School of Advanced Computing & Information Technology

Department of Computer Science, Gujarat University

MCA112 Mathematical Foundation

Assignment - Unit 1 Introduction to Set theory

1. Which of the following statements are true or false? Justify your answer.

    a. The set $A = \{a\}$ and $b = \{\{a\}\}$ are equal.

    b. $\{a\} \subset \{a\}$

    c. $\{a\} \in \{a\}$

    d. $\emptyset \subset \{a\}$

    e. $\emptyset \in \{\emptyset\}$

    f. $\{3, 4\} \subset \{1,\ 2,\ 3, \{3,4\},\ 5\}$

2. Write down the following sets into roaster form or list the elements:

    a. $A = \{x \in \mathbb{R}|\ x$ is a real number such that $x^2 = 4\}$

    b. $B = \{x \in \mathbb{N}|x$ is a prime number less than 20$\}$

    c. $C = \{x \in \mathbb{Z}|\ x$ is a positive and whose square is less than 50$\}$

    d. $D = \{x \in \mathbb{Z}|\ x$ is a positive divisor of 20$\}$

    e. $E = \{x \in \mathbb{Z}|\ x$ is a multiple of 4$\}$

3. State which of the following sets are finite and which are infinite:

    a. $A = \{x \in \mathbb{N}|x$ is a prime number$\}$

    b. $B = \{x \in \mathbb{N}|\ x$ is an even$\}$

    c. $C = \{x \in \mathbb{N}|(x + a)(x - b) = 0\}$, where a and b are arbitrary real constant

    d. $D = \{x \in \mathbb{Z}|\ x^2 = 9\}$

    e. $E = \{x \in \mathbb{Q}|3x - 1 = 0\}$

4. Write down any four properties of set operations. Explain each of its through different examples.

5. State De-Morgan's laws. Also, sketch the proof with the help of Venn diagram.

6. **(Application of a power sets in bit representation)**

Consider the set $S = \{a_1, a_2, a_3, \ldots, a_{10}\}$. Each subset of given set S is denoted by the symbol B_i, where $'i'$ represents a sequence of bit in binary digits. Determine the sequence of bit and corresponding set for the set B_{111} and B_{73}. How will you designate the set $\{a_2, a_6, a_9\}$ and $\{a_1, a_9\}$?

7. Define Cartesian product of two set. Consider $A = \{1, 2, 3\}$, $B = \{3, 5\}$ and $C = \{2, 3, 5\}$. Then compute the following:

   a. $A \times B$, $B \times A$

   b. $A \times A$, $B \times B$

   c. $A \times (B \cap C)$, $A \times (B \cup C)$

   d. $(A \times B) \cap (A \times C)$, $(A \times B) \cup (A \times C)$

8. If $A = \{a, b, c\}$, $B = \{b, d\}$ and $C = \{c, d\}$ then prove that

   a. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

   b. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   c. $A \times (B - C) = (A \times B) - (A \times C)$

   d. $A \times (B \Delta C) = (A \times B) \Delta (A \times C)$

9. **(Application in Symmetric Difference of two sets)**

   A Manufactured item can have a defect in its shape or in its weight. A sample of 100 items showed that 10 of them had defect in its shape and 12 of them in weight. Five of them had both the defects. How many of them had exactly one defect?

10. **(Application in Counting number of elements)**

    In survey of 1000 persons for usage of social media platform, it was found that 280 uses Facebook (F), 300 uses twitter (T) and 420 uses Instagram (I), 80 uses both F and T, 100 uses F and I, 50 uses T and I and 30 uses all three platforms. Answer the following:

    a. How many use at least one of these platforms?

    b. How many use none of three platforms?

    c. How many use only Facebook?

    d. How many use Facebook and Twitter but not Instagram?