DEPARTMENT OF MATHEMATICS

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Tutorial Group: B, Mon 14:00, E8A 188

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DMTH237 S113

Discrete Mathematics II

Assignment 2

Due 14:00 05/04 2013

Please sign the declaration below, and staple this sheet to the front of your solutions. Your assignment must be submitted at the Science Centre, E7A Level 1.

Your assignment must be STAPLED, please do not put it in a plastic sleeve.

PLAGIARISM Plagiarism involves using the work of another person and presenting it as one's own. For this assignment, the following acts constitute plagiarism:

a) Copying or summarizing another person's work.

b) Where there was collaborative preparatory work, submitting substantially the same final version of any material as another student.

Encouraging or assisting another person to commit plagiarism is a form of improper collusion and may attract the same penalties.

STATEMENT TO BE SIGNED BY STUDENT

1. I have read the definition of plagiarism that appears above.
2. In my assignment I have carefully acknowledged the source of any material which is not my own work.
3. I am aware that the penalties for plagiarism can be very severe.
4. If I have discussed the assignment with another student, I have written the solutions independently.

SIGNATURE...

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1. For which values of the constant $k \in \mathbb{R}$ does the system $\begin{cases} x - y = 2 \\ 3x - 3y = k \end{cases}$ have no solution? Exactly one solution? Infinitely many solutions? Explain your reasoning.

2. Solve the following (related) systems of linear equations by row reduction.

$$(a)\quad \begin{cases} x - 2y + z - 4w = 1 \\ x + 3y + 7z + 2w = 2 \\ x - 12y - 11z - 16w = 5 \end{cases} \qquad (b)\quad \begin{cases} x - 2y + z - 4w = 1 \\ x + 3y + 7z + 2w = 2 \\ x - 12y - 11z - 16w = -1 \end{cases}$$

Make sure to verify that any 'solutions' you find are indeed solutions of the given system of equations.

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3. Use the method of Gaussian elimination to find the inverse of each of the following matrices, provided the inverse exists.

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 5 \\ 0 & -3 & -8 \end{pmatrix}$

4. Suppose that $\mathcal{S} = \{A, B, \ldots, F\}$ is a set of states and $\mathcal{I} = \mathcal{O} = \{0, 1\}$ are the input and output alphabets for the Mealy machine described by the transition table below.

	Transition		Output	
	0	**1**	**0**	**1**
$\rightarrow A$	C	D	1	0
B	E	D	0	1
C	A	F	1	0
D	D	F	0	0
E	A	B	1	0
F	A	C	1	0

(a) Find the output string corresponding to the input string '0110111011001', when starting in state A.

(b) Construct a state diagram corresponding to the machine.

(c) Are there any non-accessible states? If so, remove them to get a (slightly) simpler reduced machine.

(d) With the (reduced) machine, consider the possible outputs for each input of a single '0' and a single '1'. Use the results to define the 0-equivalence classes. (At most there can be 4 classes. Why?) Number each state as 0, 1, 2 or 3 according to its 0-equivalence class.

(e) Now, employing the 0-equivalence classes, use the procedure done in lectures to determine the 1-equivalence classes of states, then the 2-equivalence classes, etc. until the k-equivalence classes have stabilised. Identify any equivalent states. Remove redundant states, describing your result as a new transition table with the reduced number of states.

(f) Put the reduced Mealy machine into 'Standard Form'.

(g) Explore what happens if you do steps (d) and (e) without first having done step (c). Is it sufficient to do step (c) at the end, and still finish with the same transition table?

(h) Convert the original Mealy machine into a Moore machine. Repeat steps (c), (d) and (e) to get a reduced machine with a minimum number of states. How does this machine compare with what you would have obtained by converting the final Mealy machine from step (e) into a Moore machine?

(i) Put the reduced Moore machine into 'Standard Form'.

5. (a) Which, if any, of the states in the following Finite State Acceptor are inaccessible?

(b) What should one do with inaccessible states?

(c) Draw a state diagram for an FSA equivalent to this one, but with the smallest possible number of states.

(d) Present the state table for the FSA in (c), in 'Standard Form'.

	0	1	
$\rightarrow A$	C	F	
B	E	C	$*$
C	E	D	$*$
D	A	B	$*$
E	E	C	
F	C	A	