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Summary

Today, concerns about the cost of toll plaza and lower efficiency are proposed more than ever. Aiming to solve the problem, we establish a model to minimize the cost while ensure safety, and increase the throughput as much as possible.

In order to simplify our model, we firstly adopt **Cellular Automaton(CA)** approach to establish a basic model, **CA Model of Traffic Flow**. With this model, we can obtain the value of throughput when the shape, size and merging pattern of any specific toll plaza are determined. To verify the reliability of this model, a real toll plaza in New Jersey is selected. The result we get by our model is in good agreement with the actual data.

Based on **CA Model of Traffic Flow**, an advanced model, **Optimal Toll Plaza Model** is proposed to find the optimal layout of the toll plaza including the shape, size and merging pattern. We define a comprehensive index E as the **performance indicator** to quantify the outcomes and make them comparable. Then, we divide our tasks into three steps. On the first step, we research three different shapes and find the isosceles trapezoid is optimal. After that, we change the size via assigning different value to L_0 (The length of the fan-in area of the toll plaza). And we find that when L_0 equals 16 units performs the best. Finally, based on the optimal shape and size, we study four kinds of merging pattern and determine the best one.

Besides parameters mentioned above, we also consider other factors (Traffic flow density, the proportion of self-driving and different toll booth) which may have impact on the solution. We draw the conclusion:

- The throughput-traffic density curve is similar to Michaelis-Menten Models
- When the proportion of self-driving vehicles increase, the throughput of fan-in area grows slightly.
- When the ratio of ETC increase, the throughput of fan-in area become larger.

Key words: CA approach, Optimal toll plaza model, performs indicator

Optimal Toll Plaza

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1. Introduction

In order to indicate the origin of barrier toll problem, the following background is worth mentioning.

1.1 Background

Currently, the explosive growth in traffic and relatively lagging transportation infrastructure have been a world-wide problem and a bottleneck for further development of large or medium-sized cities. Setting barrier toll is a widely used way to collect maintenance fee for highway. However, concerns about the running cost of toll plaza and lower efficiency caused by traffic jam are proposed more than ever. There are more than 50,000,000,000 hours delayed by congestion and over \$ 100,000 million loss in US every year ^[1]. Rational arrangement for establishing barrier toll is paid more attention by authority for the purpose of reducing cost as well as promoting efficiency. Thus optimal sets of the plaza and polls (involving shape, size and merging patterns) are in desperate need to improve the traffic congestion situation and resolve cost-benefit conflicts.

1.2 Literature review

Cellular automaton (CA) are discrete dynamical systems whose behavior is completely specified locally. Space is represented as a uniform grid, with each cell containing data. Time advances in discrete steps, and the laws of the universe are expressed in a look-up table relating each cell to nearby cells to compute its new state. The system's laws are local and uniform. The basic one-dimensional cellular automaton model for highway traffic flow is the CA rule 184th, as classified by Wolfram ^[2].

Cremer and Luding first studied how to establish a traffic flow model using deterministic cellular automaton based on the 184th cellular automaton rule, which was put forward by Wolfram. Subsequent improvements of the model have been made to simulate more realistic and complex traffic conditions. In the 1990s, Nagel and Schreckenberg put forward the famous **NaSch Model**. ^[3] CA is very suitable for the study of traffic flow problems, and the development process of CA and the traffic flow are closely related.

1.3 Restatement of the problem

To better manage the highway transportation in New Jersey, it is essential to reconsider the deployment of barrier polls. Hence modified models on both the plaza and polls are required. In order to propose the optimization solution adapting to the throughput and safety requirements of local authority, we will proceed as follows:

- Establishing a more elegant model of barrier tolls mainly from three aspects (shape, size and merging pattern)
- Determining the performance of our model when in light and heavy traffic
- Considering changes to the model when more autonomous vehicles are added to the traffic mix
- Conducting the analysis of impacts on the model causing by different proportion of three tollbooths patterns

2. Assumptions and Notations

2.1 Assumptions

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is appropriately justified.

- The highway is smooth enough to provide ideal condition for the models applied
- The operation cost of tollbooths can be neglected compared with construction cost of toll plaza and maintenance fee for the highway.
- All types of vehicles are regarded as identical cell with no difference when passing through the tollbooths.
- The delayed time refers to the time vehicles passing through the whole simulation region.
- All vehicles passing through the toll line tightly and the drivers do not want others to jump queue thus they keep a rather close distance with the one in front of itself.

2.2 Terminologies

- **Fan-in area:** a zone from the entrance of the toll plaza to the N-lane barrier polls.
- **Fan-out area:** a zone from the n-lane barrier tolls to the exit of the toll plaza.
- **Throughput:** the number of vehicles per hour passing the point where the end of the plaza joins the L outgoing traffic lanes.
- **Traffic density:** the number of vehicles passing per unit length of the motorway at moment t .

2.3 Notations

Tab.1 Notations

Symbols	Definition	Unit
V	Velocity of vehicle. The range is $\{ 1, 2, 3, \dots, V_{\max} \}$, representing the number of cell points moving forward in unit time step	Cell point/unit time step
d_n	A safety distance from the preceding vehicle	Cell point
X_n	Position of the n-th car	
V_n	Speed of the n-th car	
C_n	Whether there is a lane change condition or not	
$d_{n,other}$	A distance between an n-th vehicle and a vehicle ahead in an adjacent lane	Cell point
$d_{n,back}$	A distance between an n-th vehicle and an adjacent lane-rear vehicle	Cell point
d_{safe}	A lane-changing safety spacing	Cell point
E	<i>toll plaza traffic flow efficiency index</i>	

3. Model Overview

We use cellular automaton approach firstly to calculate the throughput of cars in different traffic environment, then we use the results gotten from the cellular automaton approach into our optimal toll plaza model.

We have three aims to approach including the shape, the size and the merging pattern. Through the optimal toll plaza model, we consider the accident prevention, the throughput and the cost. Then we separate our tasks

into three steps. On first step, we fixed other parameters and find the best shape of the plaza. On second step, we continue our work by the first step, we find the most suitable size of the plaza. On the last step, we find the most valuable merging pattern of the plaza.

At last, we find the best shape, size and merging pattern through our models.

The whole modeling process can be shown as follows:

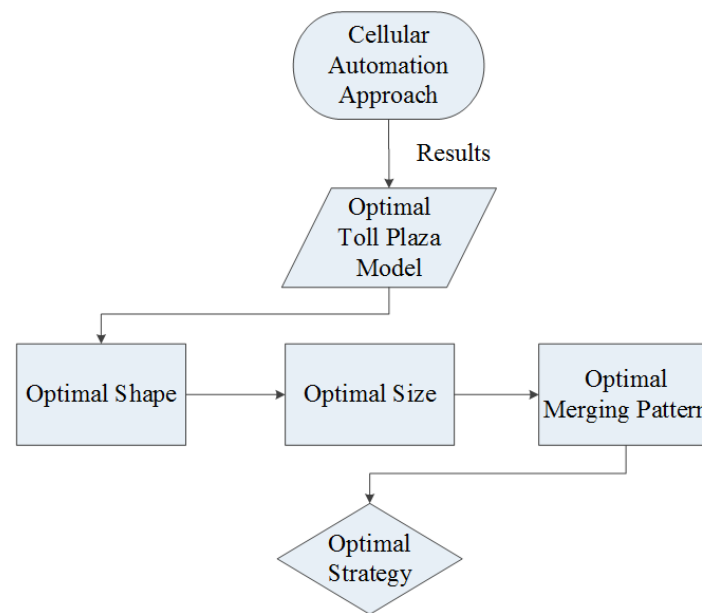


Fig.1 The modeling process

4. CA Model of Traffic Flow

4.1 Illustration of Cellular Automaton Approach

We use **Cellular Automaton (CA)** approach to study the influence of **shape, size and merging pattern** on the **cost, throughput** (number of vehicles per hour passing the point where the end of the plaza joins the L outgoing traffic lanes) and the **performance of accident prevention**.

We establish the "**CA Model of Traffic Flow**" for vehicles passing through the toll plaza.

4.2 Regulations

4.2.1 Basic regulations

In our **CA Model of Traffic Flow**, we regulate

- A cell is a small segment of length L and width equal to road. L

represents the average headway in traffic congestion .

- The whole area of the toll plaza is composed of cells.
- Each lane consists of a sequence of consecutive cells. The regime of each cell is either empty or occupied by the vehicle.
- The velocity of vehicle is V . The range is $\{ 1, 2, 3, \dots, V_{\max} \}$, representing the number of cell points moving forward in unit time step.

4.2.2 Velocity updating regulation

The velocity is updated according to the following rules:

- Acceleration. Since the driver is always expected to travel at maximum speed, if the vehicle can speed up without exceeding the speed limit V_{\max} , then it adds "1" to its speed, that is

$$V_n \rightarrow V_n + 1$$

Otherwise, the vehicle has constant speed, then

$$V_n \rightarrow V_n$$

- Since the velocity in the **CA Model** is defined as the number of cell points moving forward in unit time step. Therefore, when the number of cell points (distance) of between the vehicle and its preceding one is smaller than the current speed, the vehicle decelerates to maintain a safety distance from the preceding vehicle, thereby avoiding collision, which is called d_n , that is

$$V_n \rightarrow \min(V_n, d_n)$$

- Random slowing. Vehicles often slow for uncertain reasons (even laptops, cell phones, coffee mugs) and drivers occasionally make irrational choices. With some probability p , we have

$$V \rightarrow \max(V_n - 1, 0)$$

According to above, we can easily know the vehicle's position is updated according to the following rules:

$$X_n \rightarrow X_n + V_n$$

X_n and V_n denote the position and speed of the n th car. And d_n represents the number of vacant cell points away from the preceding vehicle, then

$$d_n = X_{n+1} - X_n - 1$$

4.2.3 Lane changing regulation

CA Model of Traffic Flow is simple in form and can describe the actual traffic flow to a certain extent. However, to describe a more complex traffic

situation, new rules must be made. Vehicles lane change is one that can not be ignored but difficult to implement. We refer to the B-man's C-model of the vehicle lane change rule of Chowdhury's **Symmetric Two-lane CA Model**,^[4] as below

$$C_n = \begin{cases} 1, & d_n < \min(v_n + 1, v_{\max}) , \quad d_{n,other} > d_n , d_{n,back} > d_{safe} \\ 0, & otherwise \end{cases}$$

C_n means whether there is a lane change condition or not, $d_{n,other}$ is a distance between an n-th vehicle and a vehicle ahead in an adjacent lane, $d_{n,back}$ stands for a distance between an n-th vehicle and an adjacent lane-rear vehicle, d_{safe} is a lane-changing safety spacing. This condition demonstrates that the vehicle encounters a block in the original lane

$$d_n < \min(v_n + 1, v_{\max})$$

And that a vehicle can achieve faster speed when in another lane equals to

$$d_{n,other} > d_n$$

Also the safety lane-changing spacing to meet the requirements equals to

$$d_{n,back} > d_{safe}$$

Then we assign the cell size to be 7.5m to match Nagel and many others^[5]. Since the maximum speed for vehicles is typically 30-40 m/s, choosing $V_{max} = 5$ makes a single time step close to 1s.

The table below shows the values of the basic parameters in this model.

Tab.2 The value of basic parameters

Basic Parameters	Meaning	Value or Shape
L_{cell}	cell length, the average head clearance when the road is blocked.	21 inch
V_{max}	maximum speed, the utmost number of cell points moving forward in a time step.	5

t	update interval, time required to update the condition of all vehicle.	1 second
L_0	The length of the fan-in area of the toll plaza	20 cells

4.2.4 Model application

In order to ensure that our model is correct and effective, we will select a real-life toll station to verify our model. Considering that we need to submit our findings to the **New Jersey Turnpike Authority**, we choose **Union Toll at Garden State Parkway**. In our letter to the New Jersey Turnpike Authority, there is a photo of the toll booth, from which we find that the toll plaza is a one-way toll plaza with five lanes both on the front and back side. And the number of toll booths is thirteen. From the website, ^[6] we can obtain the information of the toll booth and convert it to fit our size in the **CA Model of Traffic Flow**. The vertical length of the fan-in area of the plaza is approximately twenty cells, so we have L_0 of 20 in the previous table.

We utilize 24-hour traffic data from the site ^[7] of the toll station on November 17, 2015 and display it in the form of the table.

Tab.3 24-hour traffic data of Union Toll at Garden State Parkway on November 17, 2015

Hour	Traffic flow	Hour	Traffic flow	Hour	Traffic flow	Hour	Traffic flow
0 th	1120	6 th	2967	12 th	3277	18 th	3756
1 st	768	7 th	4247	13 th	3301	19 th	3254
2 ^{ed}	658	8 th	4140	14 th	3674	20 th	2817
3 th	734	9 th	3814	15 th	4294	21 st	2340
4 th	890	10 th	3457	16 th	4273	22 ^{ed}	1914
5 th	1365	11 th	3185	17 th	3687	23 th	1518

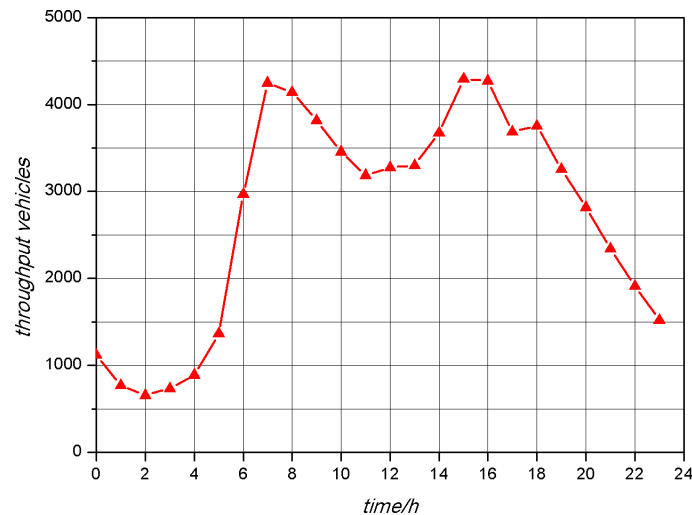


Fig.2 24-hour traffic data of Union Toll at Garden State Parkway on November17,2015

We compare the traffic flow data gained from CA simulation with the actual traffic flow, to determine the differentiation between model outcomes and realistic situation. If the difference is negligible, then we consider the model can well simulate the real traffic situation. Otherwise, the model cannot suit the real-life situation well. Then we statically collect the four hours of traffic density at the Union Toll at Garden State Parkway, (7th, 8th, 15th, 16th, **Hour**). It turns out that the actual traffic flow was 3393 and the average simulated traffic flow is 3390. The simulated data is in good agreement with the actual data, indicating that our model can successfully simulate the real traffic situation.

5 Optimal Toll Plaza Model

5.1 Model overview

To determine the optimal solution to toll plaza, we developed optimal toll plaza model based on the results of CA model, which targets solving problems relating to traffic flow. To simplify the problem, we divide the requirement into three dimensions--shape, size and merging pattern of the plaza.

5.2 Model establishment

5.2.1 Goal conversion

In the problem, we have to accomplish three goals—Maximum throughput(T), Minimum cost(C), and Accident prevention. We regard Accident prevention as a limitation, so we change three goals into two goals. The definition of two goals is shown below:

Throughput (T) -- Number of vehicles per hour passing the joint point where the end of the plaza joins the L outgoing traffic lanes

Cost (C) -- the running cost and construction fee (putting up roadblocks is included)

In order to make it comparable, we introduce a index to assess the outcomes of the model quantitatively that means we finally changed the goals into one goal:

Throughput/ Cost (T/C) -- throughput per unit cost

We assume that optimality occurs when largest throughput can be achieved per unit cost. Thus the problem can be rephrased as getting a maximum T/C ratio for the model. Here we rename the T/C ratio as *toll plaza traffic flow efficiency index*, expressed by E . Further, because the cost is proportional to the area size, so we translate one of the objectives from **Cost** into area size.

5.2.2 Influence of shape

There are so many kinds of shapes of plaza, we just take one picture to show the basic structure of the toll plaza as below:

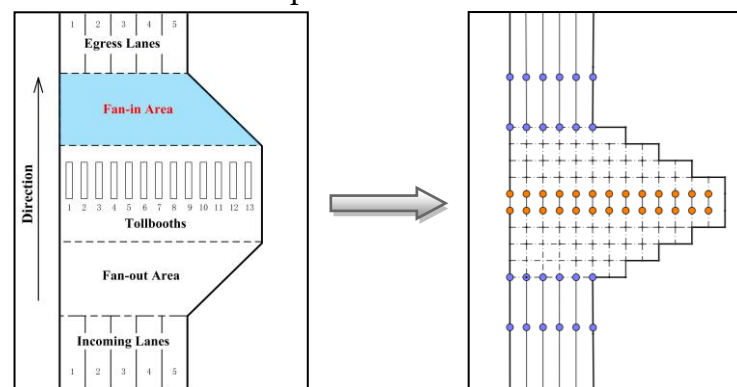


Fig.3 Toll plaza structure

The left figure in Fig.3 is a Schematic of real toll plaza, while during the CA approach, we change the area into cells like the right figure in Fig.3 to compute more easily.

Firstly, we discuss the cost and the throughput of the toll plaza with three different shapes--right trapezoid, isosceles trapezoid, rectangle. The shape

with the maximum Index E is the best. The schematic of three different shapes are showed below:

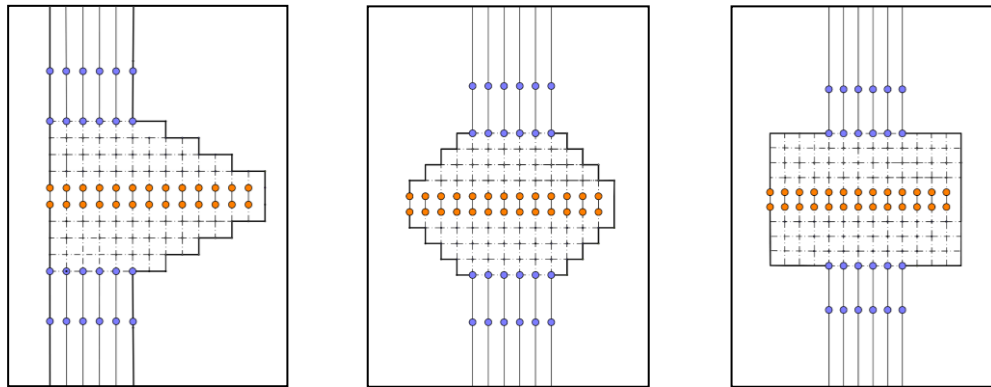


Fig.4 Three different shapes of plaza

5.2.3 Influence of size

According to our previous work, we can get the best shape of the plaza. Then based on the best shape, we discuss the influence of the plaza size.

Because the tollbooths' number(B) and the lanes' number(L) are fixed-numeric value given already, the size depends solely on L_0 . By changing L_0 from four to forty units in order to change the plaza size varies correspondingly, we can get different results with different L_0 , hence we determine the best size of the plaza.

5.2.4 Influence of merging pattern

The last step is to determine the impact of merging pattern on Index E based on optimal results of shape and size. The vehicles exiting the toll plaza area and "fanning-in" less lanes can cause "merging" phenomenon inevitably. We mainly consider 4 kinds of patterns of "merging":

- Pattern 1. No roadblock to partition the lanes-- the vehicles fan-in via synergic mechanism, entering the lanes randomly.
- Pattern 2, 3 and 4: the patterns can be showed as below:

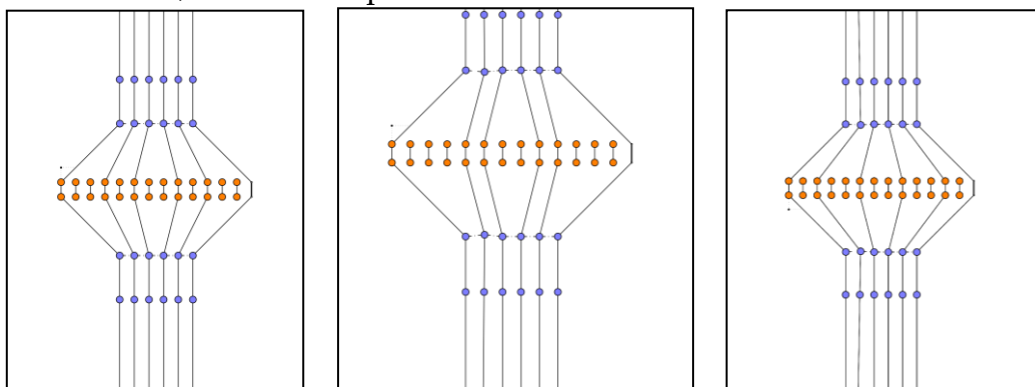


Fig.5 Merging pattern 2-4

5.3 Results

5.3.1 Optimal shape

By fixing L_0 (20) and the *merging pattern* (Pattern 1), we change the number of incoming cars with three different shapes to get the index E . The result is shown below:

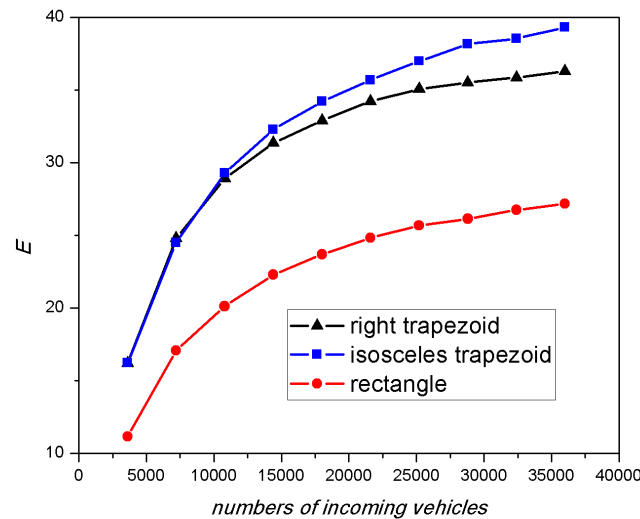


Fig.6 The performs of different shapes

It can be concluded from the figure that the isosceles trapezoid performs best while the rectangle is the worst. The area size of the rectangle is much bigger than those of right trapezoid and isosceles trapezoid, so the cost of rectangle is much higher, resulting in the lowest E among three different shapes.

What's more, when the number of incoming car is not large, the performances of right trapezoid and isosceles trapezoid are almost the same. The reason can be interpreted that both shapes are big enough to carry the vehicles. However, when the number of incoming cars increases, different results between right trapezoid and isosceles trapezoid arise. Isosceles trapezoid performs better attributing to its higher degree of symmetry than right trapezoid.

In conclusion, we think that the best shape of plaza is isosceles trapezoid, which is also the most common pattern in our daily life.

5.3.2 Optimal size

For the purpose of keeping single variable, we keep the shape (isosceles trapezoid) and merging pattern (Pattern 1) invariant while change the size of the plaza aiming to find the optimization. Because the number of lanes (L) and the number of tollbooths (B) are fixed, changing L_0 can bring change to size. Below show the results:

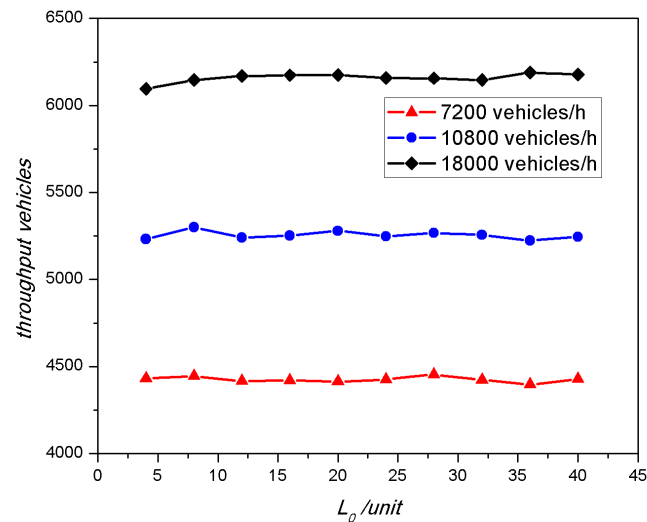


Fig.7 (a)The performs of different sizes

In Fig.7(a), we can see when the number of incoming vehicles is fixed, T fluctuates as L_0 changes. On the contrary, Fig.7(b) vividly shows that with the increase of L_0 , E decrease sharply in each situation (different numbers of incoming cars). The diagrams indicate that the bigger L_0 will lead to a lower E .

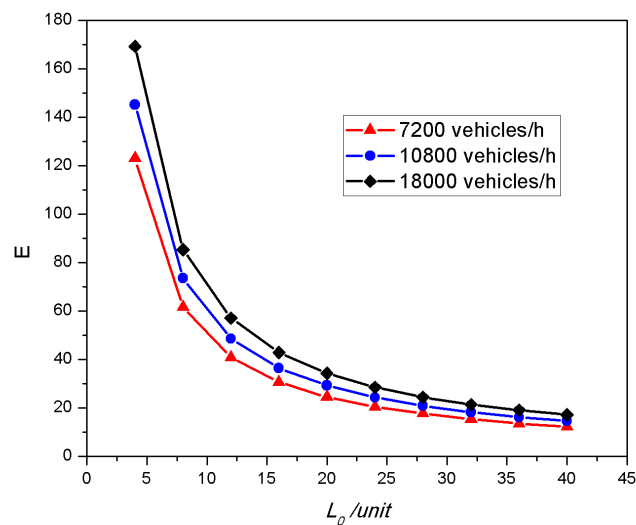


Fig.7(b) The performs of different sizes

In conclusion, if higher value of E is expected, then we should set a smaller plaza size. Yet, the size can't be so small due to the previous research [8]. We let L_0 equal to 16 (unit) for the value of E is sufficient under this situation and meets our research requirements.

5.3.3 Optimal merging pattern

We fix the optimal shape (isosceles trapezoid) and L_0 as 16 (discussed before), to solely study the pattern influence on the model by switching from Pattern 1 to Pattern 4, the outcomes are demonstrated below:

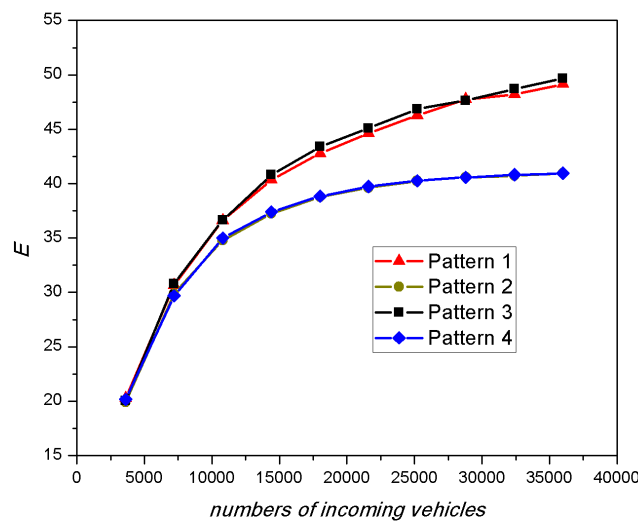


Fig.8 The performs of different merging patterns

Outcomes of Pattern 1 and Pattern 3 are close and are superior to those of Pattern 2 and Pattern 4. However, accident prevention, as a significant factor, making it unreasonable to accept Pattern1 where there's no roadblock to divide the lanes.

To summarize, we adopt Pattern 3 as the optimal pattern.

6. Model Analysis and Performance Analysis

Based on a plentitude of simulation, we find that the throughput, construction costs and safety prevention capability change when the shape, size and confluence pattern of the toll plaza are different. Obvious divergence among " fan-in" area may arise due to differentiation of merging area. In our previous work we have demonstrated how the throughput, construction costs and ability to prevent traffic accidents in confluent areas vary with the shape,

size and merging pattern.

In addition to the factors mentioned above, the throughput, construction costs and the safety prevention capacity can be affected by other factors, such as the highway traffic volume, types of driving (traditional or self-driving), charge window layouts (conventional, automated or electronic). If we are precisely the designers of the toll plaza, then our goal is to lower the cost as much as possible on condition that the vehicles pass the plaza safer and faster. To achieve this goal, the preliminary step is to understand the factors affecting the objective, as well as the sensitivity degree of the model if those factors change.

We previously combine the three factors by introducing index E and security of merging patterns. Next we continue this method to discuss impacts of the above "other factors" on achieving the target. This will be extremely important, with regard to both theoretical and engineering practicality.

6.1 The Impact of Traffic flow density on the Toll plaza

We not only discuss the impact of changes in shape, size and merging pattern on E , but also simulate the traffic density of different freeway to observe performances of our solution in light and heavy traffic. They indicate that when traffic density is light, the throughput of fan-in area increases linearly with traffic density; when traffic density is heavy, traffic density increases (even if significant) while cause few significant changes in the throughput of the fan-in area, just as the **Michaelis-Menten Model** ^[9] in biology.

This inspires us that when traffic is heavy in the region, more tollbooths should be established to cater for the excess demands. Thus increase the upper limit of traffic capacity and ease the pressure of transport.

6.2 The Impact of whether the vehicle is autopilot type on the toll plaza

Autopilot vehicles are much smarter and more responsive than ordinary vehicles, so the average speed is even greater. Whether the vehicle is autopilot or not will, in turn, affect the important parameter V_{\max} in our model. Because

as more and more autopilot vehicles are added to the flow, or the proportion of vehicles driving automatically in traffic increases, the overall speed of traffic flow will be higher. In order to maintain simulation results of our model, we should increase the V_{max} accordingly. Then we choose some bigger value of V_{max} as shown in the following figure.

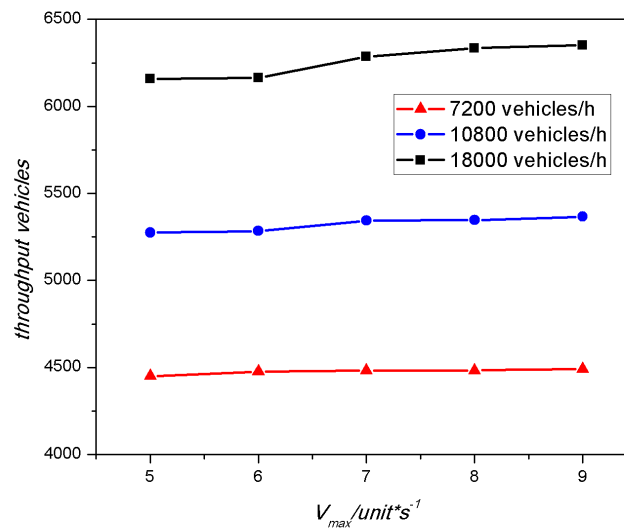


Fig.9 Different performs with V_{max} changes

According to Fig.9 we can know:

- When the proportion of vehicles driving automatically in traffic increases, the throughput of fan-in area increases in a very small scale. Because increase of the number of self-driving vehicles will increase the efficiency of the toll station but will not improve much.
- When the highway traffic density increases, the throughput of fan-in area will correspondingly increase, which is easy to understand. Additionally, when the highway traffic density is different, the throughput of fan-in area will significantly fluctuate.

6.3 The Impact of type of toll booths on the toll plaza

Type of toll booths includes conventional tollbooths, automated tollbooths, or electronic (often called ETC). ETC charges for the shortest time while automated tollbooth charges for the longest, with automated tollbooths in the midst. We re-simulate it when the proportion of the three types is different.

To simplify the study, we calculate the average time of total thirteen toll booths with different proportion of the three charging style. Here we define

“average time of the toll booths at different ratios” as t_{ave} and “delayed time of the vehicles for paying the toll” as t_i , that is

$$t_{ave} = \frac{\sum_{i=1}^{13} t_i}{13}$$

When the proportion of the three is different, we firstly get different t_{ave} , then we re-simulate our model with waiting time assigning value of 1s, 2s, 3s, 4s and 5s. There is an obvious fact that when the ETC ratio is greater, it will inevitably cause t_{ave} to decrease. Then we get a figure about the throughput of fan-in area below.

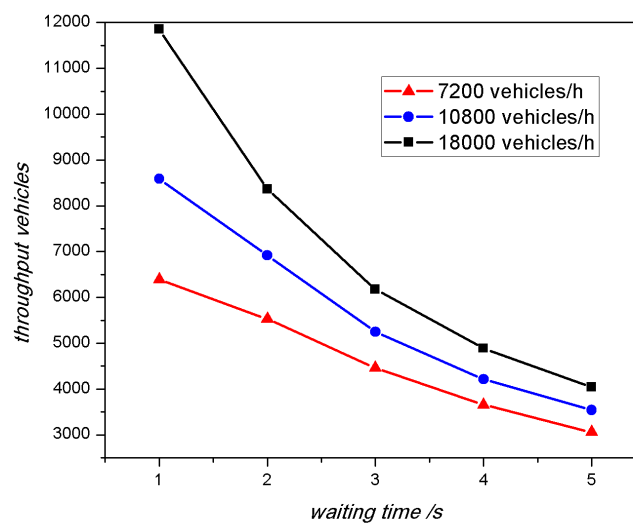


Fig.10 Different performs with waiting time changes

According to Fig.10, we can get these point that as the ratio of ETC increases, t_{ave} decreases and the throughput of fan-in area goes up, indicating that the toll booth has expanded passing-through capacity. Hence we reach a conclusion that if we want to improve the traffic capacity of toll booths, more ETC should set up.

7. Conclusion

Through the Cellular Automaton approach, we can get throughput of the tollbooth in different situations. With the help of CA approach and our optimal toll plaza model, we find the best shape of the toll plaza is Isosceles trapezoid, the best size is L_0 equals 16 unit, and the best merging pattern is pattern 3 we designed.

After analyzing the impacts **Traffic flow Size, Whether the vehicle is**

autopilot type and the type of toll booths (conventional, automated, or electronic), we can draw these conclusions:

- When the proportion of vehicles driving automatically in traffic increases, the throughput of fan-in area increases in a very small scale.
- When the highway traffic density is different, the throughput of fan-in area will be significantly different.
- As the ratio of ETC increases, t_{ave} decreases and the throughput of fan-in area becomes larger, indicating that the toll booth has increased vehicle capacity. If we want to improve the traffic capacity of toll booths, we can achieve it by setting up more ETC.
- When traffic is heavy in the region, more tollbooths should be established to cater for the excess demands. Thus increase the upper limit of traffic capacity and ease the pressure of transport.

8. Strength and Weakness

Strength:

- Valid suggestion are put forward to ease the traffic pressure around the toll plaza
- The problem is solved by the combination of suitable model. Sub-improvements of other researchers have been taken into consideration.
- The analysis of the results is interpreted by knowledge including mathematics, biology, transportation engineering etc.
- The sensitivity analysis is conducted thoroughly from the three dimensions which makes the results more convincing and universal.
- The diagrams are depicted quantitatively and vividly, making the outcomes more visible

Weakness:

- Some data and parameters are irrelevant and unavailable. In our simulation model, some parameters are set, admittedly, arbitrarily, due to lack of relevant data.
- The actual shape of the boundary is different from what we use in the simulation due to the limitation of algorithm.

Letter to the New Jersey Turnpike Authority

Dear New Jersey Turnpike Authority:

After hearing about construction problems of the shape, size and merging pattern the toll plaza to achieve the objectives of controlling cost, ensuring safety and expanding throughput as much as possible, we are writing to present some of our latest study outcomes.

We first develop our principal mode based on Cellular Automaton approach to obtain the throughput with determined size, shape and merging pattern of any specific toll plaza. In order to verify reliability and suitability of the model, we apply the real data of the toll plaza in New Jersey. To our relief, the results are in good agreement with the actual data. Next, we establish an improved model--Optimal Plaza Model, based on the Cellular Automaton approach to determine the optimal layouts of toll plaza. By introducing the comprehensive index, we quantify the outcomes making them comparable. Then we divide the objectives into three perspectives--shape, size and merging pattern to discuss the optimal layouts.

As the outside interference is uncontrollable, we conduct sensitivity analysis to consider other factors which may have impacts on the advanced model to test the stability and anti-jamming ability of the model in case extreme occasion happens. The three main factors are abnormal traffic volume (light and heavy), modes of charging window (human, automated and ETC) and vehicle driving style (traditional and self-driving). The suggestions we provide are shown as below:

Tab. 1 the Parameters and Optimal solution of Optimal Toll Plaza Model

	Parameters	Optimal solution
<i>The solution of Optimal Toll Plaza Model</i>	<i>Shape</i>	<i>Isosceles trapezoid</i>
	<i>L₀</i>	<i>20 cells</i>
	<i>Merging Pattern</i>	<i>With roadblock to partition the lanes</i>

Tab. 2 suggestions

<i>Factor we analyze</i>	<i>suggestion</i>
<i>Traffic flow density</i>	<i>If there is a heavy traffic in a toll plaza causing traffic jam, we can solve it by adding some new toll booths.</i>
<i>the proportion of different toll booths</i>	<i>If we want to improve the traffic capacity of toll booths, we can achieve it by setting up more ETC.</i>

Yours sincerely,

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