DEPARTMENT OF MATHEMATICS

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Tutorial Group: B, Mon 14:00, E8A 188

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DMTH237 S113

Discrete Mathematics II

Assignment 1

Due 14:00 22/03 2013

Please sign the declaration below, and staple this sheet to the front of your solutions. Your assignment must be submitted at the Science Centre, E7A Level 1.

Your assignment must be STAPLED, please do not put it in a plastic sleeve.

PLAGIARISM Plagiarism involves using the work of another person and presenting it as one's own. For this assignment, the following acts constitute plagiarism:

a) Copying or summarizing another person's work.

b) Where there was collaborative preparatory work, submitting substantially the same final version of any material as another student.

Encouraging or assisting another person to commit plagiarism is a form of improper collusion and may attract the same penalties.

STATEMENT TO BE SIGNED BY STUDENT

1. I have read the definition of plagiarism that appears above.
2. In my assignment I have carefully acknowledged the source of any material which is not my own work.
3. I am aware that the penalties for plagiarism can be very severe.
4. If I have discussed the assignment with another student, I have written the solutions independently.

SIGNATURE..

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1. Calculate the following products of complex numbers:

   (a) $(2-3i)\times(1+5\,i)$; (b) $\left(\frac{1}{3}-\frac{3}{8}\,i\right)\times\left(\frac{1}{2}+\frac{1}{7}\,i\right)$.

2. Express each of the following complex numbers in polar form:

   (a) i; (b) $\sqrt{3}-3i$; (c) $3+\sqrt{3}i$; (d) $-3-\sqrt{3}i$; (e) $-3-3i$.

3. For each of the complex numbers in problem 2 above, find z^2 and z^{42}.

4. Calculate the first four powers of the complex number $-1+i$, and sketch the resulting numbers in the complex plane. What do you notice? What do you expect for $(-1+i)^{43}=(-1+i)^{42+1}$?

5. Let $\mathcal{B}$ be the set of binary strings. Show that $|\mathcal{B}|=|\mathbb{N}|$, by establishing a 1–1 and onto map $\mathcal{B}\to\mathbb{N}$, or *vice versa*. [Hint: The binary expansion of each natural number except 0 begins with a $\mathsf{1}$.]

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6. Let A be a set. Prove that there is no onto function $f : A \to \mathcal{P}(A)$.

   HINT: Suppose that there is an onto function f and let $X = \{a \in A \mid a \notin f(a)\}$. Now use the 'fact' that f is onto.

7. *In this question, make sure that you follow the proper definition of the power L^n of a language L.*

   Suppose that $A = \{1, 2, 3, 4\}$. Suppose further that $L, M \subseteq A^*$ are given by $L = \{2, 23, 24, 42\}$ and $M = \{\lambda, 2, 4\}$. Determine how many elements are in each of the following.

   (a) $M^2 + L$ (b) $L\,M$ (c) $L\,M^2$.

   In each case, identify which strings can be constructed in more than one way.

8. Write down a Regular Expression for the language L consisting of all binary strings where every non-empty block of 0s has even length and every non-empty block of 1s has odd length. (Notice that the empty string is in this language.)

9. A file named randomstrings.txt can be downloaded from the 'Additional notes' webpage, part of the website for DMTH237. It contains 100 binary strings, each of length 100 characters.

   For each of the regular expressions given below, find the line-numbers of *all* the strings in that file which match the given regular expression.

   You may use whatever software you choose to answer this question; e.g., it can be done using the Find panel of most text-processing software applications, though other specialised utilities may prove to be easier to use, but may first require you to learn how to adapt to the specific language employed to denote a regular expression. You should describe briefly what software you have used, and how you have used it to determine the required line-numbers. Include a screenshot, or other graphic, to help the markers understand what you did to get your answers.

   (a) find matches to: $(0+1)^*\,(0000000000 + 1111111111)\,(0+1)^*$

   (b) find matches to: $(0+1)^*\,(01)^5\,(0+1)^*$

   (c) find matches to: $(0+1)^*\,0000000\,(0+1)^{12}\,1111111\,(0+1)^*$