DMTH237 Discrete Mathematics II — Assignment 4

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1. The coefficient of X^{10} is given by

$$2^{10}\binom{10+7-1}{10} + (-3)^{10}\binom{10+4-1}{10} = 25,088,206.$$

2. *Not attempted.*

3. The characteristic polynomial of the given equation is $\lambda^2+10\lambda+25$ with the repeated root $\lambda_1 = 5$. It follows that the general solution of the recurrence relation is given by

$$a_n = (c_1 + c_2 n)5^n.$$

4. The characteristic equation has roots $\lambda_1 = -3, \lambda_2 = 4$ and $\lambda_3 = -5$ with multiplicities $m_1 = 2, m_2 = 3$ and $m_3 = 4$, respectively. It follows that the general solution of the recurrence relation is given by

$$a_n = (c_1 + c_2 n)(-3)^n + (c_3 + c_4 n + c_5 n^2)4^n + (c_6 + c_7 n + c_8 n^2 + c_9 n^3)(-5)^n.$$

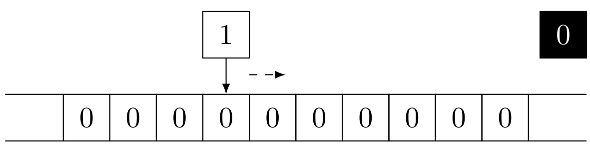
5. Cooper's machine, which calculates $f(n) = 3\lfloor \frac{1}{3}n \rfloor$, is shown below left. States [0] to [2] (inclusive) cause the Turing machine to traverse its tape, from left to right, searching for blocks of three 1's. The very last block of digits may contain one, two or three 1's. In the former two cases, all the 1's in that block are replaced with 0's (states [4] to [5]). The head is then moved to its original position (state [3]).

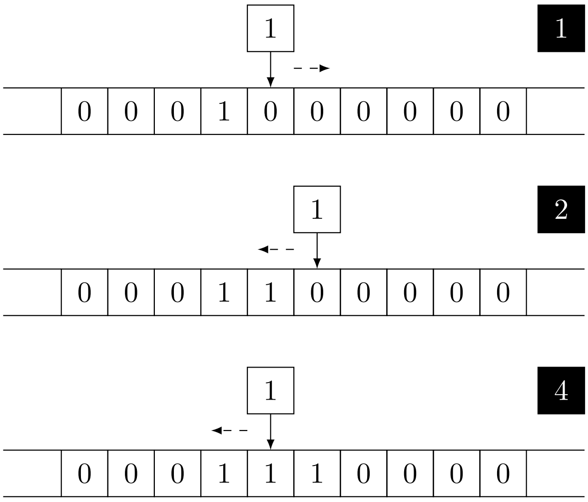
   We wish to modify Cooper's machine to calculate $f(n) = 3\lfloor \frac{1}{3}n \rfloor + 2$. We notice that states [4] and [5] remove 1's, but we want a machine which can only add them. Thus we remove states [4] and [5] and modify states [0] to [2] to add 1's in the appropriate cases. The Turing machine (below right) bears our modifications.

	0	**1**
0	0L3	1R1
1	0L4	1R2
2	0L5	1R0
3	0R6	1L3
4		0L3
5		0L4

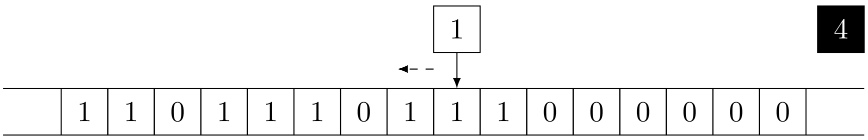
	0	**1**
0	1R1	1R1
1	1L3	1R2
2	0L3	1R0
3	0R4	1L3

6. We first prove that the base case (where $n = 0$) holds. We have the following trace, with the current state shown in the black box.

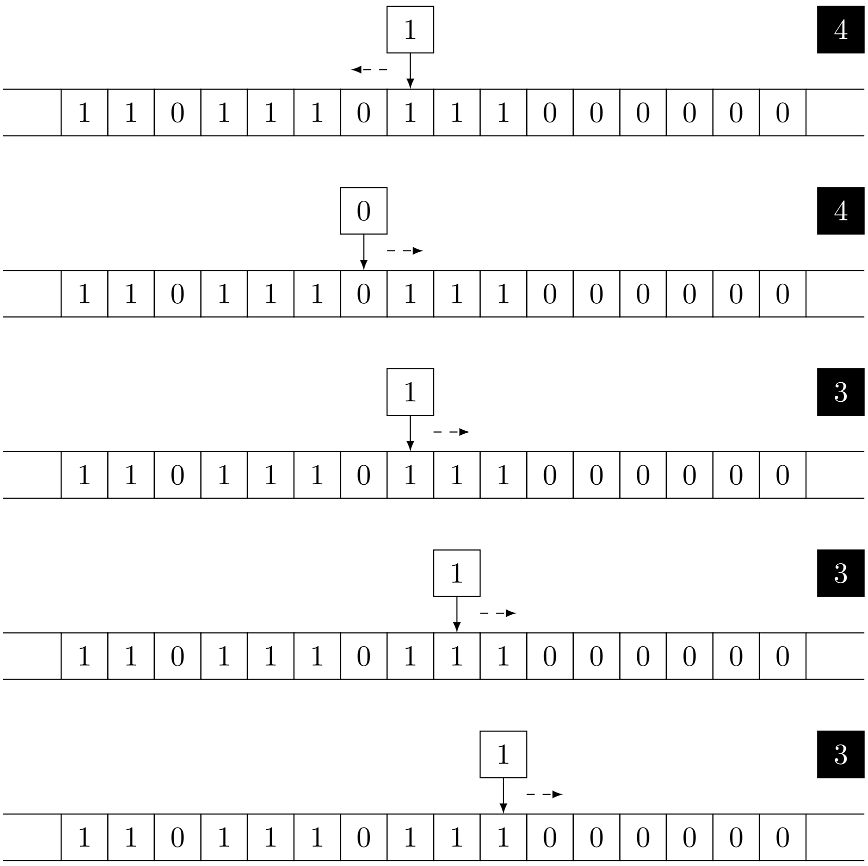


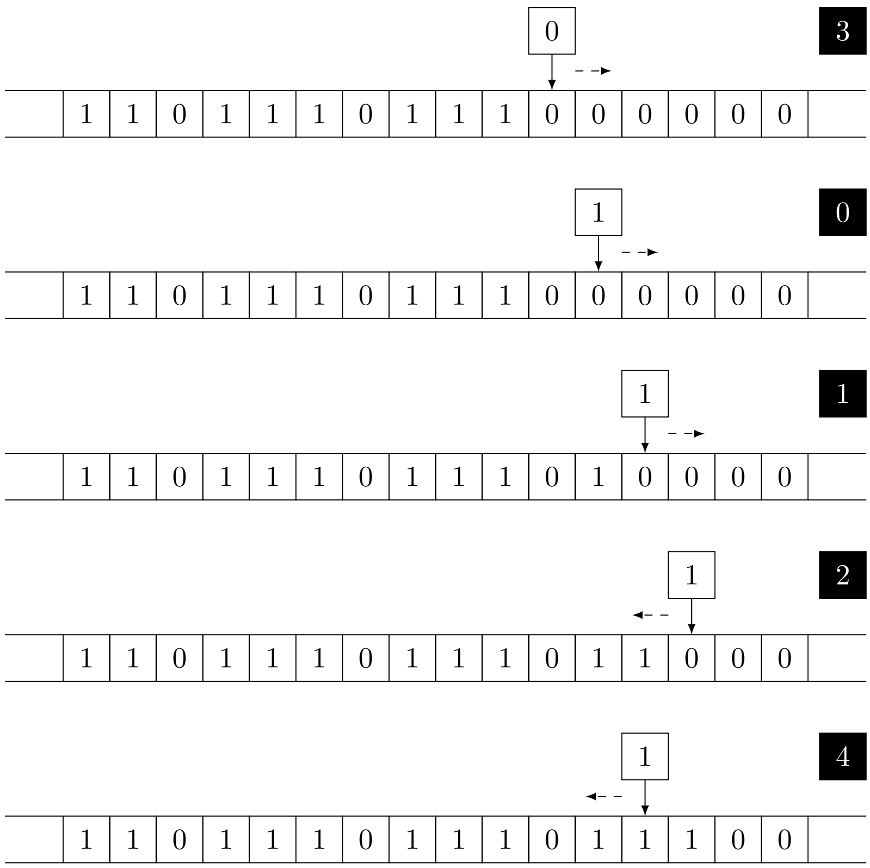


Now suppose the tape reads $0(1110)^n1110$ after $10n+3$ steps. Then, by the induction hypothesis, the Turing machine will be as shown below.



Tracing through the next 10 steps, to make $10(n+1)+3$ steps in total, we have the following.





Thus, after $10n+3$ steps the machine will be in state [4] with the tape reading $\ldots 0(1110)^n 1\underset{\uparrow}{1}10\ldots$