Compendium of Nutshell Formula

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Abstract:
This document contains essential physics formulas for quick reference.

Contents

1 QM Basics

1.1 Schrodinger Equation

$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$

1.2 Dirac Equation

1.3 Heisenberg picture

1.4 Moments

1.5 Spin

1.6 Isospin

1.7 Fermi–Dirac statistics

1.8 Bose-Einstein statistics

1.9 Fine-structure constant for electromagnetism

1.10 Compton wavelength

2 Particle Physics

2.1 Standard model Lagrangian

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \tag{1}$$

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} \tag{2}$$

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{generations}} \left[\bar{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \bar{L}_L i\gamma^\mu D_\mu L_L + \bar{e}_R i\gamma^\mu D_\mu e_R\right] \tag{3}$$

$$\mathcal{L}_{\text{Higgs}} = (D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \tag{4}$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \tag{5}$$

$$\mathcal{L}_{\text{Yukawa}} = -\left[y_u\bar{Q}_L\tilde{\phi}u_R + y_d\bar{Q}_L\phi d_R + y_e\bar{L}_L\phi e_R + \text{h.c.}\right] \tag{6}$$

2.2 Standard Model Framework

- $\mathcal{L}_{\text{Gauge}}$: The gauge field kinetic terms describe the dynamics of the gauge bosons.
  - $-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}$: Kinetic term for the $SU(2)_L$ gauge fields W^a_μ.
  - $-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$: Kinetic term for the $U(1)_Y$ gauge field B_μ.
  - $-\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$: Kinetic term for the $SU(3)_C$ gauge fields G^a_μ.
- $\mathcal{L}_{\text{Fermion}}$: The fermion kinetic terms for the Standard Model fermions (quarks and leptons).
  - Each term represents the kinetic energy of the fermions and their interaction with the gauge fields through the covariant derivative D_μ.

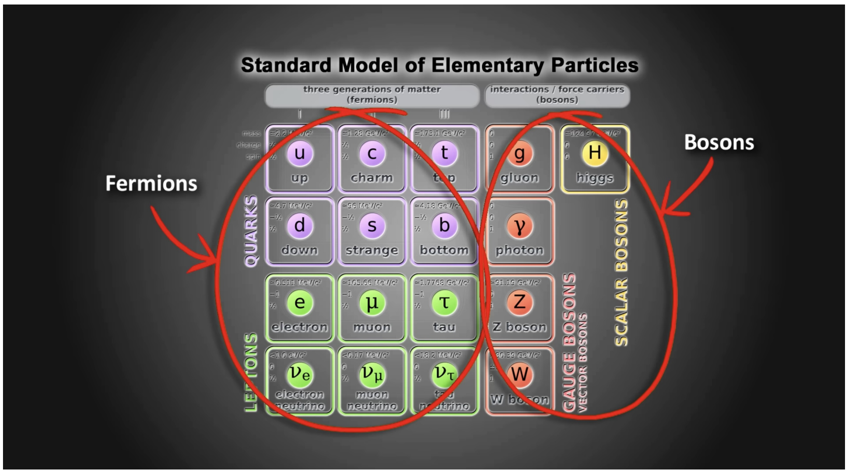

Figure 1: A description of the image.

- $\bar{Q}_L$, $\bar{u}_R$, $\bar{d}_R$, $\bar{L}_L$, and $\bar{e}_R$ represent the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, and right-handed charged leptons, respectively.

- $\mathcal{L}_{\text{Higgs}}$: The Higgs field kinetic term and its potential.
  - $(D^\mu\phi)^\dagger(D_\mu\phi)$: Kinetic term for the Higgs field ϕ, which also includes interactions with the gauge fields through the covariant derivative D_μ.
  - $V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$: The Higgs potential, responsible for spontaneous symmetry breaking and giving masses to the gauge bosons.

- $\mathcal{L}_{\text{Yukawa}}$: The Yukawa interactions between the Higgs field and fermions.
  - y_u, y_d, and y_e are the Yukawa coupling constants for the up-type quarks, down-type quarks, and charged leptons, respectively.
  - These terms provide masses to the fermions after the Higgs field acquires a vacuum expectation value (VEV).

2.3 Dark matter beyond standard model

2.4 Elementary Particles

Lepton, Hadron(Baryon, Meson), Fermion, Boson Leptons are fundamental particles that do not experience the strong nuclear force and include the electron (e^-), muon (μ^-), tau (τ^-), and their corresponding neutrinos (ν_e, ν_μ, ν_τ). Hadrons are composite particles made of quarks, categorized into baryons (such as protons and neutrons, consisting of three quarks) and mesons (such as pions, consisting of a quark and an antiquark). Fermions, which have half-integer spin and obey the Pauli exclusion principle, include both quarks and leptons, forming the building blocks of matter. Bosons are particles with integer spin that mediate the fundamental forces of nature, such as the photon for the electromagnetic force, the W and Z bosons for the weak force, and the gluon for the strong force; they also include the Higgs boson, responsible for mass via the Higgs mechanism.

2.5 Quasi Particles

Graviton, Phonon, Roton, Axion

2.6 Gell-Mann–Nishijima formula

2.7 Klein-Gordon Equation and Coefficient

2.8 Yukawa Potential

2.9 Gauge theory

3 Particle physics

3.1 Chirality

3.2 Helicity

3.3 Three channels

3.4 Flavors

3.4.1 isospin and SU(2)

3.4.2

4 Quantum Field Theory

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5 GR&Quantum Gravity

5.1 Einstein field equation

$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$

- **Holographic relationship**
- **Ampère's Law:**

6 Machine Learning

- Supervised Learning
- Unsupervised Learning
- Reinforcement Learning
- Transfer Learning
- Ensemble Learning

6.1 Concepts

6.1.1 Supervised learning

A supervised learning algorithm primarily used for classification tasks. It creates a decision boundary that maximizes the distance between different classes to improve generalization.

6.1.2 Unsupervised learning

6.2 Linear regression

A supervised learning algorithm used to model the relationship between a continuous target variable and one or more independent variables by fitting a linear equation to the data.

6.3 Support Vector Machine(SVM)

6.4 Naive Bayes

A supervised learning algorithm used for classification tasks. It assumes independence between features, which makes it fast but less accurate when features are correlated.

6.5 K-Nearest Neighbors (KNN)

A supervised learning algorithm used for both classification and regression. It determines the class of a data point based on the majority class among its nearest neighbors, but can be slow with large datasets and sensitive to outliers.

6.6 Decision Trees and Random Forests:

Decision trees use a series of questions to partition data for classification or regression. Random forests are ensembles of decision trees that reduce overfitting and increase accuracy by combining multiple decision trees using techniques like bagging and feature randomness.

7 Deep Learning

7.1 Gradient-based optimization

7.1.1 Stochastic Gradient Descent

8 Just Programming

8.1 tricks

8.2 @function

8.3 %%time

8.3.1 .copy

8.3.2 a lambda function

8.4 Monte Carlo

8.5 Markov chain

8.6 Functional programming

Functional programming aims on reducing the fequency of for/while loops and focus on the actual computation part, mainly includes three methods: map, filter, and reduce. For larger sample, functional programming could save decent amount of cpu time.

```
res = []
for i in range(len(data)):
    res.append(data[i]*2)

res = []
for n in data:
    res.append(n * 2)

res = [ n*2 for n in data]
```

8.6.1 map

```
nums = [1, 2, 3, 4]
squares = list(map(lambda x: x ** 2, nums))
print(squares)  # Output: [1, 4, 9, 16]

```

8.6.2 filter

```
nums = [1, 2, 3, 4]
even_nums = list(filter(lambda x: x % 2 == 0, nums))
```

```python
    print(even_nums)  # Output: [2, 4]

```

8.6.3 reduce

```python
    from functools import reduce
    nums = [1, 2, 3, 4]
    product = reduce(lambda x, y: x * y, nums)
    print(product)  # Output: 24

```

8.7 Decorator

```python
    def repeat(num_times):
        def decorator_repeat(func):
            def wrapper(*args, **kwargs):
                for _ in range(num_times):
                    func(*args, **kwargs)
            return wrapper
        return decorator_repeat

    @repeat(num_times=3)
    def greet(name):
        print(f"Hello, {name}!")

    greet("Alice")
#output:
#Something is happening before the function is called.
#Hello!
#Something is happening after the function is called.
```

8.8 Multi-thread in Python

9 A little bit of math

9.1 Vector, form, and tensor

10 Continuum mechanics

10.1 Navier-Stock equation