Introduction to Reinforcement Learning

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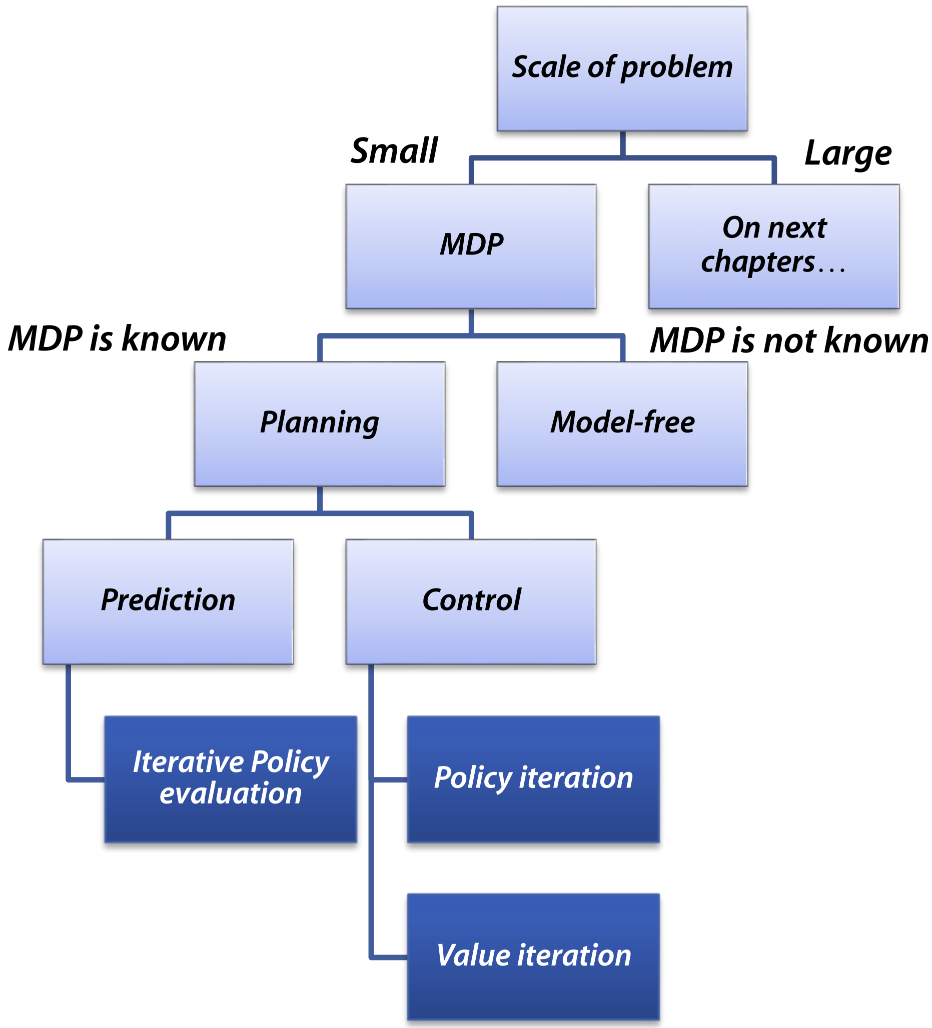
Mobile System Engineering

Categories


Scale of problem
Small
Large
MDP
On next chapters…
MDP is known
MDP is not known
Planning
Model-free
Prediction
Control
Iterative Policy evaluation
Policy iteration
Value iteration

MDP planning

- ***When all information about MDP is known, the process of using it to improve policies is called planning.***

- ***Prediction***
  - ***Finding the state-values when π is given***
- ***Control***
  - ***Finding the optimal policy π_****

What is Dynamic Programming?

Dynamic sequential or temporal component to the problem

Programming optimising a "program", i.e. a policy

- c.f. linear programming

- A method for solving complex problems
- By breaking them down into subproblems
  - Solve the subproblems
  - Combine solutions to subproblems

Requirements for Dynamic Programming

Dynamic Programming is a very general solution method for problems which have two properties:

- Optimal substructure
  - *Principle of optimality* applies
  - Optimal solution can be decomposed into subproblems
- Overlapping subproblems
  - Subproblems recur many times
  - Solutions can be cached and reused
- Markov decision processes satisfy both properties
  - Bellman equation gives recursive decomposition
  - Value function stores and reuses solutions

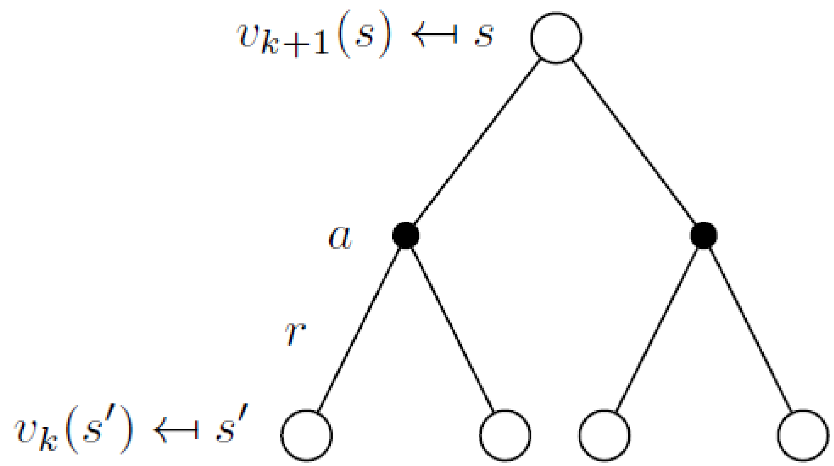
Planning by Dynamic Programming

- Dynamic programming assumes full knowledge of the MDP
- It is used for *planning* in an MDP
- For prediction:
  - Input: MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ and policy π
  - or: MRP $\langle \mathcal{S}, \mathcal{P}^{\pi}, \mathcal{R}^{\pi}, \gamma \rangle$
  - Output: value function v_{π}
- Or for control:
  - Input: MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$
  - Output: optimal value function v_*
  - and: optimal policy π_*

Iterative policy evaluation

- Problem: evaluate a given policy π
- Solution: iterative application of Bellman expectation backup
- $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_\pi$
- Using *synchronous* backups,
  - At each iteration $k+1$
  - For all states $s \in \mathcal{S}$
  - Update $v_{k+1}(s)$ from $v_k(s')$
  - where s' is a successor state of s

Iterative policy evaluation

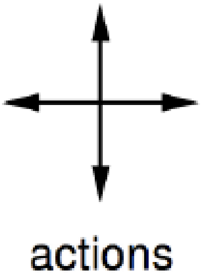


$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$

$$\mathbf{v}^{k+1} = \boldsymbol{\mathcal{R}}^{\boldsymbol{\pi}} + \gamma \boldsymbol{\mathcal{P}}^{\boldsymbol{\pi}} \mathbf{v}^k$$

Evaluating a Random Policy in the Small Gridworld

s0	*s1*	*s2*	*s3*
s4	*s5*	*s6*	*s7*
s8	*s9*	*s10*	*s11*
s12	*s13*	*s14*	*s15*



$r = -1$
on all transitions

- *Undiscounted episodic MDP ($\gamma = 1$)*
- *One terminal state s_{15}*
- *Actions leading out of the grid leave state unchanged*
- *Reward is -1 until the terminal state is reached (at the terminal state -> 0)*
- *Action follows uniform random policy*

$$\pi(n|\cdot) = \pi(e|\cdot) = \pi(s|\cdot) = \pi(w|\cdot) = 0.25$$

Iterative policy evaluation

$v(s)$			
0.0	0.0	0.0	0.0
0.0	-1.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

$v(s)$			
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

Iterative policy evaluation

state			
s0	***s1***	***s2***	***s3***
s4	***s5***	***s6***	***s7***
s8	***s9***	***s10***	***s11***
s12	***s13***	***s14***	***s15***

$v(s), k = 0$			
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

$v(s), k = 1$			
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

$v(s), k = 2$			
-2.0	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	***-1.75***
-2.0	-2.0	-1.75	0.0

$v(s), k = 3$			
-3.0	-3.0	-3.0	-3.0
-3.0	***-3.0***	***-3.0***	-2.94
-3.0	***-3.0***	-2.88	-2.4
-3.0	-2.94	-2.4	0.0

$v(s), k = \infty$			
-59.4	-57.4	-54.3	-51.7
-57.4	-54.6	-49.7	-45.1
-54.3	-49.7	-40.9	-30
-51.7	-45.1	-30	0.0

How to Improve a Policy

- Given a policy π
  - Evaluate the policy π

$$v_\pi(s) = \mathbb{E}\left[R_{t+1} + \gamma R_{t+2} + ... | S_t = s\right]$$

  - Improve the policy by acting greedily with respect to v_π

$$\pi' = \text{greedy}(v_\pi)$$

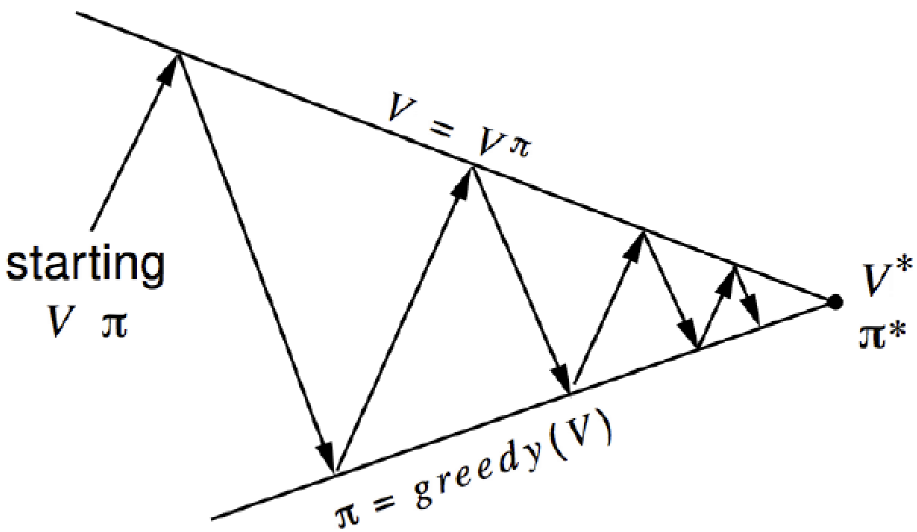
- In Small Gridworld improved policy was optimal, $\pi' = \pi^*$
- In general, need more iterations of improvement / evaluation
- But this process of policy iteration always converges to $\pi *$

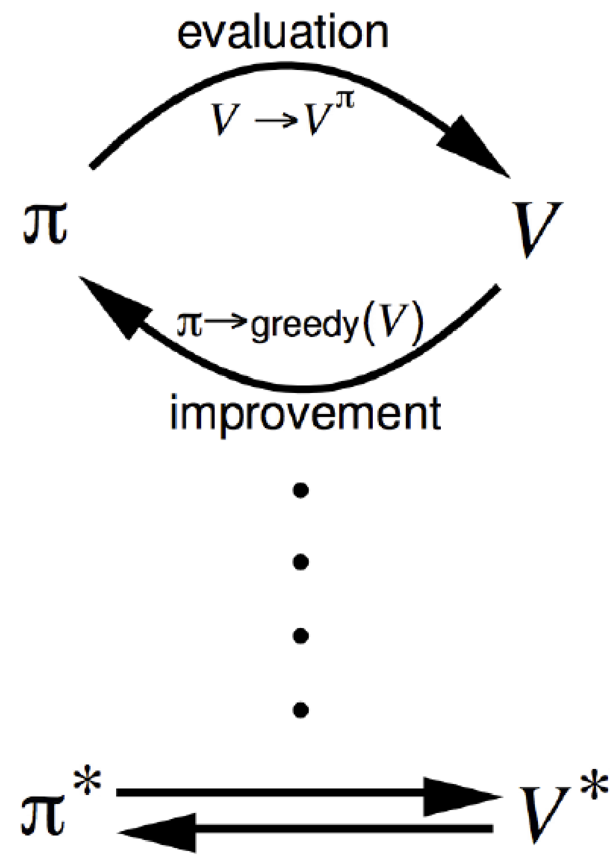
Greedy Policy

$v(s), k = \infty$			
-59.4	-57.4	-54.3	-51.7
-57.4	-54.6	-49.7	-45.1
-54.3	-49.7	-40.9	-30
-51.7	-45.1	-30	0.0

Greedy policy			
↓→	↓→	↓→	↓
↓→	↓→	↓→	↓
↓→	↓→	↓→	↓
→	→	→	F

Policy Iteration

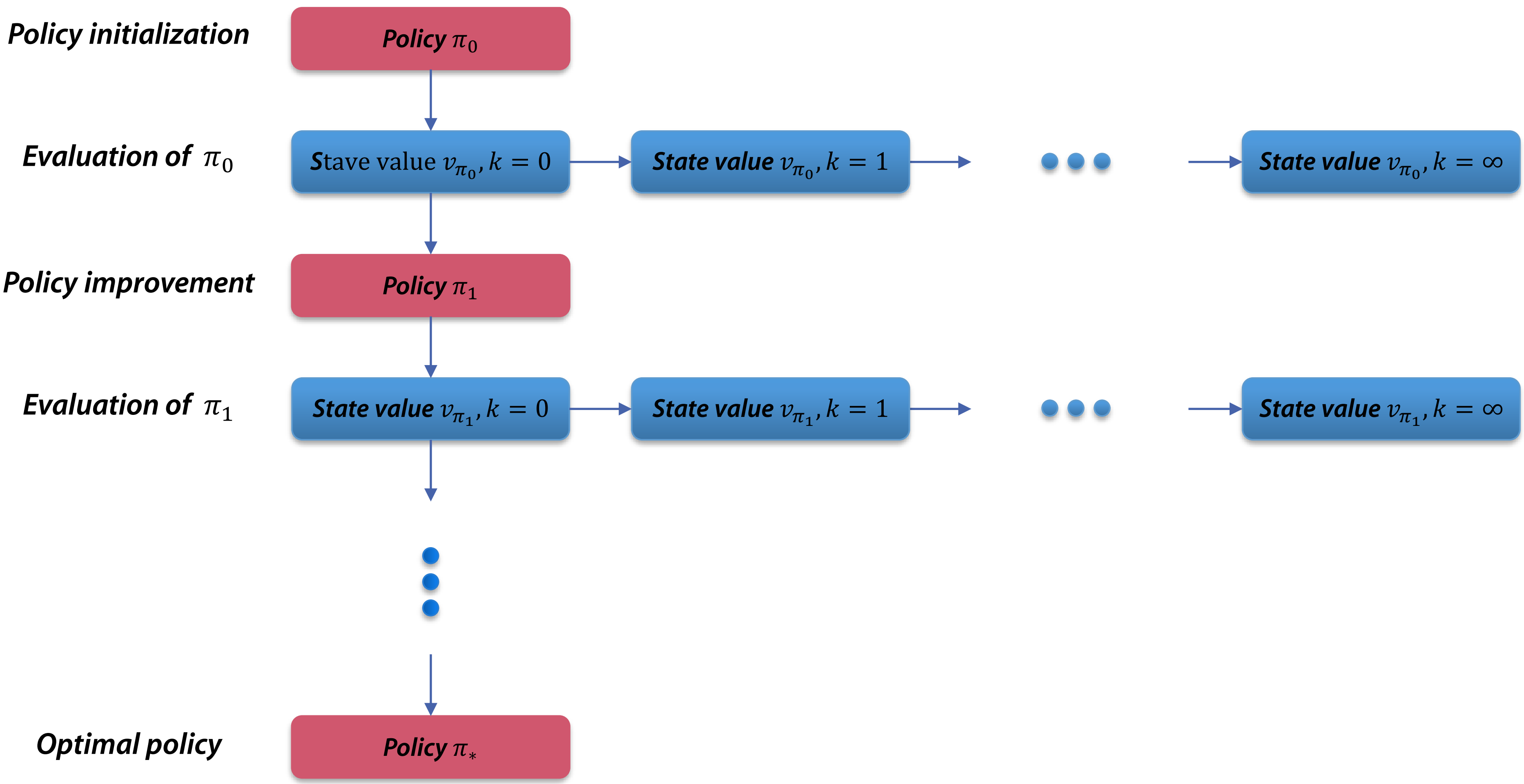




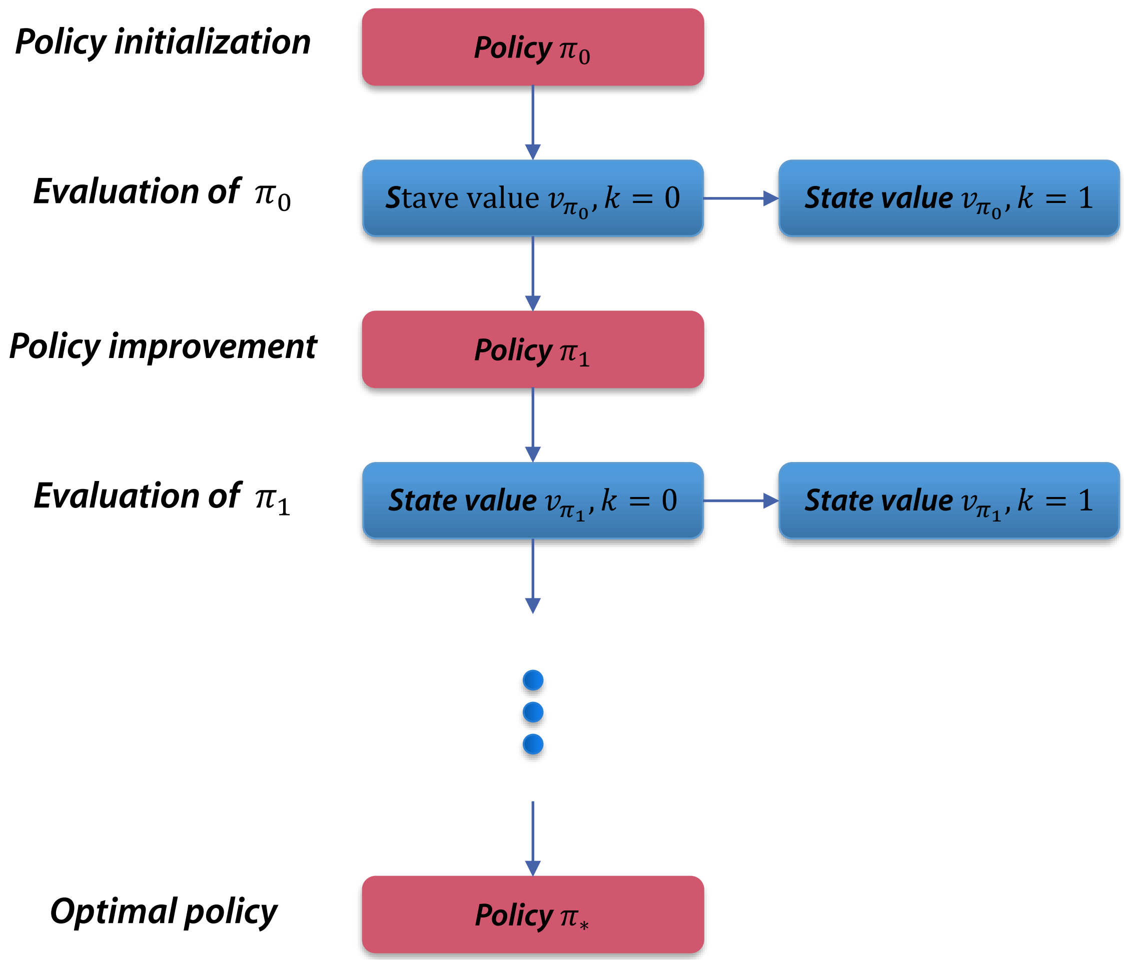
Policy evaluation Estimate v_π
Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$
Greedy policy improvement

Policy iteration



Early stopping



Early stopping

$v(s), k = 6$			
-6	-5.99	-5.96	-5.90
-5.99	***-5.95***	***-5.80***	-5.55
-5.96	***-5.80***	-5.27	-4.22
-5.90	-5.55	-4.22	0.0

$v(s), k = \infty$			
-59.4	-57.4	-54.3	-51.7
-57.4	-54.6	-49.7	-45.1
-54.3	-49.7	-40.9	-30
-51.7	-45.1	-30	0.0

Policy improvement

- Consider a deterministic policy, $a = \pi(s)$
- We can *improve* the policy by acting greedily

$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, q_\pi(s, a)$$

- This improves the value from any state s over one step,

$$q_\pi(s, \pi'(s)) = \max_{a \in \mathcal{A}} q_\pi(s, a) \geq q_\pi(s, \pi(s)) = v_\pi(s)$$

- It therefore improves the value function, $v_{\pi'}(s) \geq v_\pi(s)$

$$\begin{aligned}
v_\pi(s) \leq q_\pi(s, \pi'(s)) &= \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s\right] \\
&\leq \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma q_\pi(S_{t+1}, \pi'(S_{t+1})) \mid S_t = s\right] \\
&\leq \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 q_\pi(S_{t+2}, \pi'(S_{t+2})) \mid S_t = s\right] \\
&\leq \mathbb{E}_{\pi'}\left[R_{t+1} + \gamma R_{t+2} + \ldots \mid S_t = s\right] = v_{\pi'}(s)
\end{aligned}$$

Policy improvement

- If improvements stop,

$$q_\pi(s, \pi'(s)) = \max_{a \in \mathcal{A}} q_\pi(s, a) = q_\pi(s, \pi(s)) = v_\pi(s)$$

- Then the Bellman optimality equation has been satisfied

$$v_\pi(s) = \max_{a \in \mathcal{A}} q_\pi(s, a)$$

- Therefore $v_\pi(s) = v_*(s)$ for all $s \in \mathcal{S}$
- so π is an optimal policy

Deterministic Value Iteration

- If we know the solution to subproblems $v_*(s')$
- Then solution $v_*(s)$ can be found by one-step lookahead

$$v_*(s) \leftarrow \max_{a \in \mathcal{A}} \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_*(s')$$

- The idea of value iteration is to apply these updates iteratively
- Intuition: start with final rewards and work backwards
- Still works with loopy, stochastic MDPs

Value iteration

- $v_*(s) = \max_a \left[r_S^a + \gamma \sum_{s' \in S} P_{s,s'}^a v_*(s') \right]$
  - $r_S^a = -1$ ***for all actions***
  - $\gamma = 1$
  - $P_{s,s'}^a = 1$ ***for all actions and states***
  - $v_*(s') = 0$

- $v_*(s_5) = \max(-1 + 1.0 * 0,$
  $-1 + 1.0 * 0,$
  $-1 + 1.0 * 0,$
  $-1 + 1.0 * 0,)$
  $= -1.0$

Optimal state value			
s0	***s1***	***s2***	***s3***
s4	***s5***	***s6***	***s7***
s8	***s9***	***s10***	***s11***
s12	***s13***	***s14***	***F***

Optimal state value			
0.0	0.0	0.0	0.0
0.0	-1.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

Value iteration

State			
s0	***s1***	***s2***	***s3***
s4	***s5***	***s6***	***s7***
s8	***s9***	***s10***	***s11***
s12	***s13***	***s14***	종료

Optimal state value, $k = 0$			
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

Optimal state value, $k = 1$			
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

Optimal state value, $k = 2$			
-2.0	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	***-1.0***
-2.0	-2.0	-1.0	0.0

Optimal state value, $k = 3$			
-3.0	-3.0	-3.0	-3.0
-3.0	***-3.0***	***-3.0***	-2.94
-3.0	***-3.0***	-2.0	-1.0
-3.0	-2.0	-1.0	0.0

Optimal state value, $k = \infty$			
-6.0	-5.0	-4.0	-3.0
-5.0	-4.0	-3.0	-2.0
-4.0	-3.0	-2.0	-1.0
-3.0	-2.0	-1.0	0.0

Value iteration

- *Let's intuitively think about how many steps it would take to follow the optimal policy from the starting state to the ending point.*
  - *Result of multiple applications of the Bellman Optimal Equation*
- *Now that we have found the optimal value, we can find the optimal policy*
  - *You can move to the square with the highest optimal value.*
  - *That is, the greedy policy for optimal value*

Optimal state value, $k = \infty$			
-6.0	-5.0	-4.0	-3.0
-5.0	-4.0	-3.0	-2.0
-4.0	-3.0	-2.0	-1.0
-3.0	-2.0	-1.0	0.0