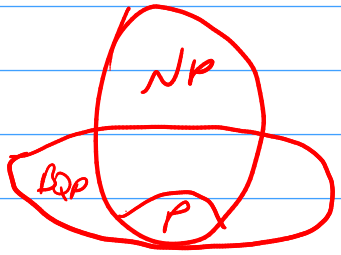
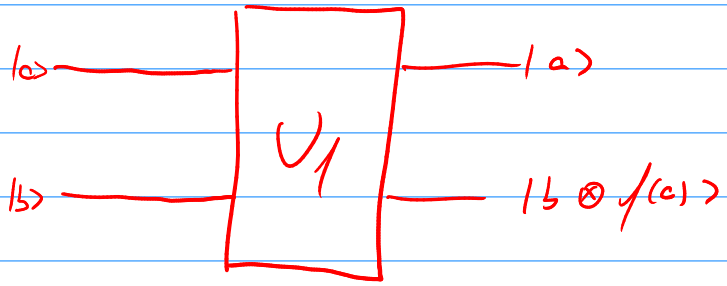
→ BQP → Problems that can be solved by a QC in Polynomial time



→ A problem for QC is how can ya verify a solution given by a QC

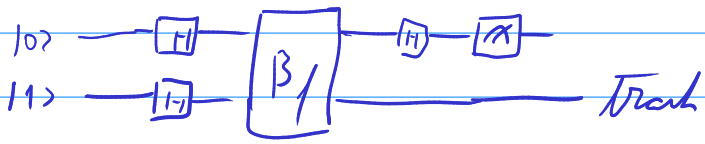
→ NISQ → used to describe things that we can do with current quantum computers



phase kickback → $|a\rangle|-\rangle \rightarrow (-1)^{f(a)}|a\rangle|-\rangle$

encode a phase

→ Deutsch's algorithm

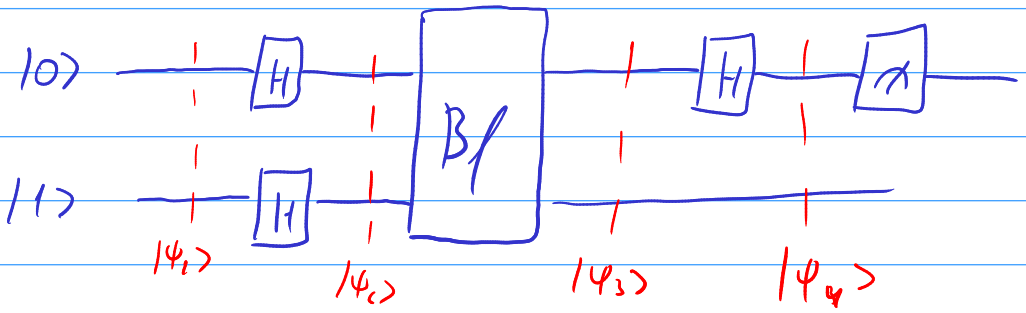


→ we have possible functions in Uf, which one is implemented and how many times I need to call the

oracle (B_f) to get the answer
→ classically → 2 times

	f_0	f_1	f_2	f_3
0	0	0	1	1
1	0	1	0	1

↳ 4 functions



$|\psi_1\rangle = |01\rangle$ $\quad |\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

$\frac{1}{2}(|0\rangle(|0\rangle-|1\rangle)+|1\rangle(|0\rangle-|1\rangle))$

$|\psi_3\rangle = \frac{1}{2}(|0\rangle(|0\oplus f(0)\rangle - |1\oplus f(0)\rangle) + |1\rangle(|0\oplus f(1)\rangle - |1\oplus f(1)\rangle)$

$= \frac{1}{2}((-1)^{f(0)}|0\rangle(|-\rangle) + (-1)^{f(1)}|1\rangle(|-\rangle))$

$|\psi_4\rangle = \frac{1}{2}((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle)$

↳ drop out the $|-\rangle$

so if we apply the H-gate on the $|a\rangle$ state

$H|a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(-1)^a|1\rangle$

and if we take away the H, we get

$$H|0\rangle \xrightarrow{H} HH|0\rangle \to |0\rangle$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\left[(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right]$$

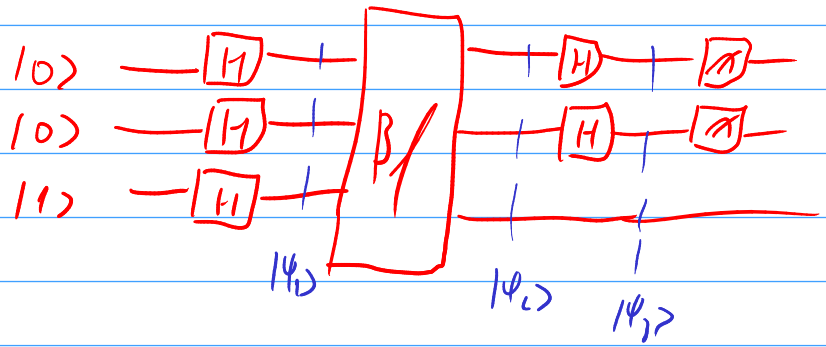
$$(-1)^{f(0)}\left[\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(-1)^{f(0)\oplus f(1)}|1\rangle\right]$$

as $H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(-1)^{a}|1\rangle\right) = |0\rangle$

$$H|\psi_3\rangle = (-1)^{f(0)}|f(0)\oplus f(1)\rangle$$

f constant $\to$ $f(0)\oplus f(1) = 0$ (f_0 and f_3)
f balanced $\to$ $f(0)\oplus f(1) = 1$ (f_1 and f_2)

Doing this single call to the oracle we can only identify if the function is constant or balanced, but not which function is inside



$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)|-\rangle$$

$$|\psi_2\rangle = \frac{1}{2}\left((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle\right)|-\rangle$$

$f \rightarrow$ returns 1 if the input corresponds to its label (f_{00} returns 1 for input 00)

$$f_{00} \rightarrow \frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$$f_{01} \rightarrow \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)$$

$$f_{10} \rightarrow \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)$$

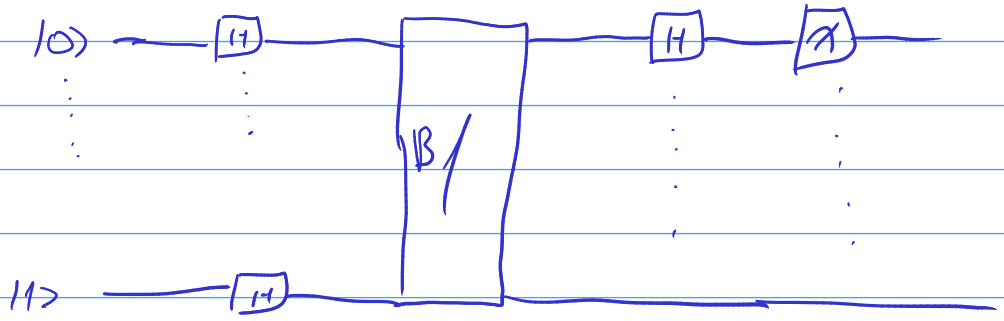
$$f_{11} \rightarrow \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)$$

Orthogonal (so there is a U transformation that maps this to basis state)

$$H|0\rangle = \frac{1}{\sqrt{2}} \sum_{b \in \{0,1\}} (-1)^{0 \cdot b} |b\rangle$$

$$H \otimes H |x_1 x_2\rangle = \left(\frac{1}{\sqrt{2}} \sum_{y_1} (-1)^{x_1 \cdot y_1} |y_1\rangle\right)\left(\frac{1}{\sqrt{2}} \sum_{y_2} (-1)^{x_2 \cdot y_2} |y_2\rangle\right)$$

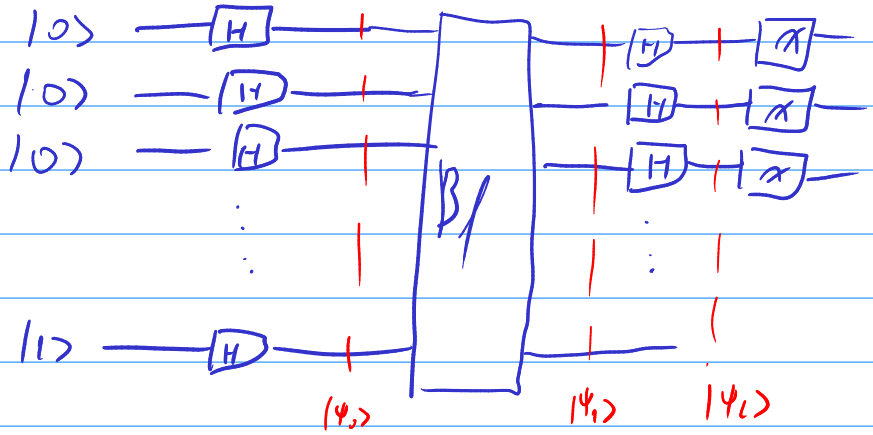
$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{x \cdot y} |y\rangle$$



if f is constant → $f(x) = 0$ or $f(x) = 1$
if f is balanced → half of values is 0 and the other half value

→ Classicly you need at least $2^{n-1}+1$ evaluations
→ Quantumly only 1 call is needed

$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{x_1 y_1 + x_2 y_2 \cdots x_n y_n} |y\rangle$$



$$|\psi_0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \; |-\rangle$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |x\rangle \; |-\rangle$$

$$|\psi_2\rangle = \left(H^{\otimes n} \otimes I\right) |\psi_1\rangle \; |-\rangle$$

$$= \left(\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} |\psi_1\rangle \right) |-\rangle$$

$$= \left(\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{f(x)} \left[\frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{x \cdot y} |y\rangle \right] \right) |-\rangle$$

$$= \sum_y \underbrace{\left(\frac{1}{2^n} \sum_x (-1)^{f(x) \oplus x \cdot y} \right)}_{\text{amplitude of } |y\rangle} |y\rangle \; |-\rangle$$

prob. of $y = 0^{\otimes n} \Rightarrow \left(\frac{1}{2^n} \sum_x (-1)^{f(x)} \right)^2$