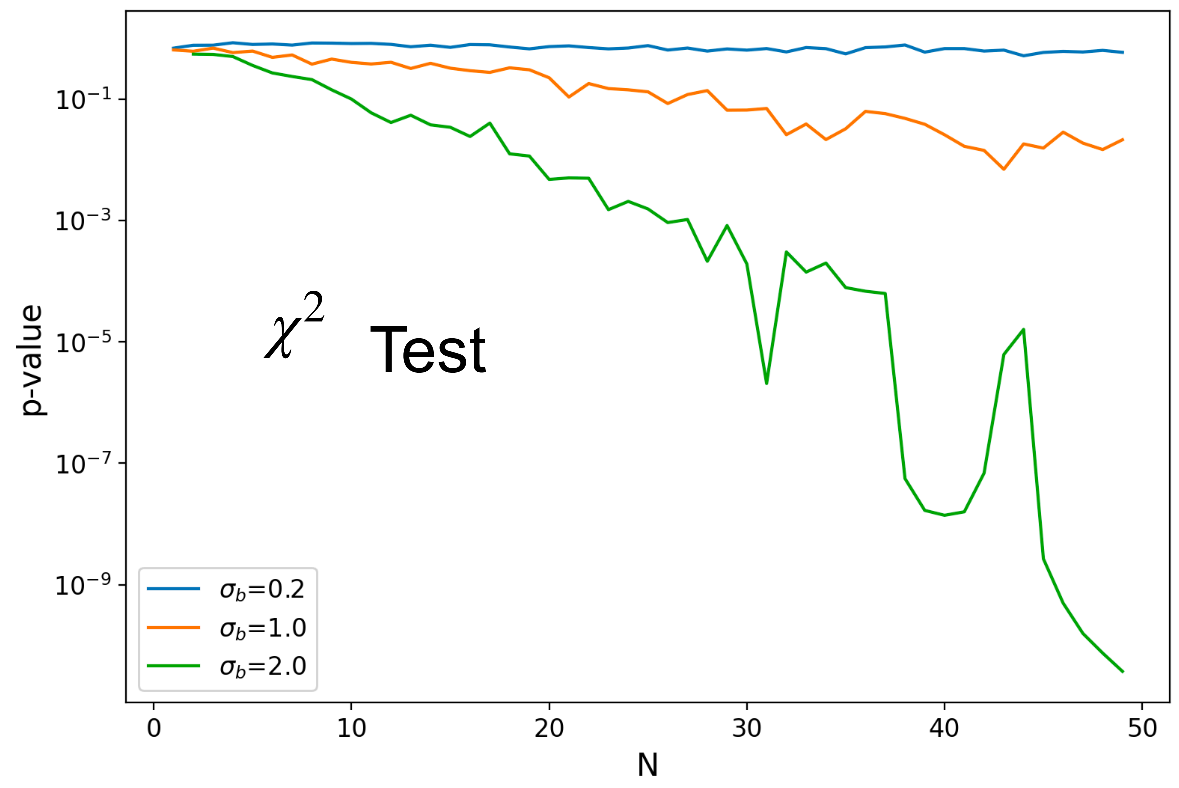
- Variation as a number of events





t-test Consistency



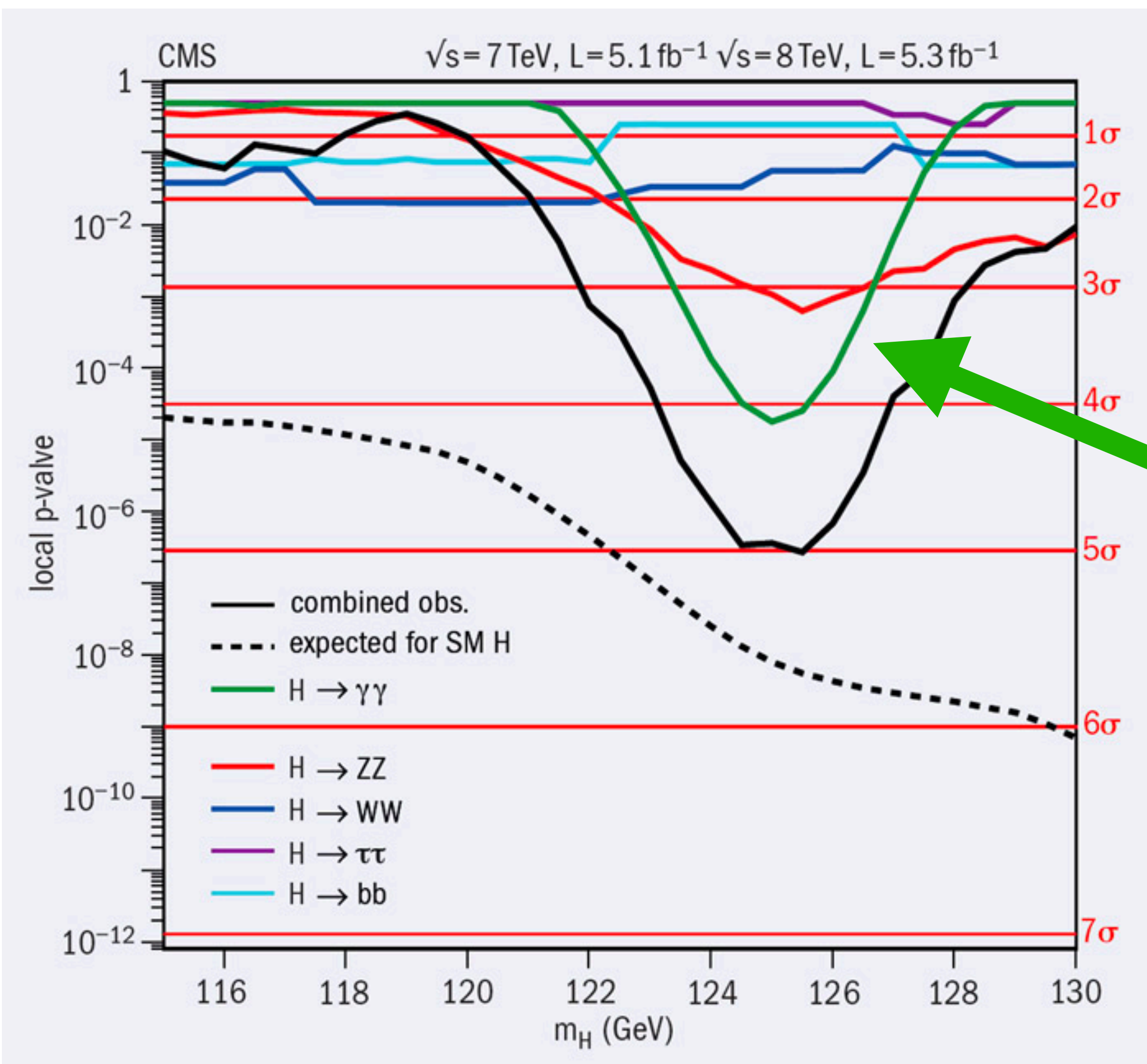


Higgs Boson

- We discovered the Higgs boson not so long ago

In this lectuere we are going to make the discovery plot

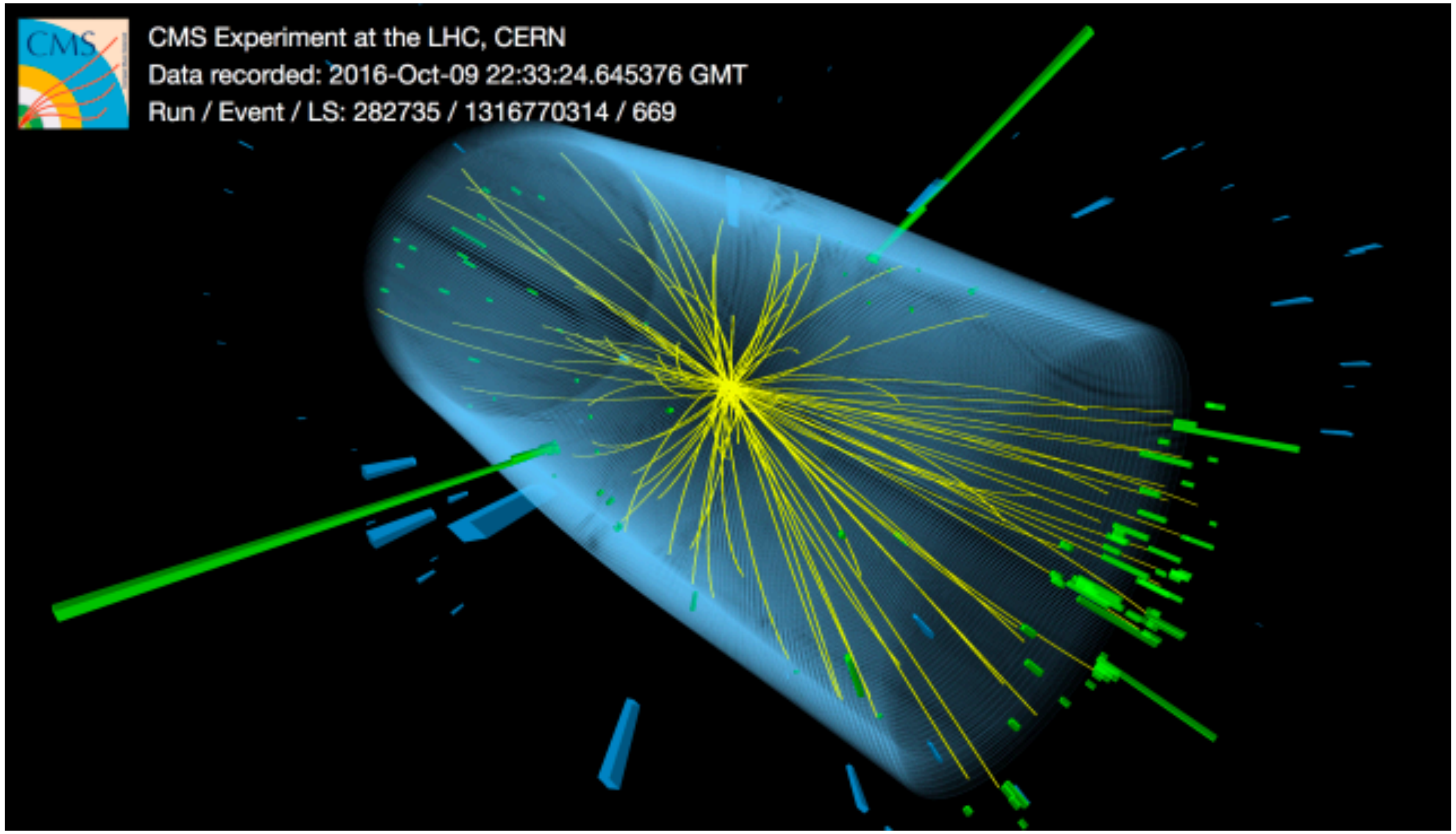
- To discovery it we made a plot like this:

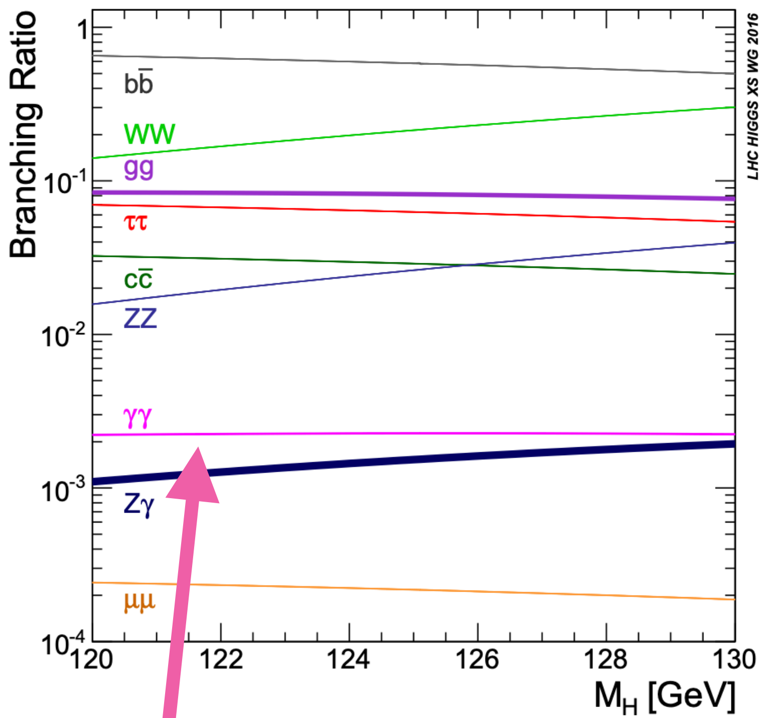


We are going to focus on his object

Higgs to two photons

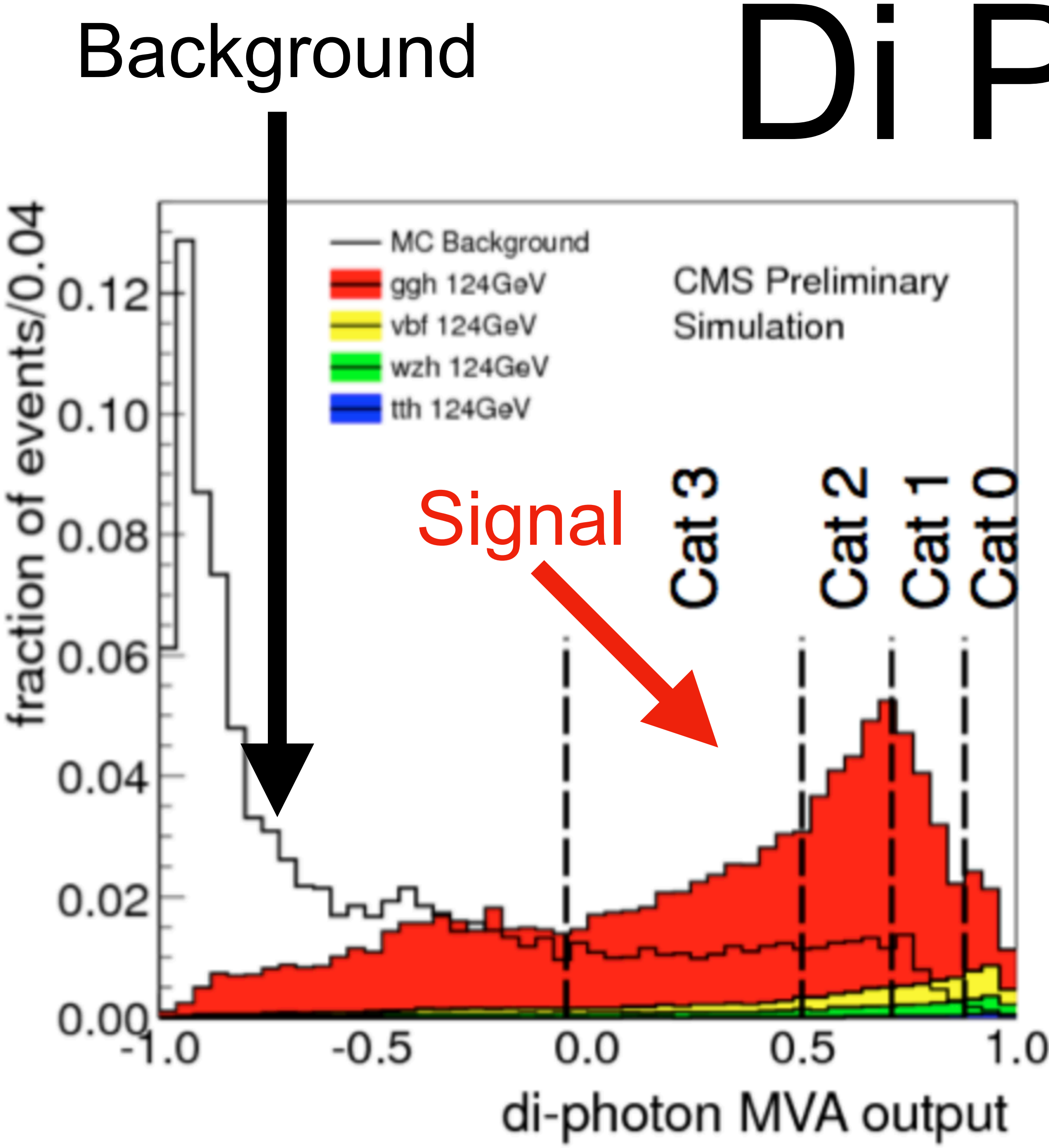
- Higgs to boson decays to nearly every other particle

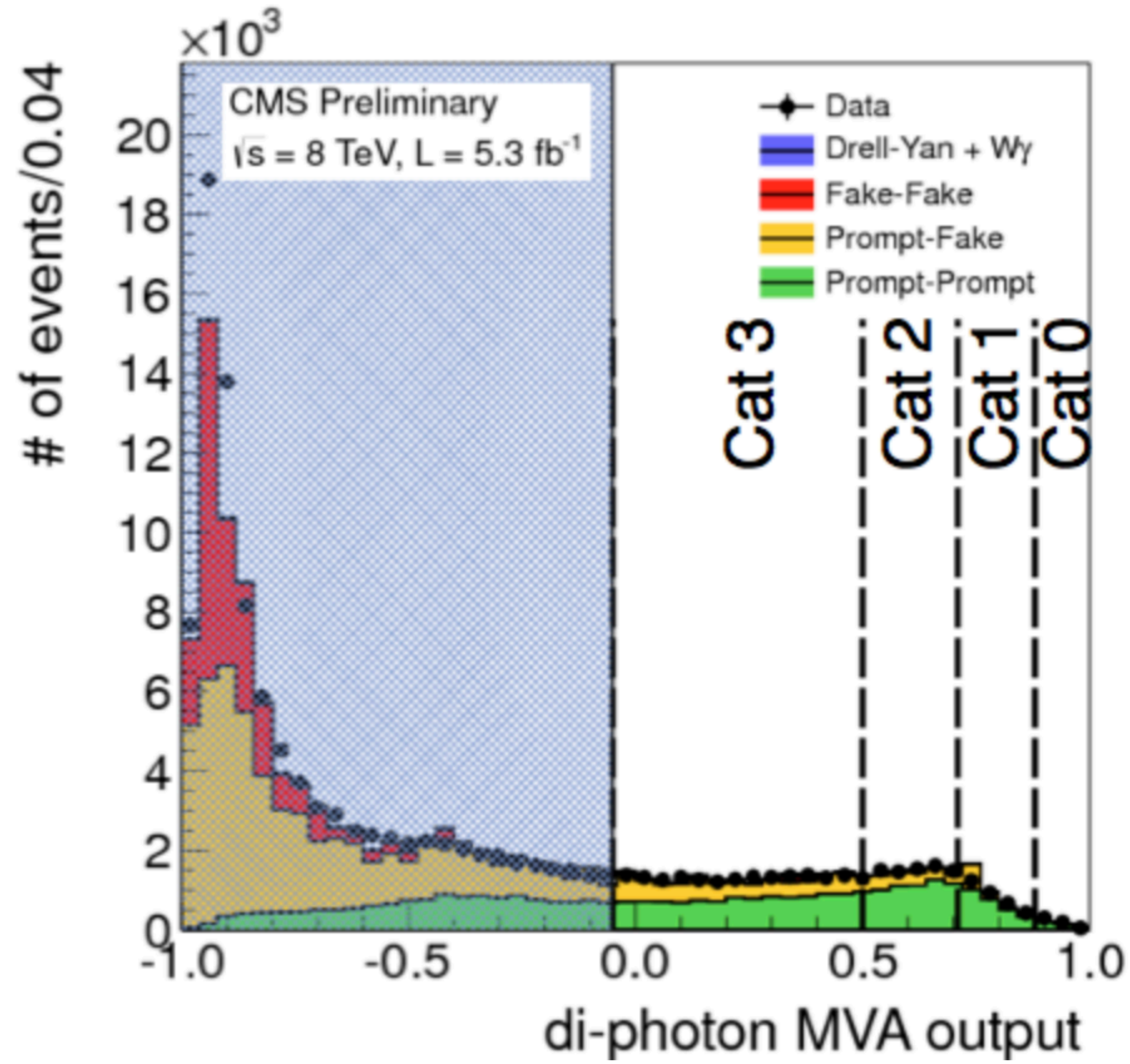




Higgs bosons to two photons occur about 0.3% of the time

Di Photon selection

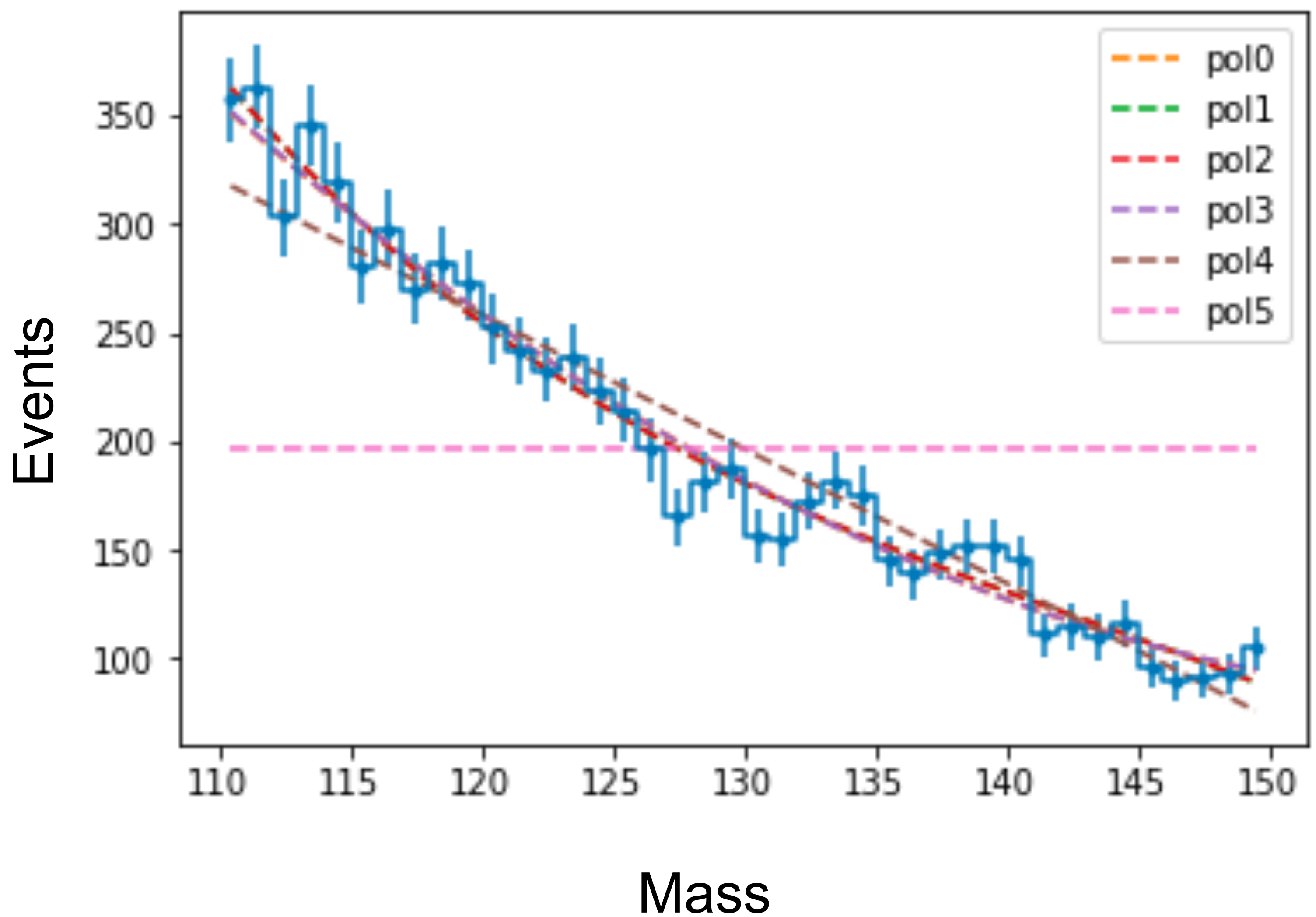




- To select di-photons we build a neural network
  - This takes into account many different Higgs properties

Building a Model

- The frequentist will look at this data and guess a model



Often when we fit, we through a barrage of functions at it
As a rule of thumb we do the “chi-by-eye” (If chi2 is good, we are ok)
A more robust is an f-test (see notes)

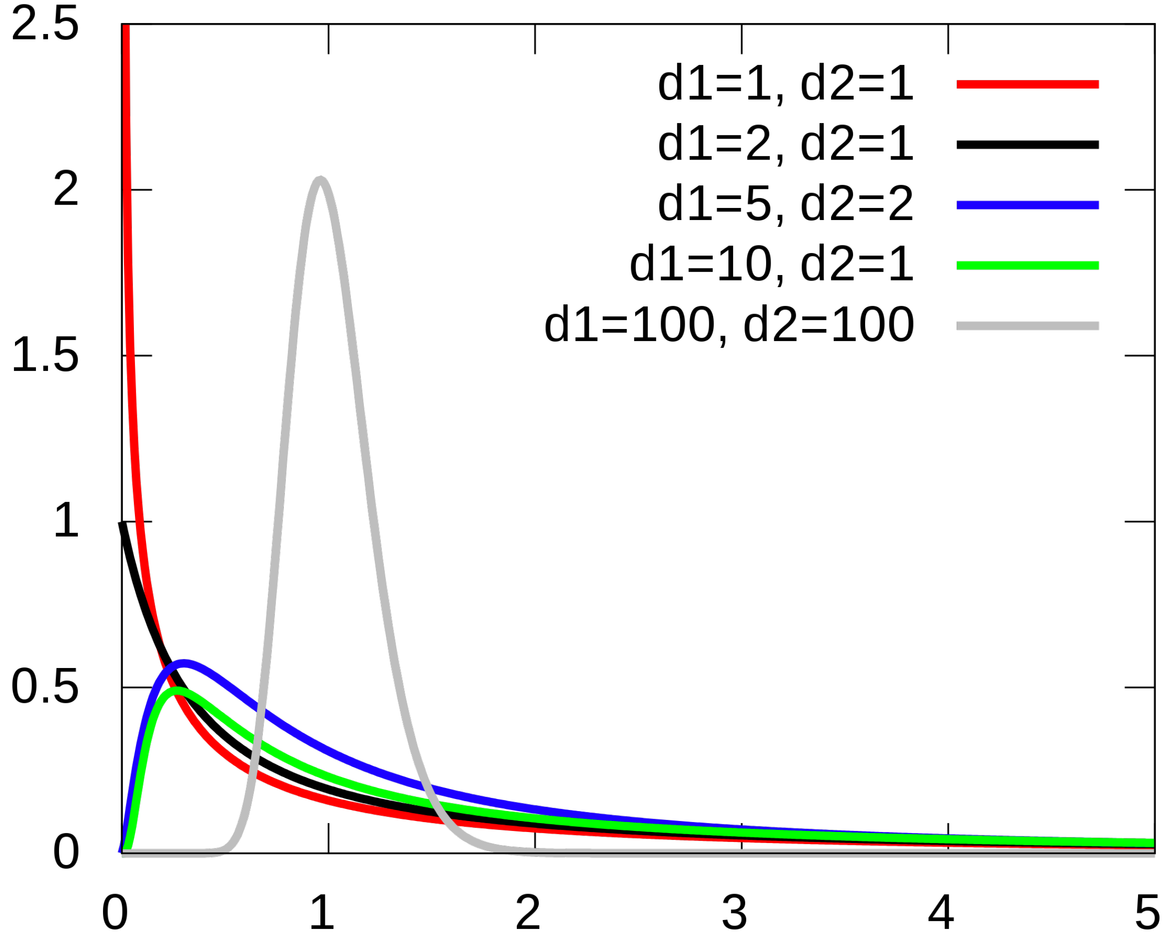
F-Statistic

- F distribution leads to a hypothesis about groupings
- $$F = \frac{\text{explained Variance}}{\text{Unexplained Variance}} = \frac{\text{Between Group Variability}}{\text{Single Group Variability}}$$
- For fitting functions as a polynomial we can write this as
- $$F = \frac{\frac{1}{\Delta_{DOF}(f,g)}\left(\sum_i \left(y_i - f(x_i)\right)^2 - \sum_i \left(y_i - g(x_i)\right)^2\right)}{\left(\frac{1}{n - p_f}\sum_i \left(y_i - f(x_i)\right)^2\right)}$$

F-Distribution

- Approaches gaussian about 1 for large N

$$f(x; d_1, d_2) = \frac{\sqrt{\frac{(d_1 x)^{d_1}\, d_2^{d_2}}{(d_1 x + d_2)^{d_1+d_2}}}}{x\,\mathrm{B}\!\left(\frac{d_1}{2}, \frac{d_2}{2}\right)}$$
$$= \frac{1}{\mathrm{B}\!\left(\frac{d_1}{2}, \frac{d_2}{2}\right)} \left(\frac{d_1}{d_2}\right)^{\frac{d_1}{2}} x^{\frac{d_1}{2}-1} \left(1 + \frac{d_1}{d_2}\,x\right)^{-\frac{d_1+d_2}{2}}$$



The F is for Fisher
He’s the dapper gent above

Now do Frequentist F-test

- n-groups of **fits** each with separate fitted likelihoods

Likelihood of a fit PolX

- Define : $\boxed{\bar{X}_j = \frac{1}{m}\sum_{i=1}^{m} X_i}$ and $\mathrm{S}_\mathrm{j}^2 = \frac{1}{\mathrm{m}-1}\sum_{\mathrm{i}=1}^{\mathrm{m}} (\mathrm{X}_\mathrm{i} - \bar{\mathrm{X}}_\mathrm{j})^2$

- Define: $\boxed{MS_B = \frac{m}{n-1}\sum_{i=1}^{n} (\bar{X}_j - \bar{X})^2}$ and $\mathrm{MS}_\mathrm{R} = \sigma^2$

Difference in likelihoods

- If $\mu_i = \mu$ or in other words are from the same distribution the

- $\frac{MS_B}{MS_R} \approx 1 = F_{n-1\ m(n-1)}$ where $F_{n-1\ m(n-1)}(x)$ is an F distrib

Higher Order Polynomial

- $$\frac{\frac{RSS_1 - RSS_2}{p_2 - p_1}}{\frac{RSS_2}{n - p_2}} \approx F_{p_2 - p_1 \; n - p_2} \text{ generically}$$
- $$\frac{\frac{\mathscr{L}_1 - \mathscr{L}_2}{p_2 - p_1}}{\frac{\mathscr{L}_2}{n - p_2}} \approx F_{p_2 - p_1 \; n - p_2}$$

Test of higher polynomial order with F-test form is called the chow test

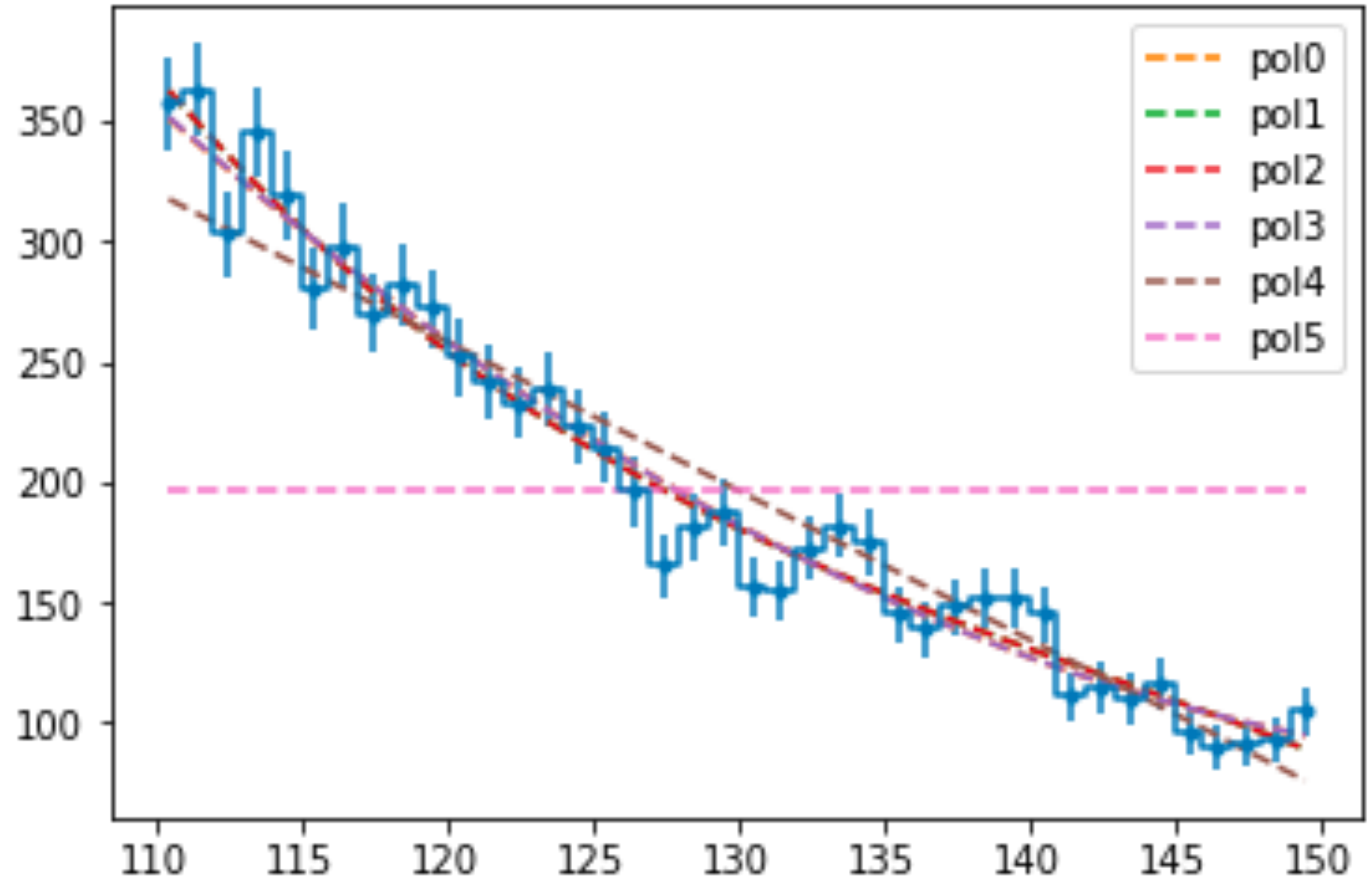
Higher Order Polynomial

- $$\frac{\frac{RSS_1 - RSS_2}{p_2 - p_1}}{\frac{RSS_2}{n - p_2}} \approx F_{p_2-p_1\ n-p_2}$$ generically

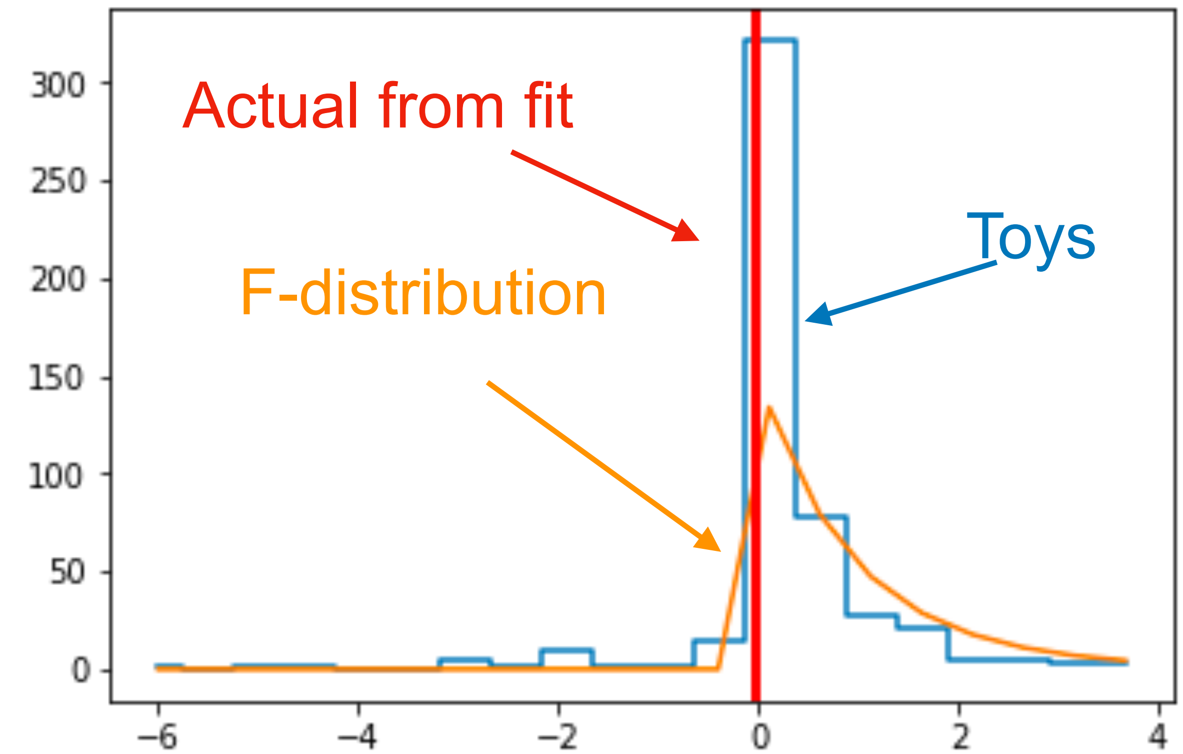
- $$\frac{\frac{\mathscr{L}_1 - \mathscr{L}_2}{p_2 - p_1}}{\frac{\mathscr{L}_2}{n - p_2}} \approx F_{p_2-p_1\ n-p_2}$$

F-test Example

- Here is an example from previous fit
- Our Actual Δ (x-axis) is consist with a high p-value



Toys: Randomly sample 3rd order dist and fit with 4th order

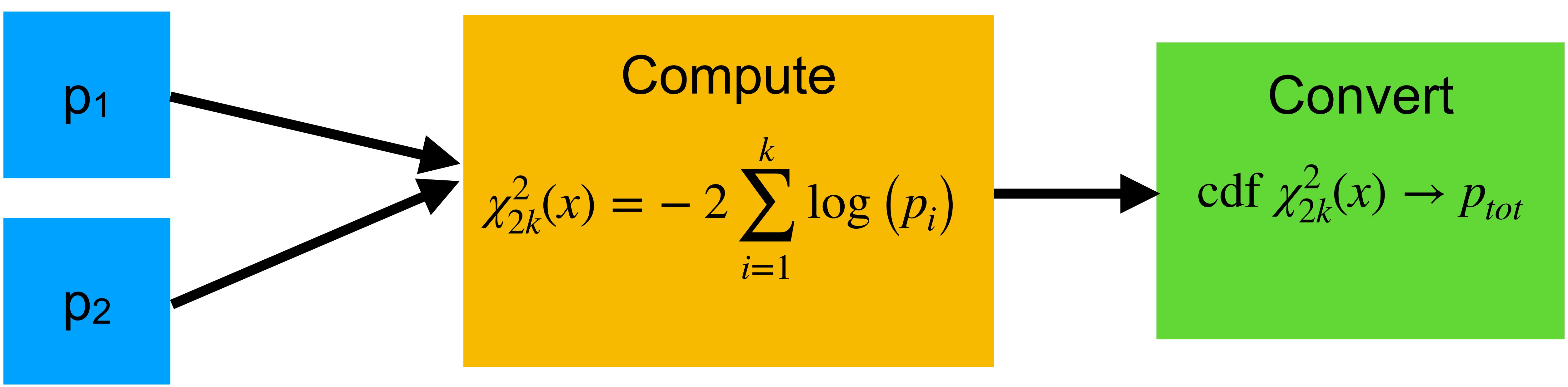


$$\frac{\frac{\mathcal{L}_1 - \mathcal{L}_2}{p_2 - p_1}}{\frac{\mathcal{L}_2}{n - p_2}}$$

For a 4th order to a 3rd order

Combining Categories

- Lets say we have *k* measurements with probability p_i
  - Each measurement is independent of each other
- We can combine categories using the following formula
  - $$\chi^2_{2k}(x) = -2\sum_{i=1}^{k}\log\left(p_i\right)$$
  - Where left is a chi2 distribution with 2k degrees of freedom



Combining Categories

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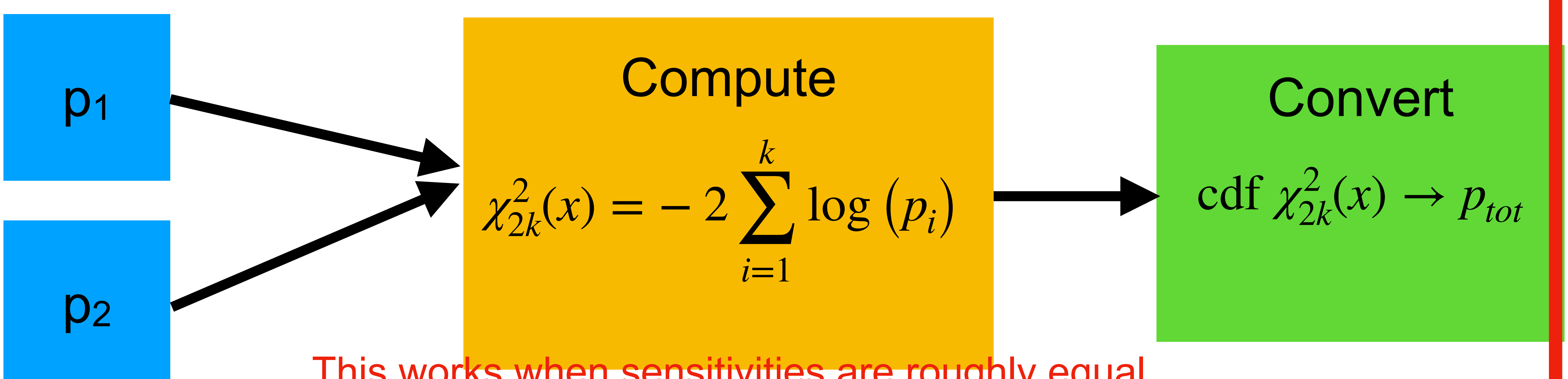
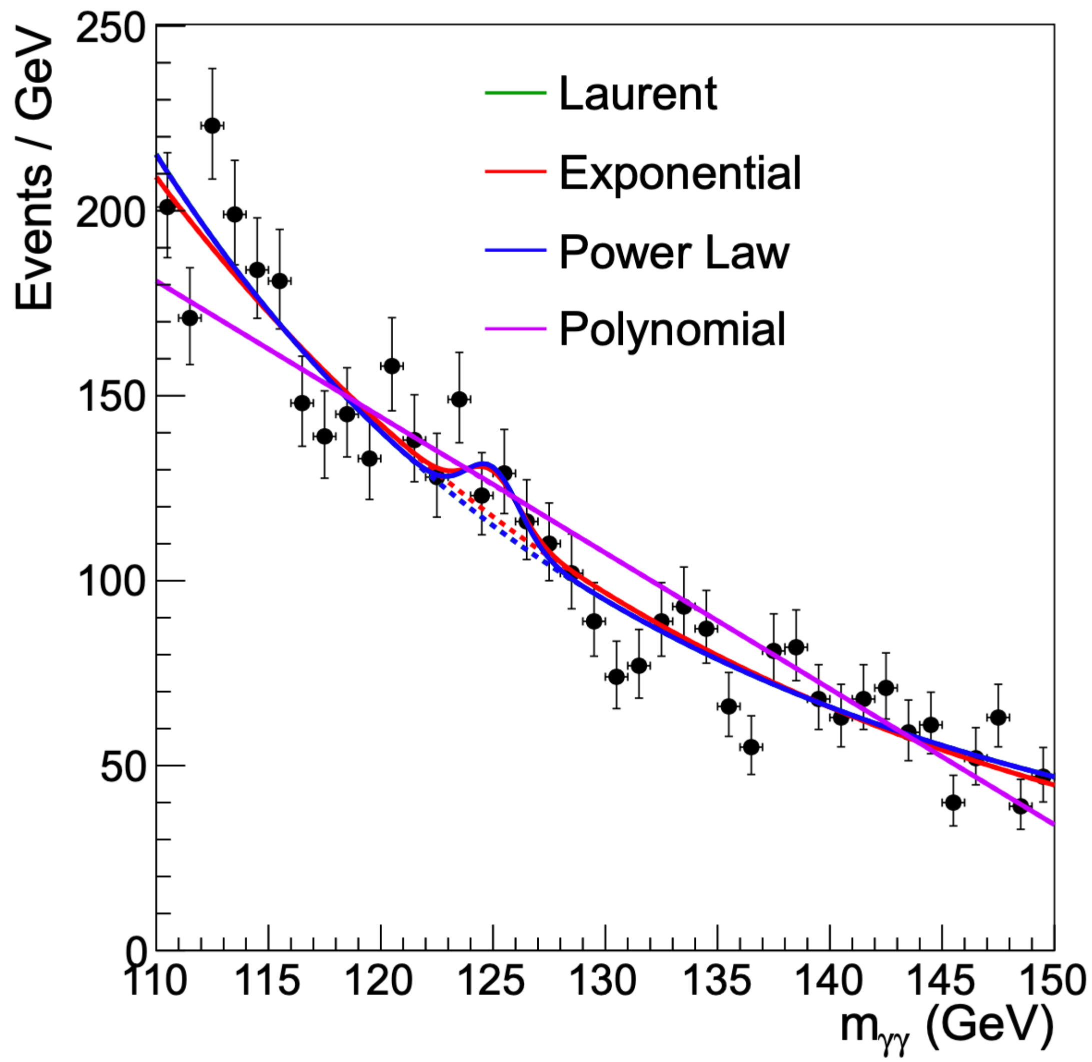
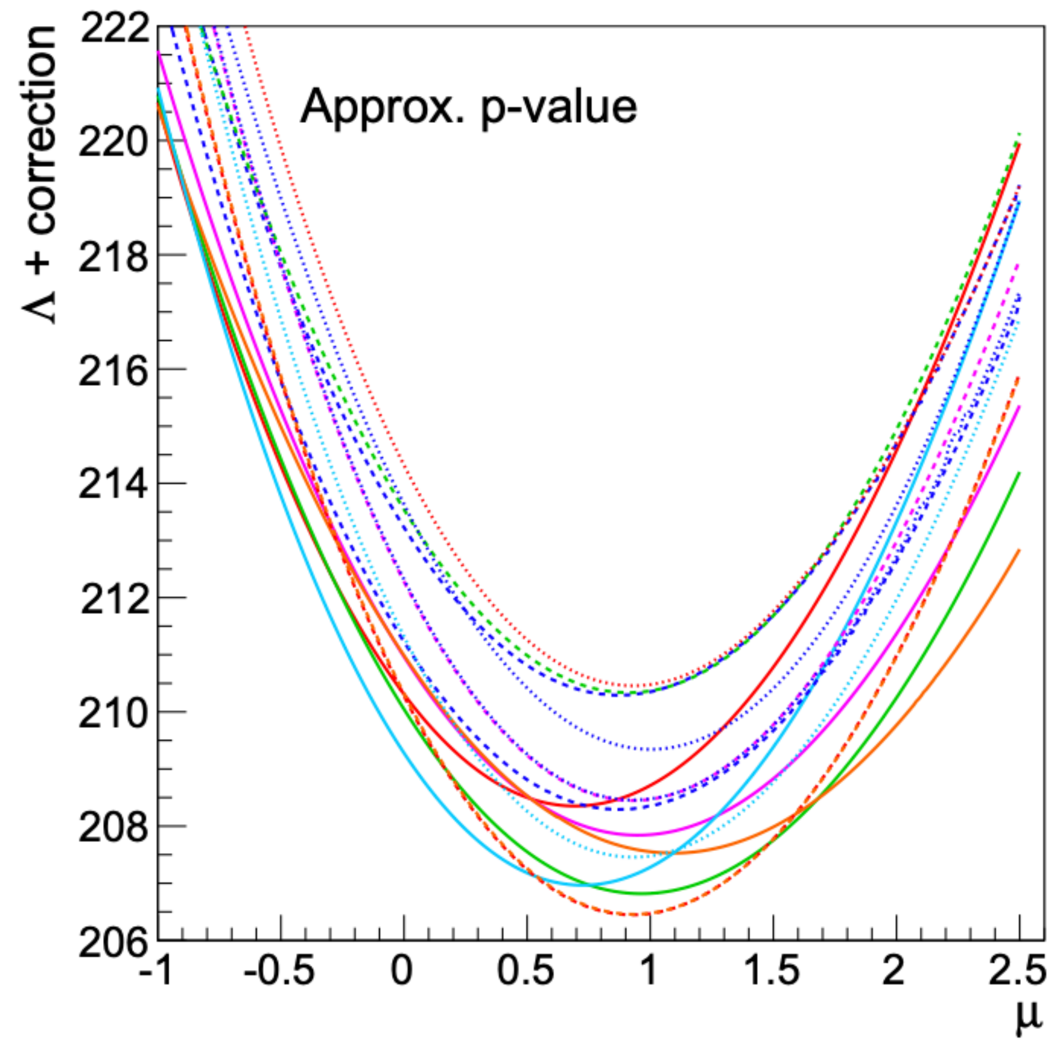


This works when sensitivities are roughly equal
More robust way is to fit all categories simulatneously

Building a Model

- Throwing a barrage of functions at the problem

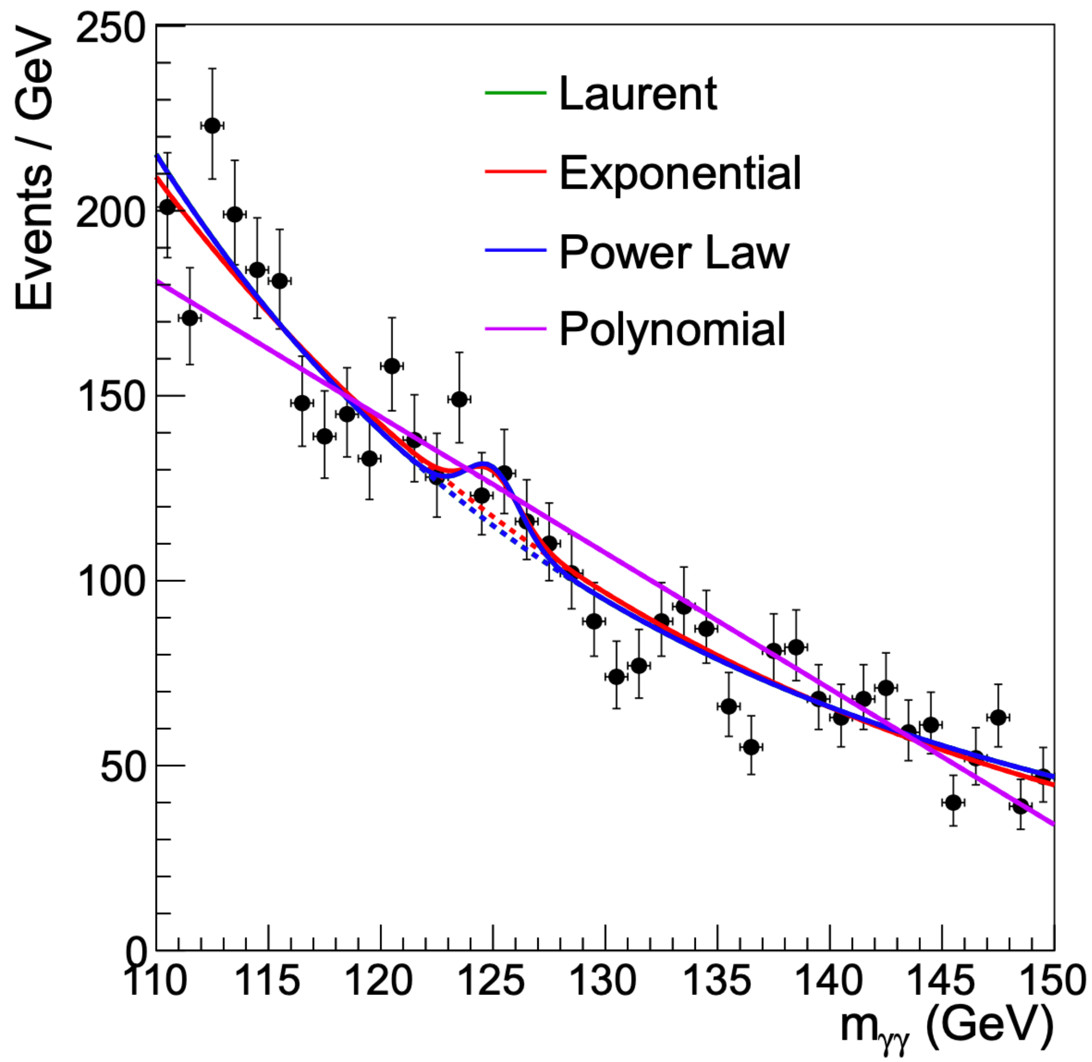


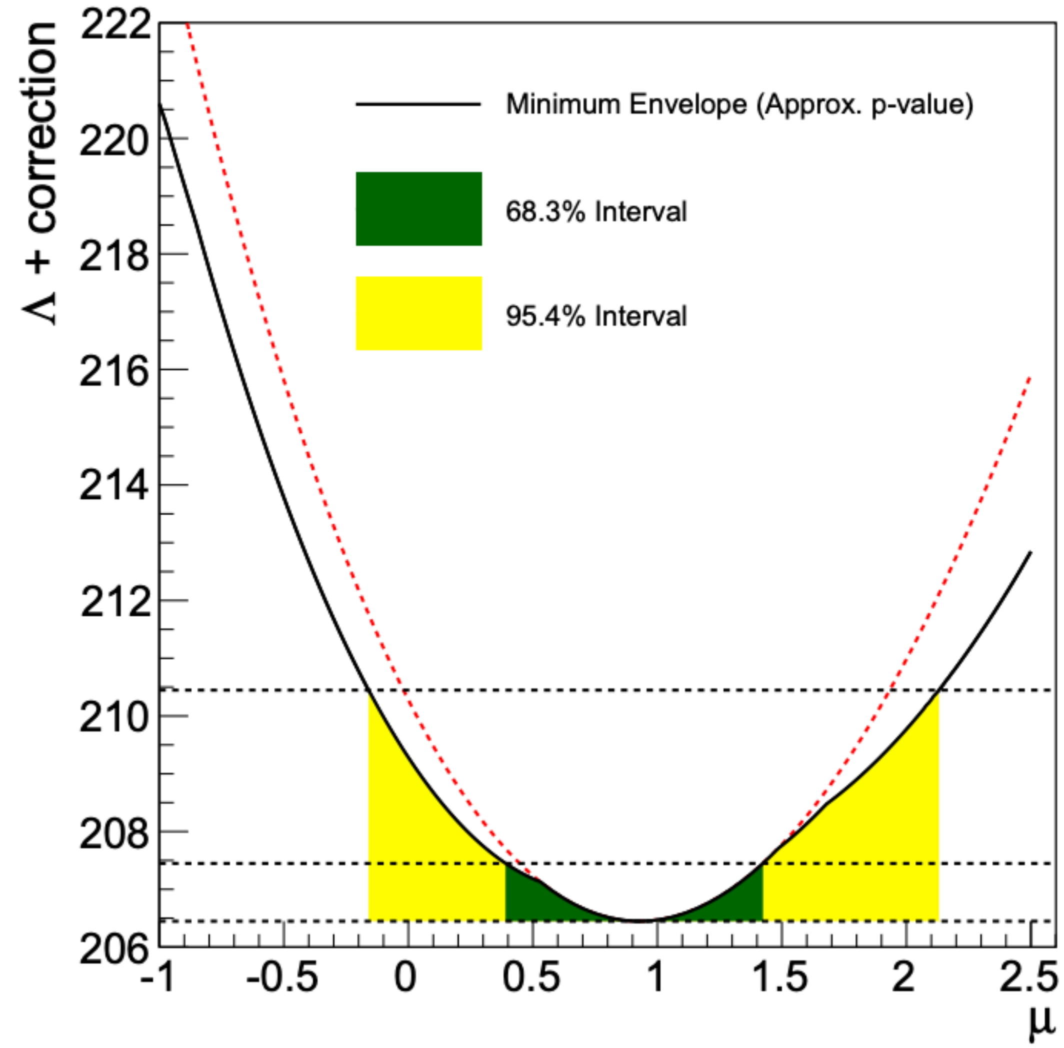


We can try a whole library of functions
The likelihood we get translates to our fit

Building a Model

- Throwing a barrage of functions at the problem

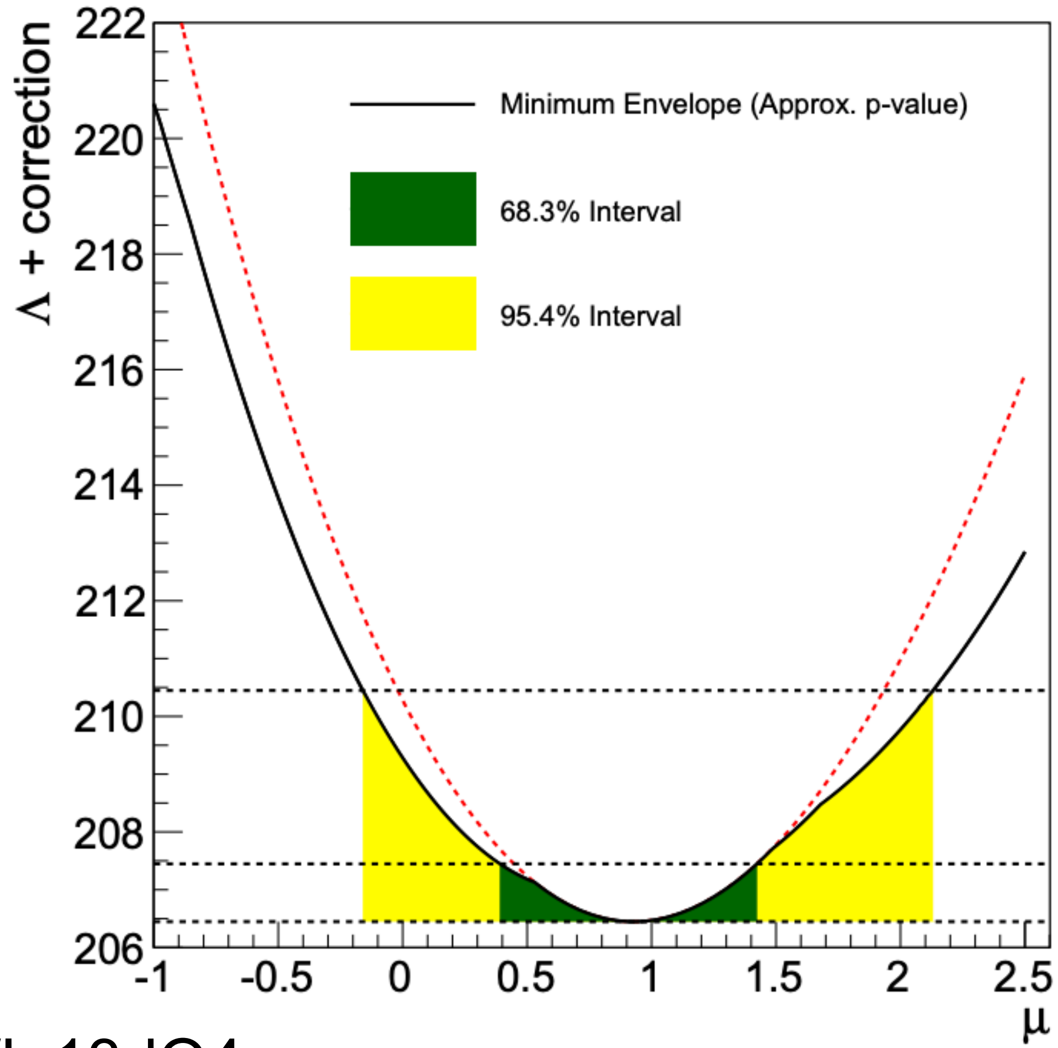




We can try a whole library of functions
The likelihood we get translates to our fit

Combined Likelihood

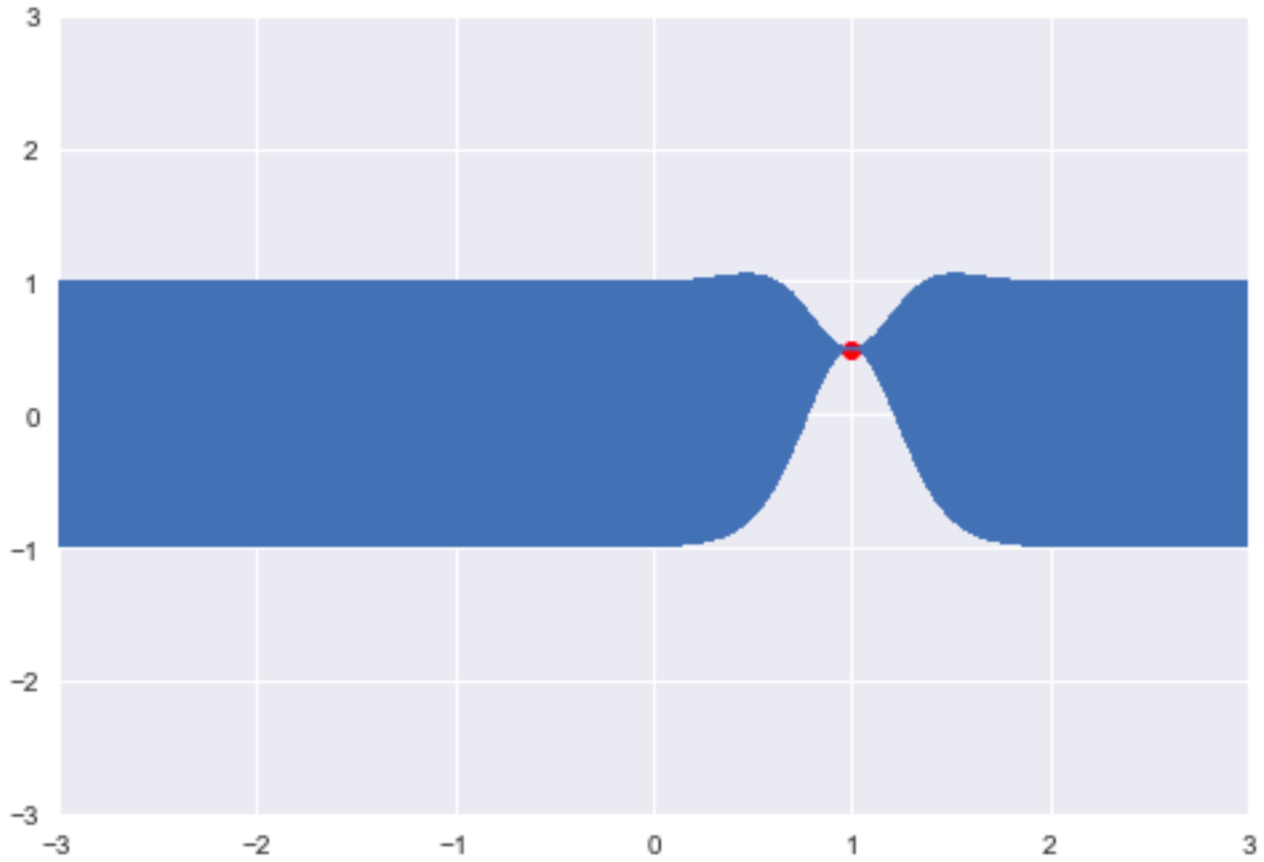
- Throwing a barrage of functions at the problem



https://www.youtube.com/watch?v=3cHWIp13dQ4

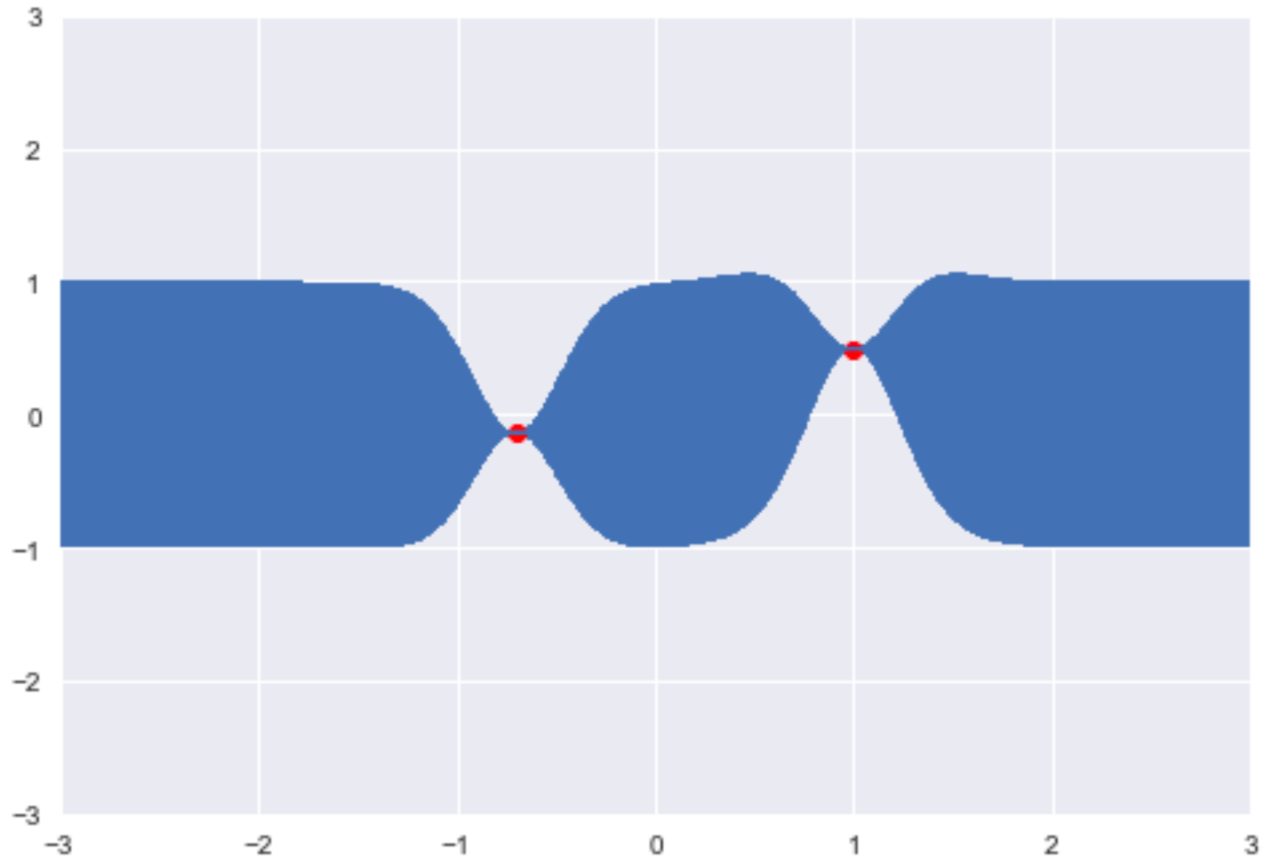
We can try a whole library of functions
The likelihood we get translates to our fit

Splines+GaussianProcesses



- Gaussian processes allow us to build function choice from the data

Splines+GaussianProcesses



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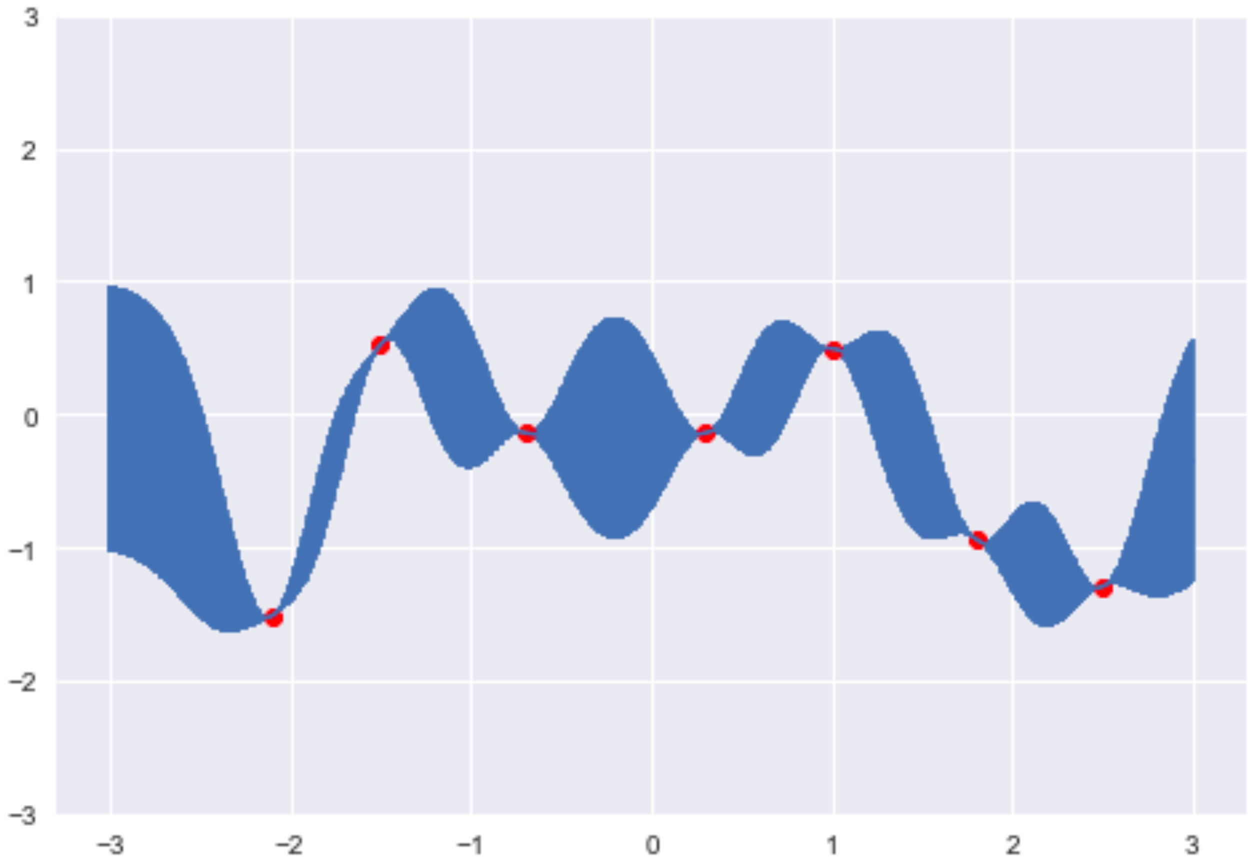
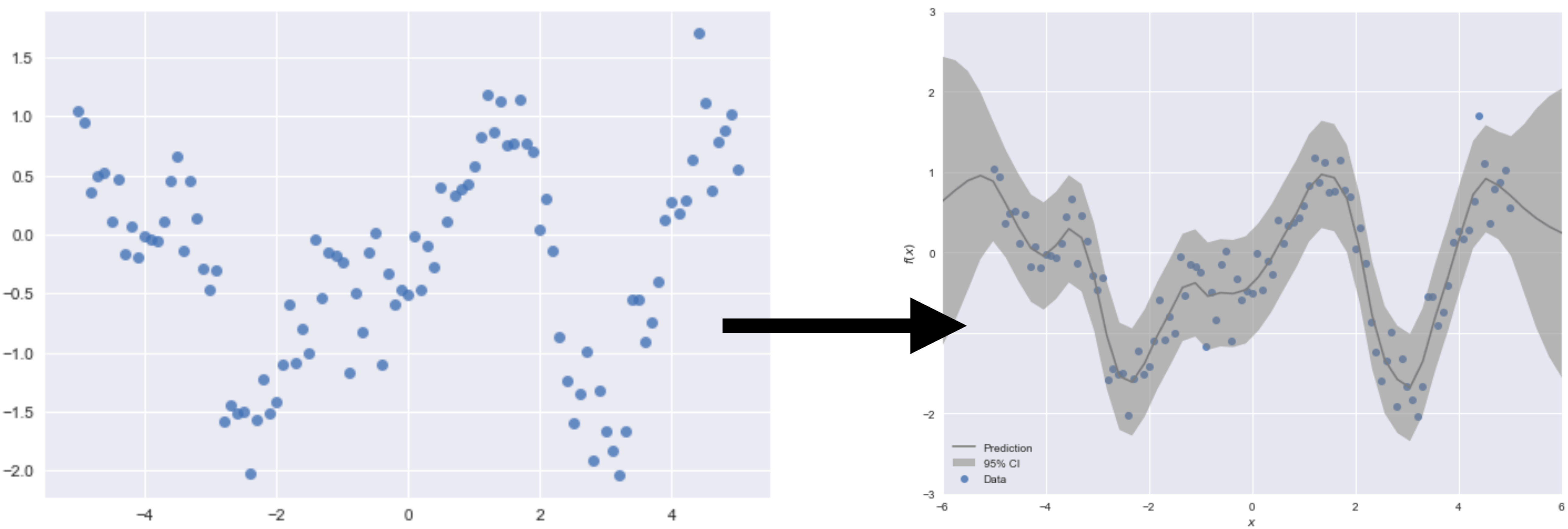
Splines+GaussianProcesses



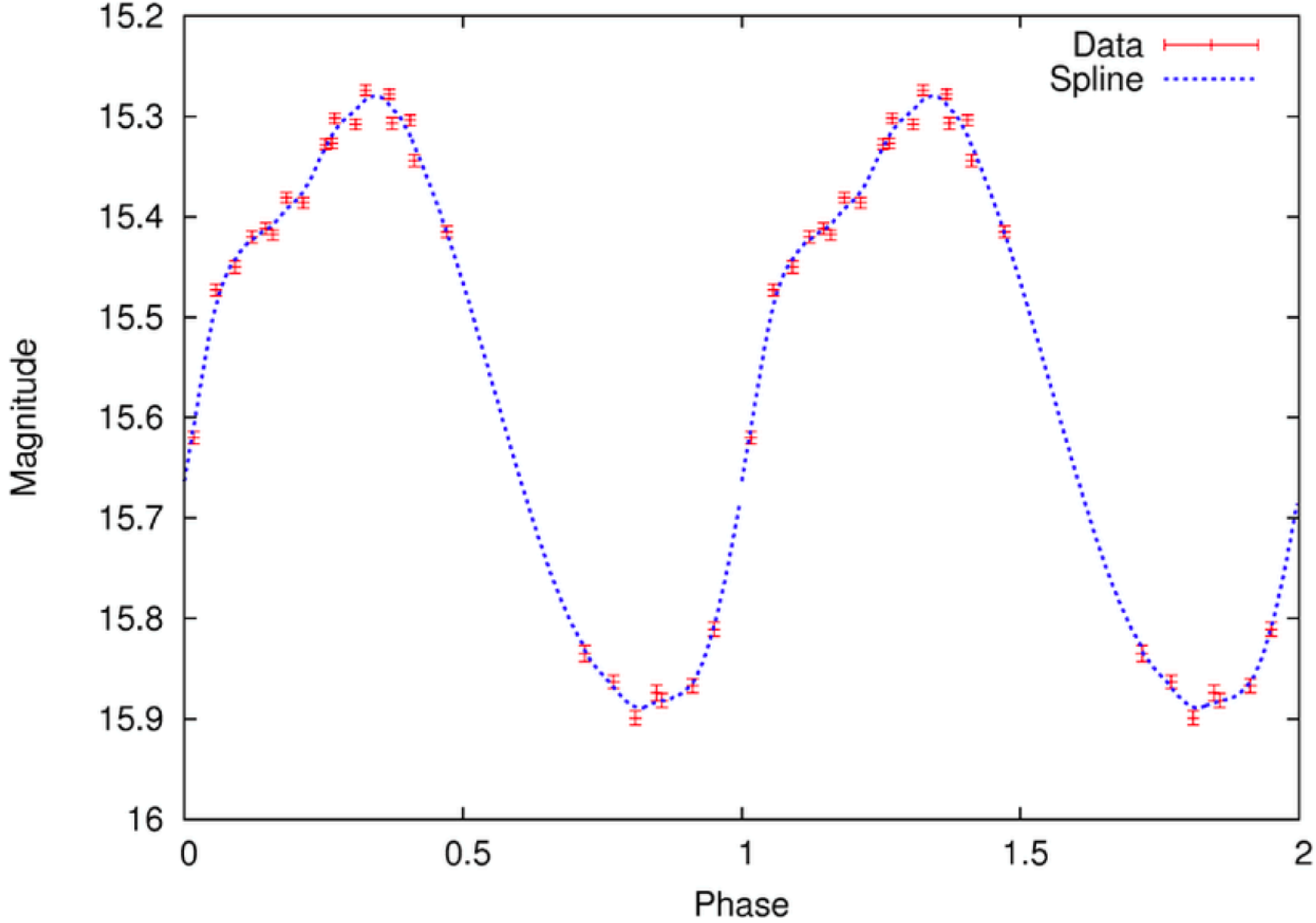
- Gaussian processes allow us to build function choice from the data

Splines+GaussianProcesses



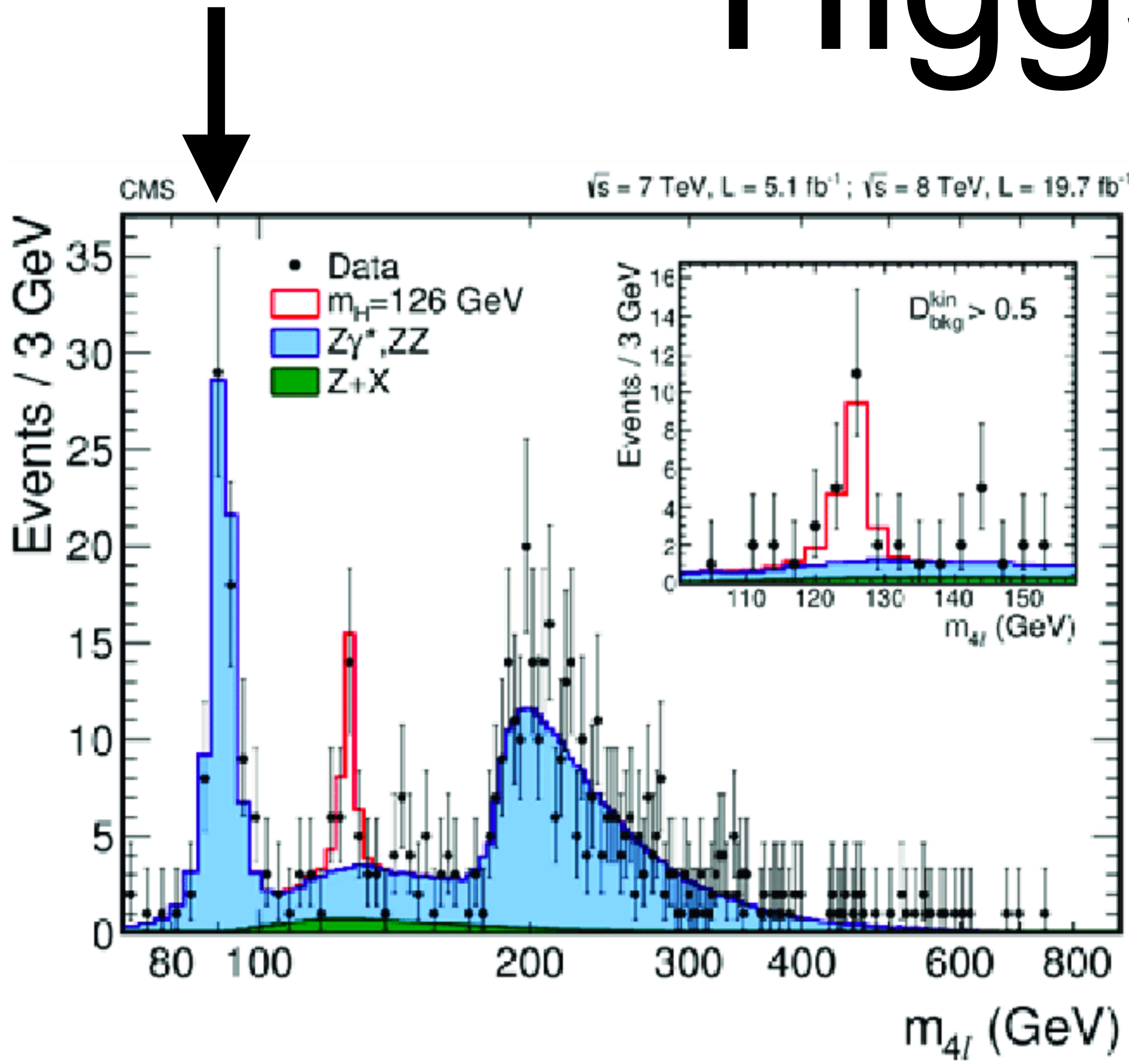
- To go from points to spline automatically

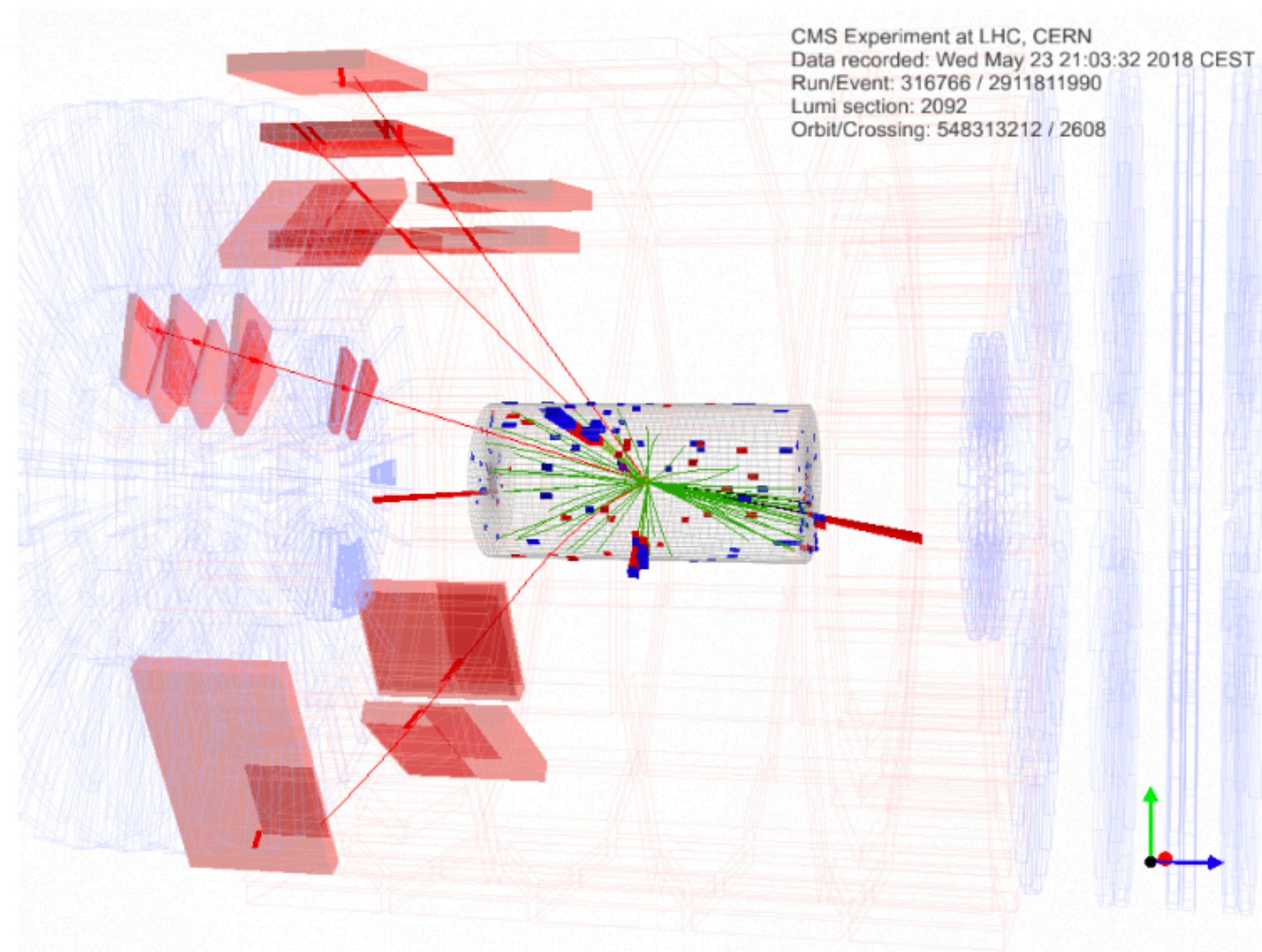
Splines+GaussianProcesses



Z-boson Peak

Higgs to 4 Leptons





- Higgs to 4 leptons aims at taking the mass of 4 leptons
  - A way to test the 4-leptons is the Z boson peak

Backup

Remind me at some point

Explain the Chow Test

Higher Order Polynomial

Test of higher polynomial order with F-test form is called the chow test

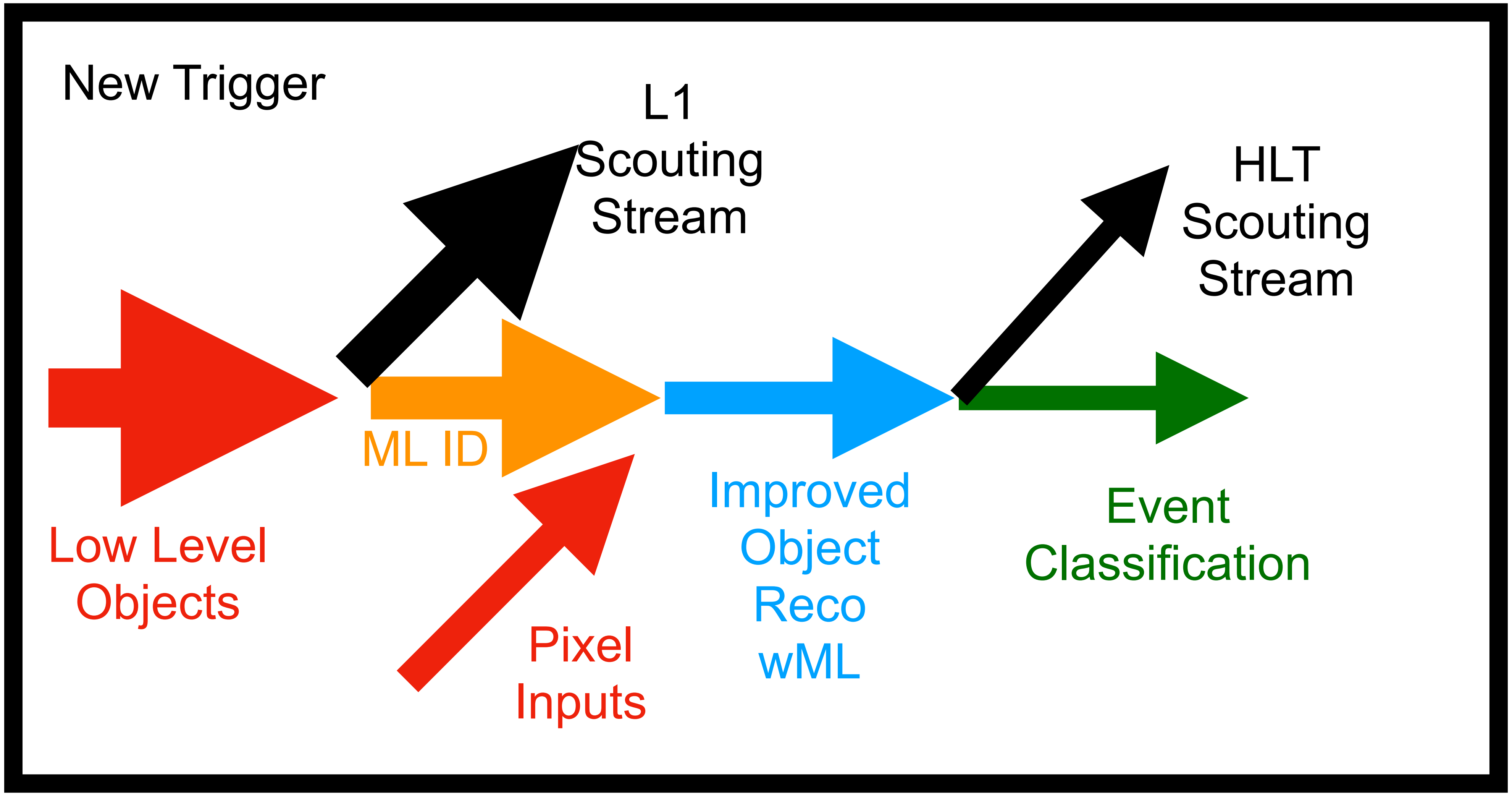
- We can evaluate this through an F-test

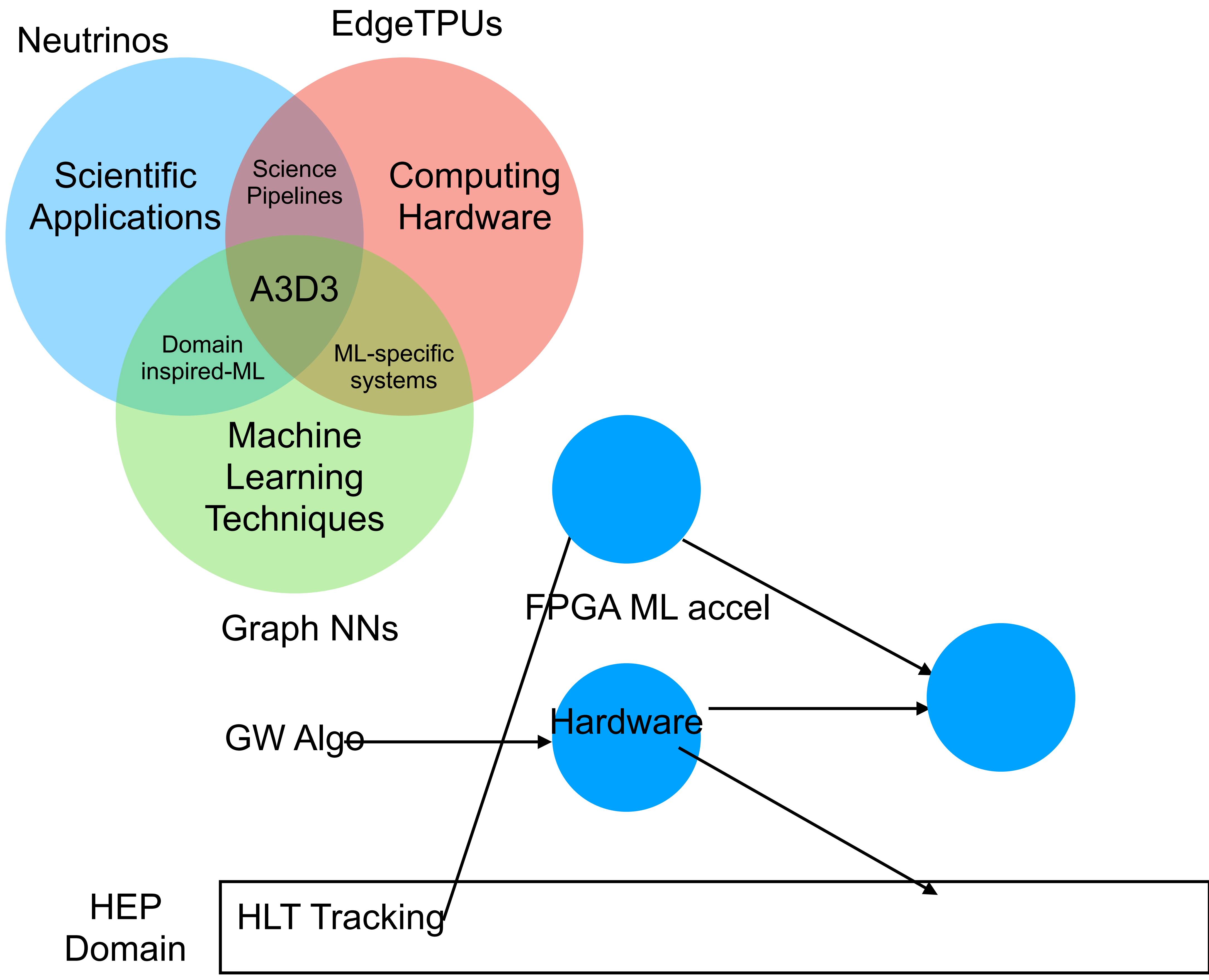
- Recall $\frac{MS_B}{MS_R} \approx 1 = F_{n-1\ m(n-1)}$

- Here we have $\frac{MS_B}{MS_R} \approx 1 = F_{p_2 - p_1\ n-p_2}$

$$F = \frac{\left(\frac{\mathrm{RSS}_1 - \mathrm{RSS}_2}{p_2 - p_1}\right)}{\left(\frac{\mathrm{RSS}_2}{n - p_2}\right)},$$

Title Text


New Trigger
L1
Scouting
Stream
HLT
Scouting
Stream
ML ID
Low Level
Objects
Pixel
Inputs
Improved
Object
Reco
wML
Event
Classification


Neutrinos
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Scientific Applications
Science Pipelines
Computing Hardware
A3D3
Domain inspired-ML
ML-specific systems
Machine Learning Techniques
Graph NNs
FPGA ML accel
GW Algo
Hardware
HEP Domain
HLT Tracking

Elastic Scatter