

# A Quantum State for N-Queens

Alexandre Silva<sup>1,2\*</sup> and Luis Hilario<sup>2,3†</sup>

<sup>1\*</sup>Computer Science, UNIVEM - Centro Universitário Eurípides de Marília, Hygino Muzy Filho, Marília, 17525-901, São Paulo, Brazil.

\*Corresponding author(s). E-mail(s): [dpbm136@gmail.com](mailto:dpbm136@gmail.com);  
Contributing authors: [luishilario@univem.edu.br](mailto:luishilario@univem.edu.br);

<sup>†</sup>These authors contributed equally to this work.

## Abstract

The abstract serves both as a general introduction to the topic and as a brief, non-technical summary of the main results and their implications. Authors are advised to check the author instructions for the journal they are submitting to for word limits and if structural elements like subheadings, citations, or equations are permitted.

**Keywords:** Quantum Computing, N-Queens, Quantum State, W-State, Optimization

## 1 Introduction

The N-Queens, is a widely spread problem in the fields of mathematics and computer science. As shown in [1][2], this problem is usually employed to teach people about optimization algorithms, backtracking methods, artificial intelligence, search algorithms, etc. Due to its simple setup but massive amount of combinations, many researchers started applying methods to translate it to quantum computing domain. In this paper, we're going to present a simplified way to generate a specific circuit for this problem, inspired by [3][4]. In our setup, we reached the constant number of qubits  $2N$  and  $O()$  in depth. Even though, the amount of qubits can be reduced to  $N$  using QAOA [5] and the depth be kept constant, our version use less classical interaction and employs a modified way to create a W-State, vanishing completely the need for checking rows and columns.

The structure of this paper is the following. In section 2 we present the basic of N-Queens problem, 3 we'll describe how our modified state works and how can be

implemented, 4 will show the complete setup along with its analysis, and finally 5 will discuss the setup.

## 2 The N-Queens Problem

The problem is described as, in a hypothetical chessboard of which, instead of the regular dimensions 8x8 and pieces displacement, you have a  $n \times n$  board with  $n$  queens placed thought the board cells. The goal of this problem, is to find all the combinations that satisfy the constraint of no two queens can attack each other.

A naive approach to solve it, is randomly test combinations of different queens in distinct sets of rows and columns. However, with a simple analysis, it's possible to check that every row must have exactly one queen, and the same for every column. Due to that, the amount of combinations to check falls from  $\prod_{i=0}^{n-1} n^2 - i$  to  $n!$ .

With its complexity being  $O(n!)$ , this problem grows really fast as the side of the board  $n$  grows. Due to that, the effort to convert this problem into quantum started to increase as the quantum computers became more popular, with QUBO [? ], QAOA [? ] and even using atoms in a lattice [? ] implementations being done. However, as the major platform for quantum computing as the current date (2025), the most part of the implementations are done using specific quantum circuits embedding the problem behavior using the available quantum gates and measurements together with post-processing classical routines to extract the solutions from the quantum system, some examples are: [? ][? ] and this very paper.

## 3 This is an example for first level head—section head

## 4 Equations

Equations in L<sup>A</sup>T<sub>E</sub>X can either be inline or on-a-line by itself (“display equations”). For inline equations use the `$...$` commands. E.g.: The equation  $H\psi = E\psi$  is written via the command `$H \psi = E \psi$`.

For display equations (with auto generated equation numbers) one can use the `equation` or `align` environments:

$$\|\tilde{X}(k)\|^2 \leq \frac{\sum_{i=1}^p \|\tilde{Y}_i(k)\|^2 + \sum_{j=1}^q \|\tilde{Z}_j(k)\|^2}{p+q}. \quad (1)$$

where,

$$\begin{aligned} D_\mu &= \partial_\mu - ig \frac{\lambda^a}{2} A_\mu^a \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c \end{aligned} \quad (2)$$

Notice the use of `\nonumber` in the `align` environment at the end of each line, except the last, so as not to produce equation numbers on lines where no equation numbers

are required. The `\label{}` command should only be used at the last line of an align environment where `\nonumber` is not used.

$$Y_{\infty} = \left(\frac{m}{\text{GeV}}\right)^{-3} \left[1 + \frac{3\ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right] \quad (3)$$

The class file also supports the use of `\mathbb{}`, `\mathscr{}` and `\mathcal{}` commands. As such `\mathbb{R}`, `\mathscr{R}` and `\mathcal{R}` produces  $\mathbb{R}$ ,  $\mathscr{R}$  and  $\mathcal{R}$  respectively (refer Subsubsection ??).

# 5 Tables

Tables can be inserted via the normal table and tabular environment. To put footnotes inside tables you should use `\footnotetext[...]` tag. The footnote appears just below the table itself (refer Tables 1 and 2). For the corresponding footnotemark use `\footnotemark[...]`

**Table 1** Caption text

Column 1	Column 2	Column 3	Column 4
row 1	data 1	data 2	data 3
row 2	data 4	data 5 <sup>1</sup>	data 6
row 3	data 7	data 8	data 9 <sup>2</sup>

Source: This is an example of table footnote. This is an example of table footnote.

<sup>1</sup>Example for a first table footnote. This is an example of table footnote.

<sup>2</sup>Example for a second table footnote. This is an example of table footnote.

The input format for the above table is as follows:

```
\begin{table}[<placement-specifier>]
\caption{<table-caption>}\label{<table-label>}%
\begin{tabular}{@{}l l l l@{}}
\toprule
Column 1 & Column 2 & Column 3 & Column 4\\
\midrule
row 1 & data 1 & data 2 & data 3 \\
row 2 & data 4 & data 5\footnotemark[1] & data 6 \\
row 3 & data 7 & data 8 & data 9\footnotemark[2]\\
\botrule
\end{tabular}
```

```

\footnotetext{Source: This is an example of table footnote.
This is an example of table footnote.}
\footnotetext[1]{Example for a first table footnote.
This is an example of table footnote.}
\footnotetext[2]{Example for a second table footnote.
This is an example of table footnote.}
\end{table}

```

**Table 2** Example of a lengthy table which is set to full textwidth

Project	Element 1 <sup>1</sup>			Element 2 <sup>2</sup>		
	Energy	$\sigma_{calc}$	$\sigma_{expt}$	Energy	$\sigma_{calc}$	$\sigma_{expt}$
Element 3	990 A	1168	$1547 \pm 12$	780 A	1166	$1239 \pm 100$
Element 4	500 A	961	$922 \pm 10$	900 A	1268	$1092 \pm 40$

Note: This is an example of table footnote. This is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote.

<sup>1</sup>Example for a first table footnote.

<sup>2</sup>Example for a second table footnote.

In case of double column layout, tables which do not fit in single column width should be set to full text width. For this, you need to use `\begin{table*}` ... `\end{table*}` instead of `\begin{table}` ... `\end{table}` environment. Lengthy tables which do not fit in textwidth should be set as rotated table. For this, you need to use `\begin{sidewaystable}` ... `\end{sidewaystable}` instead of `\begin{table*}` ... `\end{table*}` environment. This environment puts tables rotated to single column width. For tables rotated to double column width, use `\begin{sidewaystable*}` ... `\end{sidewaystable*}`.

## 6 Figures

As per the  $\text{\LaTeX}$  standards you need to use eps images for  $\text{\LaTeX}$  compilation and pdf/jpg/png images for PDF $\text{\LaTeX}$  compilation. This is one of the major difference between  $\text{\LaTeX}$  and PDF $\text{\LaTeX}$ . Each image should be from a single input .eps/vector image file. Avoid using subfigures. The command for inserting images for  $\text{\LaTeX}$  and PDF $\text{\LaTeX}$  can be generalized. The package used to insert images in  $\text{\LaTeX}$ /PDF $\text{\LaTeX}$  is the graphicx package. Figures can be inserted via the normal figure environment as shown in the below example:

```

\begin{figure}[<placement-specifier>]
\centering
\includegraphics{<eps-file>}

```

**Table 3** Tables which are too long to fit, should be written using the “sidewaystable” environment as shown here

Projectile	Element 1 <sup>1</sup>			Element <sup>2</sup>		
	Energy	$\sigma_{calc}$	$\sigma_{expt}$	Energy	$\sigma_{calc}$	$\sigma_{expt}$
Element 3	990 A	1168	1547 $\pm$ 12	780 A	1166	1239 $\pm$ 100
Element 4	500 A	961	922 $\pm$ 10	900 A	1268	1092 $\pm$ 40
Element 5	990 A	1168	1547 $\pm$ 12	780 A	1166	1239 $\pm$ 100
Element 6	500 A	961	922 $\pm$ 10	900 A	1268	1092 $\pm$ 40

Note: This is an example of table footnote this is an example of table footnote this is an example of table footnote  
this is an example of table footnote.

<sup>1</sup> This is an example of table footnote.

```
\caption{<figure-caption>}\label{<figure-label>}
\end{figure}
```



**Fig. 1** This is a widefig. This is an example of long caption this is an example of long caption this is an example of long caption this is an example of long caption

In case of double column layout, the above format puts figure captions/images to single column width. To get spanned images, we need to provide `\begin{figure*}` ... `\end{figure*}`.

For sample purpose, we have included the width of images in the optional argument of `\includegraphics` tag. Please ignore this.

## 7 Algorithms, Program codes and Listings

Packages `algorithm`, `algorithmicx` and `algpseudocode` are used for setting algorithms in  $\text{\LaTeX}$  using the format:

```
\begin{algorithm}
\caption{<alg-caption>}\label{<alg-label>}
\begin{algorithmic}[1]
. . .
\end{algorithmic}
\end{algorithm}
```

You may refer above listed package documentations for more details before setting `algorithm` environment. For program codes, the “verbatim” package is required and the command to be used is `\begin{verbatim}` ... `\end{verbatim}`.

Similarly, for listings, use the `listings` package. `\begin{lstlisting}` ... `\end{lstlisting}` is used to set environments similar to `verbatim` environment. Refer to the `lstlisting` package documentation for more details.

A fast exponentiation procedure:

```
begin
  for  $i := 1$  to 10 step 1 do
     $\text{expt}(2, i)$ ;
     $\text{newline}()$  od
where
```

Comments will be set flush to the right margin

```

proc  expt( $x, n$ )  $\equiv$ 
   $z := 1$ ;
  do if  $n = 0$  then exit fi;
    do if odd( $n$ ) then exit fi;
      comment: This is a comment statement;
       $n := n/2$ ;  $x := x * x$  od;
    {  $n > 0$  };
     $n := n - 1$ ;  $z := z * x$  od;
  print( $z$ ).
end

```

---

**Algorithm 1** Calculate  $y = x^n$

---

**Require:**  $n \geq 0 \vee x \neq 0$

**Ensure:**  $y = x^n$

```

1:  $y \leftarrow 1$ 
2: if  $n < 0$  then
3:    $X \leftarrow 1/x$ 
4:    $N \leftarrow -n$ 
5: else
6:    $X \leftarrow x$ 
7:    $N \leftarrow n$ 
8: end if
9: while  $N \neq 0$  do
10:  if  $N$  is even then
11:     $X \leftarrow X \times X$ 
12:     $N \leftarrow N/2$ 
13:  else [ $N$  is odd]
14:     $y \leftarrow y \times X$ 
15:     $N \leftarrow N - 1$ 
16:  end if
17: end while

```

---

```

for  $i := \text{maxint}$  to 0 do
begin
{ do nothing }
end;
Write( 'Case-insensitive-');
Write( 'Pascal-keywords-');

```

## 8 Cross referencing

Environments such as figure, table, equation and align can have a label declared via the `\label{#label}` command. For figures and table environments use the `\label{}` command inside or just below the `\caption{}` command. You can then use the `\ref{#label}` command to cross-reference them. As an example, consider the label declared for Figure 1 which is `\label{fig1}`. To cross-reference it, use the command `\ref{fig1}`, for which it comes up as “Figure 1”.

To reference line numbers in an algorithm, consider the label declared for the line number 2 of Algorithm 1 is `\label{algn2}`. To cross-reference it, use the command `\ref{algn2}` for which it comes up as line 2 of Algorithm 1.

### 8.1 Details on reference citations

Standard L<sup>A</sup>T<sub>E</sub>X permits only numerical citations. To support both numerical and author-year citations this template uses `natbib` L<sup>A</sup>T<sub>E</sub>X package. For style guidance please refer to the template user manual.

Here is an example for `\cite{...}`: [? ]. Another example for `\citep{...}`: [? ]. For author-year citation mode, `\cite{...}` prints Jones et al. (1990) and `\citep{...}` prints (Jones et al., 1990).

All cited bib entries are printed at the end of this article: [? ], [? ], [? ], [? ], [? ], [? ], [? ], [? ], [? ], [? ] and [? ].

## 9 Examples for theorem like environments

For theorem like environments, we require `amsthm` package. There are three types of predefined theorem styles exists—`thmstyleone`, `thmstyletwo` and `thmstylethree`

<code>thmstyleone</code>	Numbered, theorem head in bold font and theorem text in italic style
<code>thmstyletwo</code>	Numbered, theorem head in roman font and theorem text in italic style
<code>thmstylethree</code>	Numbered, theorem head in bold font and theorem text in roman style

For mathematics journals, theorem styles can be included as shown in the following examples:

**Theorem 1** (Theorem subhead) *Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text.*

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.



**Proposition 2** *Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text.*

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

*Example 1* Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

*Remark 1* Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

**Definition 1** (Definition sub head) Example definition text. Example definition text. Example definition text. Example definition text. Example definition text. Example definition text. Example definition text.

Additionally a predefined “proof” environment is available: `\begin{proof} ... \end{proof}`. This prints a “Proof” head in italic font style and the “body text” in roman font style with an open square at the end of each proof environment.

*Proof* Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text.  $\square$

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

*Proof of Theorem 1* Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text.  $\square$

For a quote environment, use `\begin{quote} ... \end{quote}`

Quoted text example. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text (refer Figure 1). Sample body text. Sample body text. Sample body text (refer Table 3).

## 10 Methods

Topical subheadings are allowed. Authors must ensure that their Methods section includes adequate experimental and characterization data necessary for others in the field to reproduce their work. Authors are encouraged to include RIIIDs where appropriate.

**Ethical approval declarations** (only required where applicable) Any article reporting experiment/s carried out on (i) live vertebrate (or higher invertebrates), (ii) humans or (iii) human samples must include an unambiguous statement within the methods section that meets the following requirements:

1. Approval: a statement which confirms that all experimental protocols were approved by a named institutional and/or licensing committee. Please identify the approving body in the methods section
2. Accordance: a statement explicitly saying that the methods were carried out in accordance with the relevant guidelines and regulations
3. Informed consent (for experiments involving humans or human tissue samples): include a statement confirming that informed consent was obtained from all participants and/or their legal guardian/s

If your manuscript includes potentially identifying patient/participant information, or if it describes human transplantation research, or if it reports results of a clinical trial then additional information will be required. Please visit (<https://www.nature.com/nature-research/editorial-policies>) for Nature Portfolio journals, (<https://www.springer.com/gp/authors-editors/journal-author/journal-author-helpdesk/publishing-ethics/14214>) for Springer Nature journals, or (<https://www.biomedcentral.com/getpublished/editorial-policies#ethics+and+consent>) for BMC.

## 11 Discussion

Discussions should be brief and focused. In some disciplines use of Discussion or ‘Conclusion’ is interchangeable. It is not mandatory to use both. Some journals prefer a section ‘Results and Discussion’ followed by a section ‘Conclusion’. Please refer to Journal-level guidance for any specific requirements.

## 12 Conclusion

Conclusions may be used to restate your hypothesis or research question, restate your major findings, explain the relevance and the added value of your work, highlight any limitations of your study, describe future directions for research and recommendations.

In some disciplines use of Discussion or 'Conclusion' is interchangeable. It is not mandatory to use both. Please refer to Journal-level guidance for any specific requirements.

**Supplementary information.** If your article has accompanying supplementary file/s please state so here.

Authors reporting data from electrophoretic gels and blots should supply the full unprocessed scans for key as part of their Supplementary information. This may be requested by the editorial team/s if it is missing.

Please refer to Journal-level guidance for any specific requirements.

**Acknowledgements.** Acknowledgements are not compulsory. Where included they should be brief. Grant or contribution numbers may be acknowledged.

Please refer to Journal-level guidance for any specific requirements.

## Declarations

Some journals require declarations to be submitted in a standardised format. Please check the Instructions for Authors of the journal to which you are submitting to see if you need to complete this section. If yes, your manuscript must contain the following sections under the heading 'Declarations':

- Funding
- Conflict of interest/Competing interests (check journal-specific guidelines for which heading to use)
- Ethics approval and consent to participate
- Consent for publication
- Data availability
- Materials availability
- Code availability
- Author contribution

If any of the sections are not relevant to your manuscript, please include the heading and write 'Not applicable' for that section.

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*Scientific Reports*: <https://www.nature.com/srep/journal-policies/editorial-policies>

BMC journals: <https://www.biomedcentral.com/getpublished/editorial-policies>

## **Appendix A    Section title of first appendix**

An appendix contains supplementary information that is not an essential part of the text itself but which may be helpful in providing a more comprehensive understanding of the research problem or it is information that is too cumbersome to be included in the body of the paper.