

Let  $X$  be a set. Let  $\mathcal{A}_1, \mathcal{A}_2 \subseteq \mathcal{P}(X)$ .

Then

$$\begin{aligned}(\cup \mathcal{A}_1) \cap (\cup \mathcal{A}_2) &= \cup_{A_2 \in \mathcal{A}_2} \{(\cup \mathcal{A}_1) \cap A_2\} \\&= \cup_{A_2 \in \mathcal{A}_2} \{ \cup_{A_1 \in \mathcal{A}_1} (A_1 \cap A_2) \} \\&= \cup_{\substack{A_1 \in \mathcal{A}_1 \\ A_2 \in \mathcal{A}_2}} (A_1 \cap A_2).\end{aligned}$$