ComputerVision

Week4 – 5

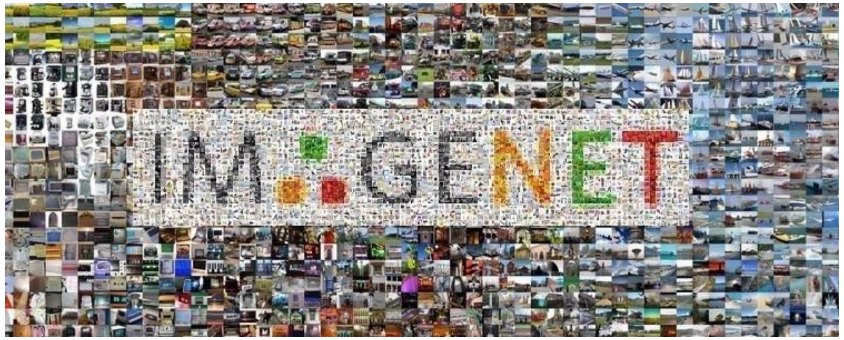
2025-2

Mobile Systems Engineering

Dankook University

Why Go Deeper?

- **The Promise of Depth in Deep Neural Networks**
  - **Deep neural networks have revolutionized visual recognition.**
  - **More layers = Better representations**:
    - Capture low-level (edges), mid-level (textures), and high-level (objects) features.
  - **Empirically proven**
    - VGG, GoogLeNet: depth correlates with improved performance on ImageNet and COCO.



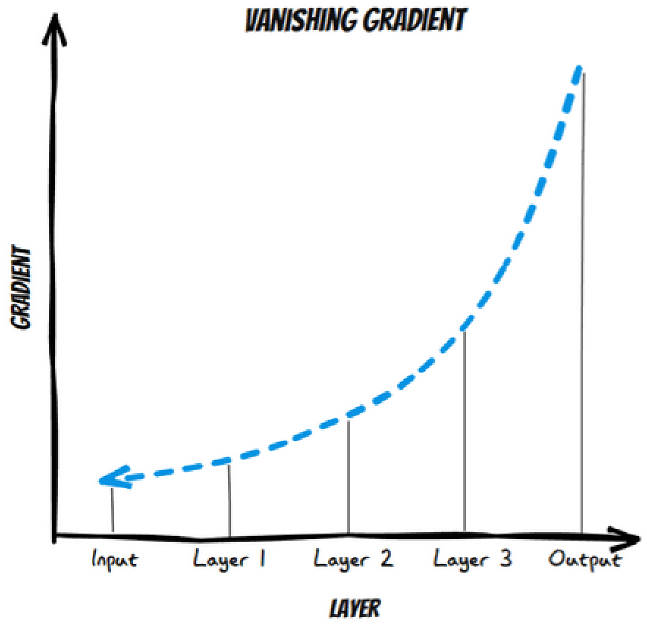
BUT... deeper networks are hard to train!

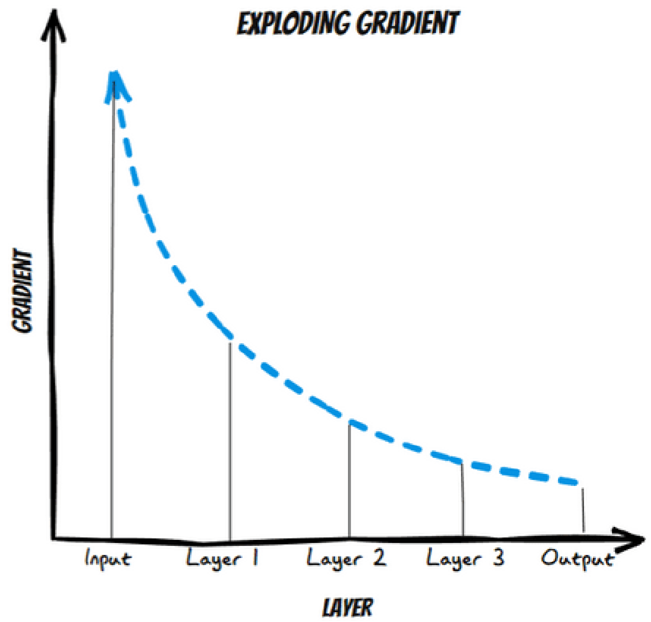
The Optimization Challenges

- **When More Layers Make Things Worse**
  - **1. Vanishing/Exploding Gradients**
    - As gradients are backpropagated, they can diminish (vanish) or blow up (explode).
    - Leads to unstable or slow training.
  - **2. Saturation of Accuracy**
    - Adding layers sometimes leads to **higher training error**, not just test error.
    - Known as the **"degradation problem"**.
  - **3. Intuition**
    - If we take a shallower network and add more layers, the deeper one should perform at least as well.
    - But in reality: deeper plain networks perform worse.

What Are Vanishing and Exploding Gradients?

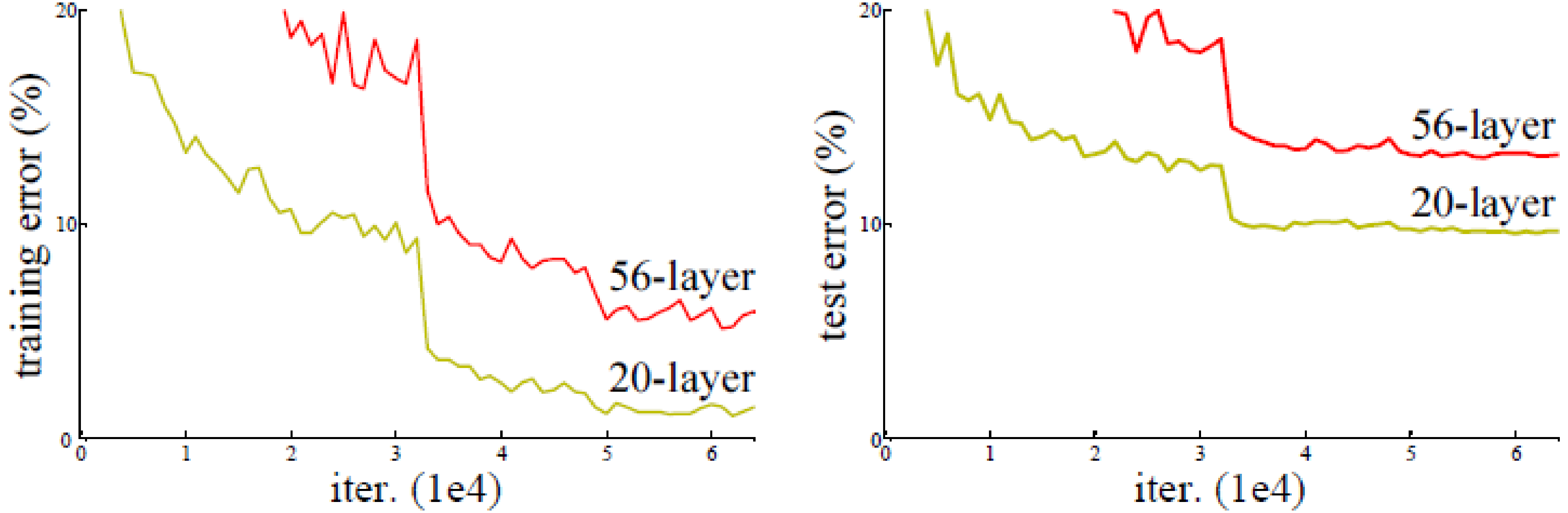
- **Vanishing and Exploding Gradients**
  - As we train deep neural networks using **backpropagation**, gradients are propagated backward through layers.
  - During this process
    - Gradients may **shrink toward zero** (vanish)
    - Or **grow excessively large** (explode)
  - Both scenarios make training **unstable or slow**.





Degradation Problem (Empirical Evidence)

- **Deeper Plain Networks Have Higher Training Error**
  - **Observation from Deep Residual Learning for Image Recognition (ResNet) – He et al. (2015)**



- Train plain networks of 20 and 56 layers on CIFAR-10
- Despite more capacity, **56-layer network performs worse** (training + test error ↑)

→ *“Indicates **optimization failure**, not overfitting!”*

Understanding the Degradation Problem

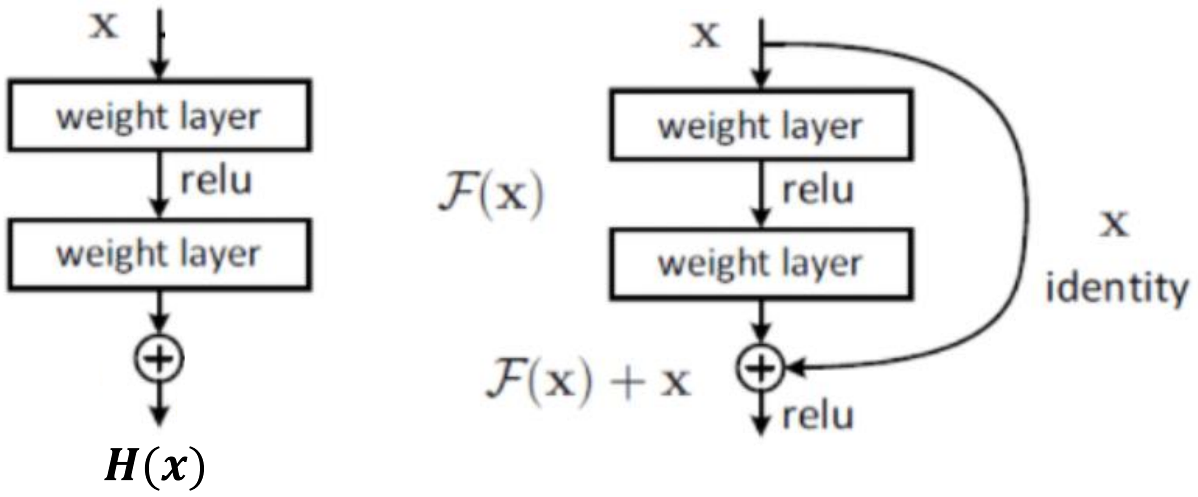
- **There Exists a Simple Solution — But It's Hard to Learn**
  - The deeper model has a **constructed solution**
    - Suppose you have a well-trained shallow network.
    - You can always create a deeper version by:
      - ✓Copying the original layers.
      - ✓Adding extra layers that perform **identity mapping** (i.e., output = input).
    - In theory, this should produce at least equal performance.
  - **So why does performance degrade in practice?**
    - It's not due to **overfitting** (since training error increases).
    - It's not due to **vanishing gradients** (since techniques like BatchNorm are applied).

Understanding the Degradation Problem

- **There Exists a Simple Solution — But It’s Hard to Learn**
  - **The real problem**
    - **Stochastic Gradient Descent (SGD) fails to find the identity mapping**.
    - The network **struggles to learn a perfect *"pass-through"* behavior through non-linear transformations**.
    - Even a seemingly trivial task — like learning to output the same input — becomes difficult for deep non-linear layers.
  - **Key Insight**
    - Depth alone is not the problem — **learning identity mappings through standard layers is**.

Need for Better Formulation

- **Residual Learning Reformulation From Learning Mappings to Learning Residuals**
  - **Key Insight**
    - Rather than learn the full mapping $\boldsymbol{H(x)}$, let's learn only what is missing from the input.
    - Define residual function: $\boldsymbol{F(x)} = \boldsymbol{H(x)} - \boldsymbol{x} \quad \Rightarrow \boldsymbol{H(x)} = \boldsymbol{F(x)} + \boldsymbol{x}$.
  - **Residual Learning Reformulation**
    - Original mapping: $\boldsymbol{H(x)}$
    - Reformulated: $\boldsymbol{H(x)} = \boldsymbol{F(x)} + \boldsymbol{x}$

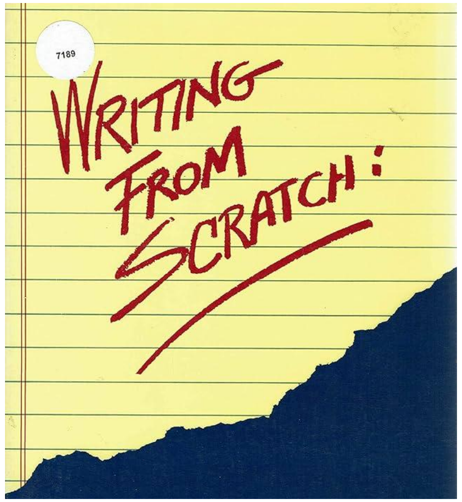


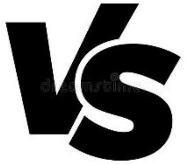
  - **Why this helps**
    - If the optimal mapping is close to identity, then
      - ✓ $\boldsymbol{F(x)}$ is close to zero → easier to learn.
    - Even if $\boldsymbol{H(x)}$ is complex, it may still be **easier to express the difference** from the input than to learn the entire function from scratch.

*"This concept is like **preconditioning** in numerical optimization — reshaping the problem to make it easier for solvers to converge."*

Analogy: Learning Perturbations

- **Residuals as Minor Adjustments to Known Inputs**
  - Think of $\boldsymbol{F}(\boldsymbol{x})$ as a **small correction or delta** to the input
    - For example, if the correct output is close to the input, the network only needs to learn the difference.
  - **Identity mapping as the "default path"**
    - The skip connection directly carries the input forward.
    - If the network doesn't need to change it much, the residual function learns small tweaks.
    - If the input needs major changes, the residual function takes full control.
  - **Example – Writing**





Summary of Motivation

- **Why Residual Learning Became a Breakthrough**
  - Deep networks are **theoretically powerful**, but practically **difficult to optimize**.

  - As depth increases, **plain networks**
    - Encounter vanishing gradients, even with tricks like BatchNorm.
    - Show **increased training error** — a clear sign of optimization issues.

  - **Residual learning provides a solution**
    - Skip connections allow gradients to flow unimpeded.
    - Identity mappings are **easy to learn** with this architecture.
    - Residual blocks make it easier for the optimizer to converge.

  - **Impact**
    - Enables networks with **>100 layers** to be trained effectively.
    - Became the foundation for
      - ✓ResNet, ResNeXt
      - ✓Faster R-CNN (backbone)
      - ✓Mask R-CNN, and more.

What Are Vanishing and Exploding Gradients?

- **What is a Computational Graph?**
  - A computational graph is a **directed acyclic graph** representing a function.
    - **Nodes:** operations (e.g., +, ×, ReLU, etc.)
    - **Edges:** flow of values (scalars, vectors)
  - Enables automatic differentiation via backpropagation
  - **Example**
    - If $\boldsymbol{z = x + y}$, and $\boldsymbol{L = sin(z)}$, then:
    - **Forward pass:** compute $\boldsymbol{z = x + y}$, then $\boldsymbol{L = sin(z)}$
    - **Backward pass:** compute $\boldsymbol{\frac{dL}{dz}, \frac{dL}{dx}, \frac{dL}{dy}}$

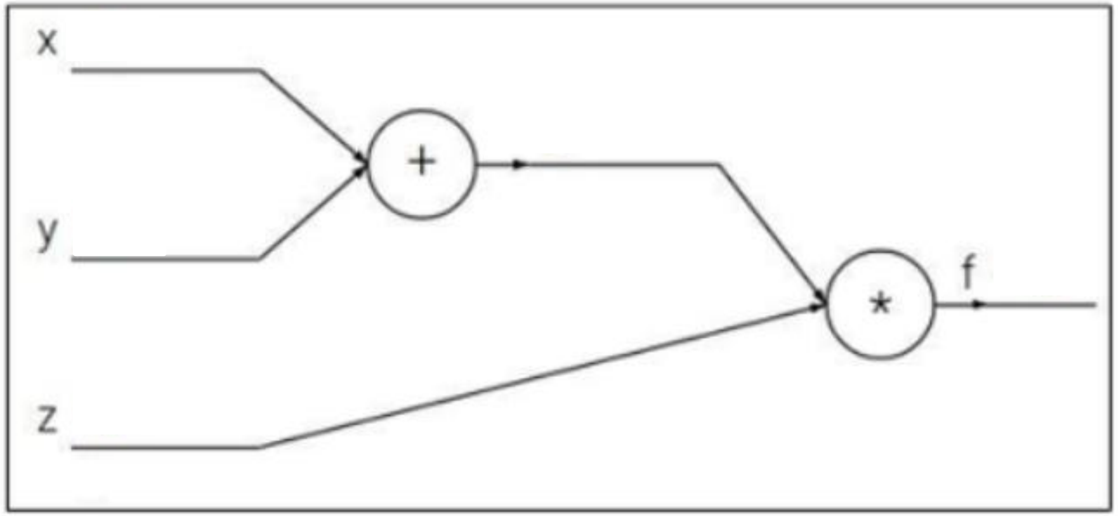
What Are Vanishing and Exploding Gradients?

- **Computational Graph Example**
  - **Function:** $\boldsymbol{L = (x + y) \cdot z}$
  - **Graph**
    - **Inputs:** $\boldsymbol{x, y, z}$
    - **Ops:** addition $\boldsymbol{x + y}$, multiplication with $\boldsymbol{z}$
    - **Final node:** loss L
  - ➡ **Forward Pass:** compute each value
    - **Intermediate variable:** $\boldsymbol{q = x + y}$ ***= −2 + 5 = 3***
    - **Final output:** $\boldsymbol{f = q \cdot z}$ **= 3 · (−4) = −12**

Backpropagation: a simple example

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



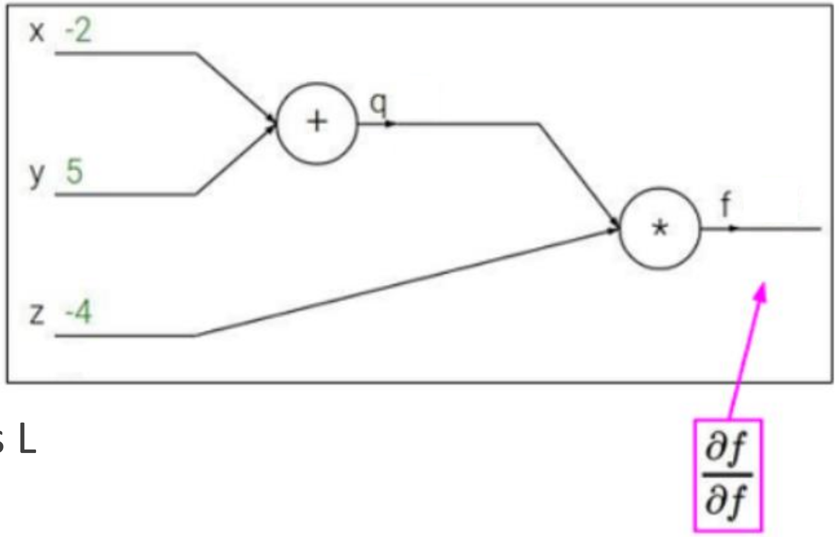
What Are Vanishing and Exploding Gradients?

- **Scalar Backpropagation**
  - **Function:** $L = (x + y) \cdot z$

Backpropagation: a simple example

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



  - **Graph**
    - **Inputs:** x, y, z / **Ops:** addition $x + y$, multiplication with z / **Final node:** loss L
  - **⟵ Backward Pass (Backpropagation)**
    - We want – $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$
    - **Step 1 –** From output to q and z: $\frac{\partial f}{\partial q} = z = -4$, $\frac{\partial f}{\partial z} = q = 3$
    - **Step 2** – From q to x and y: $\frac{\partial q}{\partial x} = 1$, $\frac{\partial q}{\partial y} = 1$
      - ✓Apply chain rule
        - ➢ $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} = -4 \cdot 1 = -4$
        - ➢ $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial y} = -4 \cdot 1 = -4$

What Are Vanishing and Exploding Gradients?

- **Vector Backpropagation**
  - **Extending Backpropagation to Matrix Operations**
    - **Goal:** Given a vector input $\boldsymbol{x}$ and a weight matrix $\boldsymbol{W}$, we aim to compute the gradient of the function

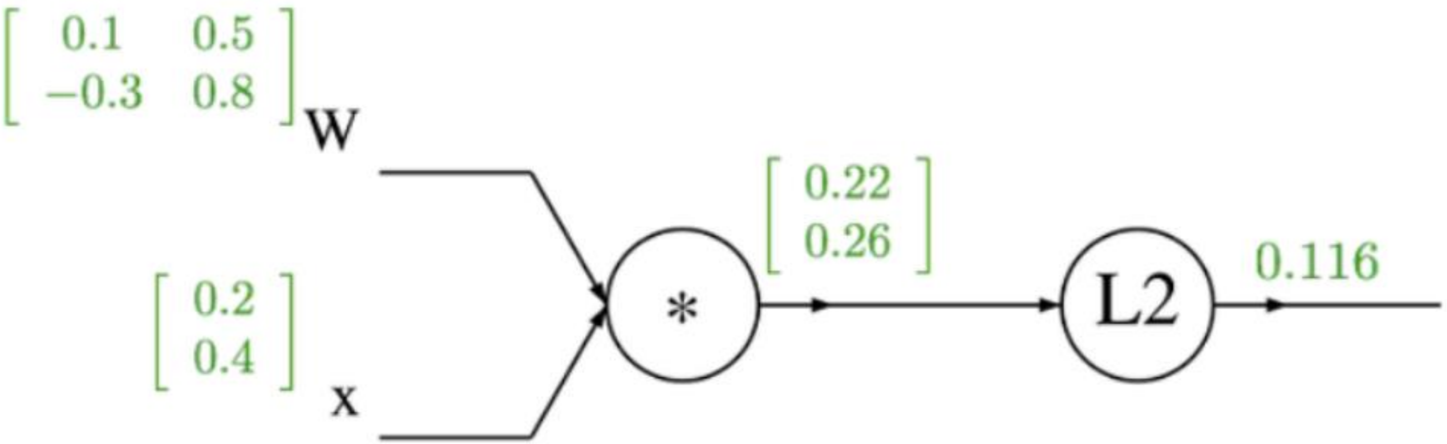
$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{W}) = \| \boldsymbol{W}\boldsymbol{x} \|^2 = \sum_{i=1}^{n} (\boldsymbol{W}\boldsymbol{x})_i^2$$

✓ $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{d}}$: input vector (column)

✓ $\boldsymbol{W} \in \mathbb{R}^{\boldsymbol{n}\times\boldsymbol{d}}$: weight matrix (trainable parameters)

✓ $\boldsymbol{q} = \boldsymbol{W}\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$: intermediate vector

✓ $\boldsymbol{f} = \| \boldsymbol{q} \|^2 = \boldsymbol{q}^{\boldsymbol{T}}\boldsymbol{q}$: scalar output

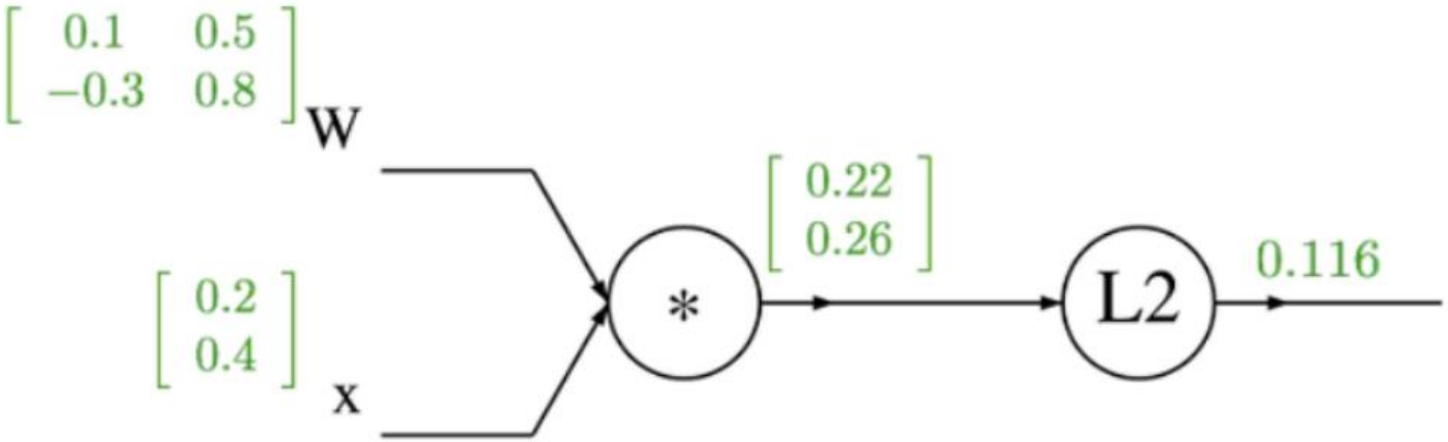


What Are Vanishing and Exploding Gradients?

- **Vector Backpropagation**
  - **Forward and Backward Pass (Step-by-Step)**
    - Step 1: Forward Pass

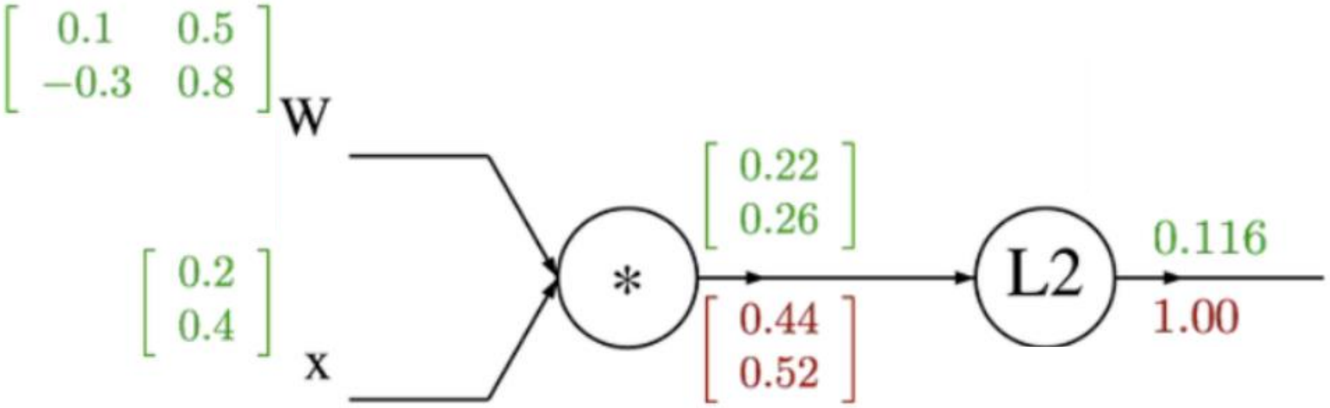
✓ $\boldsymbol{x} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \boldsymbol{W} = \begin{bmatrix} 0.1 & 0.5 \\ -0.3 & 0.8 \end{bmatrix}, \boldsymbol{q} = \boldsymbol{W}\boldsymbol{x} = \begin{bmatrix} 0.1 \cdot 0.2 + 0.5 \cdot 0.4 \\ -0.3 \cdot 0.2 + 0.8 \cdot 0.4 \end{bmatrix} = \begin{bmatrix} 0.22 \\ 0.26 \end{bmatrix}$

✓ $\boldsymbol{f} = \| \boldsymbol{q} \|^2 = 0.22^2 + 0.26^2 = 0.116$



What Are Vanishing and Exploding Gradients?

- **Vector Backpropagation**
  - **Forward and Backward Pass (Step-by-Step)**
    - **Step 2.1: Backward Pass – Gradient w.r.t. q**



✓ $f(x, W) = \| Wx \|^2 = \| q \|^2 = q^T q$

✓ $\frac{\partial f}{\partial q} = \nabla_q f = 2q = \begin{bmatrix} 0.44 \\ 0.52 \end{bmatrix}$

✓ ∇: It represents a **gradient operator** in multivariable calculus.

➢ If you have a function: $f(x_1, x_2, \dots, x_n)$, then the **gradient of** $f (\nabla f)$ is denoted by

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \cdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

➢ The gradient is a **vector** that points in the direction of **steepest increase** of the function f.

✓ **In Backpropagation**

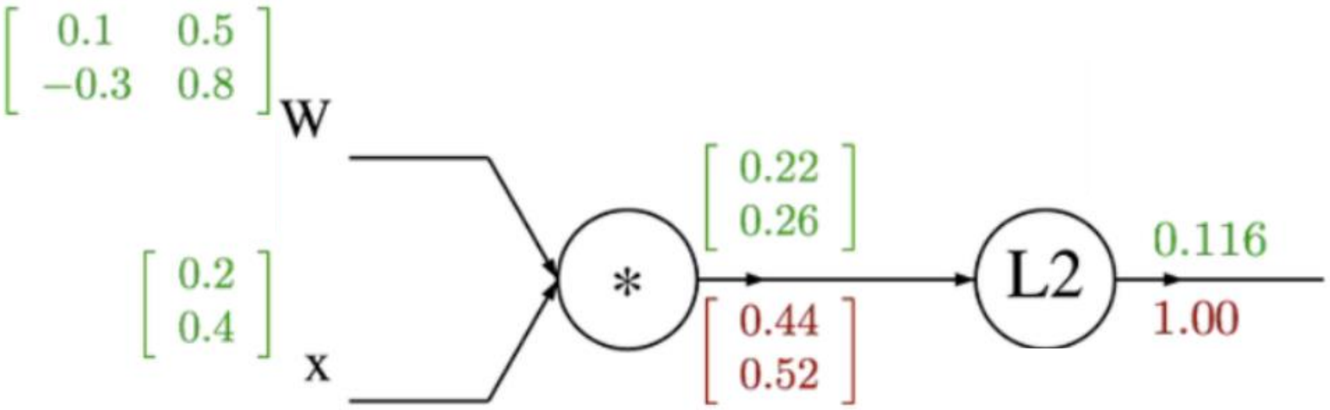
➢ $\nabla_x f$: How the scalar output f changes with respect to vector input x.

➢ $\nabla_W f$: The matrix of partial derivatives showing how each weight affects the loss.

➢ **Example:** If $f(x, y) = x^2 + y^2$, then:

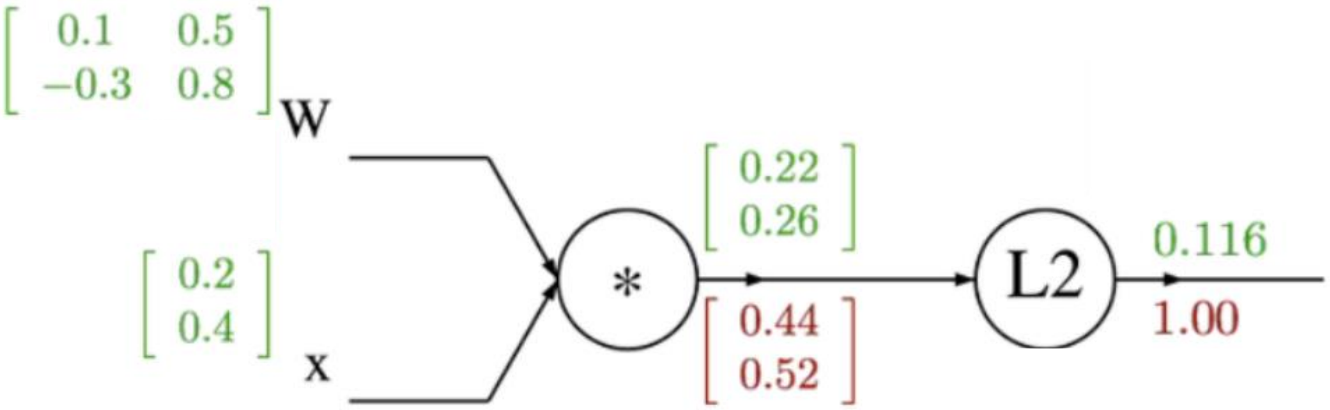
$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

What Are Vanishing and Exploding Gradients?



- **Vector Backpropagation**
  - **Forward and Backward Pass (Step-by-Step)**
    - **Step 2.1: Backward Pass – Gradient w.r.t. q**
      - ✓ $f(x, W) = \| Wx \|^2 = \| q \|^2 = q^T q$
      - ✓ $\frac{\partial f}{\partial q} = \nabla_q f = 2q = \begin{bmatrix} 0.44 \\ 0.52 \end{bmatrix}$
    - **Step 2.1: Step-by-Step Derivation**
      - ✓ Step 1: Expand the Function
        - ➢ $f(q) = qTq = \sum_{i=1}^{n} q_i^2$
      - ✓ Step 2: Differentiate Each Component
        - ➢ We compute $\frac{\partial f}{\partial q_i} = \frac{\partial}{\partial q_i} \sum_{j=1}^{n} q_j^2 = \frac{\partial}{\partial q_i} (q_1^2 + q_2^2 + \cdots + q_n^2)$
        - ➢ Since q_i^2 is the only term that depends on q_i, the derivative is $\frac{\partial f}{\partial q_i} = 2q_i$

What Are Vanishing and Exploding Gradients?



- **Vector Backpropagation**
  - **Forward and Backward Pass (Step-by-Step)**
    - **Step 2.1: Step-by-Step Derivation**
      - ✓Step 1: Expand the Function
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      - ✓Step 2: Differentiate Each Component
        - ➢We compute $\frac{\partial f}{\partial q_i} = \frac{\partial}{\partial q_i} \sum_{j=1}^{n} q_j^2 = \frac{\partial}{\partial q_i}(q_1^2 + q_2^2 + \cdots + q_n^2)$
        - ➢Since q_i^2 is the only term that depends on q_i, the derivative is $\frac{\partial f}{\partial q_i} = 2q_i$
      - ✓Step 3: Write the Gradient as a Vector
        - ➢Putting all partial derivatives together →
      - ✓**Final Result**
        - ➢$\frac{\partial}{\partial q}(q^T q) = 2q$

$$\nabla_q f = \begin{bmatrix} \frac{\partial f}{\partial q_1} \\ \frac{\partial f}{\partial q_2} \\ \cdots \\ \frac{\partial f}{\partial q_n} \end{bmatrix} = \begin{bmatrix} 2q_1 \\ 2q_2 \\ \cdots \\ 2q_n \end{bmatrix} = 2q$$

What Are Vanishing and Exploding Gradients?

- **Vector Backpropagation**
  - **Forward and Backward Pass (Step-by-Step)**
    - **Step 2.2: Backward Pass – Gradient w.r.t. W**

      ✓Using chain rule and matrix calculus: $\nabla_W f = \frac{\partial f}{\partial W} = 2q \cdot x^T$

      ✓Step-by-Step – Derivation of the Gradient

      ➢**Given Function:** $f(x, W) = ||Wx||^2 = (Wx)^T(Wx)$

      ➢Step1. Gradient w.r.t. q:

      We first rewrite f in terms of q: $f = q^T q$

      Then take the gradient of f with respect to the vector q: $\frac{\partial f}{\partial q} = \nabla_q f = 2q$

      ➢Step2. Apply the Chain Rule – To get the gradient of f with respect to W, we apply the chain rule

      $$\frac{\partial f}{\partial W} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial W} = 2q \cdot \frac{\partial q}{\partial W}$$

    - **Step 2.1: Backward Pass – Gradient w.r.t. q**

      ✓$f(x, W) = \| Wx \|^2 = \| q \|^2 = q^T q$

      ✓$\frac{\partial f}{\partial q} = \nabla_q f = 2q = \begin{bmatrix} 0.44 \\ 0.52 \end{bmatrix}$

What Are Vanishing and Exploding Gradients?

- **Vector Backpropagation**
  - **Forward and Backward Pass (Step-by-Step)**
    - ○ **Step 2.2: Backward Pass – Gradient w.r.t. W**
      - ✓Step-by-Step – Derivation of the Gradient
        - ➢Step3. Derivative of $\boldsymbol{q} = \boldsymbol{Wx}$

➢**Given Function:** $\boldsymbol{f(x, W) = ||Wx||^2 = (Wx)^T(Wx)}$

➢Step1. Gradient w.r.t. $\boldsymbol{q}$: $\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}} = \boldsymbol{\nabla_q f = \nabla_q q^T q = 2q}$

➢Step2. $\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{W}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}} \cdot \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W}} = \boldsymbol{2q} \cdot \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W}}$

Each component of q is $\boldsymbol{q_i} = \sum_{j=1}^{d} \boldsymbol{W_{ij} x_j}$

So the partial derivative is $\frac{\partial \boldsymbol{q_i}}{\partial \boldsymbol{W_{ij}}} = \boldsymbol{x_j} = \boldsymbol{\nabla_W q} = \begin{bmatrix} \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W}} \\ \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W_2}} \\ \cdots \\ \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W_n}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_1^T} \\ \boldsymbol{x_2^T} \\ \cdots \\ \boldsymbol{x_n^T} \end{bmatrix} \Rightarrow \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W}} = \boldsymbol{x^T}$

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{W_{ij}}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q_i}} \cdot \frac{\partial \boldsymbol{q_i}}{\partial \boldsymbol{W_{ij}}} = \boldsymbol{2q_i} \cdot \frac{\partial \boldsymbol{q_i}}{\partial \boldsymbol{W_{ij}}} \in \mathbb{R}^{n \times d}$$

$$= [\boldsymbol{2q_1} \quad \boldsymbol{2q_2} \quad \dots \quad \boldsymbol{2q_n}] \cdot [\boldsymbol{x_1} \quad \boldsymbol{x_2} \quad \dots \quad \boldsymbol{x_d}]^T = [\boldsymbol{2q_1} \quad \boldsymbol{2q_2} \quad \dots \quad \boldsymbol{2q_n}] \cdot \begin{bmatrix} \boldsymbol{x_1^T} \\ \boldsymbol{x_2^T} \\ \cdots \\ \boldsymbol{x_d^T} \end{bmatrix} = \begin{bmatrix} \boldsymbol{2q_1x_1} & \boldsymbol{2q_1x_2} & \dots & \boldsymbol{2q_1x_d} \\ \boldsymbol{2q_2x_1} & \boldsymbol{2q_2x_2} & \dots & \boldsymbol{2q_2x_d} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{2q_nx_1} & \boldsymbol{2q_nx_2} & \dots & \boldsymbol{2q_nx_d} \end{bmatrix}$$

$$\Rightarrow \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{W}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}} \cdot \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{W}} = \boldsymbol{2q} \cdot \frac{\partial \boldsymbol{q_i}}{\partial \boldsymbol{W_{ij}}} = \boldsymbol{2q \cdot x^T}$$

What Are Vanishing and Exploding Gradients?

- **Backpropagation in Fully Connected Layers**
  - **Gradient Computation Layer by Layer**
    - Let
      - ✓ $\boldsymbol{z}^l = \boldsymbol{W}^l \boldsymbol{a}^{l-1} + \boldsymbol{b}^l$: Linear transformation
      - ✓ $\boldsymbol{a}^l = f(\boldsymbol{z}^l)$: Activation (e.g., ReLU, sigmoid, tanh)
    - Then
      - ✓ Backpropagation equation for the gradient signal → $\boldsymbol{\delta}^l = \left(\boldsymbol{W}^{l+1}\right)^T \boldsymbol{\delta}^{l+1} \circ f'(\boldsymbol{z}^l)$
      - ✓ Gradient with respect to the weights → $\frac{\partial L}{\partial \boldsymbol{W}^l} = \boldsymbol{\delta}^l \left(\boldsymbol{a}^{l-1}\right)^T$
      - ✓ Here, $\boldsymbol{\delta}^l$ is the **error signal** passed from layer $\boldsymbol{l}+\boldsymbol{1}$ back to layer $\boldsymbol{l}$, scaled by the derivative of the activation.

- $\boldsymbol{z}^k \in \mathbb{R}^{d_k}$: Pre-activation output of layer $\boldsymbol{k}$ (i.e., before applying the activation function)
- $\boldsymbol{W}^k$: Weight matrix of the $\boldsymbol{k}$-th layer
- $\boldsymbol{a}^{k-1}$: Activation output from the previous layer
- $\boldsymbol{b}^k$: Bias vector

Term	Meaning	Mathematical Definition
$\boldsymbol{z}^k$	Pre-activation of layer $\boldsymbol{k}$	$\boldsymbol{z}^k = \boldsymbol{W}^k \boldsymbol{a}^{k-1} + \boldsymbol{b}^k$
$\boldsymbol{a}^k$	Output of layer $\boldsymbol{k}$ (post-activation)	$\boldsymbol{a}^k = f(\boldsymbol{z}^k)$
$\boldsymbol{\delta}^k$	Gradient signal at layer $\boldsymbol{k}$	$\boldsymbol{\delta}^k = \frac{\partial L}{\partial \boldsymbol{z}^k}$
$\boldsymbol{L}$	Total number of layers (depth)	$\boldsymbol{k} = 1, 2, \dots, \boldsymbol{L}$

What Are Vanishing and Exploding Gradients?

- **Backpropagation in Fully Connected Layers**
  - **Why Gradients Change Across Layers**
    - As we apply backpropagation repeatedly across many layers,

$$\boldsymbol{\delta}^{l} = \left(\boldsymbol{W}^{l+1}\right)^{\boldsymbol{T}} \boldsymbol{\delta}^{l+1} \circ \boldsymbol{f}'(\boldsymbol{z}^{l})$$

    - We are multiplying gradient vectors by weight matrices and activation derivatives at each step.
    - This means
      - ✓ If those values are **less than 1**, gradients **shrink**.
      - ✓ If they're **greater than 1**, gradients **grow**. This is the root cause of vanishing/exploding gradients.

What Are Vanishing and Exploding Gradients?

- **Vanishing Gradient — Root Cause**
  - **Why gradients shrink too much**
    - If

$$\| \boldsymbol{W^k} \| < \mathbf{1}\ \boldsymbol{and}\ | \boldsymbol{f'}(\boldsymbol{z^k}) | < \mathbf{1}$$

    - then

$$\| \boldsymbol{\delta^{\ell}} \| \rightarrow \mathbf{0}\ \boldsymbol{as}\ \boldsymbol{L} \rightarrow \infty$$

    - **This often happens when**
      - ✓You use **sigmoid** or **tanh**
      - ✓Weights are poorly initialized
      - ✓The network is very **deep**

> - $\boldsymbol{z^k} \in \mathbb{R}^{d_k}$: Pre-activation output of layer $\boldsymbol{k}$ (i.e., before applying the activation function)
> - $\boldsymbol{W^k}$: Weight matrix of the $\boldsymbol{k}$-th layer
> - $\boldsymbol{a^k} = \boldsymbol{f}(\boldsymbol{z^k})$: Output of layer $\boldsymbol{k}$ (post-activation)
> - $\boldsymbol{f'}(\boldsymbol{z^k})$: the derivative of the activation function $\boldsymbol{f}$ evaluated at $\boldsymbol{z^k}$

    - **Effect**
      - ✓Gradients are **too small to update early layers**
      - ✓Training becomes **very slow or fails entirely**

What Are Vanishing and Exploding Gradients?

- **Theoretical View – Exponential Bounds**
  - **If the Jacobian norms satisfy**
    - If

$$\left\|\frac{\partial a^k}{\partial a^{k-1}}\right\| \leq \lambda \;\; for\; all\; k$$

    - then

$$\left\|\frac{\partial L}{\partial W}\right\| \leq C \cdot \lambda^L$$

    - Where
      - ✓C: constant based on loss and inputs
      - ✓λ: upper bound on gradient propagation at each layer
      - ✓L: number of layers
    - Interpretation
      - ✓If $\lambda < 1$ → gradients vanish
      - ✓If $\lambda > 1$ → gradients explode
      - ✓If $\lambda = 1$ → gradients remain stable (ideal!)

○ k: the index of layers
○ L: the total number of layers

What Are Vanishing and Exploding Gradients?

- Think of **deep backpropagation** as simply applying the **chain rule multiple times**, just like in vector backpropagation—but layer by layer.
  - Backpropagation

$$\delta^l = \left(W^{l+1}\right)^T \delta^{l+1} \circ f'(z^l)$$

  - Gradient

$$\frac{\partial L}{\partial W^l} = \delta^l \left(a^{l-1}\right)^T$$

  - This mirrors

$$\frac{\partial f}{\partial W} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial W}$$

Error Signal **Input Influence**

- **From Simple to Deep**
  - **Start with vector form** to understand outer products and error × input structures.
  - **Extend to deep nets** using recursive formulas and activation derivatives

Recap – Summary of Motivation

- **Why Residual Learning Became a Breakthrough**
  - Deep networks are **theoretically powerful**, but practically **difficult to optimize**.
  - As depth increases, **plain networks**
    - Encounter vanishing gradients, even with tricks like BatchNorm.
    - Show **increased training error** — a clear sign of optimization issues.
  - **Residual learning provides a solution**
    - Skip connections allow gradients to flow unimpeded.
    - Identity mappings are **easy to learn** with this architecture.
    - Residual blocks make it easier for the optimizer to converge.
  - **Impact**
    - Enables networks with **>100 layers** to be trained effectively.
    - Became the foundation for:
      - ✓ResNet, ResNeXt
      - ✓Faster R-CNN (backbone)
      - ✓Mask R-CNN, and more.

What is ResNet?

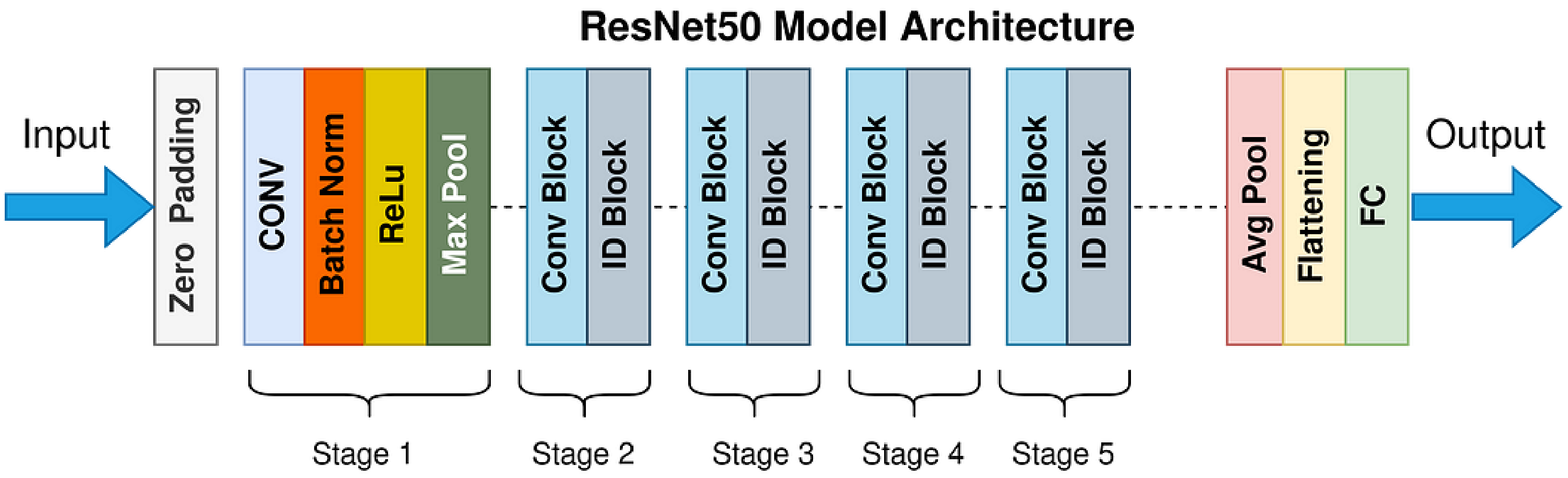
- **Deep Residual Networks – Introducing ResNet**
  - **A Breakthrough in Very Deep Neural Networks**
    - Proposed by **Kaiming He et al.** in **CVPR 2016**
    - Paper: *"Deep Residual Learning for Image Recognition"*
    - Won **ILSVRC 2015** with a **top-5 error of 3.57%**
    - Designed to enable **very deep neural networks** (e.g., 152 layers)
  - **Problem Addressed**
    - As depth increases, plain CNNs suffer from **optimization difficulties** and even **higher training error** → the **degradation problem**
  - **ResNet's solution**
    - Introduce **skip connections** and learn **residual functions** instead of direct mappings

Why Is ResNet Needed?

- **Going Deeper Isn’t Always Better**
  - **Deeper Networks Face Degradation Without the Right Design**
    - **Before ResNet:** VGG, GoogLeNet made progress by going deeper
    - **But:** ***simply adding layers doesn’t guarantee better performance!***
    - Example
      - ✓56-layer plain CNN performs worse than 20-layer CNN on CIFAR-10
  - This is **not** due to overfitting
    - It’s due to optimization issues — **gradient flow weakens**, training becomes harder
  - ResNet reformulates the learning task to address this!

Why Is ResNet Needed?

- ResNet Architecture at a Glance
  - High-Level Overview
    - Main structure
      - ✓ **Initial Layer**
        - 7×7 convolution with stride 2
        - Followed by 3×3 max pooling
      - ✓ **Four Stages of Residual Blocks**
        - Conv2_x, Conv3_x, Conv4_x, Conv5_x
        - Spatial resolution halves, channels double each stage
      - ✓ **Global Average Pooling (GAP)**
        - Aggregates features into a 1D vector
      - ✓ **Fully Connected Layer + Softmax**
        - Produces final classification scores



Example: ResNet-50 has 50 convolutional layers, mostly within residual blocks

ResNet Variants – Different Depths

- **ResNet Variants for Different Use Cases**
  - **Deeper ≠ Slower When Designed Right**

Model	Depth	Block Type	Parameters
ResNet-18	18	Basic (2×3×3 convs)	~11M
ResNet-34	34	Basic	~21M
ResNet-50	50	Bottleneck	~25M
ResNet-101	101	Bottleneck	~44M
ResNet-152	152	Bottleneck	~60M

  - **Basic block** is used in shallow networks (18/34)
  - **Bottleneck block** enables efficient deeper networks (50+)

Key Components of ResNet

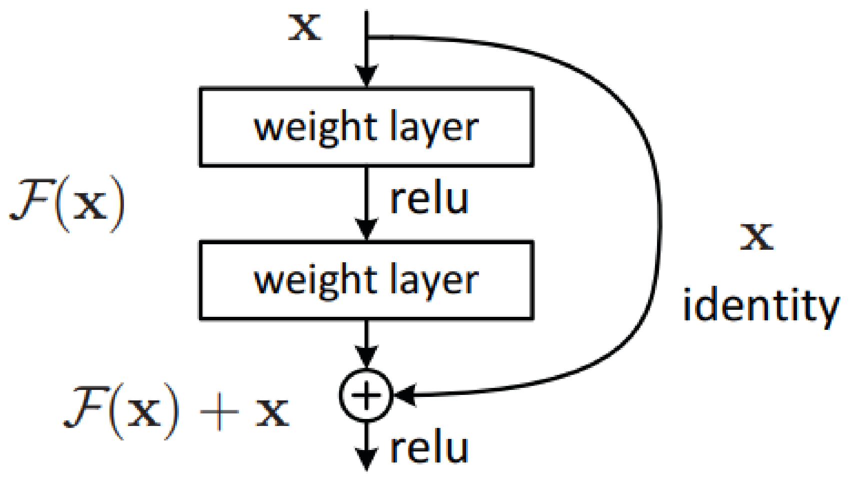
- **Components That Make ResNet Work**
  - **What's Inside the Architecture?**

Component	Role in the Network
Residual Block	Learns a residual function: $\boldsymbol{F(x)}$, and outputs $\boldsymbol{F(x)+x}$
Skip Connection	Directly passes input through identity or projection mapping
Bottleneck Block	Reduces → processes → restores dimensions via 1×1 / 3×3 / 1×1 convs
BatchNorm + ReLU	Applied after each convolution layer

- These design choices ensure effective training of **very deep** models

The first Key Components of ResNet

- **Introduction to the Residual Block**
  - **What is a Residual Block?**
    - A **Residual Block** is the core building unit of ResNet (Residual Network).
    - It was designed to address the **degradation problem** in deep networks, where adding more layers leads to **worse performance**.
    - Instead of directly learning the mapping $\boldsymbol{H(x)}$, it learns a **residual function** $\boldsymbol{F(x) = H(x) - x}$, and then reconstructs $\boldsymbol{H(x)}$ as $\boldsymbol{H(x) = F(x) + x}$



  - **Basic Structure of a Residual Block**
    - The standard formulation: $\boldsymbol{y = F(x, \{W_i\}) + x}$
      - ✓ Where $\boldsymbol{x}$ = input feature map, $\boldsymbol{F(x, \{W_i\})}$ = residual function (usually 2–3 convolutional layers with weights $\boldsymbol{\{W_i\}}$), $\boldsymbol{y}$ = output feature map
    - **Key Concept**
      - ✓ Instead of learning the full transformation, we let the stacked layers only learn the ***"difference"*** $\boldsymbol{F(x)}$ from the identity.

The first Key Components of ResNet

▪ Introduction to the Residual Block

• Why Add the Input Back?

○ **Key Idea**

✓Residual learning reformulates the desired mapping $\boldsymbol{H}(\boldsymbol{x})$ into:

$$\boldsymbol{H}(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{x}$$

→ The network only ***needs to learn the residual function*** $\boldsymbol{F}(\boldsymbol{x})$, not the full transformation $\boldsymbol{H}(\boldsymbol{x})$

○ **Why is this helpful? (Advantages)**

✓**1. Focus on what's new**

➢The layer learns ***only the part that needs to change*** (i.e., the residual difference $\boldsymbol{H}(\boldsymbol{x}) - \boldsymbol{x}$). This makes optimization easier, especially in deep networks.

➢**Naming Origin:** The term **"ResNet"** comes from the word **residual**
→ The network learns to **minimize the residual** $H(x) - x$. Hence, **Residual Network**.

The first Key Components of ResNet

- **Introduction to the Residual Block**
  - **Why Add the Input Back?**
    - **Why is this helpful? (Advantages)**
      - ✓ **2. Convergence behavior**
        - As the depth increases and the network is well-trained, the input $\boldsymbol{x}$ becomes increasingly close to the output $\boldsymbol{H(x)}$, so ***the residual*** $\boldsymbol{F(x) \to 0}$.
        - This ***stabilizes training*** and ***encourages minimal necessary updates***.
      - ✓ **3. Implementation simplicity**
        - **(1) No major change in architecture** is required.
        - **(2)** Simply add a **shortcut connection** from input to output.
        - **(3) No extra parameters** are introduced since $\boldsymbol{x}$ is reused.
        - **(4)** Aside from the **final addition operation**, the shortcut adds **almost no computational cost**.

The Second Key Components of ResNet

- Introduction to Skip (i.e., Shortcut) Connections
  - Types of Shortcut (i.e., Skip) Connections
    - When input and output dimensions do not match,
      - ✓1. Identity Mapping with Padding

$$y = F(x) + pad(x)$$

      - ✓2. Projection Mapping

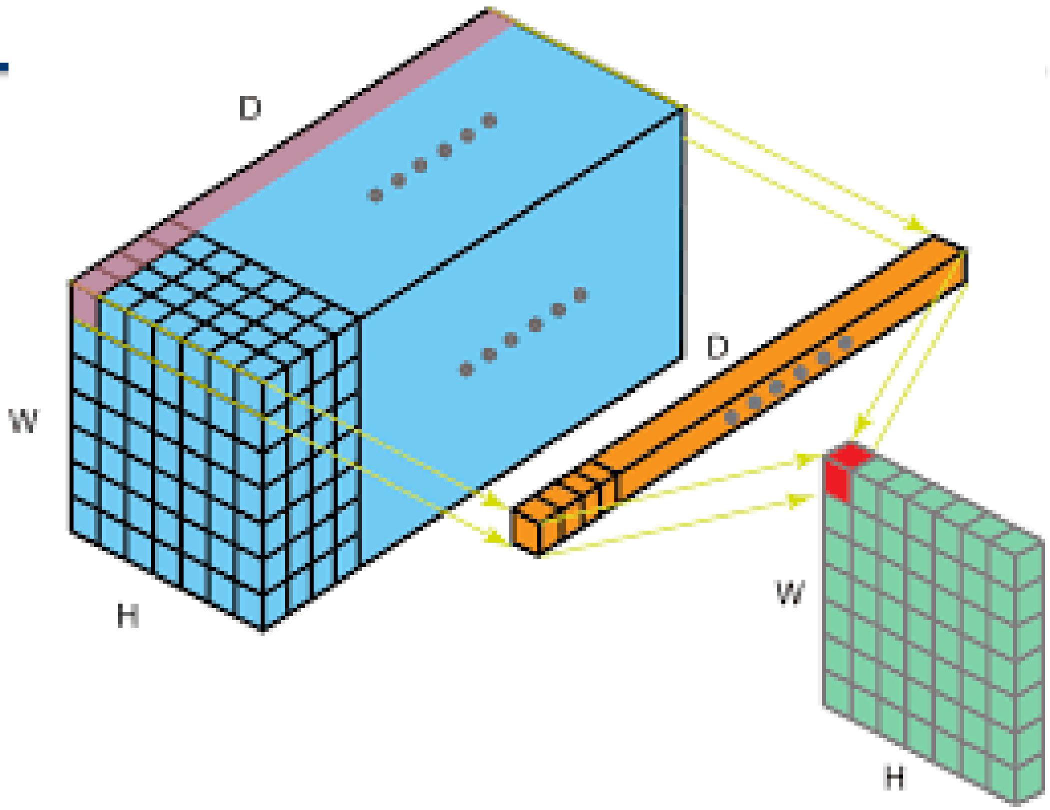
$$y = F(x) + W_s x$$

→ where W_s is a *1×1 convolution* used to match dimensions

What is 1x1 Convolution?

The Second Key Components of ResNet

- **Introduction to Skip (i.e., Shortcut) Connections**
  - **What is 1×1 Convolution?**



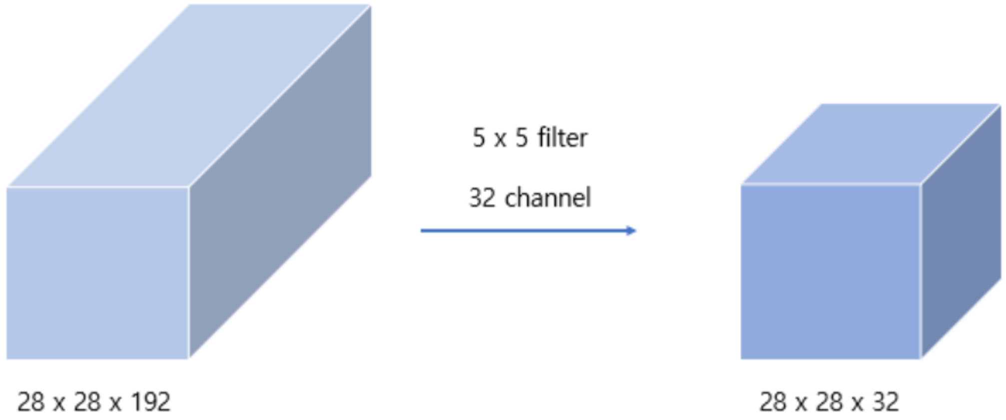
  - A **1×1 convolution** is a convolutional operation where the filter (kernel) has a spatial dimension of 1 (i.e., 1×1), but spans the full **depth (channel)** of the input.

  - Key Purposes of 1×1 Convolution

Purpose	Explanation
Channel Adjustment	Reduces or expands the number of channels without changing the spatial size (W×H)
Computational Efficiency	Reduces the number of parameters and operations, especially before expensive convolutions (e.g., 3×3, 5×5)
Adding Nonlinearity	When followed by an activation (e.g., ReLU), it increases model expressiveness
Bottleneck Design Enabler	Used in ResNet's bottleneck blocks (1×1 → 3×3 → 1×1) for compression-expansion

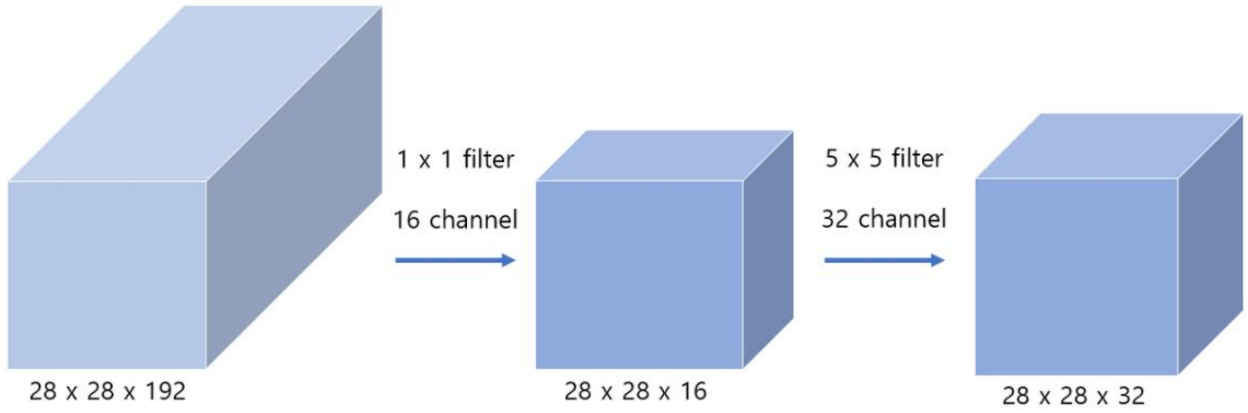
The Second Key Components of ResNet

- Introduction to Skip (i.e., Shortcut) Connections
  - **What is 1×1 Convolution?**
    - Example
      - ✓ **Scenario: 5×5 Conv on 28×28×192 → 28×28×32**
        - ➢ **Direct 5×5 convolution**
          - $\mathbf{28 \times 28 \times 32 \times 5 \times 5 \times 192 \approx 120}\boldsymbol{M}$ operations



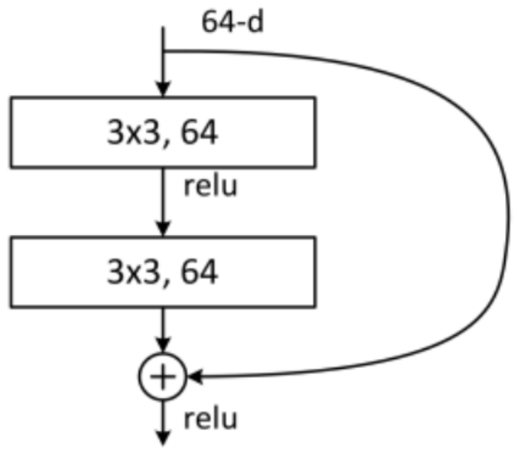
      - ✓ **With 1×1 Conv Compression**
        - ➢ Step 1. Reduce channels to 16 using 1x1 Conv
          - $\mathbf{28 \times 28 \times 16 \times 1 \times 1 \times 192 \approx 2.4}\boldsymbol{M}$
        - ➢ Step 2. Apply 5×5
          - $\mathbf{28 \times 28 \times 32 \times 5 \times 5 \times 16 \approx 10}\boldsymbol{M}$

⟶ **Total ≈ 12.4M operations** (~10× smaller)

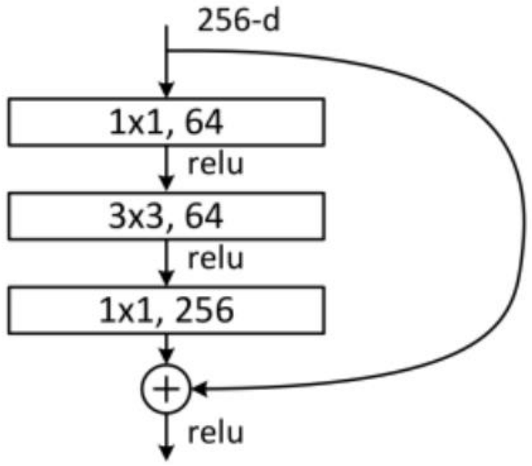


The Third Key Components of ResNet

- **Introduction to the Bottleneck Block**
  - **Residual Block Variants in ResNet Implementation**
    - **Basic Residual Block** (used in **ResNet-18, 34**)
      - ✓**Shortcut Structure:** two 3×3 convolutions
      - ✓**Shortcut type**
        - ➢ **Mostly Identity Mapping** is used **when input and output dimensions are the same.**
        - ➢ **BUT! Projection Mapping** (via 1×1 convolution) is used **when needed**
          - The number of channels increases
          - The spatial resolution changes (e.g., stride = 2)



    - **Bottleneck Block** (used in **ResNet-50, 101, 152**)
      - ✓**Shortcut Structure:** three layers consisting of 1×1 → 3×3 → 1×1
      - ✓**Shortcut type**
        - ➢**Mostly Projection Mapping** is used in most blocks, especially because
          - The bottleneck design frequently changes dimensions
          - Downsampling occurs regularly (stride = 2)
      - ✓Reduces and restores dimensions to lower computation while preserving representational power.

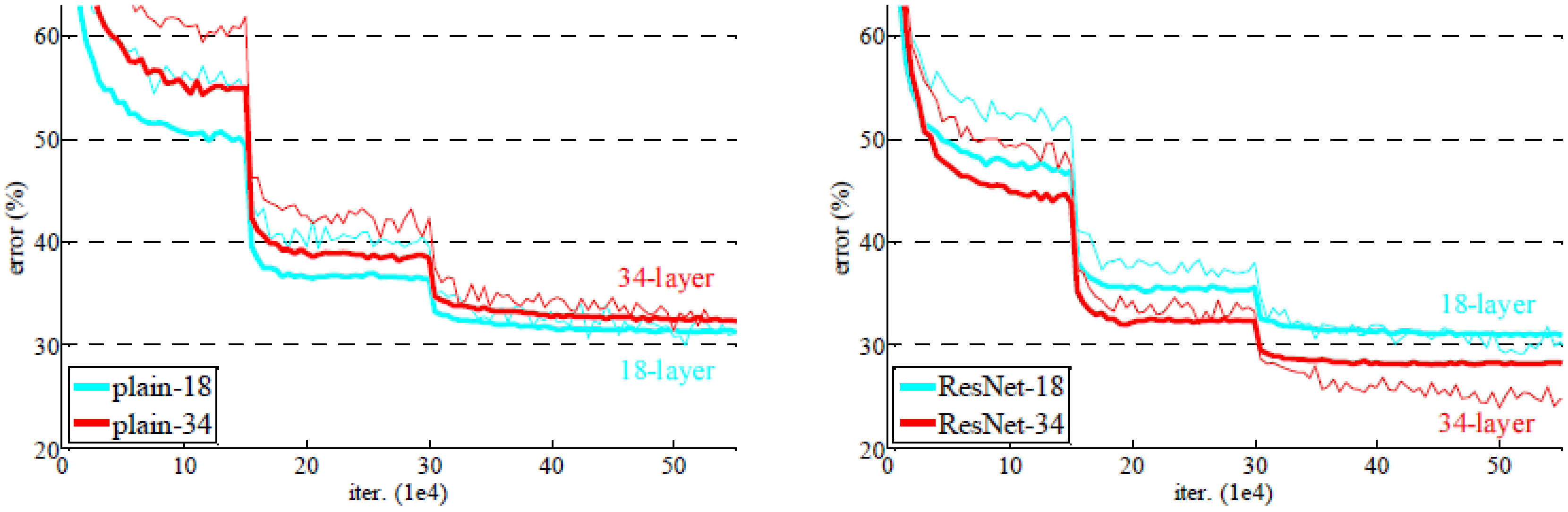


Key Components of ResNet

- **Experimental Results – Does ResNet Really Work?**
  - **Purpose**
    - To evaluate whether **shortcut connections (residual blocks)** truly improve learning in deep networks.
  - **Experimental Setup**
    - **Dataset:** ImageNet
    - **Models Compared: (1)** Plain-18 vs. Plain-34, **(2)** ResNet-18 vs. ResNet-34

Key Components of ResNet

- **Experimental Results – Does ResNet Really Work?**
  - **Observation**



  - ○ **Left: Plain Network**
    - ✓As the network **gets deeper (34-layer)**, performance **worsens**
    - ✓The **Plain-34** model has **higher training and validation error** than Plain-18
    - ✓This is a classic case of the **degradation problem**
  - ○ **Right: Residual Network**
    - ✓**ResNet-34** outperforms ResNet-18 across all iterations
    - ✓**Deeper = better performance**, as expected
    - ✓Residual learning via shortcuts enables effective optimization of deeper network

Key Components of ResNet

- **Why Residual Blocks Prevent Vanishing Gradients (i.e., Better Performance)**
  - **Forward Pass (Residual Formulation)**
    - Let the activation function be ReLU and the residual mapping be defined as:
      - ✓ $\boldsymbol{x_l + 1 = f(y_l) = f(x_l + F(x_l, W_l)) \approx x_l + F(x_l, W_l)}$
        - ➢ Assuming $\boldsymbol{f}$ is identity for simplicity.
        - ➢ Each term in the equation represents
          - $\boldsymbol{x_l}$: The input to the $\boldsymbol{l}$-th layer (or the output from the previous layer)
          - $\boldsymbol{W_l}$: The learnable weights of the $\boldsymbol{l}$-th layer
          - $\boldsymbol{F(x_l, W_l)}$: The residual function applied in the $\boldsymbol{l}$-th layer
          - $\boldsymbol{f}$: The activation function (e.g., ReLU)
          - $\boldsymbol{x_l + 1}$: The output from the $\boldsymbol{l}$-th layer (input to the next layer)

    - This leads to a general form across $\boldsymbol{L}$ layers:
      - ✓ $\boldsymbol{x_L = x_1 + \sum_{i=1}^{L-1} F(x_i, W_i)}$

Key Components of ResNet

- **Why Residual Blocks Prevent Vanishing Gradients (i.e., Better Performance)**
  - **Backward Pass (Gradient Computation)**
    - Let the loss be $\mathcal{L}$, then by the chain rule: (where the symbol $\mathcal{L}$ represents the loss function)

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial x_L}\left(1 + \frac{\partial}{\partial x_1}\sum_{i=1}^{L-1} F(x_i, W_i)\right)$$

    - The gradient splits into **two terms**
      - ✓ **(1) Identity Path (i.e., Residual Connection)**

$$\frac{\partial L}{\partial x_L} \cdot 1$$

        - ➢ This term bypasses all weight layers → gradient **flows unimpeded**
      - ✓ **(2) Residual Path**

$$\frac{\partial L}{\partial x_L} \cdot \frac{\partial}{\partial x_1}\sum_{i=1}^{L-1} F(x_i, W_i)$$

        - ➢ This term propagates through weights → may diminish or explode

Key Components of ResNet

- **Why Residual Blocks Prevent Vanishing Gradients (i.e., Better Performance)**
  - **Why This Matters**
    - **Residual connection** ensures **at least one term in the gradient is preserved** (term 1)
    - Unlike plain networks, where gradients might vanish as depth increases, ResNet ensures

$$\frac{\partial L}{\partial x_L}\ (i.e.,\quad tem\ 1) \neq 0$$

✓Even if part of the gradient shrinks, the **identity shortcut preserves signal**, avoiding full gradient collapse

- **Practical Implication**

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial x_L}\left(1 + \frac{\partial}{\partial x_1}\sum_{i=1}^{L-1} F(x_i, W_i)\right)$$

✓*In order to become gradient vanishing in real training (mini-batch SGD), the gradients in the first term become -1. BUT, the gradients vary and often don't exactly become -1.*

✓Still, **residual design avoids full cancellation**

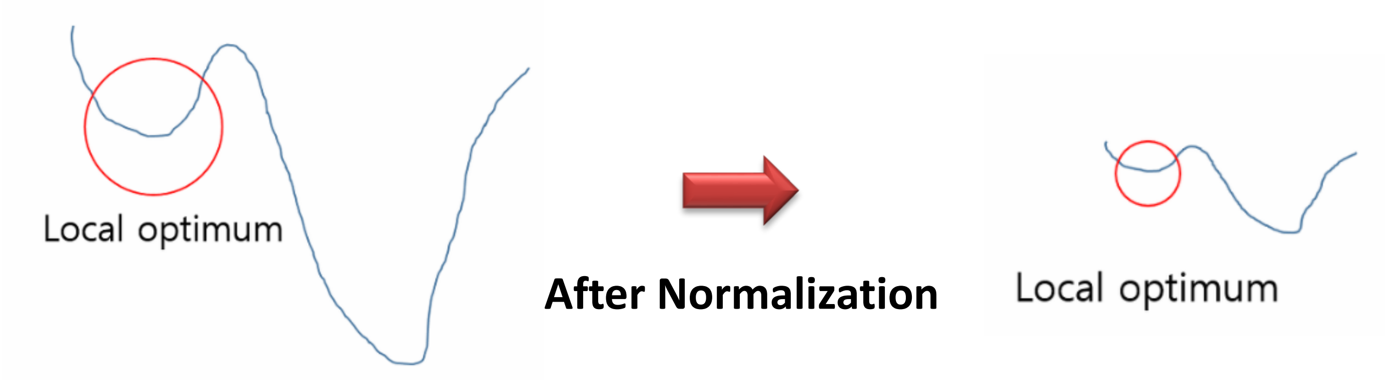
✓*Thus,* ***vanishing gradients are prevented*** *and* ***deep networks can be trained effectively***

The Fourth Key Components of ResNet

- **What is Batch Normalization?**
  - **Batch Normalization (BN) – Making Deep Networks Easier to Train**
    - BN is a technique to normalize the input of each layer so that its distribution remains stable throughout training.
  - **Why Do We Need It?**
    - Deep networks often suffer from
      - ✓ **Gradient Vanishing/Exploding**: Especially with deep networks and activation functions like sigmoid/tanh.
      - ✓ **Internal Covariate Shift**: Input distribution to each layer changes during training.

The Fourth Key Components of ResNet

- **What is Batch Normalization?**
  - **Why Do We Normalize?**
    - **Intuition Behind Normalization**
      - ✓Normalization helps the model **learn faster** and **avoid getting stuck** in bad solutions.
      - ✓During optimization, models can get trapped in **local optima** if the loss surface is highly irregular or poorly scaled.
      - ✓By normalizing, we **reshape the loss surface**, making it smoother and easier to traverse.



- **Without normalization:** Optimizer may get stuck in **sharp local minima**
- **With normalization:** Loss surface becomes **flatter**, reducing the chance of being stuck in suboptimal regions

***Goal**: Help the optimizer reach the **global optimum** by simplifying the search space.*

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Motivation – 1. The Gradient Vanishing / Exploding Problem**
    - **Why Deep Networks Are Hard to Train**
      - ✓Training deep neural networks is challenging due to the **Gradient Vanishing** or **Gradient Exploding** problem.
      - ✓Gradients become **too small** (vanish) or **too large** (explode) as they propagate backward through many layers.
      - ✓This makes it hard to update parameters effectively, resulting in
        - ➢ Poor convergence
        - ➢ High error rate
        - ➢ Inability to train deep models
    - **What Causes It?**
      - ✓Common activation functions like **sigmoid** or **tanh** squash the output to narrow ranges (e.g., [0, 1]), especially in deep layers.
      - ✓This results in
        - ➢ **Large variations in input** leading to **tiny changes in output**
        - ➢ Gradients becoming extremely small
      - ✓A typical solution is to use **ReLU** (Rectified Linear Unit), which preserves gradients better in deep models.

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Motivation – 1. The Gradient Vanishing / Exploding Problem**
    - **Workarounds (but not ideal)**
      - ✓**Change activation**: Use ReLU instead of sigmoid/tanh
      - ✓**Careful weight initialization**: Helps maintain gradient scale
      - ✓**Small learning rate**: Prevents gradients from exploding

However, these are patches, not fundamental fixes.

    - ***Core Idea: Stabilize the Entire Learning Process***
      - ✓This leads to the introduction of **Batch Normalization (BN)**
      - ✓BN normalizes inputs to each layer, so that
        - ➢The distribution of inputs stays consistent throughout training
        - ➢The network becomes **less sensitive** to initialization
        - ➢Training becomes **faster** and **more stable**
      - ✓***BN effectively prevents both gradient vanishing and exploding by keeping activations within a healthy range.***

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Motivation – 2. Internal Covariate Shift**
    - **What Is Internal Covariate Shift?**
      - ✓During training, the distribution of inputs to each layer **keeps changing** as the parameters of previous layers update.
      - ✓This shift slows down training and makes it harder for the network to converge.
    - **Covariate Shift**: When the input distribution to a model (or a layer) changes due to external factors (e.g., changing dataset).
    - **Internal Covariate Shift**: When the input distribution to a layer **changes during training** due to updates in earlier layers.

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Motivation – 2. Internal Covariate Shift**
    - **What Is Internal Covariate Shift?**
      - ✓ During training, the distribution of inputs to each layer **keeps changing** as the parameters of previous layers update.
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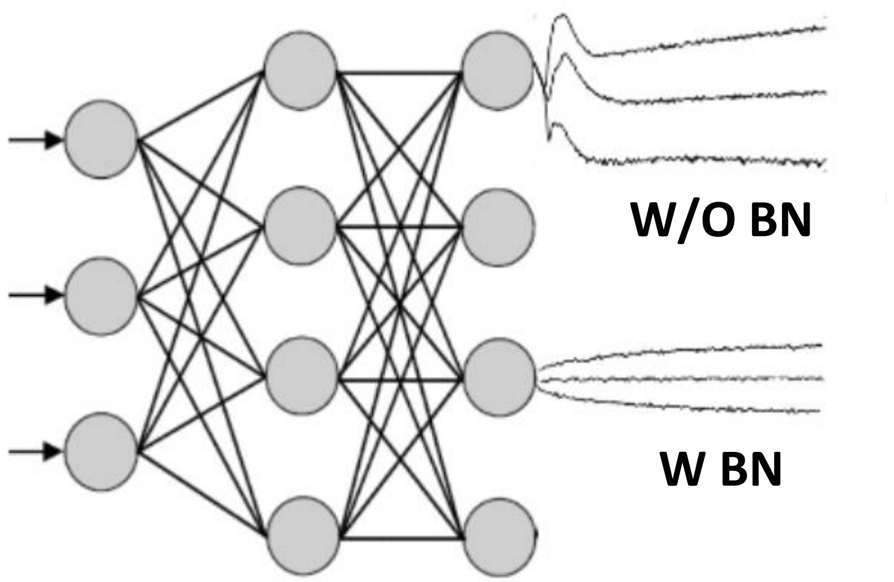
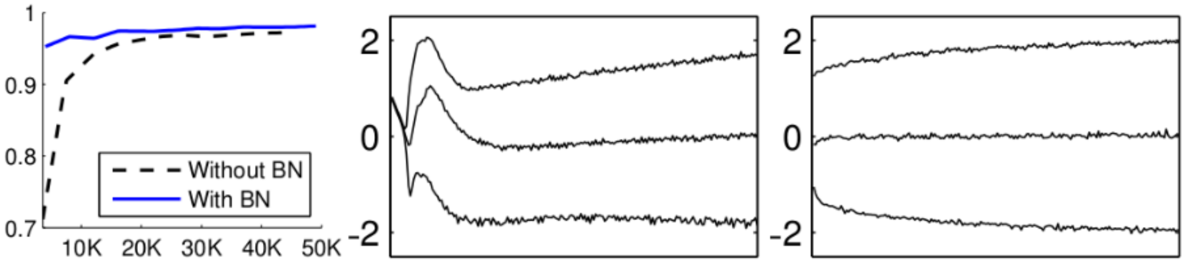
    - **Internal Covariate Shift**: When the input distribution to a layer **changes during training** due to updates in earlier layers.

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Motivation – 2. Internal Covariate Shift**
    - **Why It Matters**
      - ✓Neural networks are deep compositions of functions.
      - ✓As each layer is updated during training, the input distribution to the next layer **shifts**, making it difficult for the next layer to adapt.
      - ✓This leads to **slower convergence** and **unstable training**.



    - **Empirical Evidence**



(a) (b) Without BN (c) With BN

Batch Normalization reduces internal covariate shift ⟶ resulting in **faster training**, **better stability**, and **higher accuracy**

      - ✓The graph (right) shows how **BN stabilizes input distributions** across layers.
      - ✓With BatchNorm: input distributions become **more consistent**, helping the network learn faster and more reliably.

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **How BN Works**
    - Let $\boldsymbol{x} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_m}\}$ be a mini-batch of inputs
      - ✓ **1. Compute mean and variance**:

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i, \qquad \sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2$$

      - ✓ **2. Normalize the inputs**:

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

      - ✓ **3. Apply scale and shift**:

$$y_i = \gamma \hat{x}_i + \beta$$

        - ➢ Where
          ϵ: small constant for numerical stability
          γ: learnable **scale** parameter
          β: learnable **shift** parameter

The Fourth Key Components of ResNet

1. **Compute mean and variance:**

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i, \qquad \sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2$$

2. **Normalize the inputs:**

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

3. **Apply scale and shift:**

$$y_i = \gamma \hat{x}_i + \beta$$

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Learnable Parameters**
      - ✓**BN** introduces two learnable parameters
        - ➢ γ **(gamma)**: controls the **scale**, β **(beta)**: controls the **shift**
    - **Chain Rule Derivatives**
      - ✓Given loss L, we backpropagate through BN using the following gradients. The normalized input $\hat{x}_i$ consists of μ_B and σ_B^2.

$$\frac{\partial L}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \cdot \gamma$$

        - ➢Which means that ***we need to compute the partial derivative of the loss L with respect to μ_B and σ_B^2***.
    - **Why?**
      - ✓In Batch Normalization, we normalize the input using the batch mean μ_B and variance σ_B^2, then scale and shift it using learnable parameters γ and β.
      - ✓To update the model through backpropagation, we must compute the gradient of the loss with respect to these internal variables, including the variance.

The Fourth Key Components of ResNet

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Chain Rule Derivatives**
      - ✓Step 1. Compute the partial derivative of the loss $\boldsymbol{L}$ with respect to the **variance $\boldsymbol{\sigma_B^2}$** of a mini-batch.

$$\frac{\partial L}{\partial \sigma_B^2} = \sum_{i=1}^{m} \frac{\partial L}{\partial \widehat{x}_i} \cdot \frac{\partial \widehat{x}_i}{\partial \sigma_B^2}$$

$$= \sum_{i=1}^{m} \frac{\partial L}{\partial \widehat{x}_i} \cdot (x_i - \mu_B) \cdot \frac{\partial}{\partial \sigma_B^2} \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}}$$

$$= \sum_{i=1}^{m} \frac{\partial L}{\partial \widehat{x}_i} \cdot (x_i - \mu_B) \cdot \left(-\frac{1}{2}\right) \left(\sigma_B^2 + \epsilon\right)^{-\frac{3}{2}}$$

1. Compute mean and variance:

$$\mu_B = \frac{1}{m} \sum_{i=1}^{m} x_i \,, \qquad \sigma_B^2 = \frac{1}{m} \sum_{i=1}^{m} (x_i - \mu_B)^2$$

2. Normalize the inputs:

$$\widehat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

3. Apply scale and shift:

$$y_i = \gamma \widehat{x}_i + \beta$$

The Fourth Key Components of ResNet

1. Compute mean and variance:

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i\,, \qquad \sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2$$

2. Normalize the inputs:

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

3. Apply scale and shift:

$$y_i = \gamma \hat{x}_i + \beta$$

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Chain Rule Derivatives**
      - ✓Step 2. Compute the partial derivative of the loss $\boldsymbol{L}$ with respect to the **variance** $\boldsymbol{\mu_B}$ of a mini-batch.

$$\frac{\partial L}{\partial \mu_B} = \sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot \frac{\partial \hat{x}_i}{\partial \mu_B} + \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{\partial \sigma_B^2}{\partial \mu_B} = \sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} \cdot \frac{\partial}{\partial \mu_B}(x_i - \mu_B) + \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{\partial \sigma_B^2}{\partial \mu_B}$$

$$= \sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} \cdot -1 + \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{\partial \sigma_B^2}{\partial \mu_B}$$

$$= \left(\sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} \cdot -1\right) + \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{\partial}{\partial \mu_B}\left(\frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2\right)$$

$$= \left(\sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} \cdot -1\right) + \frac{\partial L}{\partial \sigma_B^2} \cdot \left(\frac{-2}{m}\sum_{i=1}^{m}(x_i - \mu_B)\right)$$

The Fourth Key Components of ResNet

1. Compute mean and variance:

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i, \qquad \sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2$$

2. Normalize the inputs:

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

3. Apply scale and shift:

$$y_i = \gamma \hat{x}_i + \beta$$

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Chain Rule Derivatives**
      - Step 3. Compute the partial derivative of the loss L with respect to the **input** x_i of a mini-batch.
        - **(1) Direct path through $\hat{x}_i$**

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \hat{x}_i} \cdot \frac{\partial \hat{x}_i}{\partial x_i} = \frac{\partial L}{\partial \hat{x}_i} \cdot \frac{\partial}{\partial x_i}\left(\frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}\right) = \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}}$$

- **(2) Direct path through σ_B^2**

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{\partial \sigma_B^2}{\partial x_i} = \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{\partial}{\partial x_i}\left(\frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2\right) = \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{2(x_i - \mu_B)}{m}$$

The Fourth Key Components of ResNet

1. **Compute mean and variance**:

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i\,, \qquad \sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2$$

2. **Normalize the inputs**:

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$

3. **Apply scale and shift**:

$$y_i = \gamma \hat{x}_i + \beta$$

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Chain Rule Derivatives**
      - ➢Step 3. Compute the partial derivative of the loss L with respect to the **input** x_i of a mini-batch.
        - **(3) Direct path through μ_B**

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \mu_B} \cdot \frac{\partial \mu_B}{\partial x_i} = \frac{\partial L}{\partial \mu_B} \cdot \frac{\partial}{\partial x_i}\left(\frac{1}{m}\sum_{i=1}^{m} x_i\right) = \frac{\partial L}{\partial \mu_B} \cdot \frac{1}{m}$$

- **(4) Final Expression Combining All Three Terms**

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} + \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{2(x_i - \mu_B)}{m} + \frac{\partial L}{\partial \mu_B} \cdot \frac{1}{m}$$

The Fourth Key Components of ResNet

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Chain Rule Derivatives**
      - ➢Step 4. Complete the final expression combining the results from Step1 and Step2.
        - Result from Step 1

$$\frac{\partial L}{\partial \sigma_B^2} = \sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot (x_i - \mu_B) \cdot \left(-\frac{1}{2}\right)\left(\sigma_B^2 + \epsilon\right)^{-\frac{3}{2}}$$

        - Result from Step 2

$$\frac{\partial L}{\partial \mu_B} = \sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} + \frac{\partial L}{\partial \sigma_B^2} \cdot \left(\frac{2}{m}\sum_{i=1}^{m}(x_i - \mu_B)\right)$$

        - Final Expression

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \hat{x}_i} \cdot \left(\sigma_B^2 + \epsilon\right)^{-\frac{1}{2}} + \frac{\partial L}{\partial \sigma_B^2} \cdot \frac{2(x_i - \mu_B)}{m} + \frac{\partial L}{\partial \mu_B} \cdot \frac{1}{m}$$

The Fourth Key Components of ResNet

- **Batch Normalization**
  - Backpropagation in Batch Normalization
    - **Chain Rule Derivatives**
      - ➢Step 4. Complete the final expression combining the results from Step1 and Step2.
        - Final Expression

3. Apply scale and shift:

$$\boldsymbol{y_i = \gamma \hat{x}_i + \beta}$$

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \hat{x}_i} \cdot (\sigma_B^2 + \epsilon)^{-\frac{1}{2}} + \left(\sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot (x_i - \mu_B) \cdot \left(-\frac{1}{2}\right)(\sigma_B^2 + \epsilon)^{-\frac{3}{2}}\right) \cdot \frac{2(x_i - \mu_B)}{m} + \sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot (\sigma_B^2 + \epsilon)^{-\frac{1}{2}}$$
$$+ \left(\sum_{i=1}^{m} \frac{\partial L}{\partial \hat{x}_i} \cdot (x_i - \mu_B) \cdot \left(-\frac{1}{2}\right)(\sigma_B^2 + \epsilon)^{-\frac{3}{2}}\right) \cdot \left(\frac{2}{m} \sum_{i=1}^{m} (x_i - \mu_B)\right) \cdot \frac{1}{m}$$

The final expression can be written entirely in terms of $\frac{\partial L}{\partial \hat{x}_i}$. It tells us **how much the normalized input $\hat{x}_i$ contributes to the final loss.**

$$y_i = \gamma \hat{x}_i + \beta \Rightarrow \frac{\partial L}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \cdot \frac{\partial y_i}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \cdot \gamma$$

The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Backpropagation in Batch Normalization**
    - **Chain Rule Derivatives**
      - ✓ **The final expression can be written entirely in terms of $\frac{\partial L}{\partial \hat{x}_i}$.** It tells us **how much the normalized input $\hat{x}_i$ contributes to the final loss.**

$$y_i = \gamma \hat{x}_i + \beta \Rightarrow \frac{\partial L}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \cdot \frac{\partial y_i}{\partial \hat{x}_i} = \frac{\partial L}{\partial y_i} \cdot \gamma$$

  - **Summary of the Full Process**
    - Forward Pass

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} \quad y_i = \gamma \hat{x}_i + \beta$$

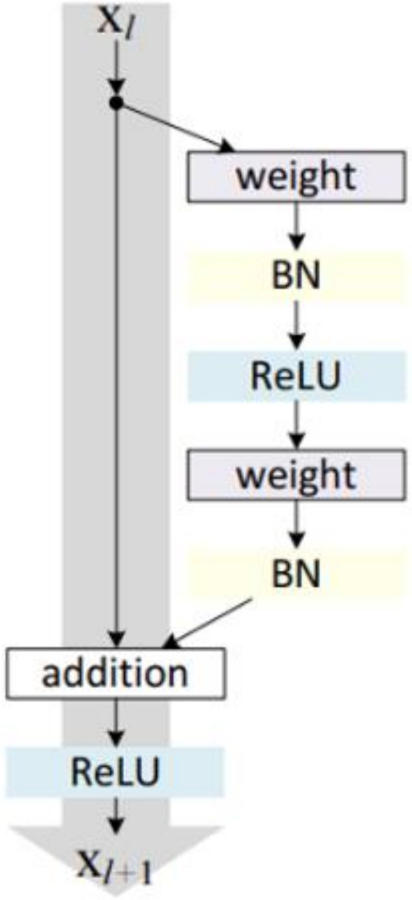
    - Backward Pass Goal

Compute $\frac{\partial L}{\partial y_i}$

We don't "observe" $\frac{\partial L}{\partial y_i}$ directly. BUT! we can compute it from $\frac{\partial L}{\partial y_i}$, which is what backprop actually gives us by using the final expression with $\frac{\partial L}{\partial \hat{x}_i}$.

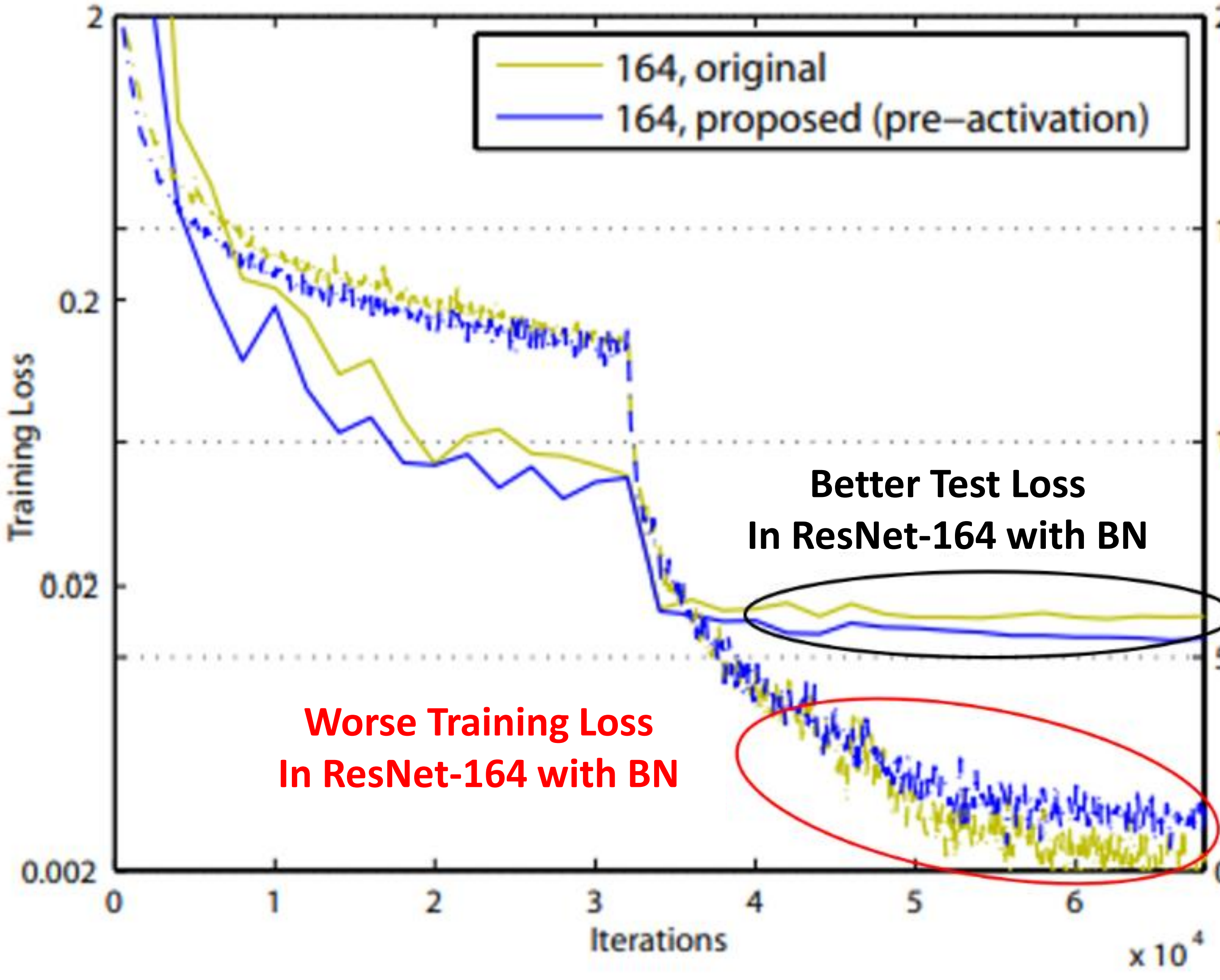
The Fourth Key Components of ResNet

- **Batch Normalization**
  - **How BN is used in ResNet**
    - In the original ResNet architecture (shown on the left), **Batch Normalization** is applied **after each convolutional layer**, before the ReLU activation.
    - The sequence inside a Residual Block is
      - ✓1. Conv (i.e., weight in the figure) → BN → ReLU
      - ✓2. Conv → BN
      - ✓3. Addition
      - ✓4. ReLU
    - **Pre-activation ResNet (Proposed Improvement)**
      - ✓In deeper networks (like ResNet-164), researchers observed that training becomes harder.
      - ✓They proposed a **pre-activation variant**, where **BN and ReLU come before convolution**.
      - ✓This leads to smoother optimization and better gradient flow.



The Fourth Key Components of ResNet

- **Batch Normalization**
  - **Performance Comparison (ResNet-164)**
    - Yellow: Original post-activation ResNet
    - Blue: Proposed pre-activation ResNet
    - **Training Loss**
      - ✓Blue curve (pre-activation) shows worse training loss (highlighted in red).
    - **Test Error**
      - ✓Blue curve generalizes better with lower test error (highlighted in black).



Indicates **less overfitting**.
Leads to **improved generalization performance**.

Summary – Key Ideas Behind ResNet

▪ Why Do We Need ResNet?

- Deeper networks offer stronger representational power.
- But **deeper ≠ better**, due to training difficulties.
- Challenges like **Vanishing/Exploding Gradients** and the **Degradation Problem** hinder performance as depth increases.

▪ Key Components of ResNet

Component	Role in the Network
Residual Block	Learns a residual function: $\boldsymbol{F}(\boldsymbol{x})$, and outputs $\boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{x}$
Skip Connection	Directly passes input through identity or projection mapping
Bottleneck Block	Reduces → processes → restores dimensions via 1×1 / 3×3 / 1×1 convs
BatchNorm + ReLU	Applied after each convolution layer

Summary – Key Ideas Behind ResNet

- **Role of Batch Normalization**
  - **Mitigates gradient vanishing/exploding**
  - Addresses **Internal Covariate Shift** → keeps input distribution stable across layers
  - Used in the sequence: Conv → BN → ReLU
    (Later, **Pre-activation**: BN → ReLU → Conv is proposed and found to perform better)

- **Takeaways**
  - **Residual learning** enables the training of very deep networks
  - ResNet architecture is not just about going deeper, but about training **effectively and reliably**
  - Still widely used as a **backbone** in modern deep learning models