Let X be a set. Let $\mathcal{A}_1, \cdots, \mathcal{A}_n \subseteq P(X)$ for some $n \in \mathbb{N}$. Assume that

$$\bigcap_{i=1}^{n-1} \left(\bigcup \mathcal{A}_i\right) = \bigcup_{\substack{A_1 \in \mathcal{A}_1 \\ \vdots \\ A_{n-1} \in \mathcal{A}_{n-1}}} (A_1 \cap \cdots \cap A_{n-1}).$$

Then

$$\bigcap_{i=1}^{n} \left(\bigcup \mathcal{A}_i\right) = \left\{\bigcap_{i=1}^{n-1} \left(\bigcup \mathcal{A}_i\right)\right\} \cap \left(\bigcup \mathcal{A}_n\right)$$

$$= \left\{\bigcup_{\substack{A_1 \in \mathcal{A}_1 \\ \vdots \\ A_{n-1} \in \mathcal{A}_{n-1}}} (A_1 \cap \cdots \cap A_{n-1})\right\} \cap \left(\bigcup \mathcal{A}_n\right)$$

$$= \bigcup_{A_n \in \mathcal{A}_n} \left\{A_n \cap \left\{\bigcup_{\substack{A_1 \in \mathcal{A}_1 \\ \vdots \\ A_{n-1} \in \mathcal{A}_{n-1}}} (A_1 \cap \cdots \cap A_{n-1})\right\}\right\}$$

$$= \bigcup_{A_n \in \mathcal{A}_n} \left\{\bigcup_{\substack{A_1 \in \mathcal{A}_1 \\ \vdots \\ A_{n-1} \in \mathcal{A}_{n-1}}} (A_1 \cap \cdots \cap A_n)\right\}$$

$$= \bigcup_{\substack{A_1 \in \mathcal{A}_1 \\ \vdots \\ A_n \in \mathcal{A}_n}} (A_1 \cap \cdots \cap A_n).$$

By induction, this statement is true for every $n \in \mathbb{N}$. $\square$.