

School of Advanced Computing & Information Technology

Department of Computer Science, Gujarat University

MCA112 Mathematical Foundation

Assignment - Unit 2 Fundamentals of Single Variable Calculus

Question 1. Write down domain and range of given functions. Discuss any four properties of its.

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|-------------------|-----------------------------------|-------------------|-------------------|
| 1) x | 2) x^2 | 3) $ x $ | 4) $[x]$ |
| 5) $\sin x$ | 6) $\cos x$ | 7) $\tan x$ | 8) $\cot x$ |
| 9) $\sec x$ | 10) $\operatorname{cosec} x$ | 11) e^x | 12) $\log x$ |
| 13) $\sin^{-1} x$ | 14) $\cos^{-1} x$ | 15) $\tan^{-1} x$ | 16) $\cot^{-1} x$ |
| 17) $\sec^{-1} x$ | 18) $\operatorname{cosec}^{-1} x$ | | |

Question 2. Check whether the following function is one-one and onto or not. Find inverse of the function if exists.

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|---------------------|--|
| 1) $f(x) = 6x$ | 2) $g(x) = 3x + 2$ |
| 3) $h(x) = 7 - x^3$ | 4) $f(x) = \frac{x+1}{x+2}$ for $x \in (-2, \infty)$ |

Question 3. Define the following terms with examples:

- | | | |
|---|---------------------|----------------------|
| 1) Function | 2) One-one function | 3) Onto function |
| 4) Even function | 5) Odd function | 6) Periodic function |
| 7) Limit | 8) Continuity | 9) Differentiability |
| 10) Anti-derivative (Indefinite Integral) | | |
| 11) Inverse function | | |

Question 4. Evaluate the following limits:

$$1) \lim_{x \rightarrow -1} \frac{x^5 + 1}{x + 1}$$

$$2) \lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$$

$$3) \lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{x-3}$$

$$4) \lim_{x \rightarrow 2} \frac{x^6 - 64}{\sqrt{x} - \sqrt{2}}$$

$$5) \lim_{x \rightarrow \infty} \frac{x^3 + 2x + 1}{4x^3 + x + 9}$$

$$6) \lim_{x \rightarrow \infty} \frac{(3x+2)^{40} (4+x)^{10}}{(3x+7)^{50}}$$

$$7) \lim_{x \rightarrow 0} \left(\frac{1-2x}{1+2x} \right)^{\frac{1}{3x}}$$

$$8) \lim_{x \rightarrow 0} \frac{(15)^x - 5^x - 3^x + 1}{x^2}$$

$$9) \lim_{x \rightarrow 0} \frac{e^{2x} - e^{-2x}}{x}$$

Question 5. Examine the continuity of the function at mentioned point:

$$1) f(x) = \begin{cases} 3 + 2x, & \text{if } x \geq 0 \\ 3 - 2x, & \text{if } x < 0 \end{cases} \quad \text{at } x = 0$$

$$2) g(x) = \begin{cases} 3 - 2x, & \text{if } x \geq 0 \\ x + 1, & \text{if } x < 0 \end{cases} \quad \text{at } x = 0$$

$$3) h(x) = \begin{cases} -2x^2, & \text{if } x \leq 0 \\ 5x + 2, & \text{if } 0 < x \leq 1 \\ 3x^2 + 4x, & \text{if } 1 < x \leq 2 \end{cases} \quad \text{at } x = 0, 1$$

Question 6. Determine the value of constant if function is continuous at mentioned point/s:

$$1) f(x) = \begin{cases} x + 1, & \text{if } x \leq 1 \\ 3 - kx, & \text{if } x > 1 \end{cases} \quad \text{at } x = 1$$

$$2) g(x) = \begin{cases} ax + 1, & \text{if } x \leq 2 \\ ax^2 - 1, & \text{if } x > 2 \end{cases} \quad \text{at } x = 2$$

$$3) \quad h(x) = \begin{cases} 2, & \text{if } x \leq 2 \\ ax - b, & \text{if } 2 < x \leq 8 \\ 15, & \text{if } x < 8 \end{cases} \quad \text{at } x = 2, 8$$

Question 7. Show that the function $f(x) = [x]$ is discontinuous at integers and continuous everywhere except integers.

Question 8. Using definition of differentiation, derive derivative of following at arbitrary point a :

1) $f_0(x) = c$, where c is constant

2) $f_1(x) = x^2$

3) $f_2(x) = x^3$

Question 9. Find derivative of following functions with respect to variable x :

1) $\tan x$ 2) $\cot x$ 3) $\sec x$ 4) $\operatorname{cosec} x$

4) $\sin x + x^4 + e^x$ 6) $\sin x \cdot \log x$

7) $\log(x \cdot e^x)$ 8) $x \cdot \cos x \cdot \log x$ 9) $\frac{e^x - 1}{e^x + 1}$ 10) $\frac{(x+1)^3}{x^2}$

Question 10. Find the derivative of following function using chain rule with respect to variable x :

1) e^{x^2} 2) $(\log x)^3$ 3) 8^{3x+2} 4) $\log\left(\frac{2x+1}{2x-1}\right)$ 5) x^x

6) $\sec(\cos e^x) \cdot \log(x^2)$ 7) $(\sin x)^x$ 8) $x^{\sin x} + (\sin x)^x$ 9) $x^y = e^{x+y}$

Question 11. Evaluate the following indefinite and definite integrals:

1) $\int 7x^2 + 2x - 8 + \frac{1}{x} - \frac{1}{x^2} dx$ 2) $\int \frac{x}{a} + \frac{a}{x} + a^x dx$

3) $\int (e^a \log x + e^{x \log a}) dx$ 4) $\int \frac{1}{1+\cos x} dx$

5) $\int (2x + 1)^7 dx$ 6) $\int e^{3x+7} dx$

7) $\int x^2(x^3 + 1)^{\frac{3}{2}} dx$ 8) $\int \frac{(2+3 \log x)^2}{x} dx$

9) $\int \frac{e^x(1+x)}{\sin^2(xe^x)} dx$ 10) $\int \frac{10x^9 + 10^x \log 10}{10^x + x^{10}} dx$

11) $\int_1^2 \frac{1}{x} dx$ 12) $\int_0^{\frac{\pi}{2}} \cos x dx$

$$13) \int_0^1 e^{3x+1} dx$$

$$14) \int_0^2 (x^2 + e^x) dx$$

$$15) \int_0^{\frac{\pi}{4}} \tan^2 x dx$$

Find the critical point, local maxima, local minima, absolute maxima and absolute minima for the following functions on given interval:

$$1) f_1(x) = 3x^4 - 4x^3 - 12x^2 + 1 \text{ on } [-2, 3]$$

$$2) f_3(x) = x - \log x \text{ on } \left[\frac{1}{2}, 2\right]$$

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