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Assignment 2
Write-Up

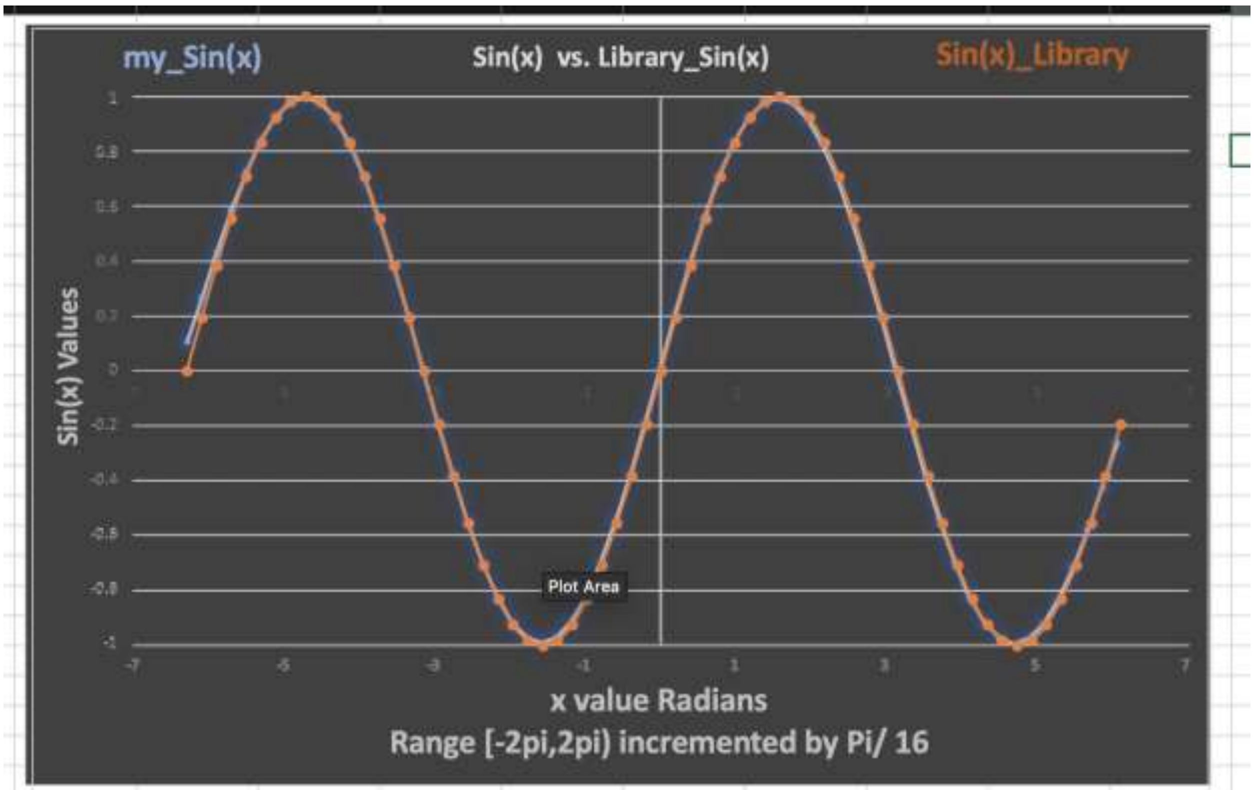
Results for my_Sin() function
Using Hornersnormal form (on 14thterm series) from the Taylor series centered about 0:

It is a lot easier to square a number than to raise it to a power, so we can simplify it by putting the formula into *Horner normal form*, by factoring out x as much as possible:

$$\sin(x) \approx \frac{x((x^2(52785432 - 479249x^2) - 1640635920)x^2 + 11511339840)}{((18361x^2 + 3177720)x^2 + 277920720)x^2 + 11511339840}.$$

This may be due to the values not being initialized at the end points independent of the approx. function

ex: my_Sin(-2pi)= my_Sin(2pi) = 0
The average difference = 0.00164123(more accurate compared to cos)



Results for my_Cos() function
Using Horners normal form (on 14th term series) from the Taylor series centered about 0:

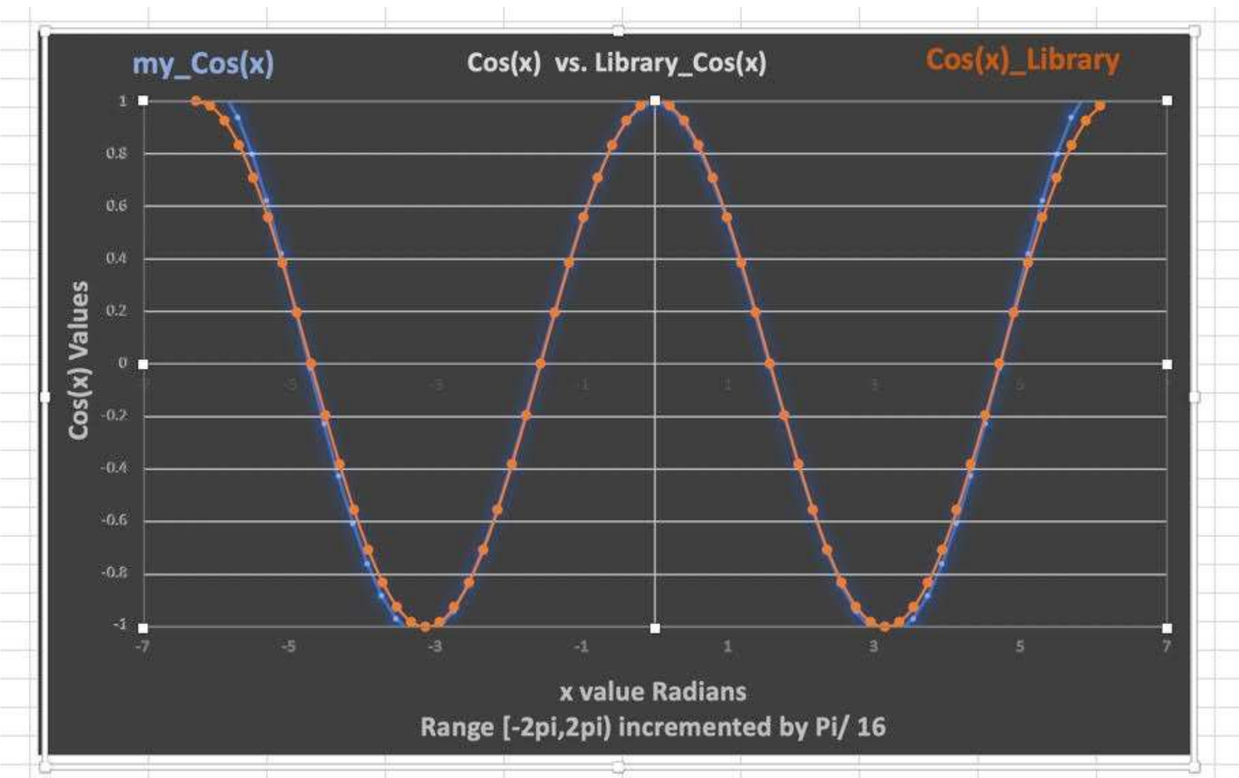
Consider the corresponding approximant for cos(x) centered around 0 written in Horner normal form:

$$\cos(x) \approx \frac{(x^2(1075032 - 14615x^2) - 18471600)x^2 + 39251520}{((127x^2 + 16632)x^2 + 1154160)x^2 + 39251520}.$$

These difference in the two functions are more apparent in the end of cosine graph where cos should evaluate to 1 or -1

This may be due to the values not being initialized at the end points independent of the approx. function
ex: my_Cos(-2pi) =my_Cos(2pi) = 1
ex: my_Cos(-pi) = my_cos(pi) = -1

This function came out fairly accurate.
The average difference = 0.00214817 (less accurate compared to sin)



Results for my_Tan() function

A Padé approximant for $\tan(x)$ is:

$$\tan(x) \approx \frac{x\,(x^8 - 990x^6 + 135135x^4 - 4729725x^2 + 34459425)}{45\,(x^8 - 308x^6 + 21021x^4 - 360360x^2 + 765765)}.$$

9th term series centered about 0

Which translates into Horner normal form:

Input:

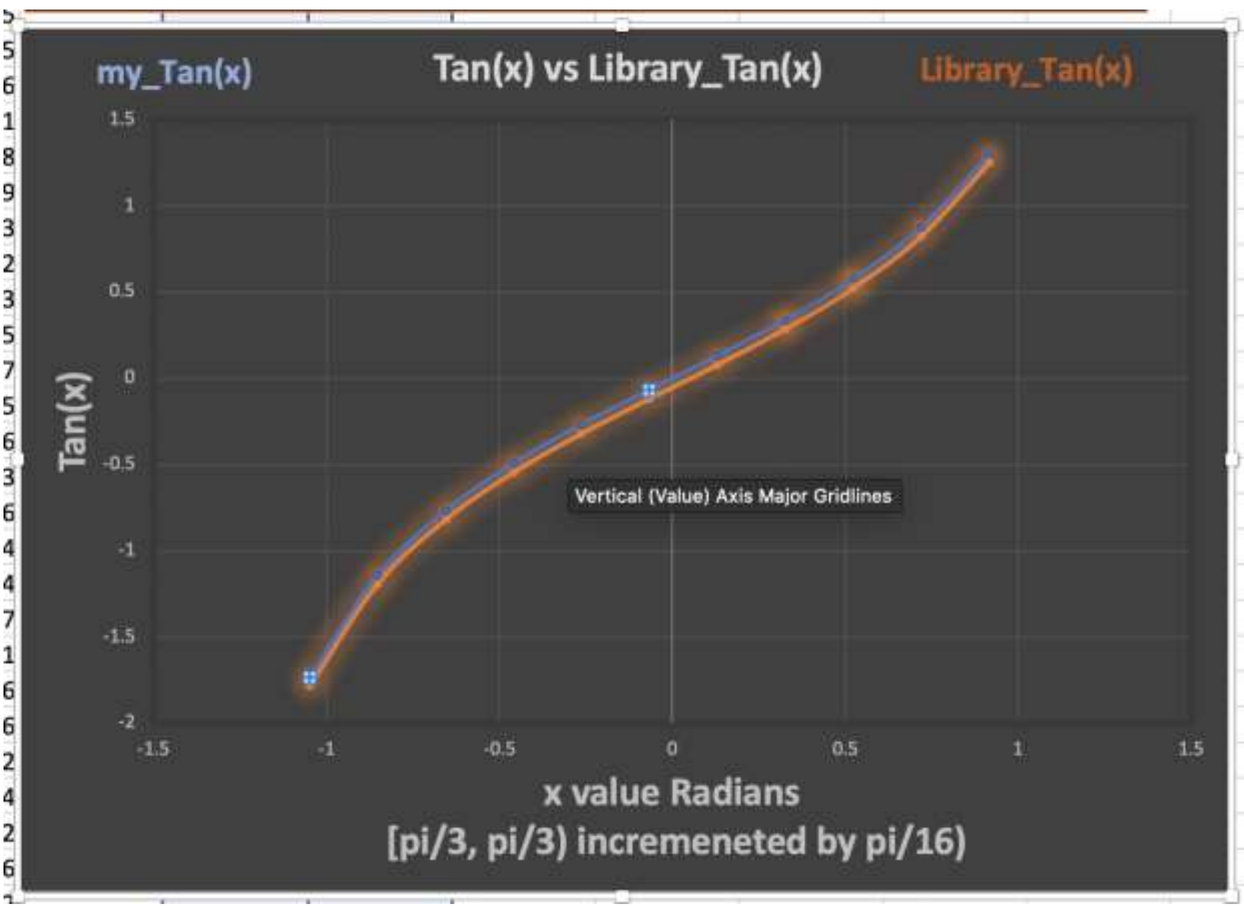
$\text{HornerForm}[x\,(x^8 - 990\,x^6 + 135\,135\,x^4 - 4\,729\,725\,x^2 + 34\,459\,425)]$

Result:

$x\,((x^2\,((x^2 - 990)\,x^2 + 135\,135) - 4\,729\,725)\,x^2 + 34\,459\,425)$

Instead I used the trigonometric identity for my_Sin(x) / my_Cos(x)   tan = sin/cos
*Note: Since the range is restricted to eliminate undefined behavior, tan = sin/cos is viable

Average difference = 0.00000000



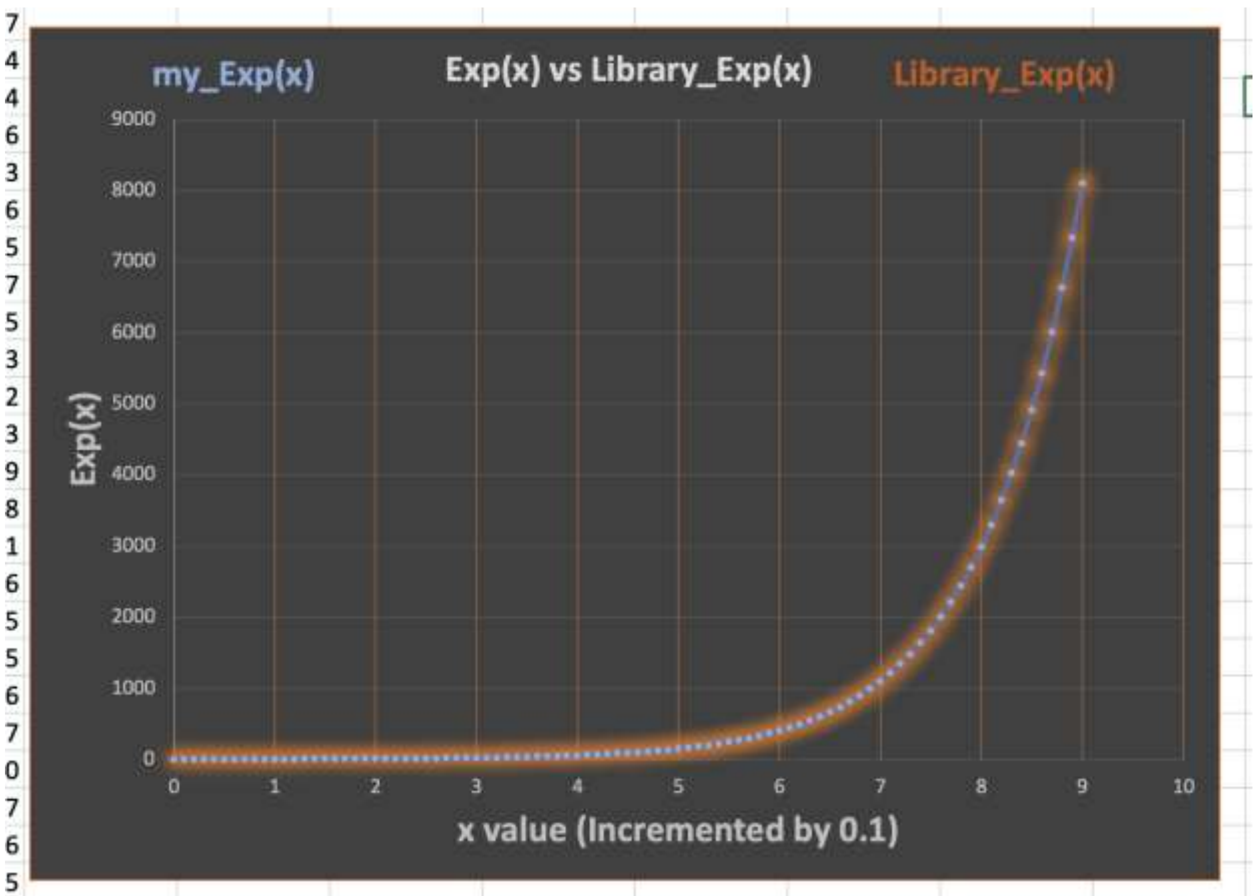
Results for my_Exp() function

Using the Taylor Series / MacLaurin Series centered about 0

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

The difference between terms is Epsilon: 10 ^ -10 instead of the lab recommended 10 ^ -9
because the higher epsilon value in my function yielded a higher precision to replicate the library exponential function.

Average difference: 0.00000000



*Note: equations above for Pade Approximations and Horners Equations are pulled from the Lab Manual for Assignment 2

Overall I am very happy with the results and can deduce that the exponential library uses the Maclaurin series to calculate its EXP() approximations similarly. The Sin and Cos function could be optimized better and the tan would not work on a value where Cos(x) = 0 (undefined) so would need to be reconfigured appropriately on a larger range.