Multiphase flows

Lecture 1

Introduction, characterization,
basic definitions, properties of phases

✓**Simultaneous** presence of different phases (gas, liquid, solid) in a domain of interest

✓**Natural** (e.g. environmental pollution problems – from local flows to global weather patterns) and **technological** systems

Examples

✓ **Solid – liquid**

1. Nature: mud flow, motion of sand
2. Human body: blood flow
3. Industry: flotation, slurry transport

✓ **Gas – solid**

1. Nature: avalanche, sand storm
2. Human body: aerosol
3. Industry: spray drying, fluidized bed, pneumatic conveying

Examples

- ✓ **Gas – liquid**

1. Nature: mist, rain
2. Industry: boiler, nuclear reactor

- ✓ **Liquid – liquid**

1. Industry: flow of emulsions

- ✓ **Three phases**

1. Industry: air-lift

Importance

- ✓ Chemical reactions
- ✓ Combustion
- ✓ Boiling and heat exchange
- ✓ Transport of materials
- ✓ Production of controlled products
- ✓ …

Importance

- A great part of anything produced in a modern society depends on a multiphase flow process

- Estimated annual turnover of **690 billion** Euros in Western Europe only

- Great relevance for Sweden (process, pharmaceutical, energy conversion, pulp and paper industry,…)

Why are multiphase flows so complex?

- ***Complicated*** and ***collective*** behaviour of a large number of interacting degrees of freedom

Complexity – some of the questions

- Individual entities or continua?
- One phase may change into another
- Even ”straightforward” quantities are defined with difficulties: size, density, viscosity…
- Shape of individual elements can be complex or change within the process
- Great separation of scales

Characterization

Ishii (1975)

Separated flows

Mixed flows

Dispersed flows

Typical regimes	Geometry	Configuration	Examples
Film flow		Liquid film in gas Gas film in liquid	Film cooling Film boiling
Annular flow		Liquid core and gas film Gas core and liquid film	Film boiling Condensors
Jet flow		Liquid jet in gas Gas jet in liquid	Atomization Jet condensor
Slug or plug flow		Gas pocket in liquid	Sodium boiling in forced convection
Bubbly annular flow		Gas bubbles in liquid Film with gas core	Evaporators with wall nucleation
Droplet annular flow		Gas core with droplets and liquid film	Steam generator
Bubbly droplet annular flow		Gas core with droplets and liquid film with gas bubbles	Boiling nuclear reactor channel
Bubbly flow		Gas bubbles in liquid	Chemical reactors
Droplet flow		Liquid droplets in gas	Spray cooling
Particulate flow		Solid particles in gas or liquid	Transportation of wheat

Terminology

- *Particles (entities, objects*…)

1. Solid particles
2. Bubbles
3. Droplets
4. …

Properties of particles

1. Geometrical (size, shape,…)
2. Mechanical (density, strength,…)
3. Thermal (condensation, evaporation,…)
4. Chemical (reactions,…)
5. Optical (reflection, refraction,…)
6. Electrical (static electricity,…)
7. …

Properties of particles – size distribution

Spherical particles – diameter as a measure of size

Non-spherical particles – equivalent diameter defined

Monodisperse distribution – particles close to a single size

Polydisperse distribution – a range of particle sizes

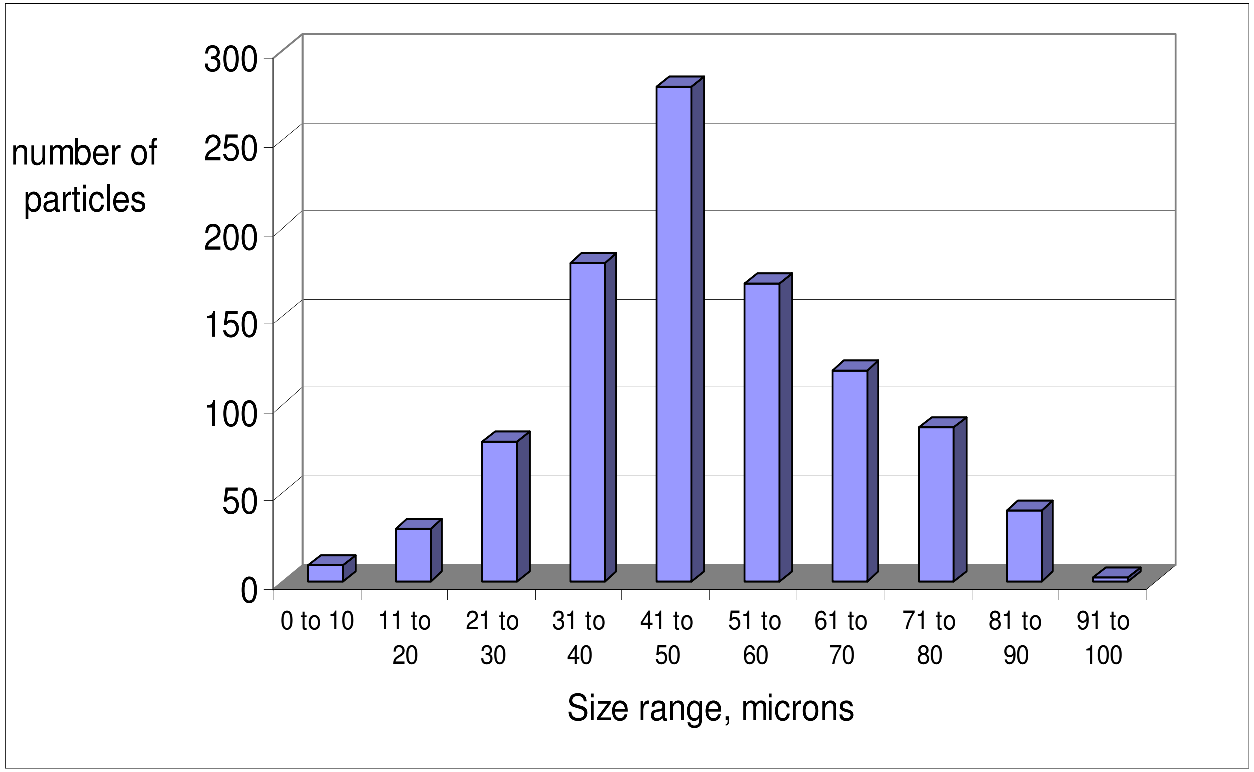
- Sampling a representative number of particles
- Choose size intervals with caution
- Count the number of particles in each size interval

Size distributions

- discrete (histograms)
- continuous

Example of a histogram

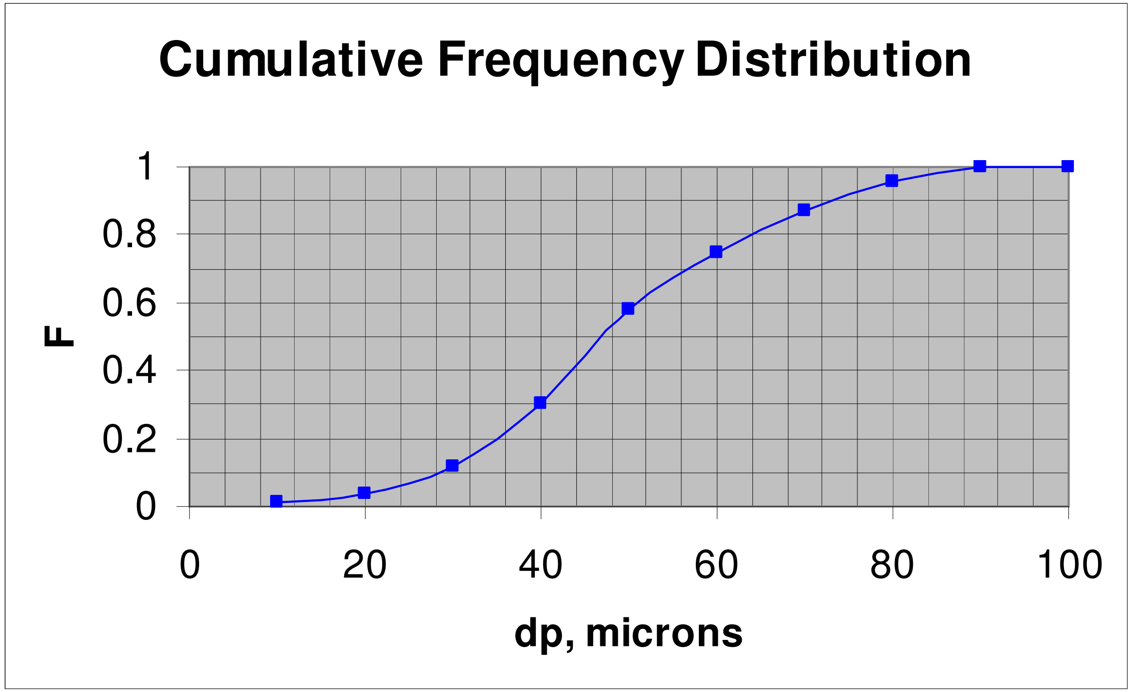
Size range, microns	number of particles
0 to 10	10
11 to 20	30
21 to 30	80
31 to 40	180
41 to 50	280
51 to 60	169
61 to 70	120
71 to 80	88
81 to 90	40
91 to 100	3



Properties of particles – continuous size distribution

Cumulative frequency distribution: F = fraction of number of particles with diameter less than or equal to a given diameter.

dp, microns	cumulative sum	F
10	10	0.01
20	40	0.04
30	120	0.12
40	300	0.3
50	580	0.58
60	749	0.749
70	869	0.869
80	957	0.957
90	997	0.997
100	1000	1



Properties of particles – continuous size distribution

Mean diameter

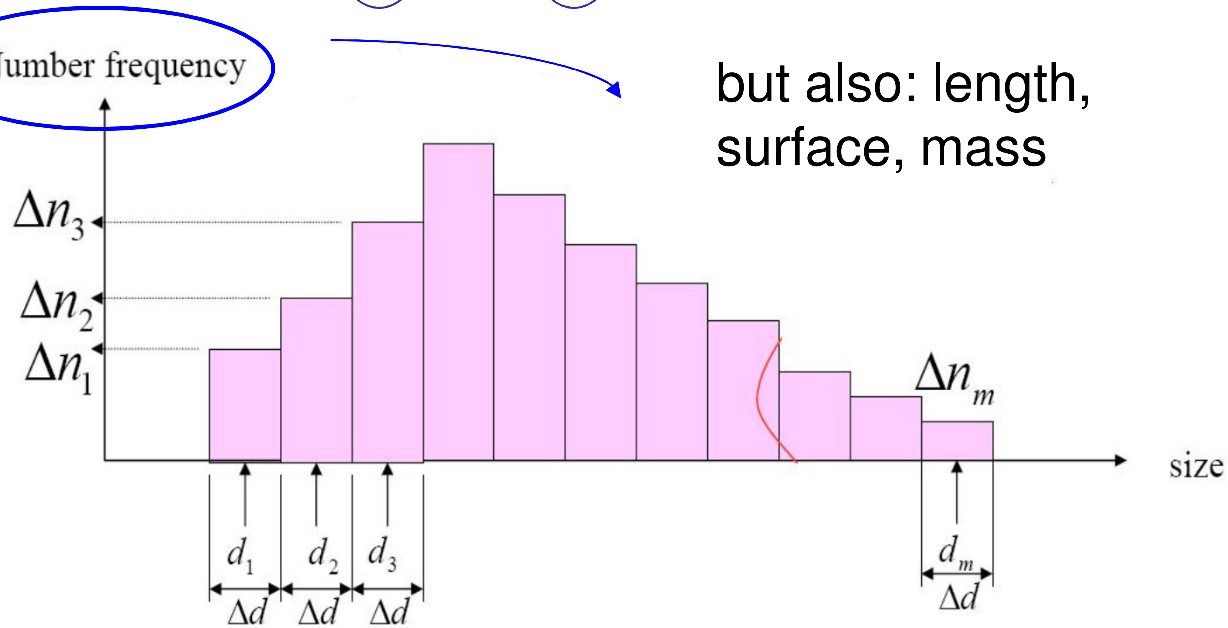
$$\bar{d} = \frac{d_1\,\Delta n_1 + d_2\,\Delta n_2 + \ldots + d_m\,\Delta n_m}{n} = d_1 \frac{\Delta n_1}{n} + d_2 \frac{\Delta n_2}{n} + \ldots + d_m \frac{\Delta n_m}{n}$$

probability that the size takes:

$$d_m - \frac{\Delta d}{2} \quad and \quad d_m + \frac{\Delta d}{2}$$

probability density

$$\frac{\Delta n_m}{n\,\Delta d} = f\left(d_m\right)$$



but also: length, surface, mass

Properties of particles – size distribution

Mean diameter

$$\overline{d} = \lim_{m\to\infty} \sum_{i=1}^{m} d_i \; f(d_i)\,\Delta d = \int_0^{\infty} d \; f(d)\,\mathrm{d}d$$

probability density

probability density function

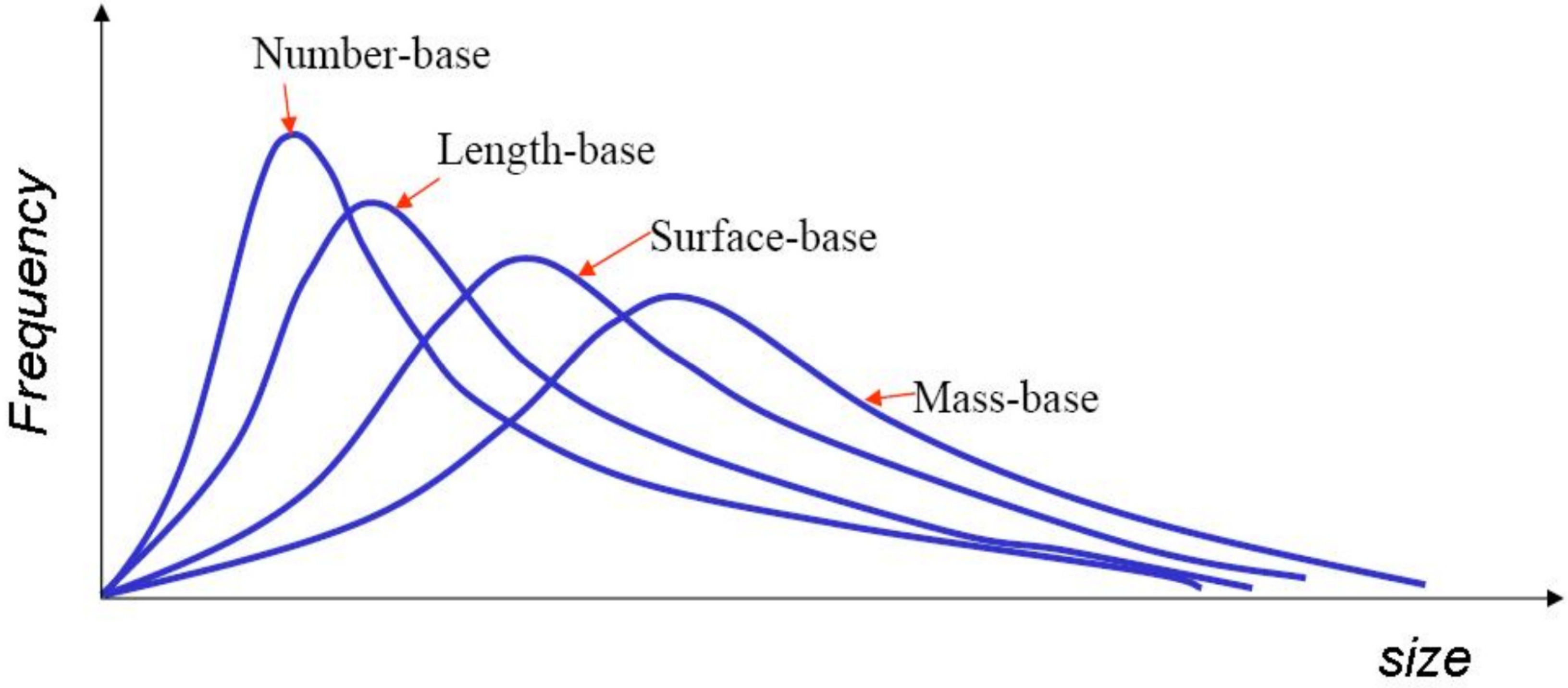
Properties of particles – size distribution

$f^{(0)}(d)$ **number-based**

$f^{(1)}(d)$ **length-based**

$f^{(2)}(d)$ **surface-based**

$f^{3}(d)$ **mass-based**



Properties of particles – size distribution

Mean diameter

$$\mu = \int_0^\infty d \; f(d)\,\mathrm{d}d \quad \Longleftarrow \quad f(d) = \left\{ f^{(0)}(d), \ldots f^{(3)}(d) \right\}$$

Variance

$$\sigma^2 = \int_0^\infty (d-\mu)^2 f(d)\,\mathrm{d}d \qquad \frac{\sigma}{\mu} < 0.1 \Longrightarrow$$

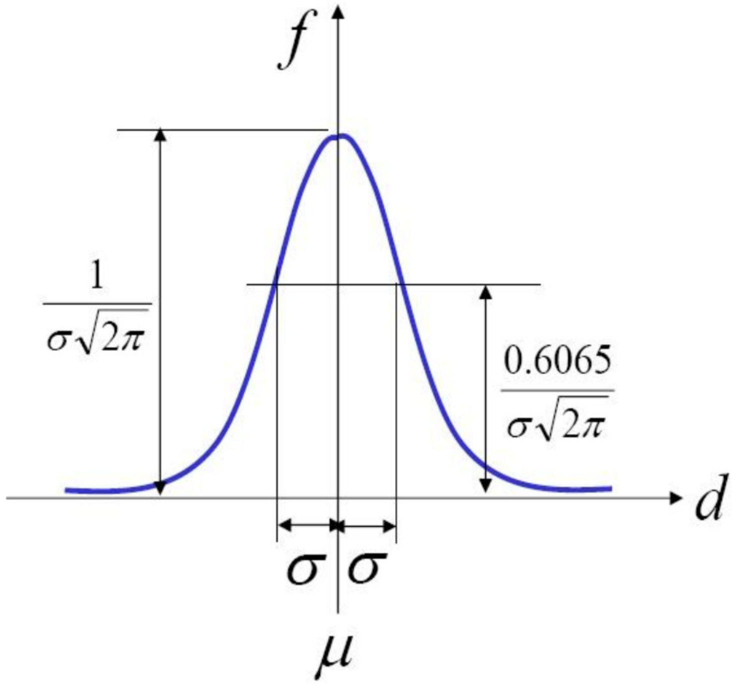
Acceptable as monodisperse

Properties of particles – size distribution

Frequently used distributions

Normal

$$f(d) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{d-\mu}{\sigma}\right)^2\right]$$



Log-normal

$$f(d) = f(x) = \frac{1}{\sigma_0\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma_0}\right)^2\right]$$

$$x = ln\, d$$

Density and volume fraction

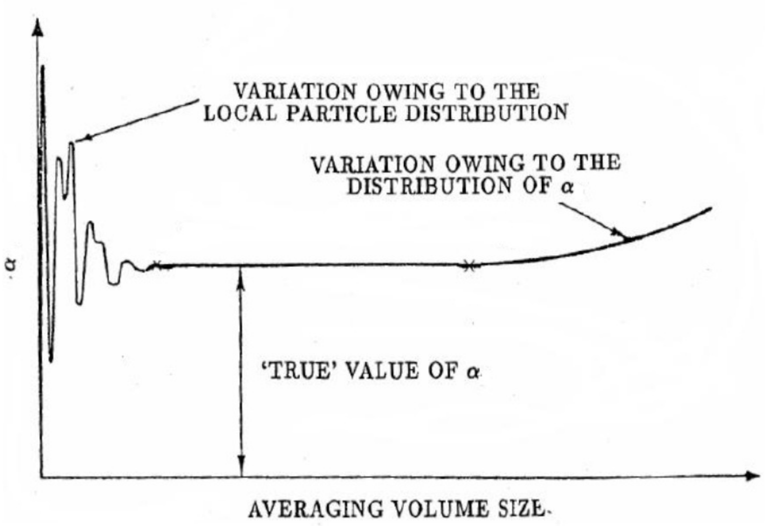
Density of continuum

$$\rho = \lim_{\delta V \to 0} \frac{\delta M}{\delta V}$$

Number density

$$n = \lim_{\delta V \to \delta V_0} \frac{\delta N}{\delta V}$$

Number of particles /unit volume



Stationary average has to be ensured!

void fraction

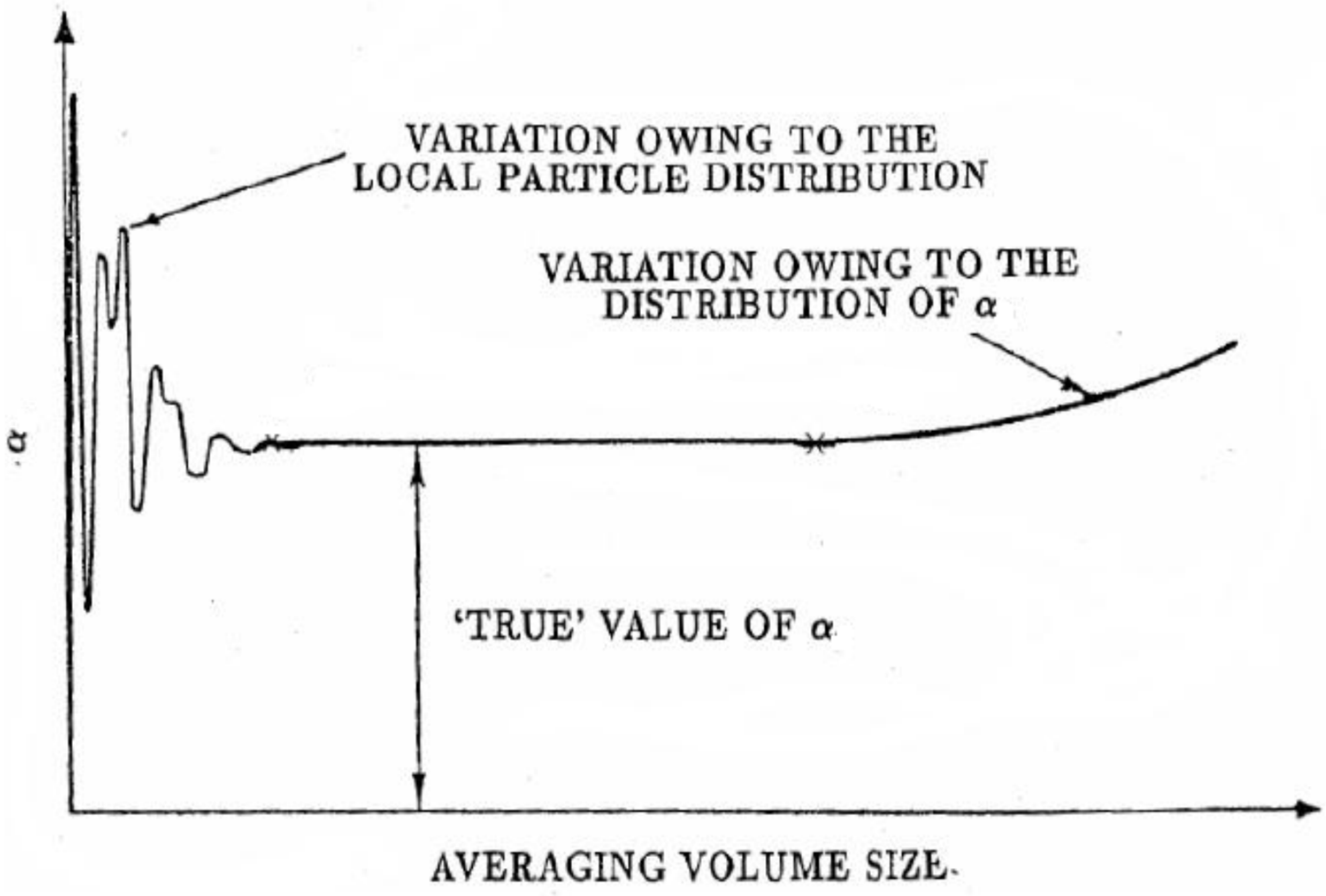
Volume fraction

$$\alpha_d = \lim_{\delta V \to \delta V_0} \frac{\delta V_d}{\delta V} \qquad \alpha_c = \lim_{\delta V \to \delta V_0} \frac{\delta V_c}{\delta V} \qquad \boxed{\alpha_c + \alpha_d = 1}$$

d – discrete

c - continuous

Separation of scales as a fundamental concept



Bulk (apparent) density

$$\overline{\rho_d} = \lim_{\delta V \to \delta V_0} \frac{\delta M_d}{\delta V}$$

Material (actual) density

$$\rho_d = \frac{\delta M_d}{\delta V_d}$$

Mixture density

$$\rho_m = \overline{\rho_c} + \overline{\rho_d} \Longleftrightarrow \rho_m = \alpha_d \rho_d + \alpha_c \rho_c$$

Mass concentration

$$C = \frac{\overline{\rho_d}}{\overline{\rho_c}} = \frac{\alpha_d \rho_d}{\alpha_c \rho_c}$$

Loading

$$z = \frac{\dot{m}_d}{\dot{m}_c} = \frac{\overline{\rho_d} v}{\overline{\rho_c} u}$$

mass flow dispersed phase/ mass flow continuous phase

Particle response time: τ_{xp}

Low Re_p – Stokes flow

$$\tau_{xp} = \frac{\rho_p d_p^2}{18\mu_f}$$

p – particle

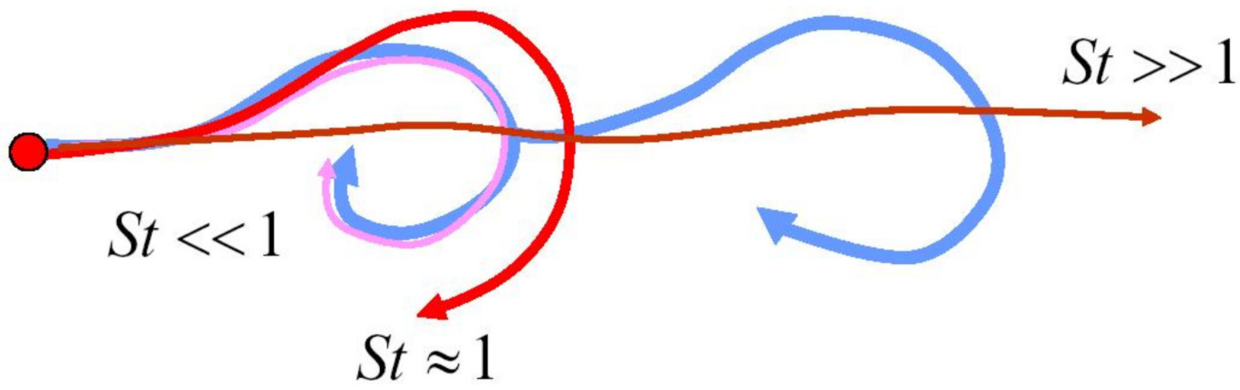
f - fluid

Stokes number

$$St = \frac{\tau_{xp}}{\tau_f}$$

Characteristic time scale of the flow

Crowe (1989)



Flow regimes: dilute vs. dense flows

dilute $$\frac{\tau_{xp}}{\tau_{coll}} < 1$$ particle motion governed by the continuous phase forces

dense $$\frac{\tau_{xp}}{\tau_{coll}} > 1$$ particle motion governed by collisions

τ_{coll} collisional time scale (time between two consecutive collisions of particles)

Governing equations in multiphase flows

- Conservation of mass, momentum and energy
- No general counterpart of the Navier-Stokes equations
- Averaging used: equations derived, but additional constitutive relations needed
- How to get a generalized set of constitutive laws for a wide variety of problems?

Averaging techniques

- Time averaging
- Volume averaging
- Ensemble averaging

Time averaging

$$\langle \ldots \rangle_t = \frac{1}{T} \int_{t-T/2}^{t+T/2} (\ldots)\, \mathrm{d}\tau$$

Separation of scales

Time scale of turbulent fluctuations	<<	Time interval for averaging	<<	Time scale of mean flow

Volume averaging

Separation of scales

- Assume there exists a length scale L_C:

$$l << L_C << L$$

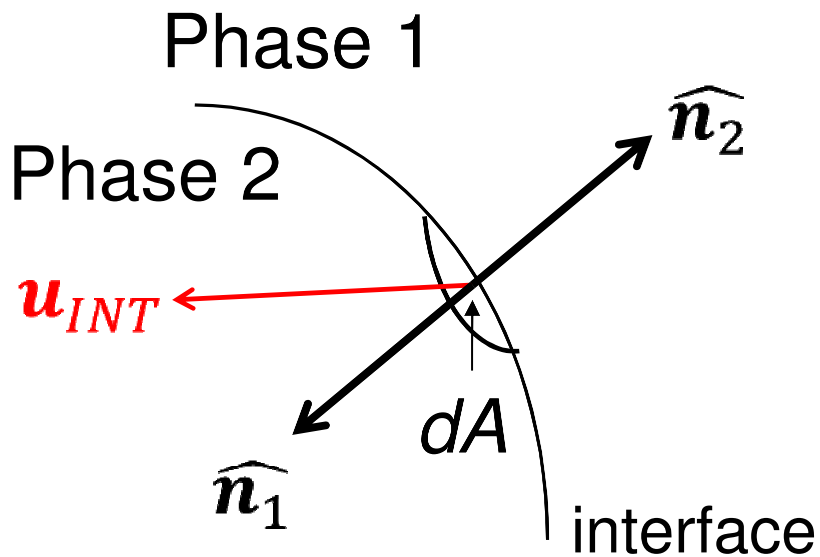
L – macroscopic length scale of the system

l – length scale associated with the distribution of phases in the system

Governing equations **valid** for each phase in the system (e.g. phase 1)

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_1 \boldsymbol{u}_1) = 0$$

$$\frac{\partial (\rho_1 \boldsymbol{u}_1)}{\partial t} + \nabla \cdot (\rho_1 \boldsymbol{u}_1 \boldsymbol{u}_1) = -\nabla p_1 + \nabla \cdot \tau_1 + \boldsymbol{F}_1$$



u_{INT}: velocity of interface

Continuity of the normal velocity field and the normal component of the stress tensor

$\widehat{\boldsymbol{n}_1}$ normal unit vector of phase 1

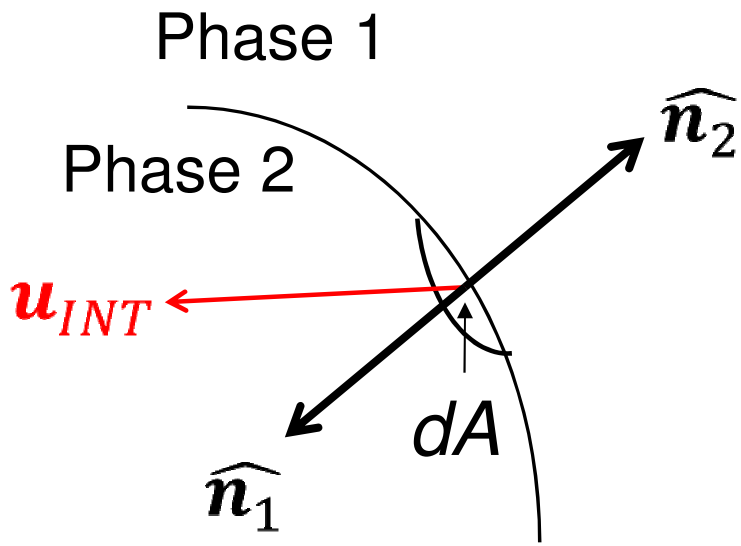
$\boldsymbol{u}_{INT}$ velocity of the interface

$\widehat{\boldsymbol{R}_{INT}} = \boldsymbol{R}_{INT}/|\boldsymbol{R}_{INT}|$ interface curvature radius vector

σ_{12} surface tension

∇_S surface gradient operator

Boundary conditions at the interface between phases 1 and 2



$$\rho_1(\boldsymbol{u}_1 - \boldsymbol{u}_{INT}) \cdot \widehat{\boldsymbol{n}_1} + \rho_2(\boldsymbol{u}_2 - \boldsymbol{u}_{INT}) \cdot \widehat{\boldsymbol{n}_2} = 0$$

Velocity (normal component) – kinematic condition

If zero interface velocity: velocity normal to the interface is continuous

Stress tensor (normal component)

$$\rho_1 \boldsymbol{u}_1(\boldsymbol{u}_1 - \boldsymbol{u}_{INT}) \cdot \widehat{\boldsymbol{n}_1} + \rho_2 \boldsymbol{u}_2(\boldsymbol{u}_2 - \boldsymbol{u}_{INT}) \cdot \widehat{\boldsymbol{n}_2} =$$

$$(-p_1 \boldsymbol{II} + \tau_1) \cdot \widehat{\boldsymbol{n}_1} + (-p_2 \boldsymbol{II} + \tau_2) \cdot \widehat{\boldsymbol{n}_2} + \frac{2\sigma_{12}}{|\boldsymbol{R}_{INT}|} \widehat{\boldsymbol{R}_{INT}}$$

Stress tensor (tangential component)

$$(\tau_1 - \tau_2) \cdot \widehat{\boldsymbol{n}_1}\, \hat{\boldsymbol{t}} = \nabla_S \sigma$$

What do we have from the previous expressions?

- For constant surface tension, absence of gravity and motion: spherical shape of the particle and from the continuity of normal stresses:

$$p_2 - p_1 = \frac{2\sigma}{R}$$

Young-Laplace equation
(R- radius of bubble/drop)

- Non-uniform surface tension: motion is always induced

However:

- the interface can have a very complex shape
- the interface can change with time – the interface should be a part of the solution (i.e. solved simultaneously with the system of equations)
- instead, we can solve the averaged equations

Averaging (*q* – any property of the system)

$$\langle q_1 \rangle = \frac{1}{V} \int_{V_1} q_1 dV$$ Partial

Volume fraction of phase 1

$$\widetilde{q_1} = \frac{1}{V_1} \int_{V_1} q_1 dV$$ Phasic

$$\widetilde{q_1} = \frac{1}{\alpha_1} \langle q_1 \rangle \quad \alpha_1 = \frac{V_1}{V}$$

$$\overline{q_1} = \frac{\int_{V_1} \rho_1 q_1 dV}{\int_{V_1} \rho_1 dV}$$ Favré

$$\overline{q_1} = \frac{\langle \rho_1 q_1 \rangle}{\alpha_1 \widetilde{\rho_1}}$$

When performing averaging on the system of governing equations, these rules apply:

$$\langle f + g \rangle = \langle f \rangle + \langle g \rangle$$

$$\langle \langle f \rangle g \rangle = \langle f \rangle \langle g \rangle$$

$$\langle C \rangle = C$$

Crucial to note

$$\langle \nabla q_1 \rangle = \nabla \langle q_1 \rangle + \frac{1}{V} \int_{A_1} q_1 \widehat{\boldsymbol{n}_1} dA$$

$$\langle \nabla \cdot \boldsymbol{q}_1 \rangle = \nabla \cdot \langle \boldsymbol{q}_1 \rangle + \frac{1}{V} \int_{A_1} \boldsymbol{q}_1 \cdot \widehat{\boldsymbol{n}_1} dA$$

$$\langle \frac{\partial q_1}{\partial t} \rangle = \frac{\partial}{\partial t} \langle q_1 \rangle - \frac{1}{V} \int_{A_1} q_1 \boldsymbol{u}_{INT} \cdot \widehat{\boldsymbol{n}_1} dA$$

After applying the averaging (and the corresponding rules discussed before)

$$\frac{\partial\langle\rho_1\rangle}{\partial t} + \nabla\cdot\langle\rho_1\boldsymbol{u}_1\rangle = \Gamma_1$$

Transfer integrals

$$\frac{\partial\langle\rho_1\boldsymbol{u}_1\rangle}{\partial t} + \nabla\cdot\langle\rho_1\boldsymbol{u}_1\boldsymbol{u}_1\rangle$$
$$= -\nabla\langle p_1\rangle + \nabla\cdot\langle\tau_1\rangle + \langle\boldsymbol{F}_1\rangle + \boldsymbol{M}_1$$

Ok to solve?

Transfer integrals

Mass and momentum interaction between the phases

$$\Gamma_1 = -\frac{1}{V}\int_{A_1} (\boldsymbol{u}_1 - \boldsymbol{u}_{INT}) \cdot \widehat{\boldsymbol{n}_1} dA$$

$$\boldsymbol{M}_1 = \frac{1}{V}\int_{A_1} (-p_1 \boldsymbol{II} + \tau_1) \cdot \widehat{\boldsymbol{n}_1} dA \cdot \widehat{\boldsymbol{n}_1}$$

$$-\frac{1}{V}\int_{A_1} \rho_1 \boldsymbol{u}_1 (\boldsymbol{u}_1 - \boldsymbol{u}_{INT}) \cdot \widehat{\boldsymbol{n}_1} dA$$

Ok to solve?

- Transfer integrals are still given in terms of integrals of the quantities over the unknown phase boundaries

- We have averages of products – we need products of averages

Example: Favré averaging for velocity and phasic for density and pressure

$$\boldsymbol{u}_1 = \overline{\boldsymbol{u}_1} + \delta \boldsymbol{u}_1$$

Decomposition of the velocity field of phase 1

The averages of products are:

$$\langle \rho_1 \boldsymbol{u}_1 \rangle = \langle \rho_1 \rangle \overline{\boldsymbol{u}_1} = \alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1}$$

$$\langle \rho_1 \boldsymbol{u}_1 \boldsymbol{u}_1 \rangle = \langle \rho_1 \rangle \, \overline{\boldsymbol{u}_1} \;\; \overline{\boldsymbol{u}_1} + \langle \rho_1 \delta \boldsymbol{u}_1 \, \delta \boldsymbol{u}_1 \rangle =$$
$$\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1} \;\; \overline{\boldsymbol{u}_1} + \langle \rho_1 \delta \boldsymbol{u}_1 \, \delta \boldsymbol{u}_1 \rangle$$

The averaged equations are (mass and momentum):

$$\frac{\partial(\alpha_1 \widetilde{\rho_1})}{\partial t} + \nabla \cdot (\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1}) = \Gamma_1$$

$$\frac{\partial}{\partial t}(\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1}) + \nabla \cdot (\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1} \;\; \overline{\boldsymbol{u}_1})$$
$$= -\nabla(\alpha_1 \, \widetilde{p_1}\,) + \nabla \cdot \langle \tau_1 \rangle + \alpha_1 \, \widetilde{\boldsymbol{F}_1} + \boldsymbol{M}_1$$
$$+\nabla \cdot \langle \tau_{\delta 1} \rangle$$

With the ***pseudo turbulent*** stress tensor:

$$\nabla \cdot \langle \tau_{\delta 1} \rangle = -\langle \rho_1 \delta \boldsymbol{u}_1 \, \delta \boldsymbol{u}_1 \rangle$$

Note:

- The tensor can indeed be connected with turbulent fluctuations

- The tensor reflects the fluctuations of phase 1 due to presence of other phase(s) – no turbulence in a classical sense.

We have the equations now, but:

- What about unclosed terms?
  - Interfacial mass transfer
  - Interfacial momentum transfer
  - Pseudo-turbulent stress tensor
- Do closure models depend on the type of averaging?

Ensemble averaging

Measurement at **a fixed time and position** for **a large number of systems** with **identical macroscopic properties and boundary conditions** and then finding a mean value

$$\langle f \rangle(\boldsymbol{r}, t) = \int_C f(\boldsymbol{r}, t; \mu)\, dm(\mu)$$

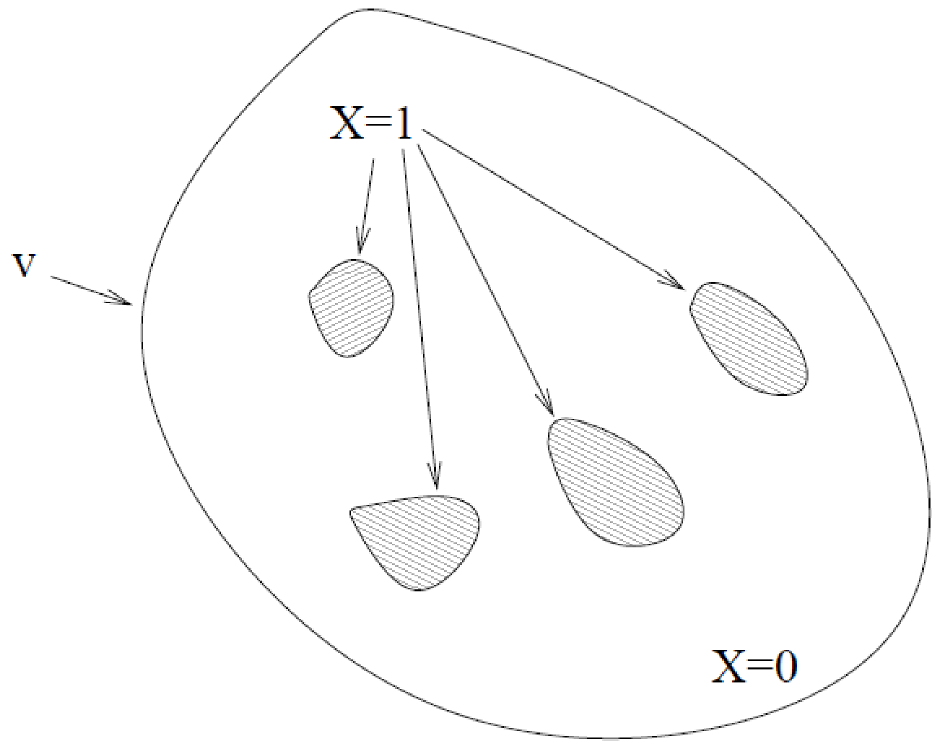
C: ensemble of systems
μ : individual member of the ensemble
dm: measure (probability to observe a system μ within *C*

Note: $\langle \nabla f \rangle = \nabla \langle f \rangle \qquad \langle \frac{\partial f}{\partial t} \rangle = \frac{\partial}{\partial t} \langle f \rangle$

- The observable quantity *f* is not associated to any particular phase (*f* is a local value <u>of any property of any phase</u> that happens to occupy point ***r*** at time *t*.
- Ensemble averaging: no paying attention to the phase that occupies the point at which the average is calculated.

Phase indicator function $X_1(\mathbf{r}, t)$

1 if phase ***1*** is present; otherwise 0



Reynolds axioms: we need partial average of f in phase 1: $\langle X_1 f \rangle$

$$\langle f + g \rangle = \langle f \rangle + \langle g \rangle; \qquad \langle \langle f \rangle g \rangle = \langle f \rangle \langle g \rangle;$$

$$\left\langle \frac{\partial f}{\partial t} \right\rangle = \frac{\partial \langle f \rangle}{\partial t}; \qquad \frac{\partial \langle X_k f_k \rangle}{\partial t} = \left\langle X_k \frac{\partial f_k}{\partial t} \right\rangle + \left\langle f_k \frac{\partial X_k}{\partial t} \right\rangle;$$

Phase indicator function satisfies:

$$\frac{\partial X_1}{\partial t} + u_{\text{int}} \nabla X_1 = 0; \qquad \alpha_1 = \langle X_1 \rangle$$

Straightforward to show:

$$\langle X_1 \nabla f \rangle = \nabla \langle X_1 f \rangle - \langle f \; \nabla X_1 \rangle$$

$$\langle X_1 \nabla \cdot f \rangle = \nabla \cdot \langle X_1 f \rangle - \langle f \cdot \; \nabla X_1 \rangle$$

$$\langle X_1 \frac{\partial}{\partial t} f \rangle = \frac{\partial}{\partial t} \langle X_1 f \rangle - \langle f \; \frac{\partial}{\partial t} X_1 \rangle$$

$$= \frac{\partial}{\partial t} \langle X_1 f \rangle + \langle f \; \boldsymbol{u}_{INT} \cdot \nabla X_1 \rangle$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \boldsymbol{u}) = 0$$

$$\frac{\partial (\rho\, \boldsymbol{u})}{\partial t} + \nabla \cdot (\rho\, \boldsymbol{u}\, \boldsymbol{u}) = \nabla \cdot \sigma + \boldsymbol{F}$$

We use the same governing equations (conservation of mass and momentum)

Note: no indices with ρ and $\boldsymbol{u}$

Derivation

Multiplying with the phase indicator function and applying the expressions from before

$$\frac{\partial\langle X_1\rho\rangle}{\partial t}+\nabla\cdot\langle X_1\rho\,\boldsymbol{u}\rangle=\langle\rho(\boldsymbol{u}-\boldsymbol{u}_{INT})\cdot\nabla X_1\rangle$$

$$\frac{\partial\langle X_1\rho\,\boldsymbol{u}\rangle}{\partial t}+\nabla\cdot\langle X_1\rho\,\boldsymbol{u}\,\boldsymbol{u}\rangle$$
$$=\nabla\cdot\langle X_1\sigma\rangle+\langle X_1\boldsymbol{F}\rangle+\langle(\rho\boldsymbol{u}(\boldsymbol{u}-\boldsymbol{u}_{INT})-\sigma)\cdot\nabla X_1\rangle$$

The same problem as with volume averaging – a similar procedure applied

Phasic and Favré averaging of any quantity *f*:

$$\widetilde{f_1} = \frac{\langle X_1 f \rangle}{\langle X_1 \rangle} = \frac{\langle X_1 f \rangle}{\alpha_1}$$

$$\overline{f_1} = \frac{\langle X_1 \rho f \rangle}{\langle X_1 \rho \rangle} = \frac{\langle X_1 \rho f \rangle}{\alpha_1 \widetilde{\rho_1}} \qquad \overline{\boldsymbol{u}_1} = \frac{\langle X_1 \rho \boldsymbol{u} \rangle}{\alpha_1 \widetilde{\rho_1}}$$

Decomposition of the velocity field

$$\delta \boldsymbol{u}_1 = \boldsymbol{u} - \overline{\boldsymbol{u}_1}$$

The momentum flux term is now:

$$\langle X_1 \rho \boldsymbol{u}\boldsymbol{u} \rangle = \alpha_1 \widetilde{\rho_1} \overline{\boldsymbol{u}_1}\, \overline{\boldsymbol{u}_1} - \tau_{\delta 1}$$

$$\tau_{\delta 1} = \langle X_1 \rho \, \delta \boldsymbol{u}_1 \, \delta \boldsymbol{u}_1 \rangle$$

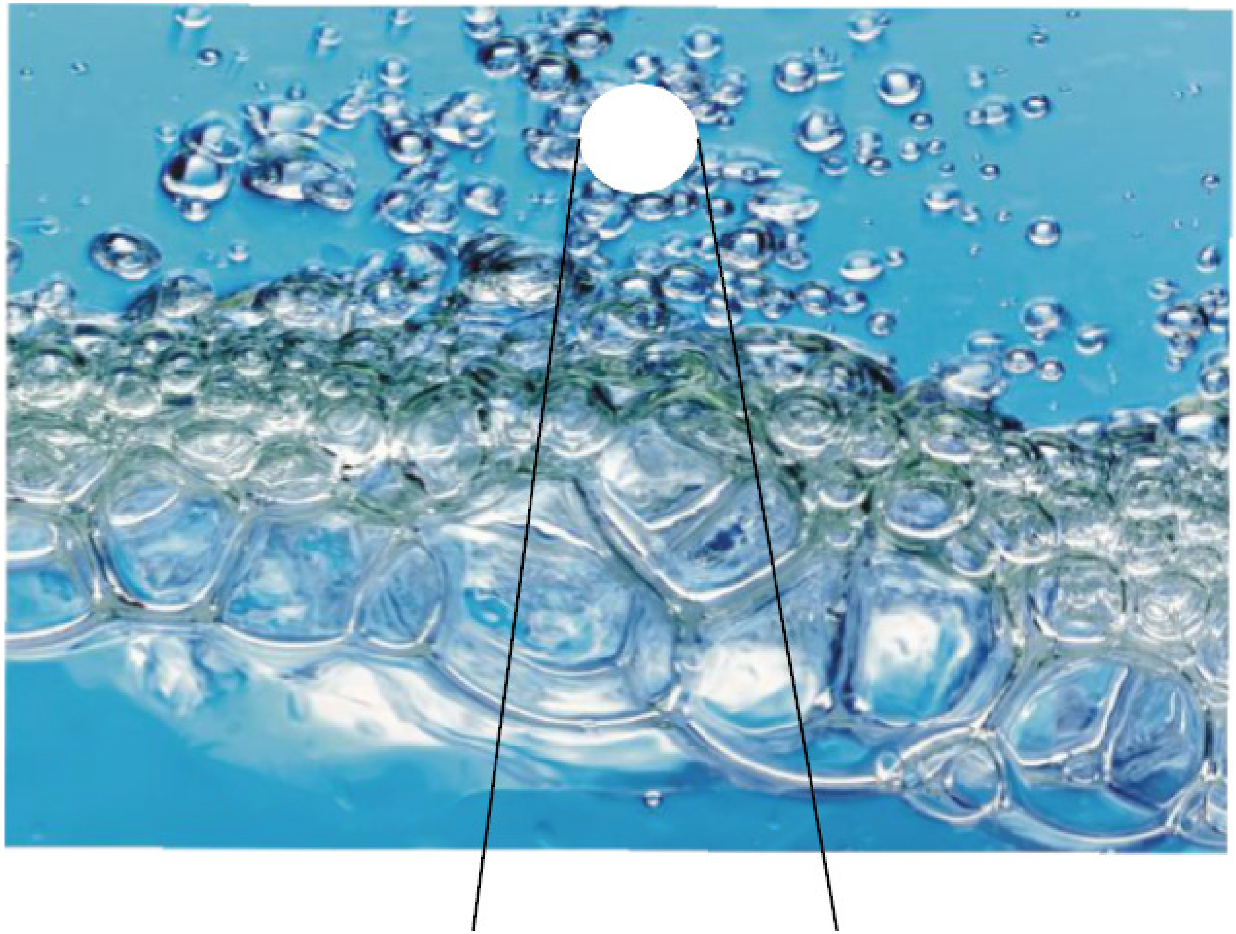
The ensemble averaged equations are now:

$$\frac{\partial(\alpha_1 \widetilde{\rho_1})}{\partial t} + \nabla \cdot (\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1}) = \Gamma_1$$

$$\frac{\partial}{\partial t}(\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1}) + \nabla \cdot (\alpha_1 \, \widetilde{\rho_1} \, \overline{\boldsymbol{u}_1} \;\; \overline{\boldsymbol{u}_1})$$
$$= -\nabla(\alpha_1 \, \widetilde{p_1}\,) + \nabla \cdot \tau_1 + \alpha_1 \, \widetilde{\boldsymbol{F}_1} \, + \boldsymbol{M}_1 + \nabla \cdot \tau_{\delta 1}$$

Same as for volume averaging but the entire different philosophy on the closures

Fundamental questions



Averaging volume

- What about other regions in this picture?
- How does the concept work?

Ergodicity hypothesis

Steady flows: *ensemble and time* averages are equivalent

Homogeneous flows: *ensemble and volume* averages are equivalent