Ways to generate entanglement between 2 parties

1: They interact one with another
2: They interact with a third one
3: They never interact

Used Basis

→ Computational basis: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$
→ Bell basis: $\{|\varphi^+\rangle, |\varphi^-\rangle, |\psi^+\rangle, |\psi^-\rangle\}$

→ they all are orthogonal on 4D in the Hilbert space

$$|\varphi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad |\varphi^-\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\varphi^+\rangle + |\varphi^-\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) + \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

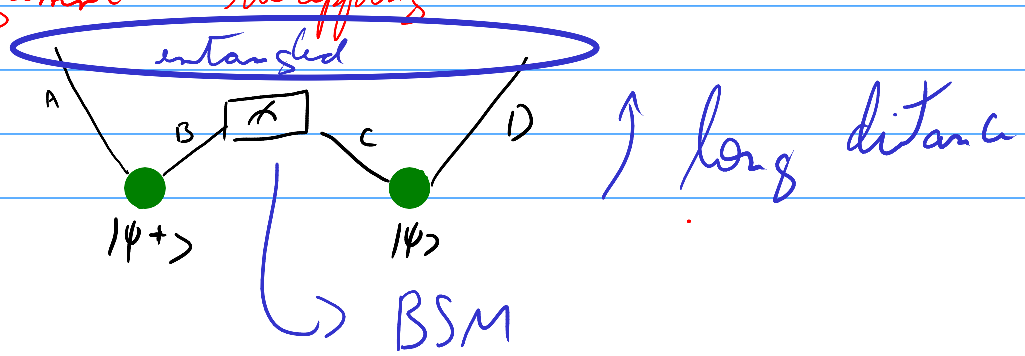
$$\frac{1}{\sqrt{2}}|00\rangle + \cancel{\frac{1}{\sqrt{2}}|11\rangle} + \frac{1}{\sqrt{2}}|00\rangle - \cancel{\frac{1}{\sqrt{2}}|11\rangle}$$

$$|\varphi^+\rangle + |\varphi^-\rangle = \frac{2}{\sqrt{2}}|00\rangle$$

$$|00\rangle = \frac{\sqrt{2}}{2}\left(|\varphi^+\rangle + |\varphi^-\rangle\right) = \frac{1}{\sqrt{2}}\left(|\varphi^+\rangle + |\varphi^-\rangle\right)$$

this way we can express the Bell basis in terms of the computational basis

Entanglement swapping



→ If we have 2 initial Bell State, and we do a measurement using AD a BC in the Bell Basis we'll have something like:

$$|\psi^+\rangle_{AB}|\psi^+\rangle_{CD} = \frac{1}{2}\Big[|\psi^+\rangle_{AD}|\psi^+\rangle_{BC} + |\psi^-\rangle_{AD}|\psi^-\rangle_{BC} +$$
$$|\phi^+\rangle_{AD}|\phi^+\rangle_{BC} + |\phi^-\rangle_{AD}|\phi^-\rangle_{BC}\Big]$$

→ So when we measure the 2 parties get the same Bell State (A, D), for that we just need to measure BC to collapse AD, then the two parties has never interacted with each other

BBM92 + Entanglement Swapping

→ Now Alice and Bob are A and D
→ Then we do the same thing as before

How many Errors can be corrected

$$R_{sp} = 1 - \underbrace{f\,H(QBER)}_{EC} - \underbrace{H(QBER)}_{PA}$$

Key generation rate

efficiency

Shannon Entropy

$f = 1 \rightarrow$ efficient
$f > 1 \rightarrow$ less efficient

$$QBER_{CO}(f=1) = 0{,}11$$

cut off value for QBER

You'll lose too many qubits if QBER > 0,11

Factors that affect the long distance Protocol
-> Errors during the creation of the entangled pairs
-> Photons may be lost during the path of BSM
-> Photons being not detected
-> BSM fail

BSM = Bell State mean..

P. of an entan. be created
$P = \chi^2$

P. of photons reach BSM
$P = 10^{-\alpha/10}$

P. of photons be detected
$P = \eta_0^2$

P. of BSM success
$P = \frac{1}{2}\eta_0^2$

$$R_{S/} = \underbrace{\chi^2 \cdot 10^{-\alpha/10} \cdot \eta_0^2 \cdot \frac{1}{2}\eta_0^2}$$

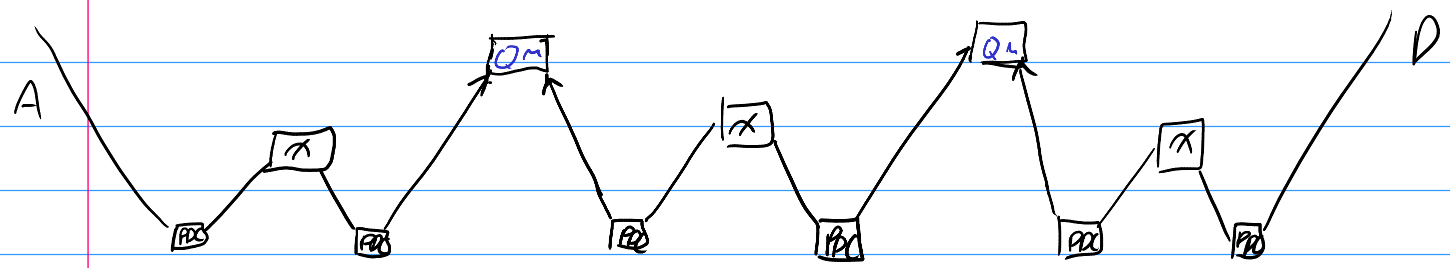
total of lost qubits

$R = R_{S/}\, R_{SP}$ <- I have to wait too much time to have a qubit

Increasing the distance

Concatenated entanglement Swapping

QM = Quantum memory



PDC = Parametric down conversion

→ More qubits are lost with the:

Quantum Memory (Read and Write)

→ Every pair that comes in a BSM must be keep till all the others has succeed

→ Now just $\frac{1}{2}$ of the BSM must be successfull

Quantum Repeaters

→ Quantum Relay + Quantum memories