let $X$ be a set. let $A$ and $B_i$ are subsets of $X$ where $i \in I$ for some index set $I$. Then $A \cap (\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A \cap B_i)$.

proof.

$A \cap B_i \subset A$ and $A \cap B_i \subset \cup B_i$.

$\Rightarrow A \cap B_i \subset A \cap (\cup B_i)$.

$\Rightarrow \cup(A \cup B_i) \subset A \cap (\cup B_i)$. $\therefore$ (RHS) $\subseteq$ (LHS).

On the other hand,

If $x \in A \cap (\bigcup_{i \in I} B_i)$, then $x \in A$ and $x \in \bigcup_{i \in I} B_i$. So $x \in A$ and $x \in B_{i_0}$ for some $i_0 \in I$. Hence $x \in A \cap B_{i_0}$, and so $x \in \bigcup_{i \in I} (A \cap B_i)$. Thus (LHS) $\subseteq$ (RHS). $\square$.