Formula Sheet for Financial Mathematics

SIMPLE INTEREST

$$I = Prt$$

- I is the amount of interest earned
- P is the principal sum of money earning the interest
- *r* is the simple annual (or nominal) interest rate (usually expressed as a percentage)
- t is the interest period *in years*

$$S = P + I$$

$$S = P(1 + rt)$$

- S is the future value (or maturity value). It is equal to the principal plus the interest earned.

COMPOUND INTEREST

$$FV = PV(1 + i)^n$$

$$i = \frac{j}{m}$$

j = nominal annual rate of interest
m = number of compounding periods
i = periodic rate of interest

$$PV = FV(1 + i)^{-n} \quad \text{OR} \quad PV = \frac{FV}{(1+i)n}$$

ANNUITIES

Classifying rationale	**Type of annuity**	
Length of conversion period relative to the payment period	**Simple annuity** - when the interest compounding period is the same as the payment period (C/Y = P/Y). For example, a car loan for which interest is compounded monthly and payments are made monthly.	**General annuity** - when the interest compounding period does NOT equal the payment period (C/Y ≠ P/Y). For example, a mortgage for which interest is compounded semi-annually but payments are made monthly.
Date of payment	**Ordinary annuity** – payments are made at the END of each payment period. For example, OSAP loan payment.	**Annuity due** - payments are made at the BEGINNING of each payment period. For example, lease rental payments on real estate.
Payment schedule	**Deferred annuity** – first payment is delayed for a period of time.	**Perpetuity** – an annuity for which payments continue forever. (Note: payment amount ≤ periodic interest earned)

Beginning date and end date	**Annuity certain** – an annuity with a fixed term; both the beginning date and end date are known. For example, installment payments on a loan.	**Contingent annuity** - the beginning date, the ending date, or both are unknown. For example, pension payments.

ORDINARY SIMPLE annuity

FV_n = PMT $\left[\frac{(1+i)^n-1}{i}\right]$

Note: $\left[\frac{(1+i)^n-1}{i}\right]$ is called the **compounding** or **accumulation factor for annuities** (or the accumulated value of one dollar per period).

PV_n = PMT $\left[\frac{1-(1+i)^{-n}}{i}\right]$

ORDINARY GENERAL annuity

FV_g = PMT $\left[\frac{(1+p)^n-1}{p}\right]$

PV_g = PMT $\left[\frac{1-(1+p)^{-n}}{p}\right]$

***First, you must calculate $\boldsymbol{p}$ (equivalent rate of interest per payment period) using $\boldsymbol{p = (1+i)^c - 1}$ where $\boldsymbol{i}$ is the periodic rate of interest and **c** is the number of interest conversion periods per payment interval.

c = $\frac{\#\ of\ interest\ conversion\ periods\ per\ year}{\#\ of\ payment\ periods\ per\ year}$

c = $\frac{\mathrm{C/Y}}{\mathrm{P/Y}}$

CONSTANT GROWTH annuity

size of *n*th payment = PMT $(1+k)^{n-1}$

k = constant rate of growth

PMT = amount of payment

n = number of payments

sum of periodic constant growth payments = PMT $\left[\frac{(1+k)^n-1}{k}\right]$

FV = PMT $\left[\frac{(1+i)^n-(1+k)^n}{i-k}\right]$

$\left[\frac{(1+i)^n-(1+k)^n}{i-k}\right]$ is the **compounding factor** for constant – growth annuities.

PV = PMT $\left[\frac{1-(1+k)^n(1+i)^{-n}}{i-k}\right]$

$\left[\frac{1-(1+k)^n(1+i)^{-n}}{i-k}\right]$ is the **discount factor** for constant – growth annuities.

PV = n (PMT)(1 + i)$^{-1}$ [This formula is used when the constant growth rate and the periodic interest rate are the same.]

<u>SIMPLE annuity DUE</u>

FV_n(due) = PMT $\left[\frac{(1+i)^n-1}{i}\right](1+i)$

PV_n(due) = PMT $\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$

<u>GENERAL annuity DUE</u>

FV_g = PMT $\left[\frac{(1+p)^n-1}{p}\right](1+i)$

PV_g = PMT $\left[\frac{1-(1+p)^{-n}}{p}\right](1+i)$

Note that you must first calculate ***p (equivalent rate of interest per payment period) using ***p* = (1+*i*)^c–1** where ***i*** is the periodic rate of interest and **c** is the number of interest conversion periods per payment interval.

<u>ORDINARY DEFERRED ANNUITIES</u> <u>or</u> **<u>DEFERRED ANNUITIES DUE:</u>**

Use the same formulas as ordinary annuities (simple or general) OR annuities due (simple or general). Adjust for the **period of deferment** – period between "now" and the starting point of the term of the annuity.

<u>ORDINARY SIMPLE PERPETUITY</u>

PV = $\frac{PMT}{i}$

<u>ORDINARY GENERAL PERPETUITY</u>

PV = $\frac{PMT}{p}$ where $p = (1+i)^c - 1$

SIMPLE PERPETUITY DUE

PV (due) = PMT + $\frac{PMT}{i}$

GENERAL PERPETUITY DUE

PV (due) = PMT + $\frac{PMT}{p}$ where $p = (1+i)^c - 1$

AMORTIZATION involving SIMPLE ANNUITIES:

Amortization refers to the method of repaying both the principal and the interest by a series of equal payments made at equal intervals of time.

If the payment interval and the interest conversion period are *equal* in length, the problem involves working with a simple annuity. Most often the payments are made at the *end* of a payment interval meaning that we are working with an *ordinary simple annuity.*

The following formulas apply:

$$PV_n = PMT\left[\frac{1-(1+i)^{-n}}{i}\right] \qquad FV_n = PMT\left[\frac{(1+i)^{n}-1}{i}\right]$$

Finding the outstanding principal balance using the **retrospective method**:

Outstanding balance = FV of the original debt ─ FV of the payments made

Use $FV = PV(1+i)^n$ to calculate the FV of the original debt.

Use $FV_n = PMT\left[\frac{(1+i)^{n}-1}{i}\right]$ to calculate the FV of the payments made