

Practical 3.3

- In this assignment the homework and the practical are presented as one integrated exercise.
- You must be able to do and understand all calculations with your **calculator and with Excel.**

Exercise 1

Answer the following Question.

The Energy Information Administration reported that the mean retail price per gallon of regular grade gasoline was \$2.30. Suppose that the standard deviation was \$0.1 and that the retail price per gallon has a bell-shaped distribution.

- a. What percentage of regular gasoline sold between \$2.20 and \$2.40 per gallon?
- b. What percentage of regular gasoline sold between \$2.20 and \$2.50 per gallon?
- c. What percentage of regular gasoline sold more than \$2.50 per gallon?

- Do the question on your own and THEN check the MEMO below.
- It helps to draw a graph!

MEMO:

- a. \$2.20 is one standard deviation below the mean and \$2.40 is one standard deviation above the mean. The empirical rule says that approximately 68% of gasoline sales should be in the price range.
- b. Part (a) shows that approximately 68% of the gasoline sales are between \$2.20 and \$2.40. Since the bell-shaped distribution is symmetric, approximately half of 68%, or 34%, of the gasoline sales should be between \$2.20 and the mean price of \$2.30. \$2.50 is two standard deviations above the mean price of \$2.30. The empirical rule says that approximately 95% of the gasoline sales should be within two standard deviations of the mean. Thus, approximately half of 95%, or 47.5%, of the gasoline sales should be between the mean price of \$2.30 and \$2.50. The percentage of gasoline sales between \$2.20 and \$2.50 should be approximately $34\% + 47.5\% = 81.5\%$.
- c. \$2.50 is two standard deviations above the mean and the empirical rule says that approximately 95% of the gasoline sales should be within two standard deviations of the mean. Thus, $1 - 95\% = 5\%$ of the gasoline should be more than two standard deviations from the mean. Since the bell-shaped distribution is symmetric, we expected half of 5%, or 2.5%, would be more than \$2.50.

Exercise 2

- Answer questions (a) to (d) and **THEN** check the MEMO below.

Many families in California are using backyard structures for home offices, art studios, hobby areas as well as additional storage. Suppose that the mean price for a customized wooden, shingled backyard structure is \$3100. Assume that the standard deviation is \$1200.

- What is the z -score for a backyard structure costing \$2300?
- What is the z -score for a backyard structure costing \$4900?
- Interpret the z -scores in parts (a) and (b). Comment on whether either should be considered an outlier
- If the cost of a backyard shed-office combination built in Albany, California, is \$13000, should this structure be considered an outlier? Explain.

- Make sure that you also know how to calculate the z -scores in Excel 2010.

MEMO:

a.

$$z = \frac{x - \mu}{\sigma} = \frac{2300 - 3100}{1200} = -.67$$

b.

$$z = \frac{x - \mu}{\sigma} = \frac{4900 - 3100}{1200} = 1.50$$

- c. \$2300 is .67 standard deviations below the mean. \$4900 is 1.50 standard deviations above the mean. Neither is an outlier.

d.

$$z = \frac{x - \mu}{\sigma} = \frac{13000 - 3100}{1200} = 8.25$$

\$13,000 is 8.25 standard deviations above the mean. This cost is an outlier.

Exercise 3

A service station recorded the following frequency distribution for the number of gallons of gasoline sold per car in a sample of 680 cars.

| Gasoline (gallons) | Frequency |
|--------------------|------------|
| 0 – 4 | 74 |
| 5 – 9 | 192 |
| 10 – 14 | 280 |
| 15 – 19 | 105 |
| 20 – 24 | 23 |
| 25 – 29 | 6 |
| Total | 680 |

Compute the mean, variance and standard deviation for these grouped data. If the service station expects to service about 120 cars on a given day, estimate the total number of gallons of gasoline that will be sold.

- The solutions are shown in Excel in the steps below.
- Make sure that you also know how to calculate all the answers with your calculator.

1. Type the following information in an Excel spread sheet. (See Fig 5.6)

Figure 5.6

| | A | B | C |
|---|------------------|------------------|-----------------|
| 1 | Gasoline | | |
| 2 | (gallons) | Frequency | Midpoint |
| 3 | | <i>f</i> | <i>M</i> |
| 4 | 0-4 | 74 | 2 |
| 5 | 5-9 | 192 | 7 |
| 6 | 10-14 | 280 | |
| 7 | 15-19 | 105 | |
| 8 | 20-24 | 23 | |
| 9 | 25-29 | 6 | |

2. After selecting cells C4:C5 in Fig 5.6 drag the *fill handle* down to cell C9. All midpoints are now copied into cells C6:C9. (See Fig 5.7)
3. Calculate the average

$$\bar{x} = \frac{\sum fM}{n}$$

by entering the formula: `=B4*C4` into cell D4. (See Fig. 5.7)

Figure 5.7

| | A | B | C | D |
|---|------------------|------------------|-----------------|------------------|
| 1 | Gasoline | | | |
| 2 | (gallons) | Frequency | Midpoint | |
| 3 | | <i>f</i> | <i>M</i> | <i>fM</i> |
| 4 | 0-4 | 74 | 2 | =B4*C4 |
| 5 | 5-9 | 192 | 7 | |
| 6 | 10-14 | 280 | 12 | |
| 7 | 15-19 | 105 | 17 | |
| 8 | 20-24 | 23 | 22 | |
| 9 | 25-29 | 6 | 27 | |

Select cell D4 and double click the *fill handle* or drag the *fill handle* down to cell D9.

4. Calculate the average in cell B12. (See Fig. 5.8)

Figure 5.8 Formula Sheet

| | A | B | C | D |
|----|-----------|-------------|----------|-------------|
| 1 | Gasoline | | | |
| 2 | (gallons) | Frequency | Midpoint | |
| 3 | | <i>f</i> | <i>M</i> | <i>fM</i> |
| 4 | 0-4 | 74 | 2 | =B4*C4 |
| 5 | 5-9 | 192 | 7 | =B5*C5 |
| 6 | 10-14 | 280 | 12 | =B6*C6 |
| 7 | 15-19 | 105 | 17 | =B7*C7 |
| 8 | 20-24 | 23 | 22 | =B8*C8 |
| 9 | 25-29 | 6 | 27 | =B9*C9 |
| 10 | | =SUM(B4:B9) | | =SUM(D4:D9) |
| 11 | | | | |
| 12 | Average | =D10/B10 | | |

Figure 5.8 Value Sheet

| | A | B | C | D |
|----|-----------|-----------|----------|-----------|
| 1 | Gasoline | | | |
| 2 | (gallons) | Frequency | Midpoint | |
| 3 | | <i>f</i> | <i>M</i> | <i>fM</i> |
| 4 | 0-4 | 74 | 2 | 148 |
| 5 | 5-9 | 192 | 7 | 1344 |
| 6 | 10-14 | 280 | 12 | 3360 |
| 7 | 15-19 | 105 | 17 | 1785 |
| 8 | 20-24 | 23 | 22 | 506 |
| 9 | 25-29 | 6 | 27 | 162 |
| 10 | | 680 | | 7305 |
| 11 | | | | |
| 12 | Average | 10.743 | | |

5. After calculating the average, it is now possible to calculate the variance

$$s^2 = \frac{\sum f_i (M_i - \bar{x})^2}{n}$$

in columns E, F and G and in cell B13. (See Fig. 5.9 and let $\bar{x} = \mathbf{xbar}$.)

Figure 5.9 Formula Sheet

| | A | B | C | D | E | F | G |
|----|-----------|--------------|----------|-----------|-------------------|---------------------|----------------------|
| 1 | Gasoline | | | | | | |
| 2 | (gallons) | Frequency | Midpoint | | | | |
| 3 | | <i>f</i> | <i>M</i> | <i>fM</i> | <i>(M - xbar)</i> | <i>(M - xbar)^2</i> | <i>f(M - xbar)^2</i> |
| 4 | 0-4 | 74 | 2 | 148 | =C4-\$B\$12 | =E4*E4 | =B4*F4 |
| 5 | 5-9 | 192 | 7 | 1344 | =C5-\$B\$12 | =E5*E5 | =B5*F5 |
| 6 | 10-14 | 280 | 12 | 3360 | =C6-\$B\$12 | =E6*E6 | =B6*F6 |
| 7 | 15-19 | 105 | 17 | 1785 | =C7-\$B\$12 | =E7*E7 | =B7*F7 |
| 8 | 20-24 | 23 | 22 | 506 | =C8-\$B\$12 | =E8*E8 | =B8*F8 |
| 9 | 25-29 | 6 | 27 | 162 | =C9-\$B\$12 | =E9*E9 | =B9*F9 |
| 10 | | 680 | | 7305 | | | =SUM(G4:G9) |
| 11 | | | | | | | |
| 12 | Average | 10.743 | | | | | |
| 13 | Variance | =G10/(B10-1) | | | | | |

Figure 5.9 Value Sheet

| | A | B | C | D | E | F | G |
|----|-----------|-----------|----------|-----------|-------------------|---------------------|----------------------|
| 1 | Gasoline | | | | | | |
| 2 | (gallons) | Frequency | Midpoint | | | | |
| 3 | | <i>f</i> | <i>M</i> | <i>fM</i> | <i>(M - xbar)</i> | <i>(M - xbar)^2</i> | <i>f(M - xbar)^2</i> |
| 4 | 0-4 | 74 | 2 | 148 | -8.743 | 76.434 | 5656.107 |
| 5 | 5-9 | 192 | 7 | 1344 | -3.743 | 14.007 | 2689.422 |
| 6 | 10-14 | 280 | 12 | 3360 | 1.257 | 1.581 | 442.662 |
| 7 | 15-19 | 105 | 17 | 1785 | 6.257 | 39.154 | 4111.219 |
| 8 | 20-24 | 23 | 22 | 506 | 11.257 | 126.728 | 2914.744 |
| 9 | 25-29 | 6 | 27 | 162 | 16.257 | 264.302 | 1585.809 |
| 10 | | 680 | | 7305 | | | 17399.963 |
| 11 | | | | | | | |
| 12 | Average | 10.743 | | | | | |
| 13 | Variance | 25.626 | | | | | |

6. The standard deviation is calculated in cell B14. (See Fig 5.10)

Figure 5.10 Formula Sheet

| | A | B |
|----|-----------------|--------------|
| 12 | Average | =D10/B10 |
| 13 | Variance | =G10/(B10-1) |
| 14 | Std.dev. | =SQRT(B13) |

Figure 5.10 Value Sheet

| | A | B |
|----|-----------------|--------|
| 12 | Average | 10.743 |
| 13 | Variance | 25.626 |
| 14 | Std.dev. | 5.062 |

Note:

If the service station expects to service 120 cars on a given day, then the estimate of total gallons sold would be: $(10.743)(120) = 1289.16$