DEPARTMENT OF MATHEMATICS

NAME: Nassif-Haynes Christian

Student Id: 42510023

Tutorial Group: B, Mon 14:00, E8A 188

Tutor: Mitch Buckley

DMTH237 S113

Discrete Mathematics II

Assignment 3

Due 14:00 03/05 2013

Please sign the declaration below, and staple this sheet to the front of your solutions. Your assignment must be submitted at the Science Centre, E7A Level 1.

Your assignment must be STAPLED, please do not put it in a plastic sleeve.

PLAGIARISM Plagiarism involves using the work of another person and presenting it as one's own. For this assignment, the following acts constitute plagiarism:

a) Copying or summarizing another person's work.

b) Where there was collaborative preparatory work, submitting substantially the same final version of any material as another student.

Encouraging or assisting another person to commit plagiarism is a form of improper collusion and may attract the same penalties.

STATEMENT TO BE SIGNED BY STUDENT

1. I have read the definition of plagiarism that appears above.
2. In my assignment I have carefully acknowledged the source of any material which is not my own work.
3. I am aware that the penalties for plagiarism can be very severe.
4. If I have discussed the assignment with another student, I have written the solutions independently.

SIGNATURE...

---

1. Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters ?, $>$, $<$, !, $+$, and $=$.

   (a) How many different passwords are available for this computer system?

   (b) How many of these passwords contain at least one occurrence of at least one of the six special characters?

   (c) Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes one millisecond for a hacker to check each possible password.

2. How many strings of 9 digits have each of the digits 1, 3, and 7 appearing at least once?

First downloaded: 29/4/2013 at 22:45::25

3. Let n be a fixed natural number. Show that

$$\sum_{r=0}^{m} \binom{n+r-1}{r} = \binom{n+m}{m}$$

   (a) using a combinatorial argument;

   (b) by induction (on m).

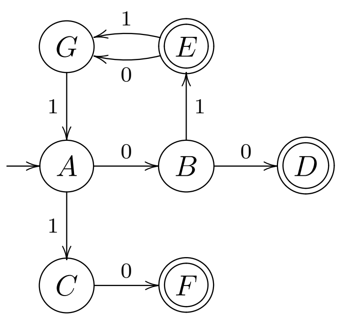
4. In how many ways can you choose 7 different numbers, no two consecutive, from $\{1, 2, 3, \ldots, 50\}$?

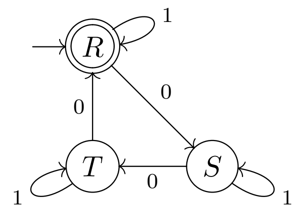
   HINT: Let the 7 numbers be $a_1 < a_2 < \cdots < a_7$. Let $u_1 = a_1$, $u_2 = a_2 - a_1$, $u_3 = a_3 - a_2$, etc., $u_7 = a_7 - a_6$, $u_8 = 50 - a_7$, and find $u_1 + u_2 + \cdots + u_8$. What are the conditions on the various u_j, $1 \le j \le 8$?

5. The machine on the left below is a deterministic machine that accepts the language

$$\mathcal{L} = \big(0101 + 0111\big)^*(00 + 01 + 10)\,,$$

   while the machine on the right is a deterministic machine that accepts the language $\mathcal{M}$ consisting of all binary strings in which the number of 0s is divisible by 3.





   Create a deterministic machine which accepts the language $\overline{\mathcal{L}}$ (all strings *except* those in $\mathcal{L}$) and a deterministic machine that accepts the language $\overline{\mathcal{M}}$. Next construct a non-deterministic machine that accepts the language $\overline{\mathcal{L}} + \overline{\mathcal{M}} = \overline{\mathcal{L} \cap \mathcal{M}}$. Convert this machine to a deterministic machine, and hence produce a machine that accepts all strings in the language $\mathcal{L} \cap \mathcal{M}$.

   Note the similarity of this exercise to that required for the project, with this one being a little bit easier. Work on this exercise first, then apply what you have learned to complete the task for your assignment. Both are due in week 8, immediately after the mid-semester break.