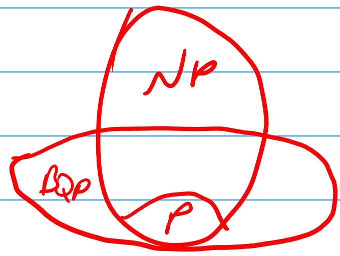
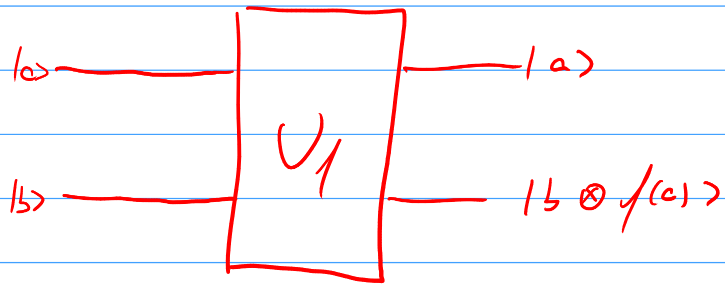
→ BQP → Problems that can be solved by a QC in Polynomial time



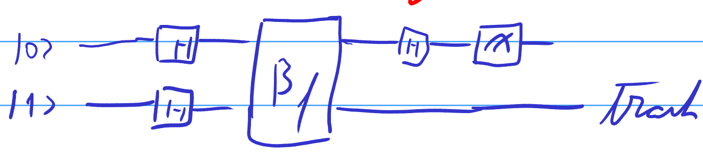
→ A problem for QC is how can you verify a solution given by a QC

→ NISQ → used to describe things that we can do with current quantum computers



phase kickback → $|a\rangle|-\rangle \longrightarrow \underbrace{(-1)^{f(a)}}_{\text{induced a phase}}|a\rangle|-\rangle$

→ Deutsch's algorithm

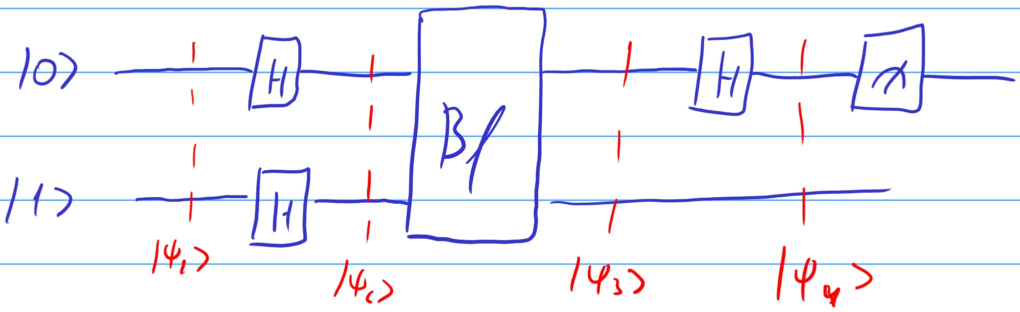


→ we have possible functions in Bf, which one is implemented and how many times I need to call the

Oracle (Bf) to get the answer
→ Classically → 2 times

	f_0	f_1	f_2	f_3
0	0	0	1	1
1	0	1	0	1

↩ 4 functions



$|\psi_1\rangle = |01\rangle$ $\qquad |\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\,\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$

$$\frac{1}{2}\Big(|0\rangle(|0\rangle-|1\rangle) + |1\rangle(|0\rangle-|1\rangle)\Big)$$

$$|\psi_3\rangle = \frac{1}{2}\Big(|0\rangle\big(|0\oplus f(0)\rangle - |1\oplus f(0)\rangle\big) + |1\rangle\big(|0\oplus f(1)\rangle - |1\oplus f(1)\rangle\big)\Big)$$

$$= \frac{1}{2}\Big((-1)^{f(0)}|0\rangle(|-\rangle) + (-1)^{f(1)}|1\rangle(|-\rangle)\Big)$$

$$|\psi_4\rangle = \frac{1}{2}\Big((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\Big)$$

↳ drop out the $|-\rangle$

so if we apply the H-gate on the $|c\rangle$ state

$$H|c\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(-1)^c|1\rangle$$

and if we take away the H, we get

$$H|0\rangle \xrightarrow{H} HH|0\rangle \to |0\rangle$$

$$|\varphi_3\rangle = \frac{1}{\sqrt{2}}\left[(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right]$$

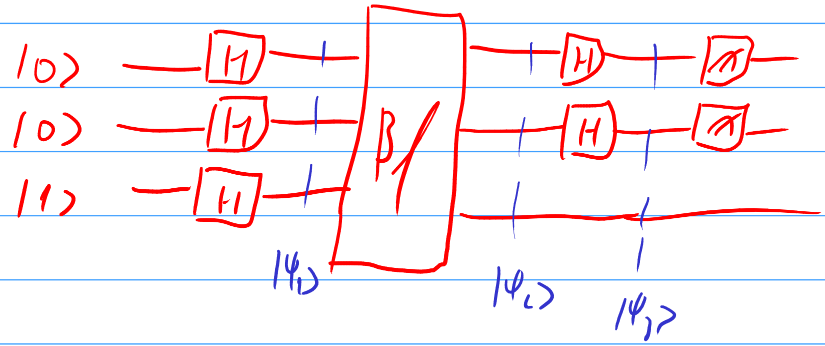
$$(-1)^{f(0)}\left[\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(-1)^{f(0)\oplus f(1)}|1\rangle\right]$$

as $H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(-1)^{a}|1\rangle\right) = |a\rangle$

$$H|\varphi_3\rangle = (-1)^{f(0)}|f(0) \oplus f(1)\rangle$$

f constant $\to f(0) \oplus f(1) = 0$ (f_0 and f_3)
f balanced $\to f(0) \oplus f(1) = 1$ (f_1 and f_2)

Doing this single call to the oracle we can only identify if the function is constant or balanced, but not which function is inside



$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)|-\rangle$$

$$|\psi_2\rangle = \frac{1}{2}\Big((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle\Big)\,|-\rangle$$

$f \rightarrow$ returns 1 if the input corresponds to its label (f_{00} returns 1 for input 00)

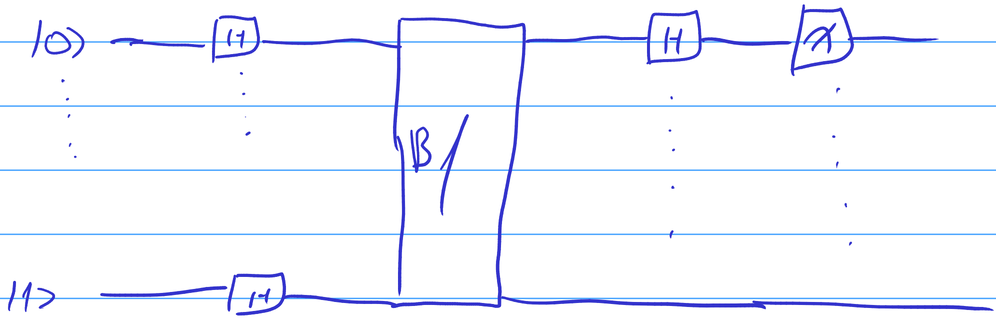
$$\left.\begin{aligned}
f_{00} &\rightarrow \frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\\
f_{01} &\rightarrow \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle + |11\rangle\right)\\
f_{10} &\rightarrow \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right)\\
f_{11} &\rightarrow \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right)
\end{aligned}\right\}$$

Orthogonal (so there is a U transformation that maps them to basis states)

$$H|a\rangle = \frac{1}{\sqrt{2}} \sum_{b \in \{0,1\}} (-1)^{a \cdot b} |b\rangle$$

$$H \otimes H |x_1 x_2\rangle = \left(\frac{1}{\sqrt{2}} \sum_{y_1} (-1)^{x_1 \cdot y_1} |y_1\rangle\right)\left(\frac{1}{\sqrt{2}} \sum_{y_2} (-1)^{x_2 \cdot y_2} |y_2\rangle\right)$$

$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y\in\{0,1\}^n} (-1)^{x\cdot y} |y\rangle$$



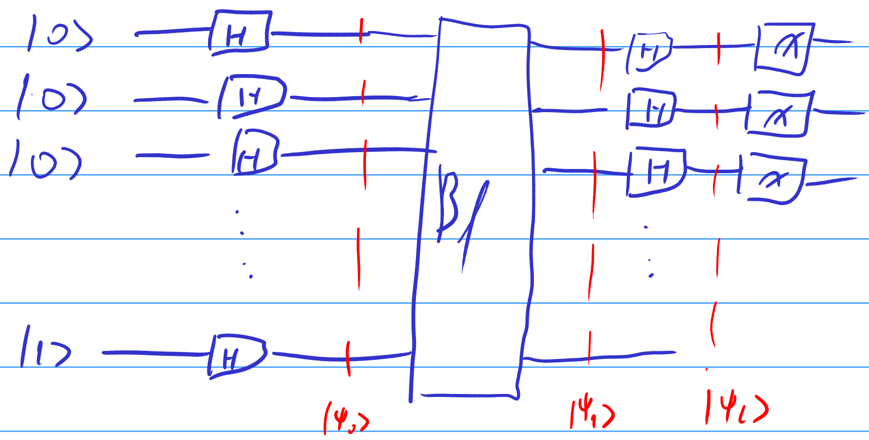
if f is constant → f(x) = 0 or f(x) = 1

if f is balanced → half of values is 0 and the other half values

→ Classically you need at least $2^{n-1}+1$ evaluations

→ Quantumly only 1 call is needed

$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y\in\{0,1\}^n} (-1)^{x_1y_1 + x_2y_2 \cdots x_ny_n} |y\rangle$$



$$|\psi_0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle\, |-\rangle$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} |x\rangle \quad |-\rangle$$

$$|\psi_2\rangle = \left(H^{\otimes n} \otimes I\right) |\psi_1\rangle \, |-\rangle$$

$$= \left(\frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} |\psi_1\rangle \right) |-\rangle$$

$$= \left(\frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} (-1)^{f(x)} \left[\frac{1}{\sqrt{2^n}} \sum_{y\in\{0,1\}^n} (-1)^{xy} |y\rangle \right] \right) |-\rangle$$

$$= \sum_y \underbrace{\left(\frac{1}{2^n} \sum_x (-1)^{f(x) \oplus x\cdot y} \right)}_{\text{amplitude for } |y\rangle} |y\rangle \; |-\rangle$$

prob. of $y = 0^{\otimes n}$ $\Rightarrow \left(\frac{1}{2^n} \sum_x (-1)^{f(x)} \right)^2$

→ The return of f is extracted in the ancn. qubit, but the result of DJ algorithm is from the mea. of the first n qubits. So $f(0) = f(1) = 0$ say that the return value of an oracle will always be 0