MAT 242 Written Homework #6

ASU ID: 1228859049
Kaushik Karumudi

1.A, 6.4, 6.A

1. Let W be the subspace by $\left\{ \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ -4 \\ -7 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 3 \\ -9 \end{bmatrix} \right\}$

Note that this basis is not orthogonal

a) Find the orthogonal projection of $\begin{bmatrix} 3 \\ 1 \\ 9 \\ 3 \end{bmatrix}$ into W

$$A = \begin{bmatrix} -1 & -5 & 6 \\ 0 & 0 & 0 \\ -2 & -4 & 3 \\ -2 & -7 & -9 \end{bmatrix}; \quad \vec{u} = \begin{bmatrix} 3 \\ 1 \\ 9 \\ 3 \end{bmatrix}$$

$$\vec{p} = A(A^TA)^{-1}(A^T\vec{u})$$

$$\vec{p} = \begin{bmatrix} -1 & -5 & 6 \\ 0 & 0 & 0 \\ -2 & -4 & 3 \\ -2 & -7 & -9 \end{bmatrix} \begin{bmatrix} 9 & 27 & 18 \\ 27 & 90 & 81 \\ 18 & 81 & 126 \end{bmatrix}^{-1} \begin{bmatrix} -27 \\ -72 \\ -18 \end{bmatrix}$$

$$\vec{p} = \begin{bmatrix} -1 & -5 & 6 \\ 0 & 0 & 0 \\ -2 & -4 & 3 \\ -2 & -7 & -9 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\boxed{\vec{p} = \begin{bmatrix} 3 \\ 0 \\ 9 \\ 3 \end{bmatrix}}$$

b) Find an orthogonal basis for W:

$$NEW_1 = OLD_1 = \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix}$$

$$NEW_2 = OLD_2 - \frac{OLD_2 \cdot NEW_1}{NEW_1 \cdot NEW_1} \cdot NEW_1$$

$$NEW_2 = \begin{bmatrix} -5 \\ 0 \\ -4 \\ -7 \end{bmatrix} - \frac{27}{9} \cdot \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \\ -1 \end{bmatrix}$$

$$NEW_3 = OLD_3 - \frac{OLD_3 \cdot NEW_1}{NEW_1 \cdot NEW_1} \cdot NEW_1 - \frac{OLD_3 \cdot NEW_2}{NEW_2 \cdot NEW_2} \cdot NEW_2$$

$$NEW_3 = \begin{bmatrix} -6 \\ 0 \\ 3 \\ -9 \end{bmatrix} - \frac{18}{9} \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix} - \frac{27}{9} \begin{bmatrix} -2 \\ 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix}$$

$$\text{Orthogonal basis} = \left\{ \begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix} \right\}$$

$$|NEW_1| = \sqrt{NEW_1 \cdot NEW_1} = 3$$
$$|NEW_2| = \sqrt{NEW_2 \cdot NEW_2} = 3$$
$$|NEW_2| = \sqrt{NEW_3 \cdot NEW_3} = 3$$

$$\text{Orthonormal basis} = \left\{ \frac{1}{3}\begin{bmatrix} -1 \\ 0 \\ -2 \\ -2 \end{bmatrix}, \frac{1}{3}\begin{bmatrix} -2 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \frac{1}{3}\begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix} \right\}$$

2. Find the best-fitting curve of each type below, using $(-5,-146)$, $(-2,-17)$, $(-1,-6)$, and $(3,38)$ for data points

a) General parabolas: $y = ax^2 + bx + c$

$$\begin{aligned} 25a - 5b + c &= -146 \\ 4a - 2b + c &= -17 \\ a - b + c &= -6 \\ 9a + 3b + c &= 38 \end{aligned}$$

$$A = \begin{bmatrix} 25 & -5 & 1 \\ 4 & -2 & 1 \\ 1 & -1 & 1 \\ 9 & 3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -146 \\ -17 \\ -6 \\ 38 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = (A^TA)^{-1}(A^TB)$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 723 & -107 & 39 \\ -107 & 39 & -5 \\ 39 & -5 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -3382 \\ 884 \\ -131 \end{bmatrix} = \begin{bmatrix} -3.444 \\ 15.866 \\ 20.665 \end{bmatrix}$$

$$\boxed{y = -3.444x^2 + 15.866x + 20.665}$$

b) Parabolas symmetric about the y-axis: $y = ax^2 + c$

$$25a - 5b + c = -146$$
$$4a - 2b + c = -17$$
$$a - b + c = -6$$
$$9a + 3b + c = 38$$

$$A = \begin{bmatrix} 25 & 1 \\ 4 & 1 \\ 1 & 1 \\ 9 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -146 \\ -17 \\ -6 \\ 38 \end{bmatrix}$$

$$\begin{bmatrix} a \\ c \end{bmatrix} = (A^T A)^{-1} (A^T B)$$

$$\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 723 & 39 \\ 39 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -3382 \\ -131 \end{bmatrix} = \begin{bmatrix} -6.141 \\ 27.123 \end{bmatrix}$$

$$\boxed{y = -6.141x^2 + 27.123}$$

3. Find the Least Square Solution to the following system of linear equations:

$$\begin{aligned} x \quad\quad - z &= 1 \\ -2x + y + 4z &= -4 \\ 2y + 5z &= -7 \\ -2x - 2y - 4z &= 10 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 4 \\ 0 & 2 & 5 \\ -2 & -2 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -4 \\ -7 \\ 10 \end{bmatrix}$$

$$\hat{x} = (A^TA)^{-1}(A^TB)$$

$$\hat{x} = \begin{bmatrix} 9 & 2 & -1 \\ 2 & 9 & 22 \\ -1 & 22 & 58 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -11 \\ -38 \\ -92 \end{bmatrix}$$

$$\boxed{\hat{x} = \begin{bmatrix} -3.8462 \\ 9.2308 \\ -5.1538 \end{bmatrix}}$$

$$A\hat{x} = b$$

$$\begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 4 \\ 0 & 2 & 5 \\ -2 & -2 & -4 \end{bmatrix} \begin{bmatrix} -3.8462 \\ 9.2308 \\ -5.1538 \end{bmatrix} = \boxed{\begin{bmatrix} 1.3076 \\ -3.6920 \\ -7.3074 \\ 9.8460 \end{bmatrix} \approx \begin{bmatrix} 1 \\ -4 \\ -7 \\ 10 \end{bmatrix}}$$