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Continuous-time periodic square wave 连续时间方波

$$x(t)=\begin{cases}1 & |t|<T_1\\ 0 & T_1<|t|<\frac{T}{2}\end{cases}\qquad a_k=\frac{1}{T}\int_{-T_1}^{T_1}e^{-jk\omega_0 t}\,dt=\begin{cases}\frac{2T_1}{T} & k=0\\ -\frac{e^{-jk\omega_0T_1}-e^{jk\omega_0T_1}}{jk\omega_0T} & k\neq 0\end{cases}=\begin{cases}\frac{2T_1}{T} & k=0\\ \frac{\sin(k\omega_0T_1)}{k\pi} & k\neq 0\end{cases}$$

等比数列

通项公式：$a_n=a_1q^{n-1}$

两项关系：$a_n=a_mq^{n-m}$

求和公式：$S_n=a_1\frac{1-q^n}{1-q}=\frac{a_1-a_nq}{1-q}$

Discrete-time periodic square wave 离散时间方波 $x[n]=1$ for $-N_1\le n\le N_1$

When $k=0,\pm N,\pm 2N,\ldots,$ $a_k=\frac{1}{N}\sum_{n=<N>}x[n]e^{-jk\omega_0 n}=\frac{1}{N}\sum_{n=-N_1}^{N_1}e^{-jk\frac{2\pi}{N}n}=\frac{1}{N}\sum_{n=-N_1}^{N_1}1=\frac{2N_1+1}{N}$

When $k\neq 0,\pm N,\pm 2N,\ldots,$ $a_k=\frac{1}{N}\sum_{n=<N>}x[n]e^{-jk\omega_0 n}=\frac{1}{N}\sum_{n=-N_1}^{N_1}e^{-jk\omega_0 n}=\frac{1}{N}e^{jk\frac{2\pi}{N}N_1}\frac{1-e^{-jk\frac{2\pi}{N}(2N_1+1)}}{1-e^{-jk\frac{2\pi}{N}}}=\frac{1}{N}\frac{\sin\left(\frac{2k\pi\left(N_1+\frac{1}{2}\right)}{N}\right)}{\sin\left(\frac{k\pi}{N}\right)}$

化简技巧$1-e^{-jk\frac{2\pi}{N}}=e^{-jk\frac{2\pi}{2N}}\left(e^{jk\frac{2\pi}{2N}}-e^{-jk\frac{2\pi}{2N}}\right)$

Even and odd decomposition

$$\mathcal{E}v\{x[t]\}=\frac{1}{2}\{x(t)+x(-t)\}$$

$$\mathcal{O}d\{x[t]\}=\frac{1}{2}\{x(t)-x(-t)\}$$

奇分解围绕原点旋转除 2，偶分解围绕 y 轴变换除 2

系统性质的判据	输入输出关系	单位冲激响应
Memoryless	Output at $t=t_0$ depends only on the value of input at $t=t_0$	$h(t)=0$ for $t\neq 0$
Invertible	There exist an inverse system	$\exists h_1[n]\ s.t.\ h[n]*h_1[n]=\delta[n]$
Causality	Output at this time only depends on values of the input at the present time and in the past	$h[n]=0$ for $n<0$
Stability	BIBO	$\int_{-\infty}^{\infty}\lvert h(\tau)\rvert d\tau<\infty$
Time-invariance	Let $x_1(t)=x(t-t_0)$ Check $y_1(t)=?\,y(t-t_0)$	Must satisfy
Linearity	Let $x_1(t),x_2(t),x_3(t)=x_1(t)+x_2(t)$ Check $y_3(t)=?\,ay_1(t)+by_2(t)$	Must satisfy

Convolution 卷积

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=-\infty}^{\infty}x[n-k]h[k]$$

$$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau\int_{-\infty}^{+\infty}x(t-\tau)h(\tau)d\tau$$

一个函数的t换成τ，另一个函数的t换成$t-\tau$；如果结果以原点为界，记得写成$u[n]u(t)$的形式；$x(\tau)$的图像正常，$h(t-\tau)$的图像关于y轴翻转后，原点为t

Singularity functions

$u_k(t)$求导；$u_{-k}(t)$积分；$u_1(t)$是 unit doublet

Impulse train

$$x(t)=\sum_{k\in\mathbb{R}}\delta(t-kT)\overset{FS}{\leftrightarrow}a_k=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\delta(t)e^{-jk\omega_0 0}\,dt=\frac{1}{T}$$

Properties of convolution 卷积的性质

$x(t)*\delta(t)=x(t)$

$x(t)*\delta(t-t_0)=x(t-t_0)$

$x(t-t_1)*\delta(t-t_2)=x(t-t_1-t_2)$

$x(t)*\delta'(t)=x'(t)$

$x(t)*u(t)=x^{-1}(t)=\int_{-\infty}^{t}x(\tau)d\tau$

$x(t)*h(t)=x'(t)*h^{-1}(t)$

Euler's formula 欧拉公式

$e^{j\theta}=\cos\theta+j\sin\theta$

$\cos\theta=\frac{e^{j\theta}+e^{-j\theta}}{2}$

$\sin\theta=\frac{e^{j\theta}-e^{-j\theta}}{2j}$

Fourier Series 傅里叶级数

$x(t)=e^{st}\xrightarrow{LTI}y(t)=H(s)e^{st}$ $\sum_k a_ke^{s_kt}\xrightarrow{LTI}\sum_k a_kH(s_k)e^{s_kt}$	$x[n]z^n\xrightarrow{LTI}y[n]=H(z)z^n$ $\sum_k a_kz_k^n\xrightarrow{LTI}\sum_k a_kH(z_k)z_k^n$
$H(s)=\int_{-\infty}^{\infty}h(t)e^{-st}\,dt$ $H(j\omega)=\int_{-\infty}^{\infty}h(t)e^{-j\omega t}\,dt$	$H(z)=\sum_{k=-\infty}^{\infty}h[k]z^{-k}$ $H(e^{j\omega})=\sum_{n=-\infty}^{\infty}h[n]e^{-j\omega n}$
$x(t)=\sum_{k=-\infty}^{\infty}a_ke^{jk\omega_0t}$ $a_k=\frac{1}{T}\int_T x(t)e^{-jk\omega_0t}\,dt$	$x[n]=\sum_{k=<N>}a_ke^{jk\omega_0n}$ $a_k=\frac{1}{N}\sum_{n=<N>}x[n]e^{-jk\omega_0n}$

诱导公式

$\sin(2k\pi+\alpha)=\sin\alpha$

$\cos(2k\pi+\alpha)=\cos\alpha$

$\sin(-\alpha)=-\sin\alpha$

$\cos(-\alpha)=\cos\alpha$

$\sin(\pi-\alpha)=\sin\alpha$

$\cos(\pi-\alpha)=-\cos\alpha$

$\sin(\pi+\alpha)=-\sin\alpha$

$\cos(\pi+\alpha)=-\cos\alpha$

$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$

$\mathrm{tg}(k\pi+\alpha)=\mathrm{tg}\,\alpha$

$\mathrm{tg}(-\alpha)=-\mathrm{tg}\,\alpha$

$\mathrm{tg}(\pi-\alpha)=-\mathrm{tg}\,\alpha$

$\mathrm{tg}(\pi+\alpha)=\mathrm{tg}\,\alpha$

$\mathrm{tg}\left(\frac{\pi}{2}-\alpha\right)=\mathrm{ctg}\,\alpha$

$\mathrm{tg}\left(\frac{\pi}{2}+\alpha\right)=-\mathrm{ctg}\,\alpha$

TABLE 17.1 The method of undetermined coefficients for selected equations of the form

$$ay''+by'+cy=G(x).$$

If $G(x)$ has a term that is a constant multiple of . . .	And if	Then include this expression in the trial function for y_p.
e^{rx}	r is not a root of the auxiliary equation	Ae^{rx}
	r is a single root of the auxiliary equation	Axe^{rx}
	r is a double root of the auxiliary equation	Ax^2e^{rx}
$\sin kx$, $\cos kx$	ki is not a root of the auxiliary equation	$B\cos kx+C\sin kx$
px^2+qx+m	0 is not a root of the auxiliary equation	Dx^2+Ex+F
	0 is a single root of the auxiliary equation	Dx^3+Ex^2+Fx
	0 is a double root of the auxiliary equation	$Dx^4+Ex^3+Fx^2$

和差化积公式与积化和差公式

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$

自变量变换

Systematic approach: $x(t) \to x(t+\beta) \to x(\alpha t+\beta)$

Real, even, odd properties of a signal

If $x(t)$ is real, then from the synthesis equation:

$$\sum_{k\in\mathbb{R}} a_k e^{jk\omega_0 t} = \sum_{k\in\mathbb{R}} a_k^* e^{-jk\omega_0 t} = \sum_{-k\in\mathbb{R}} a_{-k}^* e^{jk\omega_0 t} \Rightarrow a_k = a_{-k}^*$$

If $x(t)$ is real and even, then $x(t) = x(-t) \Rightarrow a_k = a_{-k}$; $a_{-k} = a_{-k}^* \Rightarrow a_k = a_k^*$

Hence a_k is real and even.

Similarly, if $x(t)$ is real and odd, then $a_k = -a_{-k}$ and $a_k = -a_k^*$, a_k is purely imaginary and odd.

N^th order linear constant-coefficient differential equation

Expression: $\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$

Solution: $y(t) = y_p(t) + y_h(t)$, where $y_h(t) = A_1 e^{s_1 t} + \cdots + A_N e^{s_N t}$

Causal and LTI ⇔ Initial rest (i.e. if $x(t) = 0$ for $t < t_0$ then $y(t) = 0$ for $t < t_0$)

N^th order linear constant-coefficient difference equation

Expression: $\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$

Solution: $y[n] = y_p[n] + y_h[n]$, where $y_h[n] = A_1 z_1^n + \cdots + A_N z_N^n$

Causal and LTI ⇔ Initial rest (i.e. if $x[n] = 0$ for $n < n_0$ then $y[n] = 0$ for $n < n_0$)

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$, $y(t)$: Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k, b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$a_k = a^*_{-k}$; $\Re e\{a_k\} = \Re e\{a_{-k}\}$; $\Im m\{a_k\} = -\Im m\{a_{-k}\}$; $\lvert a_k\rvert = \lvert a_{-k}\rvert$; $\sphericalangle a_k = -\sphericalangle a_{-k}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real]; $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\Re e\{a_k\}$; $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$, $y[n]$: Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	a_k, b_k: Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k$ (viewed as periodic with period mN)
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k]$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$a_k = a^*_{-k}$; $\Re e\{a_k\} = \Re e\{a_{-k}\}$; $\Im m\{a_k\} = -\Im m\{a_{-k}\}$; $\lvert a_k\rvert = \lvert a_{-k}\rvert$; $\sphericalangle a_k = -\sphericalangle a_{-k}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ [x[n] real]; $x_o[n] = \mathcal{O}d\{x[n]\}$ [x[n] real]	$\Re e\{a_k\}$; $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

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System Properties (Laplace Transform)

Causality	ROC is a right-half plane
Causality (Rational $H(s)$)	ROC is the right-half plane to the right of the rightmost pole
Anticausality	ROC is a left-half plane (to the left of the leftmost pole)
Stability	ROC includes the entire $j\omega$-axis

Causal LTI system with rational $H(s)$: all poles lie in the left-half of the s-plane

System Properties (Z-Transform)

Causality	ROC is the exterior of a circle including infinity
Causality (Rational $H(s)$)	ROC is the exterior of a circle outside the outermost pole + 分子 z 阶数不能比分母大
Stability	ROC includes the unit circle

Causal LTI system with rational $H(s)$: all poles lies inside the

$e^{j\frac{3\pi}{4}} = \frac{1}{\sqrt{2}}(-1+j)$, $e^{j\frac{\pi}{2}} = j$, $e^{j\frac{\pi}{4}} = \frac{1}{\sqrt{2}}(1+j)$

$e^{j\pi} = 1$, $e^{j0} = 1$

$e^{-j\frac{3\pi}{4}} = \frac{-1}{\sqrt{2}}(1+j)$, $e^{-j\frac{\pi}{2}} = -j$, $e^{-j\frac{\pi}{4}} = \frac{1}{\sqrt{2}}(1-j)$

Euler's formula 欧拉公式

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$
$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

诱导公式

$\sin(2k\pi+\alpha) = \sin\alpha$
$\cos(2k\pi+\alpha) = \cos\alpha$
$\sin(-\alpha) = -\sin\alpha$
$\cos(-\alpha) = \cos\alpha$
$\sin(\pi-\alpha) = \sin\alpha$
$\cos(\pi-\alpha) = -\cos\alpha$
$\sin(\pi+\alpha) = -\sin\alpha$
$\cos(\pi+\alpha) = -\cos\alpha$
$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$
$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$
$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$
$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$
$\mathrm{tg}(k\pi+\alpha) = \mathrm{tg}\,\alpha$
$\mathrm{tg}(-\alpha) = -\mathrm{tg}\,\alpha$
$\mathrm{tg}(\pi-\alpha) = -\mathrm{tg}\,\alpha$
$\mathrm{tg}(\pi+\alpha) = \mathrm{tg}\,\alpha$
$\mathrm{tg}\left(\frac{\pi}{2}-\alpha\right) = \mathrm{ctg}\,\alpha$
$\mathrm{tg}\left(\frac{\pi}{2}+\alpha\right) = -\mathrm{ctg}\,\alpha$

Nyquist Sampling Theorem

Let $x(t)$ be a band-limited signal within $\pm\omega_M$, then $x(t)$ is uniquely determined by its samples $x(nT)$ if $\omega_s > 2\omega_M$, where $\omega_s = \frac{2\pi}{T}$.
Nyquist rate: $2\omega_M$

Z-Transform Z 变换

$$X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$
$$x[n] = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$$

Z 变换特殊峰的判断

信号为正，零点有峰
信号为负，无穷有峰

Fourier Transform 傅里叶变换

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k\delta(\omega-k\omega_0)$$
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

时域频域卷积相乘关系

$$y(t) = h(t)*x(t) \overset{\mathcal{F}}{\leftrightarrow} Y(j\omega) = H(j\omega)X(j\omega)$$
$$r(t) = s(t)p(t) \overset{\mathcal{F}}{\leftrightarrow} R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\theta)P\big(j(\omega-\theta)\big)d\theta$$

Convolution 卷积

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \int_{-\infty}^{+\infty} x(t-\tau)h(\tau)d\tau$$

一个函数的t换成τ，另一个函数的t换成$t-\tau$；如果结果以原点为界，记得写成$u[n]u(t)$的形式；$x(\tau)$的图像正常，$h(t-\tau)$的图像关于y轴翻转后，原点为t

Laplace Transform 拉普拉斯变换

$$X(s) \triangleq \int_{-\infty} x(t)e^{-st}dt$$
$$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$$

Sinc Function

$$\mathrm{sinc}\,\theta = \frac{\sin\pi\theta}{\pi\theta}$$

等比数列

通项公式：$a_n = a_1q^{n-1}$
两项关系：$a_n = a_mq^{n-m}$
求和公式：$S_n = a_1\frac{a-q^n}{1-q} = \frac{a_1-a_nq}{1-q}$

Other Transform Pairs

$u_n(t) = \frac{d^n}{dt^n}\delta(t) \overset{\mathcal{L}}{\leftrightarrow} s^n, u^{-n}(t) = \frac{1}{s^n}$

$e^{-at}\cos\omega_0t\,u(t) \overset{\mathcal{L}}{\leftrightarrow} \frac{s+a}{(s+a)^2+\omega_0^2}$

$e^{-at}\sin\omega_0t\,u(t) \overset{\mathcal{L}}{\leftrightarrow} \frac{\omega_0}{(s+a)^2+\omega_0^2}$

$\left(\frac{\sin t}{\pi t}\right)^2 \overset{\mathcal{F}}{\leftrightarrow}$三角形底$-2$到$2$，高$\frac{1}{\pi}$

Fourier Series 傅里叶级数

$x(t) = e^{st} \xrightarrow{LTI} y(t) = H(s)e^{st}$ $\sum_k a_ke^{s_kt} \xrightarrow{LTI} \sum_k a_kH(s_k)e^{s_kt}$	$x[n]z^n \xrightarrow{LTI} y[n] = H(z)z^n$ $\sum_k a_kz_k^n \xrightarrow{LTI} \sum_k a_kH(z_k)z_k^n$
$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt$ $H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$	$H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$ $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$
$x(t) = \sum_{k=-\infty}^{\infty} a_ke^{jk\omega_0t}$ $a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0t}\,dt$	$x[n] = \sum_{k=\langle N\rangle} a_ke^{jk\omega_0n}$ $a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0n}$

Even and odd decomposition

$$\mathcal{E}v\{x[t]\} = \frac{1}{2}\{x(t)+x(-t)\}$$
$$\mathcal{O}d\{x[t]\} = \frac{1}{2}\{x(t)-x(-t)\}$$

奇分解围绕原点旋转除 2，偶分解围绕 y 轴变换除 2

和差化积公式与积化和差公式

$\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha-\sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha-\cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$
$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$
$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$

采样与恢复的过程

$$p(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT) \overset{\mathcal{F}}{\leftrightarrow} \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-k\frac{2\pi}{T}\right) = \omega_s\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_s)$$
$$p(t)x(t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2\pi}X(j\omega)*P(j\omega) = \frac{\omega_s}{2\pi}\sum_{k=-\infty}^{\infty}X\big(j(\omega-k\omega_s)\big) = \frac{1}{T}\sum_{k=-\infty}^{\infty}X\big(j(\omega-k\omega_s)\big)$$

采样后信号的频率响应幅值变为$1/T$倍，因此恢复信号时要通过一个增益为T的 BPF，BPF 的截止频率最好为$\frac{\omega_s}{2}$

Hilbert Transform

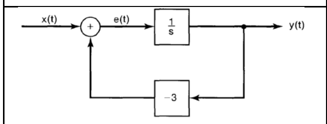
$$\frac{1}{\pi t} \overset{\mathcal{F}}{\leftrightarrow} -j\cdot\mathrm{sign}\,\omega, \cos\omega_0t \xrightarrow{h(t)} \sin\omega_0t$$

Quick Reference Checklist 快速检查单

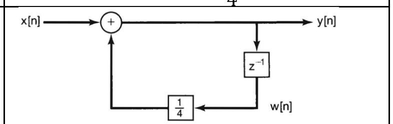
Signal	Transform	ROC	Pole
$e^{-at}u(t)$	$\frac{1}{s+a}$	$Re\{s\} > -a$	$-a$
$e^{-at}u(-t)$	$-\frac{1}{s+a}$	$Re\{s\} < -a$	$-a$
$e^{at}u(t)$	$\frac{1}{s-a}$	$Re\{s\} > a$	a
$e^{at}u(-t)$	$-\frac{1}{s-a}$	$Re\{s\} < a$	a
$a^nu[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$	a
$a^nu[-n-1]$	$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$	a
$(-a)^nu[n]$	$\frac{1}{1+az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$	$-a$
$(-a)^nu[-n-1]$	$-\frac{1}{1+az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$	$-a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$Re\{s\} > -a$	$-a(\times 2)$
$na^nu[n]$	$\frac{1}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$	$a(\times 2)$

Basic BD Pairs

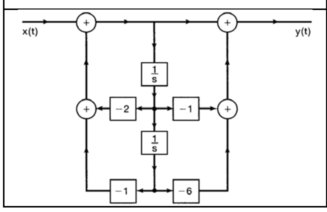
$$H(s) = \frac{1}{s+3}$$



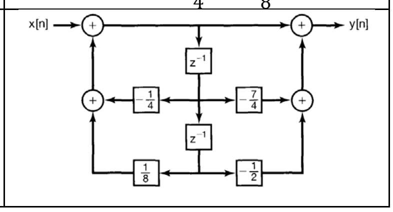
$$H(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$



$$H(s) = \frac{s^2-s-6}{s^2+2s+1}$$



$$H(z) = \frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$



Parseval's Relation (Aperiodic Sig.)	Initial/Final Value Theorem	Initial Value Theorem	Group Delay
$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2 d\omega$	$x(0^+) = \lim_{s\to\infty} sX(s)$, $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$	If $x[n]=0$ for $n<0$, $x[0] = \lim_{z\to\infty} X(z)$.	$\tau(\omega) = -\frac{d}{d\omega}\{\sphericalangle H(j\omega)\}$

Properties of Fourier Transform 傅里叶变换的性质

Property	Aperiodic signal	Fourier transform
	$x(t)$	$X(j\omega)$
	$y(t)$	$Y(j\omega)$
Linearity	$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
Time Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time Reversal	$x(-t)$	$X(-j\omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t)*y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t}x(t)dt$	$\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega)=X^*(-j\omega) \\ Re\{X(j\omega)\}=Re\{X(-j\omega)\} \\ Im\{X(j\omega)\}=-Im\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert=\lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega)=-\sphericalangle X(-j\omega)\end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t)=Ev\{x(t)\}$ $x_o(t)=Od\{x(t)\}$ $x(t)$ real	$Re\{X(j\omega)\}$ $jIm\{X(j\omega)\}$

Basic Fourier Transform Pairs

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_0 t}$	$2\pi\sum_{k=-\infty}^{\infty}a_k\delta(\omega-k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$a_1=1$ $a_k=0,\ otherwise$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$a_1=a_{-1}=\frac{1}{2}$ $a_k=0,\ otherwise$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$a_1=-a_{-1}=\frac{1}{2j}$ $a_k=0,\ otherwise$
$x(t)=1$	$2\pi\delta(\omega)$	$a_0=1,\ a_k=0,$ $k\neq 0$
Periodic square wave $x(t)=\begin{cases}1, & \lvert t\rvert<T_1\\ 0, & T_1<\lvert t\rvert\le\frac{T}{2}\end{cases}$ And $x(t+T)=x(t)$	$\sum_{k=-\infty}^{\infty}\frac{2\sin k\omega_0 T_1}{k}\delta(\omega-k\omega_0)$	$\frac{\omega_0 T_1}{\pi}\mathrm{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right)=\frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$	$a_k=\frac{1}{T}$ for all k
$x(t)=\begin{cases}1, & \lvert t\rvert<T_1\\ 0, & \lvert t\rvert>T_1\end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	
$\frac{\sin Wt}{\pi t}$	$X(j\omega)=\begin{cases}1, & \lvert\omega\rvert<W\\ 0, & \lvert\omega\rvert>W\end{cases}$	
$\delta(t)$	1	
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	
$\delta(t-t_0)$	$e^{-j\omega t_0}$	
$e^{-at}u(t), Re\{a\}>0$	$\frac{1}{a+j\omega}$	
$te^{-at}u(t), Re\{a\}>0$	$\frac{1}{(a+j\omega)^2}$	
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t), Re\{a\}>0$	$\frac{1}{(a+j\omega)^n}$	

Properties of Laplace Transform 拉普拉斯变换的性质

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	
	$x_1(t)$	$X_1(s)$	
	$x_2(t)$	$X_2(s)$	
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1\cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	At least $R_1\cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R\cap Re\{s\}>0$

Laplace Transform Pairs

#	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$1/s$	$\mathcal{Re}\{s\}>0$
3	$-u(-t)$	$1/s$	$\mathcal{Re}\{s\}<0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathcal{Re}\{s\}>-a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathcal{Re}\{s\}<-a$
8	$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$\mathcal{Re}\{s\}>-a$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$\mathcal{Re}\{s\}<-a$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathcal{Re}\{s\}>0$
12	$[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathcal{Re}\{s\}>0$

Properties of Z-Transform

Property	Signal	z-Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	At least $R_1\cap R_2$
Tiime shifting	$x[n-n_0]$	$z^{-n_0}X(z)$	R except for possible origin
Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
	$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R
Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R
Time expansion	$x_{(k)}[n]=\begin{cases}x[r], & n=rk\\ 0, & n\neq rk\end{cases}$	$X(z^k)$	$R^{\frac{1}{k}}$
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	At least $R_1\cap R_2$
First difference	$x[n]-x[n-1]$	$(1-z^{-1})X(z)$	At least $R\cap\{\lvert z\rvert>0\}$
Accumuation	$\sum_{k=-\infty}^{n}x[k]$	$\frac{1}{1-z^{-1}}X(z)$	At least $R\cap\{\lvert z\rvert>1\}$
Differentiation in the z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

Z-Transform Pairs

Signal	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert>-1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert<-1$
$\delta[n-m]$	z^{-m}	All z, except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert>\lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert<\lvert a\rvert$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$

$\log(1+u)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}u^n}{n},\ \lvert u\rvert<1$

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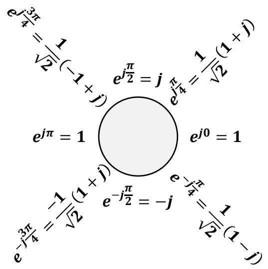
System Properties (Z-Transform)

Causality	ROC is the exterior of a circle including infinity
Causality (Rational $H(s)$)	ROC is the exterior of a circle outside the outermost pole + 分子 z 阶数不能比分母大
Stability	ROC includes the unit circle

Causal LTI system with rational $H(s)$: all poles lies inside the unit circle + 分子 z 阶数不能比分母大



Nyquist Sampling Theorem

Let $x(t)$ be a band-limited signal within $\pm\omega_M$, then $x(t)$ is uniquely determined by its samples $x(nT)$ if $\omega_s > 2\omega_M$, where $\omega_s = \frac{2\pi}{T}$.

Nyquist rate: $2\omega_M$

诱导公式

$\sin(2k\pi+\alpha)=\sin\alpha$
$\cos(2k\pi+\alpha)=\cos\alpha$
$\sin(-\alpha)=-\sin\alpha$
$\cos(-\alpha)=\cos\alpha$
$\sin(\pi-\alpha)=\sin\alpha$
$\cos(\pi-\alpha)=-\cos\alpha$
$\sin(\pi+\alpha)=-\sin\alpha$
$\cos(\pi+\alpha)=-\cos\alpha$
$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$
$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$
$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$
$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$
$\mathrm{tg}(k\pi+\alpha)=\mathrm{tg}\,\alpha$
$\mathrm{tg}(-\alpha)=-\mathrm{tg}\,\alpha$
$\mathrm{tg}(\pi-\alpha)=-\mathrm{tg}\,\alpha$
$\mathrm{tg}(\pi+\alpha)=\mathrm{tg}\,\alpha$
$\mathrm{tg}\left(\frac{\pi}{2}-\alpha\right)=\mathrm{ctg}\,\alpha$
$\mathrm{tg}\left(\frac{\pi}{2}+\alpha\right)=-\mathrm{ctg}\,\alpha$

Euler's formula 欧拉公式

$$e^{j\theta}=\cos\theta+j\sin\theta$$
$$\cos\theta=\frac{e^{j\theta}+e^{-j\theta}}{2}$$
$$\sin\theta=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$

和差化积公式与积化和差公式

$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$
$\cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$
$\cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$
$\sin\alpha\sin\beta=-\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)]$

Convolution 卷积

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=-\infty}^{\infty}x[n-k]h[k]$$
$$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau\int_{-\infty}^{+\infty}x(t-\tau)h(\tau)d\tau$$

一个函数的t换成τ，另一个函数的t换成$t-\tau$；如果结果以原点为界，记得写成$u[n]u(t)$的形式；$x(\tau)$的图像正常，$h(t-\tau)$的图像关于y轴翻转后，原点为t

Z 变换特殊峰的判断

信号为正，零点有峰
信号为负，无穷有峰

Group Delay, Phase Delay

Phase delay: $\tau_p(\omega_0)=-\frac{\theta(\omega_0)}{\omega_0}$

Group delay: $\tau_g(\omega)=-\frac{d\theta(\omega)}{d\omega}$

$$y_a(t)=a\left(t-\tau_g(\Omega_C)\right)\cos\Omega_C\left(t-\tau_p(\Omega_C)\right)$$

Hilbert Transform

$$\frac{1}{\pi t}\overset{\mathcal{F}}{\leftrightarrow}-j\cdot\mathrm{sign}\,\omega,\ \cos\omega_0 t\overset{h(t)}{\longrightarrow}\sin\omega_0 t$$

Sinc Function

$$\mathrm{sinc}\,\theta=\frac{\sin\pi\theta}{\pi\theta}$$

2*N*-Point DFT using a single *N*-Point DFT

$v[n]$长$2N$，$g[n]=v[2n],h[n]=v[2n+1]$

则$V[k]=G[\langle k\rangle_N]+W_{2N}^{k}H[\langle k\rangle_N]$

系统性质的判据	输入输出关系	单位冲激响应
Memoryless	Output at $t=t_0$ depends only on the value of input at $t=t_0$	$h(t)=0$ for $t\neq 0$
Invertible	There exist an inverse system	$\exists h_1[n]\ s.t.\ h[n]*h_1[n]=\delta[n]$
Causality	Output at this time only depends on values of the input at the present time and in the past	$h[n]=0$ for $n<0$
Stability	BIBO	$\int_{-\infty}^{\infty}\lvert h(\tau)\rvert d\tau<\infty$
Time-invariance	Let $x_1(t)=x(t-t_0)$ Check $y_1(t)=?\,y(t-t_0)$	Must satisfy
Linearity	Let $x_1(t),x_2(t),x_3(t)=x_1(t)+x_2(t)$ Check $y_3(t)=?\,ay_1(t)+by_2(t)$	Must satisfy

圆周卷积算线性卷积

1. 补 0 法：两序列都补到$M+N-1$长，算圆周卷积即可
2. Overlap-add：将$x[n]$切段，每段与$h[n]$用补 0 法算圆周卷积，最后相加时要有$N-1$长度的 overlap
3. Overlap-save：相加时不重叠，而是切断时重叠$N-1$长度；最前面补$N-1$个 0，相加时每段舍去前$N-1$

Z-变换对快速检查单

Signal	Transform	ROC	Pole
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$	a
$a^n u[-n-1]$	$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$	a
$(-a)^n u[n]$	$\frac{1}{1+az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$	$-a$
$(-a)^n u[-n-1]$	$-\frac{1}{1+az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$	$-a$
$na^n u[n]$	$\frac{1}{(1-az^{-1})^2}$	$\lvert z\rvert>\lvert a\rvert$	$a(\times 2)$

Time-Domain Sampling 时域采样

$x_a(t)\overset{CTFT}{\longleftrightarrow}X_a(j\Omega)$, $x(t)=x_a(t)p(t)\overset{CTFT}{\longleftrightarrow}X(j\Omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_a\left(j(\Omega-k\Omega_s)\right)$

$x[n]=x(t)|_{t=nT}\overset{F}{\leftrightarrow}X(e^{j\omega})=X(j\Omega)|_{\Omega=\frac{\omega}{T}}=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_a\left(j\frac{\omega-2k\pi}{T}\right)$

采样后信号的频率响应幅值变为$1/T$倍，因此恢复信号时要通过一个增益为T的 BPF，BPF 的截止频率最好为$\frac{\omega_s}{2}$；Ω轴上的Ω_s对应ω轴上的2π

连续离散公式：时域$t=nT$，频域$\omega=\Omega T$，任何地方$f=1/T$

时域采样导致频域周期化：$F_{out}=F_{in}\pm kF_s$，$\Omega_{out}=\Omega_{in}\pm k\Omega_s$

带通采样

$\Omega_H=M(\Delta\Omega)\Rightarrow$

$\Omega_T=2(\Delta\Omega)=\frac{2\Omega_H}{M}$

恢复使用 gain 为T的 $\Omega_L\leq|\Omega|\leq\Omega_H$的 BPF

Fourier-Domain Sampling 频域采样

频域采N个样（不关心$x[n]$周期是不是N），时域信号就以N为周期

$$y[n]=\sum_{m=-\infty}^{\infty}x[n+mN],0\leq n\leq N-1$$

多画几个周期找规律！

Geometric Series 等比数列

通项公式：$a_n=a_1q^{n-1}$

两项关系：$a_n=a_mq^{n-m}$

求和公式：$S_n=a_1\frac{a-q^n}{1-q}=\frac{a_1-a_nq}{1-q}$

信号的分解

奇偶分解	$x_{ev}(t)=\frac{1}{2}\left(x(t)+x(-t)\right)$ $x_{od}(t)=\frac{1}{2}\left(x(t)-x(-t)\right)$	$X_{ev}[k]=\frac{1}{2}(X[k]+X[\langle -k\rangle_N])$ $X_{od}[k]=\frac{1}{2}(X[k]-X[\langle -k\rangle_N])$
实虚分解	$x_{re}(t)=\frac{1}{2}\left(x(t)+x^*(t)\right)$ $x_{im}(t)=\frac{1}{2j}\left(x(t)-x^*(t)\right)$	$X_{re}[k]=\frac{1}{2}(X[k]+X^*[k])$ $X_{im}[k]=\frac{1}{2}(X[k]-X^*[k])$
共轭对称分解	$x_{cs}(t)=\frac{1}{2}\left(x(t)+x^*(-t)\right)$ $x_{ca}(t)=\frac{1}{2}\left(x(t)-x^*(-t)\right)$	$X_{cs}[k]=\frac{1}{2}(X[k]+X^*[\langle -k\rangle_N])$ $X_{ca}[k]=\frac{1}{2}(X[k]-X^*[\langle -k\rangle_N])$

信号的范数、能量、功率

$\|x\|_p=\left(\sum_{n=-\infty}^{\infty}|x[n]|^p\right)^{\frac{1}{p}}$, $\|x\|_\infty=|x|_{max}$

$\|X\|_p=\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^p\right)^{\frac{1}{p}}$, $\|X\|_\infty=\max|X(e^{j\omega})|$

$MSE=\frac{1}{N}\sum_{i=0}^{N-1}(|y[n]-x[n]|)^2=\frac{1}{N}(\|y[n]-x[n]\|_2)^2$

Total energy: $\epsilon_x=\sum_{n=-\infty}^{\infty}|x[n]|^2$ (Energy signal: finite energy)

Average power: $P_x=\lim_{K\to\infty}\frac{1}{2K+1}\sum_{k=-K}^{K}|x[n]|^2$ (Power signal: finite power)

Passive system: $\epsilon_y\leq\epsilon_x<\infty$, lossless system: $\epsilon_y=\epsilon_x<\infty$

Circular Time-Shift/Reversal

原：0 1 2 3 4
移：3 4 0 1 2
反：0 4 3 2 1

DFT Matrix Relation

$\boldsymbol{x}=D_N^{-1}\boldsymbol{X}$, $\boldsymbol{X}=D_N\boldsymbol{x}$

$$D_N=\begin{bmatrix}W_N^0 & W_N^0 & W_N^0\\ W_N^0 & W_N^1 & W_N^2\\ W_N^0 & W_N^2 & W_N^4\end{bmatrix},\ D_N^{-1}=\frac{1}{N}\begin{bmatrix}W_N^0 & W_N^0 & W_N^0\\ W_N^0 & W_N^{-1} & W_N^{-2}\\ W_N^0 & W_N^{-2} & W_N^{-4}\end{bmatrix},\ D_4=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -j & -1 & j\\ 1 & -1 & 1 & -1\\ 1 & j & -1 & -j\end{bmatrix}$$

离散三角信号的周期性

$\omega N=2k\pi\Rightarrow N=\frac{2k\pi}{\omega}$

离散三角信号的周期性小结论

1. 周期N必须是整数
2. 角频率相差$2k\pi$的信号完全相同
3. 任何数列的最大频率为π（folding frequency），且对称频率（0.6π和1.4π）信号相同

DFT Geometric Symmetry

Geometric symmetry: $x[n]=x[N-1-n],\theta(\omega)=-\frac{N-1}{2}\omega+\beta$ (type1 odd, type2 even)

Geometric anti-symmetry: $x[n]=-[N-1-n],\theta(\omega)=-\frac{N-1}{2}\omega+\frac{\pi}{2}+\beta$ (type3 odd, type4 even)，中心系数为 0

变换

CTFT	DTFT	DFT	Z-transform
时域连续非周期 频域连续非周期	时域离散非周期 频域连续周期	时域离散周期 频域离散周期	
$X_a(j\Omega)=\int_{-\infty}^{\infty}x_a(t)e^{-j\Omega t}dt$ $x_a(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X_a(j\Omega)e^{j\Omega t}d\Omega$	$X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}$ $x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}d\omega$	$X[k]=\sum_{n=0}^{N-1}x[n]W_N^{kn}$ $x[n]=\frac{1}{N}\sum_{k=0}^{N-1}X[k]W_N^{-kn}$	$G(z)=\sum_{n=-\infty}^{\infty}g[n]z^{-n}$ $g[n]=\frac{1}{2\pi j}\oint_C G(z)z^{n-1}dz$
$\int_{-\infty}^{\infty}\lvert x_a(t)\rvert^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X_a(j\Omega)\rvert^2d\Omega$	$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\omega})\rvert d\omega$	$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X[k]\rvert^2$	
	$x[-n]\leftrightarrow X(e^{-j\omega})$ $x^*[-n]\leftrightarrow X^*(e^{j\omega})$ $x^*[n]\leftrightarrow X^*(e^{-j\omega})$	$x[\langle -n\rangle_N]\leftrightarrow X[\langle -k\rangle_N]$ $x[\langle -n\rangle_N]\leftrightarrow X^*[k]$ $x^*[n]\leftrightarrow X^*[\langle -k\rangle_N]$	$x[-n]\leftrightarrow X(z^{-1})$ with inverted R

Theorems

Property	Signal	DTFT	DFT	Z-Transform	ROC
Linearity	$ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$	$aX[k]+bY[k]$	$aX_1(z)+bX_2(z)$	At least $R_1\cap R_2$
Time Shifting	$x[n-n_0], x[\langle n-n_0\rangle_N]$	$e^{-j\omega n_0}X(e^{j\omega})$	$W_N^{kn_0}X[k]$	$z^{-n_0}X(z)$	R except for possible origin
Frequency Shifting (Z-Domain Scaling)	$e^{j\omega_0 n}x[n], W_N^{-k_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$	$X[\langle k-k_0\rangle_N]$	$X(e^{-j\omega_0}z)$	R
Time Expansion	$x_{(k)}[n]=x\left[\frac{n}{k}\right]$ $if\ n=mk\ else\ 0$	$X(e^{jk\omega})$		$X(z^k)$	$R^{\frac{1}{k}}$
Convolution	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$	$X[k]Y[k]$	$X(z)Y(z)$	At least $R_1\cap R_2$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi}X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	$\frac{1}{N}\sum_{m=0}^{N-1}X[m]Y[\langle k-m\rangle_N]$	$\frac{1}{2\pi j}\oint_C X(v)H\left(\frac{z}{v}\right)v^{-1}dv$	At least R_1R_2
Differencing in Time	$x[n]-x[n-1]$	$(1-e^{-j\omega})X(e^{j\omega})$		$(1-z^{-1})X(z)$	At least $R\cap\{\lvert z\rvert>0\}$
Accumulation	$\sum_{k=-\infty}^{n}x[k]$	$\frac{1}{1-e^{-j\omega}}X(e^{j\omega})$		$\frac{1}{1-z^{-1}}X(z)$	At least $R\cap\{\lvert z\rvert>1\}$
Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$		$-z\frac{dX(z)}{dz}$	R

DTFT Transform Pairs

Signal	Transform
$\delta[n]$	1
$\mu[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$
1	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-\omega_0+2\pi k)$
$\alpha^n\mu[n],(\lvert\alpha\rvert<1)$	$\frac{1}{1-\alpha e^{-j\omega}}$
$(n+1)\alpha^n\mu[n],(\lvert\alpha\rvert<1)$	$\frac{1}{(1-\alpha e^{-j\omega})^2}$
$h_{LP}[n]=\frac{\sin\omega_c n}{\pi n}$	$H_{LP}(e^{j\omega})=\begin{cases}1, & 0\le\lvert\omega\rvert\le\omega_c\\0, & \omega_c<\lvert\omega\rvert\le\pi\end{cases}$
$\cos\omega_0 n$	$\pi\sum_{l=-\infty}^{\infty}\{\delta(\omega-\omega_0-2\pi l)+\delta(\omega+\omega_0-2\pi l)\}$
$\sin\omega_0 n$	$\frac{\pi}{j}\sum_{l=-\infty}^{l=\infty}\{\delta(\omega-\omega_0-2\pi l)-\delta(\omega+\omega_0-2\pi l)\}$

Z-Transform Pairs

Signal	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert>-1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert<-1$
$\delta[n-m]$	z^{-m}	All z, except 0 or ∞
$a^nu[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$
$-a^nu[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$
$na^nu[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert>\lvert a\rvert$
$-na^nu[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert<\lvert a\rvert$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$

P-Series Test

$\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for $p\ge1$

Initial Value Theorem

If $x[n]=0$ for $n<0$, $x[0]=\lim_{z\to\infty}X(z)$.

DFT 关键公式

$$\frac{1}{N}\sum_{n=0}^{N}e^{j\frac{2\pi}{N}(k-l)n}=\begin{cases}1, & k=l+rn\\0, & k\ne l\end{cases}$$

$$W_N=e^{-j\frac{2\pi}{N}},\quad \cos\left[\frac{2\pi}{N}rn\right]=\frac{1}{2}(W_N^{rn}+W_N^{-rn})$$

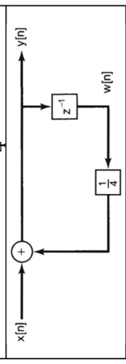
$$\frac{1}{1-\alpha e^{-j\omega}}=\frac{1}{1-\alpha e^{-j\omega}}\cdot\frac{1-\alpha e^{j\omega}}{1-\alpha e^{j\omega}}$$
$$=\frac{1-\alpha\cos\omega-j\alpha\sin\omega}{1-2\alpha\cos\omega+\alpha^2}$$
$$=\frac{1}{\sqrt{1-2\alpha\cos\omega+\alpha^2}}\measuredangle\arctan\left(-\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)$$

DFT Transform Pairs

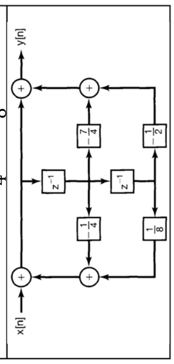
Signal	Transform
$\delta[n]$	1
$\delta[n-m]$	W_N^{km}
$\cos\left(\frac{2\pi}{N}rn\right)$	$\begin{cases}\frac{N}{2}, & k=r\ or\ N-r\\0, & otherwise\end{cases}$

Basic BD Pairs

$$H(z)=\frac{1}{1-\frac{1}{4}z^{-1}}$$



$$H(z)=\frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$



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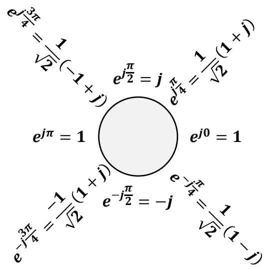
System Properties (Z-Transform)

Causality	ROC is the exterior of a circle including infinity
Causality (Rational $H(s)$)	ROC is the exterior of a circle outside the outermost pole + 分子 z 阶数不能比分母大
Stability	ROC includes the unit circle

Causal LTI system with rational $H(s)$: all poles lies inside the unit circle + 分子 z 阶数不能比分母大



Nyquist Sampling Theorem

Let $x(t)$ be a band-limited signal within $\pm\omega_M$, then $x(t)$ is uniquely determined by its samples $x(nT)$ if $\omega_s > 2\omega_M$, where $\omega_s = \frac{2\pi}{T}$.

Nyquist rate: $2\omega_M$

诱导公式

$\sin(2k\pi+\alpha) = \sin\alpha$
$\cos(2k\pi+\alpha) = \cos\alpha$
$\sin(-\alpha) = -\sin\alpha$
$\cos(-\alpha) = \cos\alpha$
$\sin(\pi-\alpha) = \sin\alpha$
$\cos(\pi-\alpha) = -\cos\alpha$
$\sin(\pi+\alpha) = -\sin\alpha$
$\cos(\pi+\alpha) = -\cos\alpha$
$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$
$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$
$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$
$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$
$\mathrm{tg}(k\pi+\alpha) = \mathrm{tg}\,\alpha$
$\mathrm{tg}(-\alpha) = -\mathrm{tg}\,\alpha$
$\mathrm{tg}(\pi-\alpha) = -\mathrm{tg}\,\alpha$
$\mathrm{tg}(\pi+\alpha) = \mathrm{tg}\,\alpha$
$\mathrm{tg}\left(\frac{\pi}{2}-\alpha\right) = \mathrm{ctg}\,\alpha$
$\mathrm{tg}\left(\frac{\pi}{2}+\alpha\right) = -\mathrm{ctg}\,\alpha$

Euler's formula 欧拉公式

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$
$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

和差化积公式与积化和差公式

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$
$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$
$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$

Convolution 卷积

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \int_{-\infty}^{+\infty} x(t-\tau)h(\tau)d\tau$$

一个函数的t换成τ，另一个函数的t换成$t-\tau$；如果结果以原点为界，记得写成$u[n]u(t)$的形式；$x(\tau)$的图像正常，$h(t-\tau)$的图像关于y轴翻转后，原点为t

Z 变换特殊峰的判断

信号为正，零点有峰
信号为负，无穷有峰

Group Delay, Phase Delay

Phase delay: $\tau_p(\omega_0) = -\frac{\theta(\omega_0)}{\omega_0}$

Group delay: $\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$

$$y_a(t) = a\left(t-\tau_g(\Omega_c)\right)\cos\Omega_c\left(t-\tau_p(\Omega_c)\right)$$

Hilbert Transform

$\frac{1}{\pi t} \overset{\mathcal{F}}{\leftrightarrow} -j\cdot\mathrm{sign}\,\omega$, $\cos\omega_0 t \overset{h(t)}{\longrightarrow} \sin\omega_0 t$

Sinc Function

$\mathrm{sinc}\,\theta = \frac{\sin\pi\theta}{\pi\theta}$

$2N$-Point DFT using a single N-Point DFT

$v[n]$长$2N$，$g[n] = v[2n], h[n] = v[2n+1]$
则$V[k] = G[\langle k\rangle_N] + W_{2N}^k H[\langle k\rangle_N]$

系统性质的判据	输入输出关系	单位冲激响应
Memoryless	Output at $t=t_0$ depends only on the value of input at $t=t_0$	$h(t)=0$ for $t\neq 0$
Invertible	There exist an inverse system	$\exists h_1[n]\ s.t.\ h[n]*h_1[n] = \delta[n]$
Causality	Output at this time only depends on values of the input at the present time and in the past	$h[n]=0$ for $n<0$
Stability	BIBO	$\int_{-\infty}^{\infty}\lvert h(\tau)\rvert d\tau < \infty$
Time-invariance	Let $x_1(t) = x(t-t_0)$ Check $y_1(t) =?\, y(t-t_0)$	Must satisfy
Linearity	Let $x_1(t), x_2(t), x_3(t) = x_1(t)+x_2(t)$ Check $y_3(t) =?\, ay_1(t)+by_2(t)$	Must satisfy

圆周卷积算线性卷积

1. 补 0 法：两序列都补到$M+N-1$长，算圆周卷积即可
2. Overlap-add：将$x[n]$切段，每段与$h[n]$用补 0 法算圆周卷积，最后相加时要有$N-1$长度的 overlap
3. Overlap-save：相加时不重叠，而是切断时重叠$N-1$长度；最前面补$N-1$个 0，相加时每段舍去前$N-1$

Z-变换对快速检查单

Signal	Transform	ROC	Pole
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$	a
$a^n u[-n-1]$	$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$	a
$(-a)^n u[n]$	$\frac{1}{1+az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$	$-a$
$(-a)^n u[-n-1]$	$-\frac{1}{1+az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$	$-a$
$na^n u[n]$	$\frac{1}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$	$a(\times 2)$

Time-Domain Sampling 时域采样

$x_a(t) \overset{CTFT}{\longleftrightarrow} X_a(j\Omega)$，$x(t) = x_a(t)p(t) \overset{CTFT}{\longleftrightarrow} X(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(j(\Omega-k\Omega_s)\right)$

$x[n] = x(t)|_{t=nT} \overset{F}{\leftrightarrow} X(e^{j\omega}) = X(j\Omega)|_{\Omega=\frac{\omega}{T}} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(j\frac{\omega-2k\pi}{T}\right)$

采样后信号的频率响应幅值变为$1/T$倍，因此恢复信号时要通过一个增益为T的 BPF，BPF 的截止频率最好为$\frac{\omega_s}{2}$；Ω轴上的Ω_s对应ω轴上的2π

连续离散公式：时域$t=nT$，频域$\omega=\Omega T$，任何地方$f=1/T$

时域采样导致频域周期化：$F_{out} = F_{in} \pm kF_s$，$\Omega_{out} = \Omega_{in} \pm k\Omega_s$

带通采样

$\Omega_H = M(\Delta\Omega) \Rightarrow$
$\Omega_T = 2(\Delta\Omega) = \frac{2\Omega_H}{M}$
恢复使用 gain 为T的$\Omega_L \le |\Omega| \le \Omega_H$的 BPF

Fourier-Domain Sampling 频域采样

频域采N个样（不关心$x[n]$周期是不是N），时域信号就以N为周期

$$y[n] = \sum_{n=-\infty}^{\infty} x[n+mN], 0 \le n \le N-1$$

多画几个周期找规律!

Geometric Series 等比数列

通项公式：$a_n = a_1 q^{n-1}$
两项关系：$a_n = a_m q^{n-m}$
求和公式：$S_n = a_1\frac{a-q^n}{1-q} = \frac{a_1-a_n q}{1-q}$

信号的分解

奇偶分解	$x_{ev}(t) = \frac{1}{2}\left(x(t)+x(-t)\right)$ $x_{od}(t) = \frac{1}{2}\left(x(t)-x(-t)\right)$	$X_{ev}[k] = \frac{1}{2}(X[k]+X[\langle -k\rangle_N])$ $X_{od}[k] = \frac{1}{2}(X[k]-X[\langle -k\rangle_N])$
实虚分解	$x_{re}(t) = \frac{1}{2}\left(x(t)+x^*(t)\right)$ $x_{im}(t) = \frac{1}{2j}\left(x(t)-x^*(t)\right)$	$X_{re}[k] = \frac{1}{2}(X[k]+X^*[k])$ $X_{im}[k] = \frac{1}{2}(X[k]-X^*[k])$
共轭对称分解	$x_{cs}(t) = \frac{1}{2}\left(x(t)+x^*(-t)\right)$ $x_{ca}(t) = \frac{1}{2}\left(x(t)-x^*(-t)\right)$	$X_{cs}[k] = \frac{1}{2}(X[k]+X^*[\langle -k\rangle_N])$ $X_{ca}[k] = \frac{1}{2}(X[k]-X^*[\langle -k\rangle_N])$

信号的范数、能量、功率

$\|x\|_p = \left(\sum_{n=-\infty}^{\infty}|x[n]|^p\right)^{\frac{1}{p}}$，$\|x\|_\infty = |x|_{max}$

$\|X\|_p = \left(\frac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{j\omega})|^p\right)^{\frac{1}{p}}$，$\|X\|_\infty = \max|X(e^{j\omega})|$

$MSE = \frac{1}{N}\sum_{i=0}^{N-1}(|y[n]-x[n]|)^2 = \frac{1}{N}(\|y[n]-x[n]\|_2)^2$

Total energy: $\epsilon_x = \sum_{n=-\infty}^{\infty}|x[n]|^2$ (Energy signal: finite energy)

Average power: $P_x = \lim_{K\to\infty}\frac{1}{2K+1}\sum_{k=-K}^{K}|x[n]|^2$ (Power signal: finite power)

Passive system: $\epsilon_y \le \epsilon_x < \infty$, lossless system: $\epsilon_y = \epsilon_x < \infty$

Circular Time-Shift/Reversal

原：0 1 2 3 4
移：3 4 0 1 2
反：0 4 3 2 1

DFT Matrix Relation

$\boldsymbol{x} = D_N^{-1}\boldsymbol{X}$, $\boldsymbol{X} = D_N\boldsymbol{x}$

$$D_N = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 \\ W_N^0 & W_N^2 & W_N^4 \end{bmatrix}, D_N^{-1} = \frac{1}{N}\begin{bmatrix} W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} \\ W_N^0 & W_N^{-2} & W_N^{-4} \end{bmatrix}, D_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

离散三角信号的周期性

$\omega N = 2k\pi \Rightarrow N = \frac{2k\pi}{\omega}$

离散三角信号的周期性小结论

1. 周期N必须是整数
2. 角频率相差$2k\pi$的信号完全相同
3. 任何数列的最大频率为π（folding frequency），且对称频率（0.6π和1.4π）信号相同

DFT Geometric Symmetry

Geometric symmetry: $x[n] = x[N-1-n], \theta(\omega) = -\frac{N-1}{2}\omega + \beta$ (type1 odd, type2 even)

Geometric anti-symmetry: $x[n] = -[N-1-n], \theta(\omega) = -\frac{N-1}{2}\omega + \frac{\pi}{2} + \beta$ (type3 odd, type4 even)，中心系数为 0

Order

$$\omega_c = \frac{\omega_s+\omega_p}{2}$$
$$\Delta\omega = 2\pi\frac{\Delta f}{F_T} \Longrightarrow M$$
$$\Longrightarrow N = 2M+1$$

$$h_{LP}[n] = \frac{\sin[\omega(n-M)]}{\pi(n-M)}\cdot w[n-M]$$
$$h_{HP}[n] = \begin{cases} 1-\frac{\omega_c}{\pi} & n=0 \\ -\frac{\sin(\omega_c n)}{\pi n} & |n|>0 \end{cases}$$
$$h_{BP}[n] = \frac{\sin(\omega_{c2}n)-\sin(\omega_{c1}n)}{\pi n}, |n|\ge 0$$
$$h_{BS}[n] = \begin{cases} 1-\frac{\omega_{c2}-\omega_{c1}}{\pi n} & n=0 \\ \frac{\sin(\omega_{c2}n)-\sin(\omega_{c1}n)}{\pi n} & |n|\ge 0 \end{cases}$$

$$w_R[n] = \begin{cases} 1 & -M\le n\le M \\ 0 & |n|>M \end{cases}$$
$$w_{Hann}[n] = \frac{1}{2}\left[1+\cos(\frac{2\pi n}{2M+1})\right]$$
$$w_{Hamming}[n] = \left[0.54+0.46\cos(\frac{2\pi n}{2M+1})\right]$$
$$w_{Blackman}[n] = \left[0.42+0.5\cos(\frac{2\pi n}{2M+1})+0.08\cos(\frac{4\pi n}{2M+1})\right]$$

变换

CTFT	DTFT	DFT	Z-transform
时域连续非周期 频域连续非周期	时域离散非周期 频域连续周期	时域离散周期 频域离散周期	
$X_a(j\Omega)=\int_{-\infty}^{\infty}x_a(t)e^{-j\Omega t}dt$ $x_a(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X_a(j\Omega)e^{j\Omega t}d\Omega$	$X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}$ $x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})e^{j\omega n}d\omega$	$X[k]=\sum_{n=0}^{N-1}x[n]W_N^{kn}$ $x[n]=\frac{1}{N}\sum_{k=0}^{N-1}X[k]W_N^{-kn}$	$G(z)=\sum_{n=-\infty}^{\infty}g[n]z^{-n}$ $g[n]=\frac{1}{2\pi j}\oint_C G(z)z^{n-1}dz$
$\int_{-\infty}^{\infty}\lvert x_a(t)\rvert^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X_a(j\Omega)\rvert^2d\Omega$	$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(e^{j\omega})\rvert^2d\omega$	$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X[k]\rvert^2$	
	$x[-n]\leftrightarrow X(e^{-j\omega})$ $x^*[-n]\leftrightarrow X^*(e^{j\omega})$ $x^*[n]\leftrightarrow X^*(e^{-j\omega})$	$x[\langle -n\rangle_N]\leftrightarrow X[\langle -k\rangle_N]$ $x[\langle -n\rangle_N]\leftrightarrow X^*[k]$ $x^*[n]\leftrightarrow X^*[\langle -k\rangle_N]$	$x[-n]\leftrightarrow X(z^{-1})$ with inverted R

Butterworth Filter

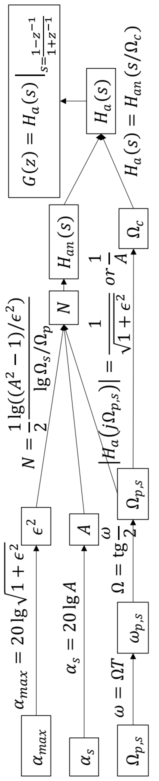
$$|H_a(j\Omega)|^2=\frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

Theorems

Property	Signal	DTFT	DFT	Z-Transform	ROC
Linearity	$ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$	$aX[k]+bY[k]$	$aX_1(z)+bX_2(z)$	At least $R_1\cap R_2$
Time Shifting	$x[n-n_0], x[\langle n-n_0\rangle_N]$	$e^{-j\omega n_0}X(e^{j\omega})$	$W_N^{kn_0}X[k]$	$z^{-n_0}X(z)$	R except for possible origin
Frequency Shifting (Z-Domain Scaling)	$e^{j\omega_0 n}x[n], W_N^{-k_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$	$X[\langle k-k_0\rangle_N]$	$X(e^{-j\omega_0}z)$	R
Time Expansion	$x_{(k)}[n]=x\left[\frac{n}{k}\right]$ if $n=mk$ else 0	$X(e^{jk\omega})$		$X(z^k)$	$R^{\frac{1}{k}}$
Convolution	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$	$X[k]Y[k]$	$X(z)Y(z)$	At least $R_1\cap R_2$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi}X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	$\frac{1}{N}\sum_{m=0}^{N-1}X[m]Y[\langle k-m\rangle_N]$	$\frac{1}{2\pi j}\oint_C X(v)H\left(\frac{z}{v}\right)v^{-1}dv$	At least R_1R_2
Differencing in Time	$x[n]-x[n-1]$	$(1-e^{-j\omega})X(e^{j\omega})$		$(1-z^{-1})X(z)$	At least $R\cap\{\lvert z\rvert>0\}$
Accumulation	$\sum_{k=-\infty}^{n}x[k]$	$\frac{1}{1-e^{-j\omega}}X(e^{j\omega})$		$\frac{1}{1-z^{-1}}X(z)$	At least $R\cap\{\lvert z\rvert>1\}$
Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$		$-z\frac{dX(z)}{dz}$	R

DTFT Transform Pairs

Signal	Transform
$\delta[n]$	1
$\mu[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$
1	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-\omega_0+2\pi k)$
$\alpha^n\mu[n], (\lvert\alpha\rvert<1)$	$\frac{1}{1-\alpha e^{-j\omega}}$
$(n+1)\alpha^n\mu[n], (\lvert\alpha\rvert<1)$	$\frac{1}{(1-\alpha e^{-j\omega})^2}$
$h_{LP}[n]=\frac{\sin\omega_c n}{\pi n}$	$H_{LP}(e^{j\omega})=\begin{cases}1, & 0\le\lvert\omega\rvert\le\omega_c\\0, & \omega_c<\lvert\omega\rvert\le\pi\end{cases}$
$\cos\omega_0 n$	$\pi\sum_{l=-\infty}^{\infty}\{\delta(\omega-\omega_0-2\pi l)+\delta(\omega+\omega_0-2\pi l)\}$
$\sin\omega_0 n$	$\frac{\pi}{j}\sum_{l=-\infty}^{l=\infty}\{\delta(\omega-\omega_0-2\pi l)-\delta(\omega+\omega_0-2\pi l)\}$



Z-Transform Pairs

Signal	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert>-1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert<-1$
$\delta[n-m]$	z^{-m}	All z, except 0 or ∞
$a^nu[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$
$-a^nu[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$
$na^nu[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert>\lvert a\rvert$
$-na^nu[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert<\lvert a\rvert$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert>1$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert>r$

P-Series Test

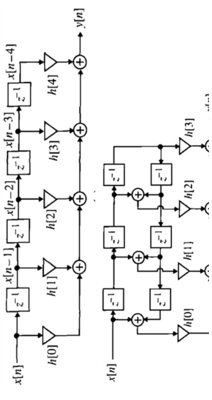
$\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for $p\ge 1$

IIR Filter Design

①对比两个α选窗
②$\Delta\omega$算M
③$N=2M+1$
④ $h[n]=h_{LP}[n-M]w[n-M]$

$$\frac{1}{1-\alpha e^{-j\omega}}=\frac{1}{1-\alpha e^{-j\omega}}\cdot\frac{1-\alpha e^{j\omega}}{1-\alpha e^{j\omega}}=\frac{1-\alpha\cos\omega-j\alpha\sin\omega}{1-2\alpha\cos\omega+\alpha^2}=\frac{1}{\sqrt{1-2\alpha\cos\omega+\alpha^2}}\measuredangle\arctan\left(-\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)$$

FIR Direct Form & Linear Phase



DFT 关键公式

$$\frac{1}{N}\sum_{n=0}^{N}e^{j\frac{2\pi}{N}(k-l)n}=\begin{cases}1, & k=l+rm\\0, & k\ne l\end{cases}$$

$$W_N=e^{-j\frac{2\pi}{N}},\quad \cos\left[\frac{2\pi}{N}rm\right]=\frac{1}{2}(W_N^{rm}+W_N^{-rm})$$

DFT Transform Pairs

Signal	Transform
$\delta[n]$	1
$\delta[n-m]$	W_N^{km}
$\cos\left(\frac{2\pi}{N}rn\right)$	$\begin{cases}\frac{N}{2}, k=r \text{ or } N-r\\0, otherwise\end{cases}$
$\sin(\frac{2\pi}{N}rn)$	$\begin{cases}\frac{N}{2j}, k=r\\-\frac{N}{2j}, k=N-1\\0, otherwise\end{cases}$

IIR Direct Form II

$$H(z)=\frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}},\quad H(z)=\frac{1}{1-\frac{1}{4}z^{-1}}$$