Quantum Oracles - How to Transform Classical Problems into Quantum ones

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Abstract

Using quantum oracles and other effects, like superposition, *5 mini-projects* were done. The main goal of these projects was to answer if it's possible to bring some problems to the quantum realm and if such translation worth it. After finishing the tests, it was possible to see that, some of the implementations show descent results. However, classical computing is still a fundamental part of quantum algorithms.

1   Introduction

Now days, isn't difficult to hear someone talking about quantum computers, and how these machines will change the world. Nonetheless, the major fraction of these comments come from extrapolations and science fiction present in popular movies and series around the world. In this paper, I'm going to show that quantum computing it's not a magical trick to solve everything, but a tool for solving a distinct group of problems.

To do that, *5 mini-projects* were implemented using qiskit, an open source framework from IBM, exploiting quantum effects and using classical/quantum algorithms to reach the expected outcomes. After that, the results will be here compared with their classical counterparts. Such mini-projects are the following: Quantum File explorer3.1, miles to kilometers conversion3.2, Hanoi towers3.3, Buckshot Roulette 3.4 and QRAM 3.5. All these implementations can be found at my GitHub repository.

2   Oracles

Based on the idea of *Oracle Turing Machines* [1][2][3][4], the Oracles are mathematical modeling tools, used to abstract outlying parts of an algorithms into a black-box, making the algorithm analysis much easier. These machines could also be seen as a function, getting an input x e returning $f(x)$ with time-complexity $O(1)$. Because of these unreachable characteristics for real life, this model can't be implemented, being used only for formal description of decision problems.

However, in quantum computing, it's possible to implement something similar to that, taking advantage of your inner structure and quantum effects to gain a *Speed-up* relative to its classical counterparts, such Speed-up can be seen, for example, in the Deutsch–Jozsa algorithm [5]. Furthermore, the Quantum Oracles have a fundamental role determining the circuit complexity. Some approaches for that are: *depth*, calculating the longest path in the circuit that some information must pass through and *gate counting*, summing up how many gates were applied in the final circuit. Nevertheless, these approaches are very dependent of the *QPU* topology, differing from each *backend* used during the *transpilation* process. To solve this problem, a well-known technique is to pack some parts of the circuit into Oracles,

and then describing the complexity based on how many times they are called, it's also known as *query complexity* [6] [4].

2.1 Types of Oracles

Using the base description of Quantum Oracles, we can classify them based on their structures and how the data is processed.

2.1.1 Phase Oracle

The Phase Oracle, is the most well-known format. Some Algorithms, like Deutsch–Jozsa, Grover, Simon and Bernstein–Vazirani, use it to take advantage over Classical approaches.

2.1.1.1 Default Behavior

As its main characteristic, the Phase Oracle adds a global phase to the circuit, using quantum effects like *Phase Kickback* (basically the phase pass all the way through CNOT's target and is applied in the control Qubit), to change values in superposition.
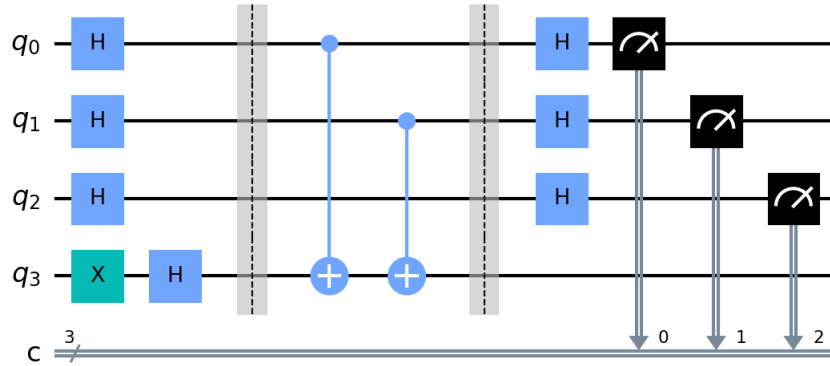

Figure 1: Phase Oracle using Phase-Kickback Example

In the Figure 1, a π phase was added in the auxiliary Qubit ($q3$), taking advantage of the phase $|-\rangle$, which will be the responsible to modify the values in the unitary matrix. In this setup, the $CNOT$ gates act slightly different from its default behavior, this way, the gate invert the value of the target Qubit and, due the π phase, it also acts like a Z gate applied in the control Qubit. So, after applying $CNOT |-\rangle |+\rangle$ (little-endian), the state becomes $\frac{1}{\sqrt{2}}(|0\rangle |-\rangle - |1\rangle |-\rangle)$, and after removing the superposition, the final state is: $\frac{1}{\sqrt{2}}(|+\rangle |1\rangle - |-\rangle |1\rangle)$. This way, the control Qubit is changed due the *Phase Kickback* and its state is flipped from $|0\rangle$ to $|1\rangle$. Taking this effect as and advantage, we can use it to encode some binary values inside these oracles and use them to do some calculations.

2.1.1.2 Minimal Oracle Version

Furthermore, There's another layout possible for implementing a Phase Oracle. Once this one only applies a phase in some bit-strings, the auxiliary Qubit can be removed, and the phase can be provided by CZ gates (or any other gate which can apply a phase for certain bit-strings). Doing that, it's possible to create a phase oracle keep its unitary and reversible nature, in the format of a minimal Oracle.
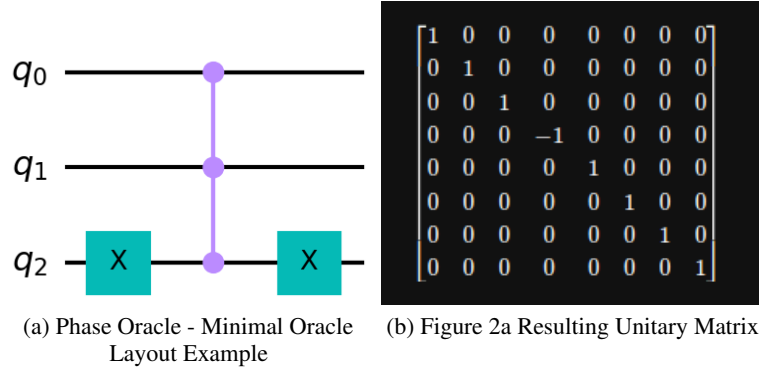
(a) Phase Oracle - Minimal Oracle
Layout Example

(b) Figure 2a Resulting Unitary Matrix

Figure 2: Phase Oracle Minimal Version

In the example above 2, a MCP gate was applied adding a π phase and another two X gates were used to invert the Qubits we want to have the value 0 as a trigger for the MCP phase application. Following the little-endian format the final encoded bit-string is: 011_2 or 3_{10}.

Looking at its unitary matrix 2b, it's possible to see the 8×8 identity matrix with a -1 value in the column relative to the bit-string 011_2, representing the encoded value.

2.1.2 Boolean Oracle

The Boolean Oracle has a similar structure with the Phase Oracle. However, this time a phase isn't applied, acting this way a regular boolean function, mapping inputs to outputs.
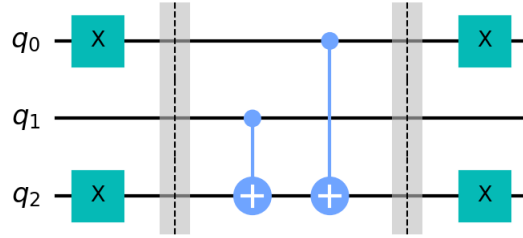

Figure 3: Boolean Oracle Example

The example in the Figure 3, is a well-known balanced Oracle for the Deutsch-Jozsa's algorithm. Nevertheless, to implement it, the Oracle must be transformed into a Phase Oracle.

2.1.3 Minimal Oracle

As previously mentioned, the Minimal Oracle is a function which its essence is unitary and reversible, so no additional Qubits are required. Therefore, this format can be either Boolean or Phase Oracle, depending on its implementation.
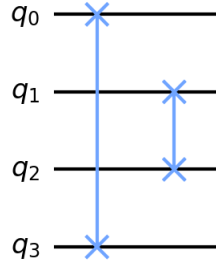
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Figure 4: Boolean Minimal Oracle example

The example 4 shows an Boolean Oracle which two $SWAP$ gates were used to invert the Qubits values order. Doing this way, the final matrix keeps its unitary and reversible properties, once the $SWAP$ gates act in both Qubits together, mirroring its action.

2.1.4 QFT(Quantum Fourier Transform)

The QFT is, in a nutshell, a quantum algorithm based on the Discrete Fourier Transform, used for quantum states period finding, projecting values from the computational basis onto the X basis (also know as Fourier basis).

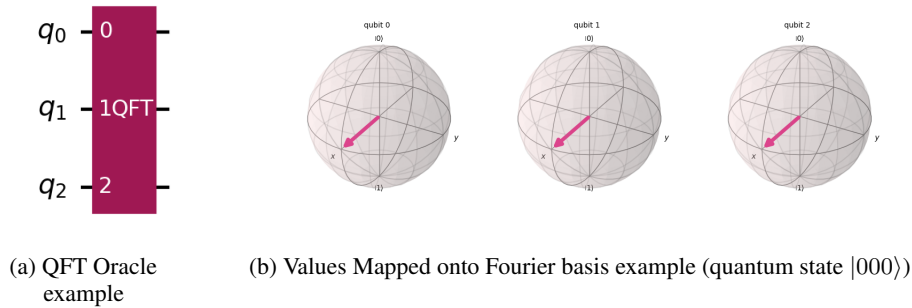Even though it's an algorithm by itself, its application is done in the format of Oracles in quantum circuits.

(a) QFT Oracle example

(b) Values Mapped onto Fourier basis example (quantum state $|000\rangle$)

Figure 5: QFT Oracle

2.1.5 Other kinds of Oracles

In addition to the oracles mentioned above, it's possible come across with different mentions of Oracles, like the Simons's, Deutsch-Jozsa's, etc. However, these are just implementations of models already shown in this paper. Also, the most relevant for the following projects are the Phase and Boolean ones, due that, there's no use to keep discussing about many others sub-categories of Oracles here.

3 Development

3.1 File Explorer

Imagine a quantum computer, but instead of one of those we already have in real life, imagine a different kind, very similar to those we have at home, with a quantum operational system, quantum files, etc. This imaginary version could be thought as a hybrid computer as well, taking advantage of both quantum and classical capabilities. Using this idea, the first project implements a file explorer for this ideal system.

3.1.1 Algorithms Used

3.1.1.1 Grover

The Standard algorithm for search problems is the Grover's algorithm. This one, realizes searches on unsorted "databases" (bit-string in this case) with $O(\sqrt{2^n})$, being n the number of Qubits. Its tenets are based on amplitude amplification, using a Phase Oracle to mark some bit-strings and then using a Diffuser to amplify the probability to find these values after measuring it.
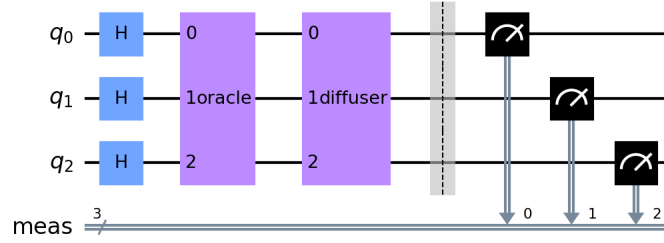

Figure 6: Example Grover's algorithm with 3 Qubits

For building the circuit, is needed a joint of $Oracle + Diffuser$ applied k time, which $k \approx \frac{\pi}{4\sqrt{\frac{a}{2^n}}} - \frac{1}{2}$, being a the number of values marked by the Oracle. Once we want to find just a single file, the use of this relation is not required, being need to apply the algorithm once to find the bit-string.

Even this being the best option for bit-string finding, in this project different rotations were tested trying to find a better superposition which increases the chances to find the value we want.
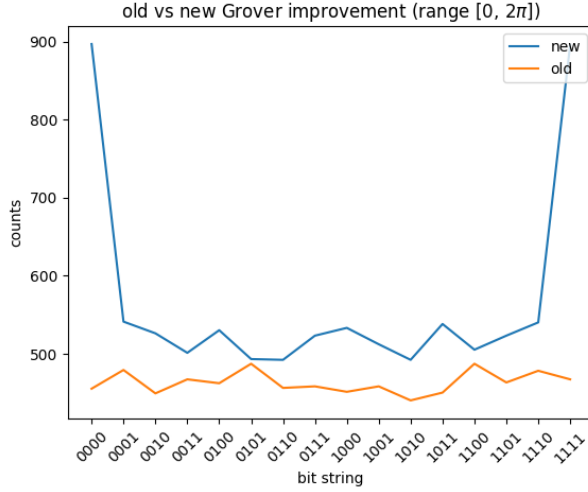

Figure 7: Comparing the Standard Grover's algorithm with the version modifying the rotation angles $[0, 2\pi]$ for each 4 bits bit-string

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Using specific rotations for each bit-string, it's possible to improve the outcomes, and even stand out the default rotation.
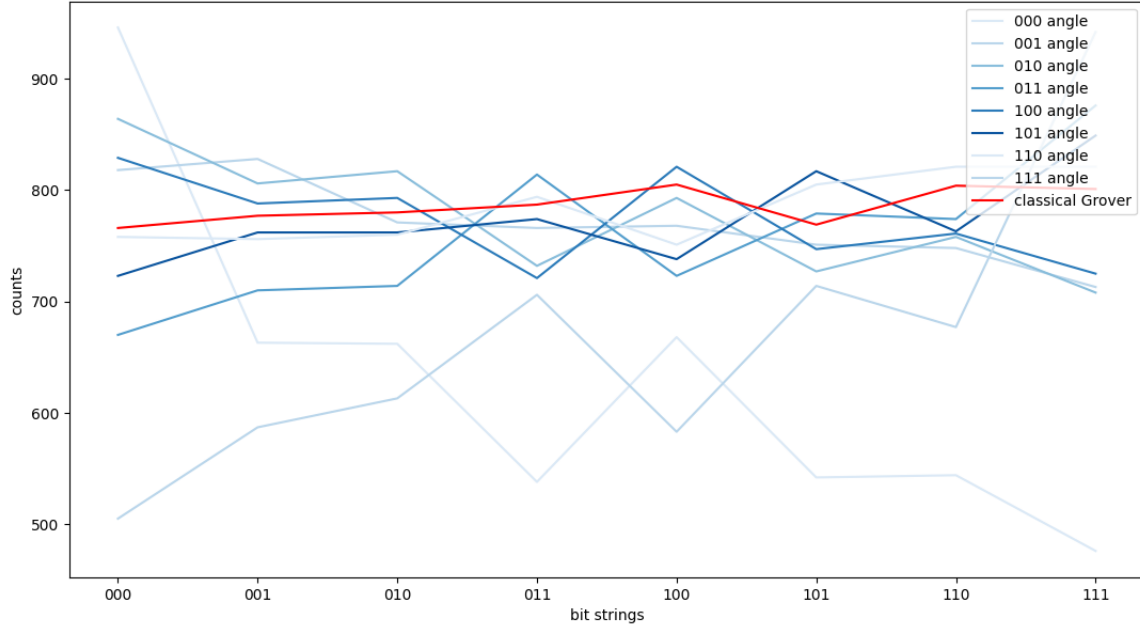

Figure 8: Test Using the best angles of each bit-string in different ones

However, using the best angle of a distinct bit-string as the source of superposition in the algorithm, isn't enough to overcome the classical Grover implementation, and also the probability distribution becomes irregular. So, for a general purpose implementation, the default rotation is the best one for the most part of the cases. This behavior is even weird with 2 Qubits, being the H gate rotation the best ever for that.

(a) Results for Grover with 2 Qubits (default rotation) (b) Results for Grover with 2 Qubits (modified rotation)
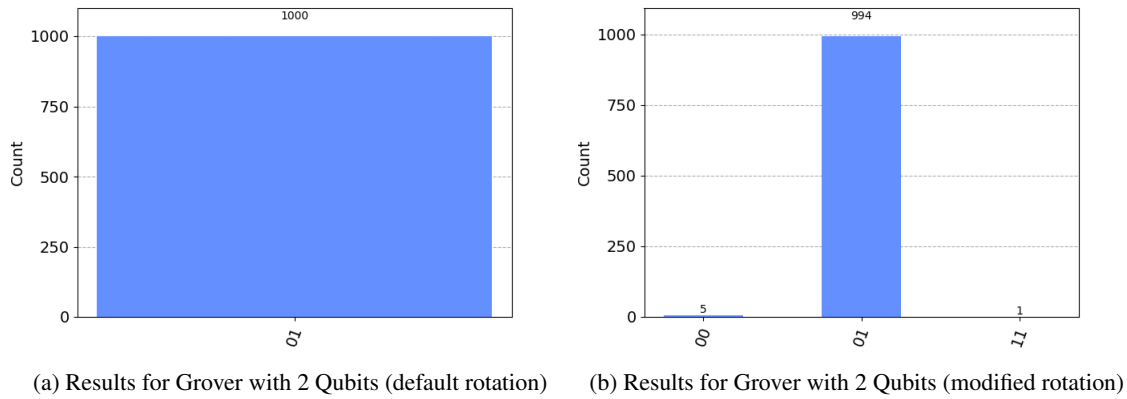
Figure 9: Comparing Grover with 2 qubits using the default and modified rotation

For our purpose in this project, the hybrid approach was used. So during the circuit setup, the searched value pass through a Hash-Table will optimal angles. This way, the algorithm can achieve better outcomes at the end of the execution, finding, this way, the file we want.

3.1.1.2 Sets Difference

Overlapping two distinct Phase Oracles, each one with a range of values encoded. The remaining Phases, are the difference of the sets encoded by them [7].
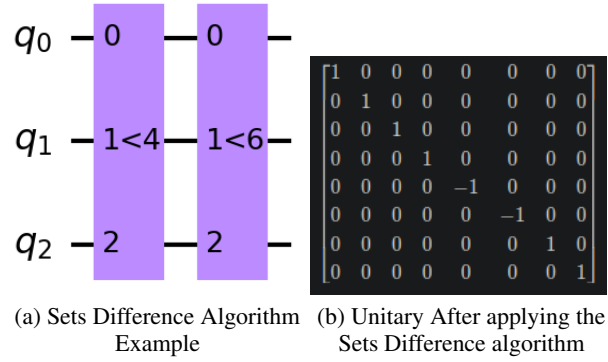
(a) Sets Difference Algorithm (b) Unitary After applying the
Example   Sets Difference algorithm

Figure 10: Sets Difference example

In the last example 10a, the set $\{000, 001, 010, 0110\}$ was encoded in the first Oracle and $\{000, 001, 010, 011, 100, 101\}$ in the second one.

After Overlapping the phases 10b, only the values $\{100, 101\}$ keep the -1 value. This way, we can do the set operation just applying some oracles with the values range we want.

3.1.2 Final Solution

As a solution for this problem, a hash function was made, mapping the path of some file p to a specific bit-string b, $F : p \rightarrow b$. With this function, we can use a set returned values and encode them into a Phase Oracle, creating a sort of quantum Look-Up-Table for files in that machine. Moreover, a second Oracle containing all the bit-strings, with the exception of the searched one, is applied for difference of sets algorithm, this way the phases will be overlapped, and only the one we want will pass to the Grover's algorithm.
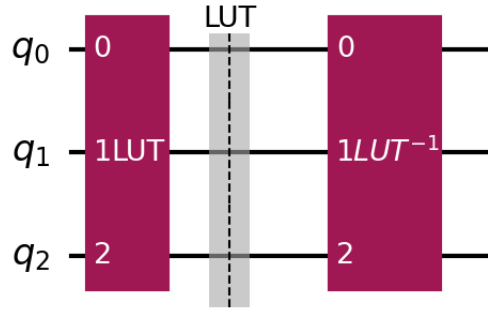

Figure 11: Look-up-Tables applied

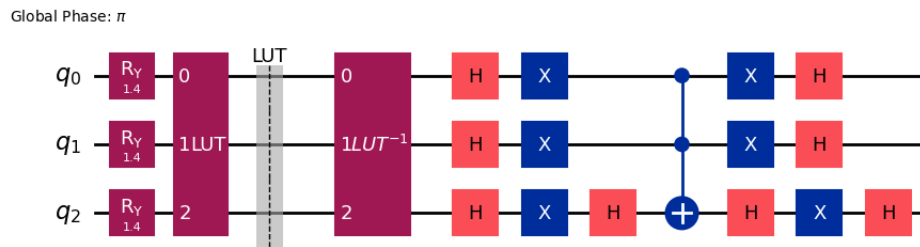Finally, the modified version of the Grover's algorithm is applied.

Figure 12: File explorer final circuit

This setup, maximizes the probability to find the queried file, presenting the following distribution after 1000 shots.
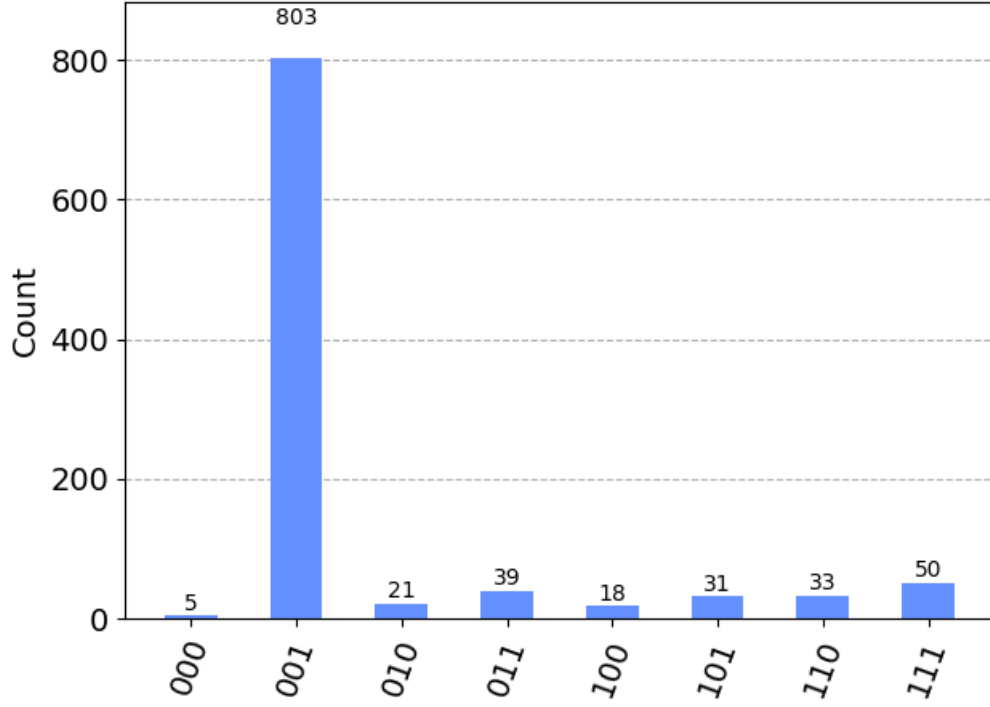
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Figure 13: File Explorer Results after running with Qiskit-AER

3.1.3 Results

For this specific ideal case, it is, probably, one of the best known ways to do the search between "quantum files".

However, it's not the best alternative as a way to speed-up classical searches. It happens because storing a large Look-Up-Table for encoded files and another one for keeping track of each optimal angle for every bit-string contained between 2^n combinations, could be really costly and slow. Also, it would be required a hash function with a tiny probability of collision.

An alternative for that could be using the standard Grover's algorithm, but it still the need to encode the hard drive files to a big table. Also, comparing the classical tree approach with Grover's, their worst cases scenarios are $O(log(n))$ and $O(\sqrt{n})$, which implies that the first one still faster than the later. As a last point, the quantum version for this problem, could present the wrong file after searching due errors and the quantum randomness.

Keeping that in mind, the final algorithm is a right choice for a quantum system as describe at the beginning of this section, but it can't overcome the best algorithm in a classical computer.

3.2 Miles to Kilometers Conversion

The second problem that was tested here, was the conversion from miles to kilometers. This idea came out after finding a quantum algorithm capable of calculate the Fibonacci sequence, which is an essential piece for this very project.

3.2.1 Used Algorithms

3.2.1.1 Quantum Fibonacci

The quantum version of Fibonacci was presented in [8]. This paper shows that, using a quantum circuit with all bit-string in superposition, and then apply controlled rotations to remove values that contains consecutive ones, is possible to approximate the result of the *n*-th Fibonacci sequence value.
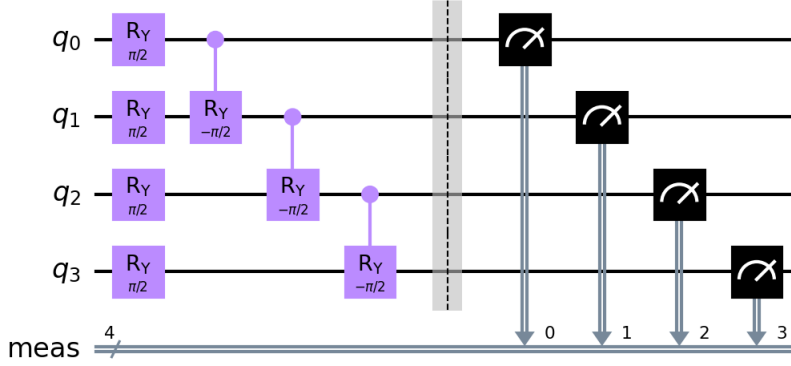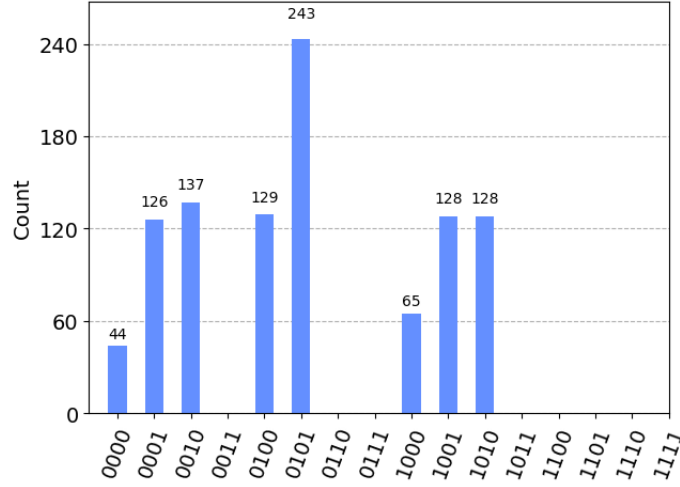
Figure 14: Quantum Fibonacci Example

Figure 15: Quantum Fibonacci result - F(4)

After running the circuit k times, the amount of unique bit-string that appeared during the experiment represent the n-th Fibonacci value, which n is the total of Qubits were used in the circuit. So, for the example 14 above, the position n we want, is 4, and the result after counting the bars of which values are $\neq 0$ give us the result of $F(4) = 8$.

It may seem wrong, however, the quantum setup differ slightly from the classical one, once the first value is 2 instead of 1, this way, the sequence is: $F(1) = 2, F(2) = 3, F(3) = 5, F(4) = 8, F(5) = 13, F(6) = 21, ...$, which implies that it worked for this example.

3.2.1.2 Approximating Miles to Kilometers

To approximate the value from miles to kilometers, an old approach for that is using the Fibonacci sequence. This method, uses the relation $F_{km} = F_{milhas}(n+1)$, which F is the classical Fibonacci algorithm ($F(1) = 1$ and $F(2) = 2$), and n is a known position in this very sequence. Following this equation, the relative kilometers values will be given in the next n ($n+1$).

miles	Km	Real(Km) value
1	2	1.60934
2	3	3.21869
3	5	4.82803
5	8	8.04672

Table 1: Approximated miles-Km values

9

For values that aren't present in the sequence, they can be generated combining multiple values which are there. For example, to transform 10 miles to kilometers, we could use the values 8 and 2, which are known in the sequence, and then used the previous cited method for each part. So, for this example, $8 = F(5)$ and $2 = F(2)$, so $F(5+1) + F(2+1) = F(6) + F(3) = 13 + 3 = 16Km$ giving a close value of $\approx 16.0934Km$.

3.2.2 Implementation

Using the formulation above, the final algorithm takes place using a Hybrid approach, following the structure bellow:

Algorithm 1 Quantum Conversion algorithm
---
parts = $BreakInputIntoFibonacciParts(input)$
for parte in parts **do**
    Apply the Oracle embedding $F(part)$
    Measure the Oracle Qubits
    Reset these Qubits
end for
Check the outcomes
Multiply the given result for its relative i

In this algorithm, is needed a preprocessing routine, using a classical algorithm, to split the input number into smaller parts that can be calculated using the sequence. Also, to reduce the amount of Oracle applications to decrease the quantum noise, this processing returns a tuple for each number containing n and the total of applications needed p, this way, we can apply each Oracle once and after retrieving the outcomes, the final result is giving by: $\sum_{i=0}^{m} o_i p_i$, being m the total of parts and o the given output.
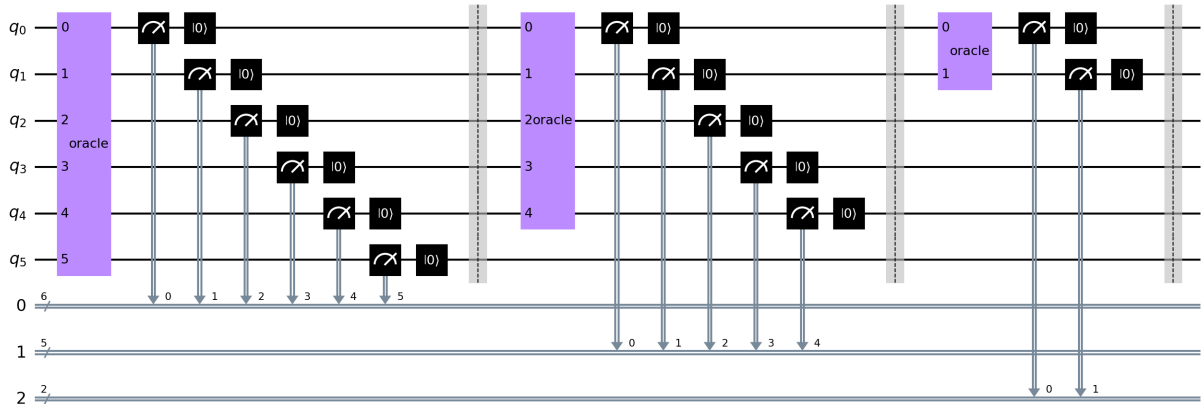

Figure 16: Final Miles to Kilometers Circuit

3.2.3 Thoughts on the Results

With this method, is possible to evaluate some input values. However, there are some points which make it unfeasible.

Execution Time Each part to be calculated, takes too much *shots*, taking around $500 - 10000$ to get better results, therefore increasing the execution time

Errors Due the high depth of this kind of circuit, this setup is likely to present wrong results due multi-Qubit operations in real hardware

Impression After running the algorithm for many different values, it's clear that, for large numbers, the output deviates a lot from what the expected was

Classical Intervention Once it requires both pre and post-processing using classical algorithms, the usefulness of the quantum version decreases
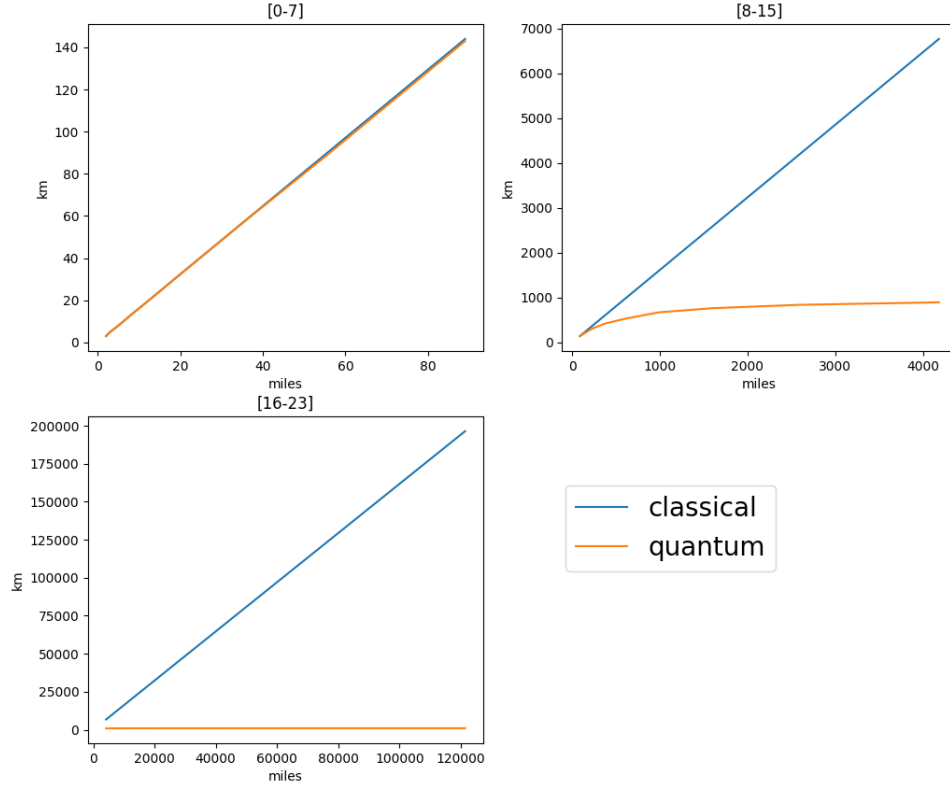
Figure 17: Comparing the precision between classical and quantum versions

Thus, this protocol can be used for small numbers. However, to be useful as its classical version, it should be more resilient to errors and precision issues, also it needs to be more independent from classical computations, once most of the work is based on classical routines.

Consequently, this final algorithm isn't good as the classical one, and probably isn't useful enough for real applications. Nonetheless, it's possibly a starting point for different approaches for real implementations.

3.3 Hanoi Towers

For the Hanoi Towers implementation, it was designed a way to encode the discs positions in the tower using their binary values and the Phase Oracle as a storage device.

3.3.1 Implementation

This project, requires $(\lfloor \log_2 x \rfloor + 1) * 3$ Qubits, being x the number of disks. The Qubits layout follows this sequence: $|t_{n-1}t_{n-2}...t_0\rangle \, |a_{n-1}a_{n-2}...a_0\rangle \, |s_{n-1}s_{n-2}...s_0\rangle$, where s,a,t are the first, second and third towers respectively.

To prepare the state, a Phase Oracle is prepared encoding each disc number (from 1 to x) in the first rod (Qubits s) using the global phase π. After that, $SWAP$ operations are done following the classically preprocessed steps, moving bit-by-bit from one rod to another.
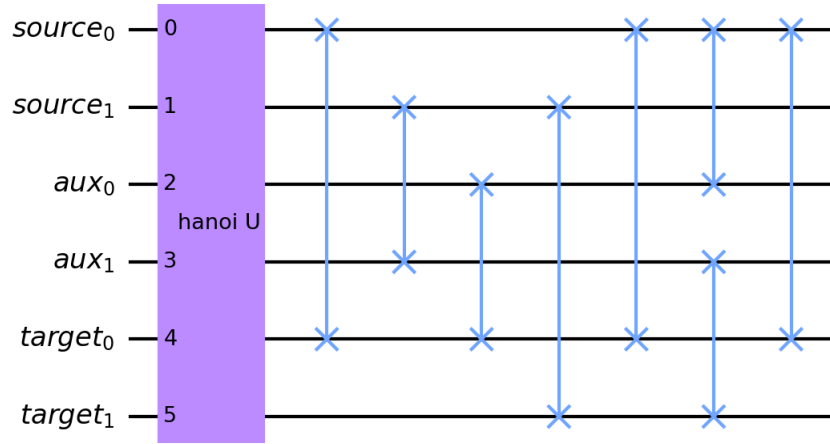
11

Figure 18: Quantum Hanoi Tower with 3 disks

As cited previously, the movements are preprocessed before playing. This way,the quantum setup acts like a classical player, executing step-by-step his strategy.

Also, as we're playing with phases, it's possible to apply phase amplification algorithm, like Grover's, to evaluate the tower and get the resulting values.
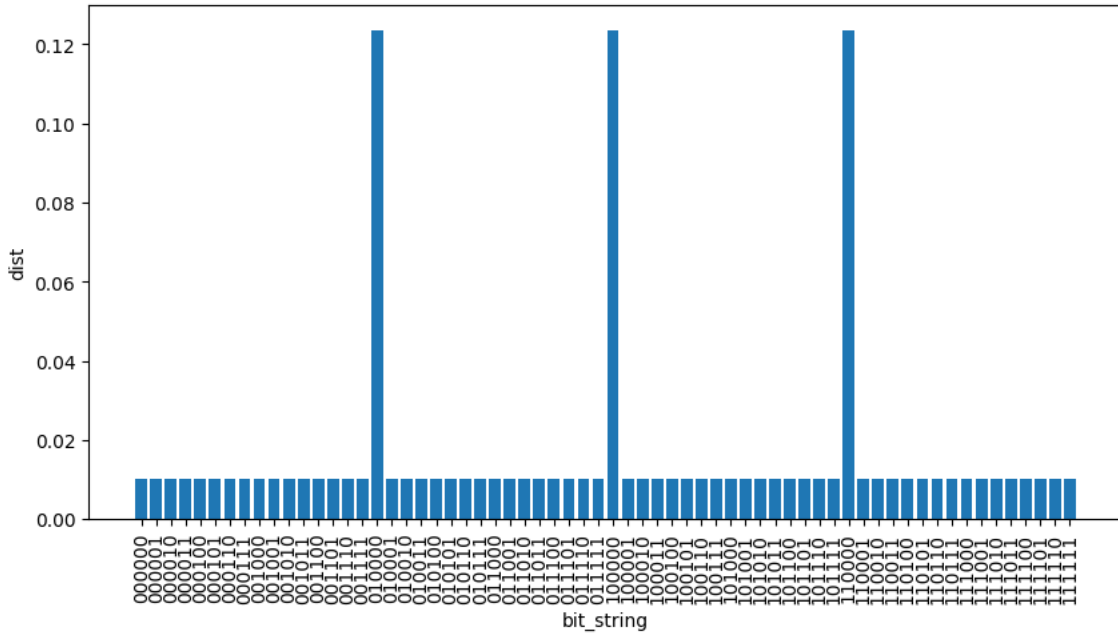

Figure 19: Quantum Hanoi Tower with 3 disks - evaluation using Grover's algorithm

As shown in 19, after evaluating the circuit, it's possible to see 3 high probability bit-strings, which represent the encoded disks. These bit-string are 010000, 100000 and 110000, so the operations moved successfully the disks from the source rod (rightmost 2 bits) to the target rod (leftmost 2 bits).

3.3.2 Thoughts on the Results

Once it followed the same sequence of implementation as the classical one, requiring, inclusively, preprocessing. This version is equivalent to the classical algorithm, keeping in mind that for a classical computer, it's also required to precompute a movement, and then play it.

3.4 Buckshot Roulette

Buckshot Roulette is a computer game made by Mike Klubnika, based on the idea of modifying the infamous Russian Roulette.

During the game, you are challenged to play against a demon (dealer). If you win, you'll receive a prize, otherwise the game will restart and you can try again.

In this very project, we analyzed the first game match, trying to find the best strategy to maximize the player's winnings. The reason for choosing the first match, is because of its simplicity, being away from power-ups that could complicate the analysis, and also because this one shows the base dynamics for the entire game, so doing well in this one, is the first step to beat the whole game.

3.4.1 Game Dynamics

In the first round, 2 fake bullets and 1 real are put into the shotgun. Then, the player starts his play, choosing between shooting himself or his opponent. If the player shoots himself and it's a real bullet, he loses one life point, otherwise he gets another chance to choose. However, if he chooses to shot his opponent, if it's real he takes away one life point from him, otherwise his opponent receives the gun.

This dynamics is followed in the entire game for both player and dealer, only being slight modified when power-ups are gained.

3.4.2 Modeling the Game Strategies

The round dynamics can be modeled as a binary tree, using a some game theory ideas to understand it better.
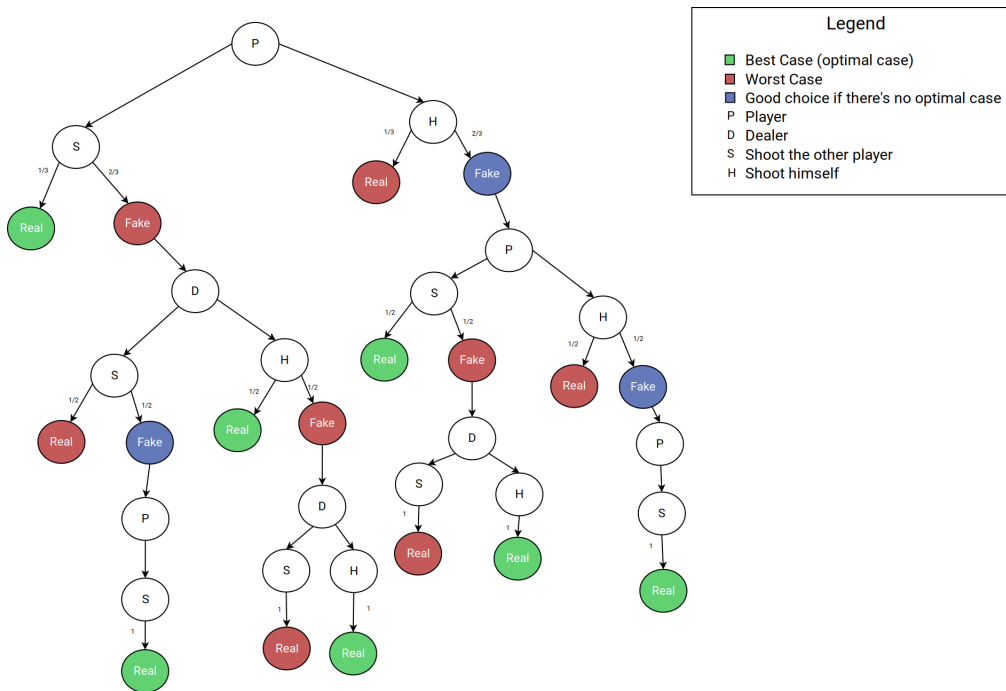

Figure 20: Buckshot Roulette tree diagram

Nessa estrutura, é previsto que o jogador seja um agente racional, e o dealer uma máquina com ações aleatórias. Assim, o jogador sempre visa o seu próprio benefício, enquanto o dealer age pela sorte.Tal comportamento pode ser visto nas folhas da árvore do qual, sempre que o player é o próximo jogador, sua ação é apenas atirar no adversário, enquanto o dealer ainda possui a possibilidade de entregar o jogo atirando em si próprio, mesmo havendo apenas uma bala na arma e, pela lógica do jogo, ser uma bala verdadeira.
Seguindo essa estrutura, podemos simular os possíveis caminhos e verificar a melhor estratégia.
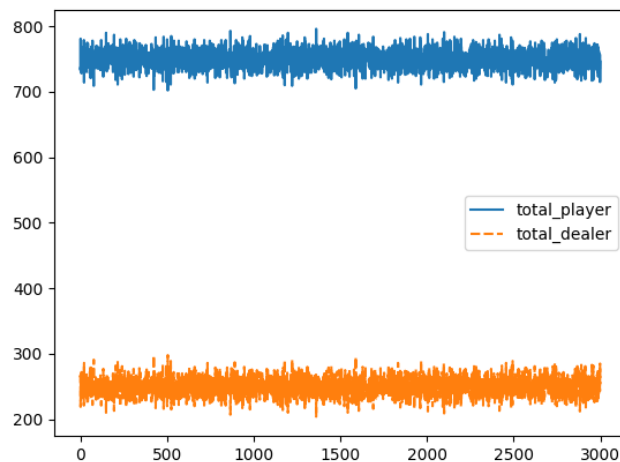

Figure 21: Buckshot Roulette clássico - melhor estratégia

Após testar os caminhos possíveis, o melhor resultado obtido foi esse apresentado acima em 21. Com um pouco de investigação, foi possível entender que essa estratégia se baseia no jogador começar atirando no dealer. Isso acontece, pois, ao seguir tal caminho, ele tem uma chance a menos de perder a rodada ao atirar em si mesmo logo no começo da partida.

rodada	ação	resultado da ação	resultado da partida
1	player atira no dealer	real	player ganha
1	player atira no dealer	fake	-
2	dealer atira no player	real	dealer ganha
2	dealer atira no player	fake	-
2	dealer atira nele mesmo	real	player ganha
2	dealer atira nele mesmo	fake	-
3	player atira no dealer	real	player ganha
3	dealer atira no player	real	dealer ganha
3	dealer atira nele mesmo	real	player ganha

Table 2: melhor estratégia - possíveis resultados

3.4.3   Versão quântica

Para a versão quântica, um circuito foi modelado imitando o funcionamento do game. Nesse algoritmo, um Oracle foi usado para cada jogador, implementando internamente sua estratégia.
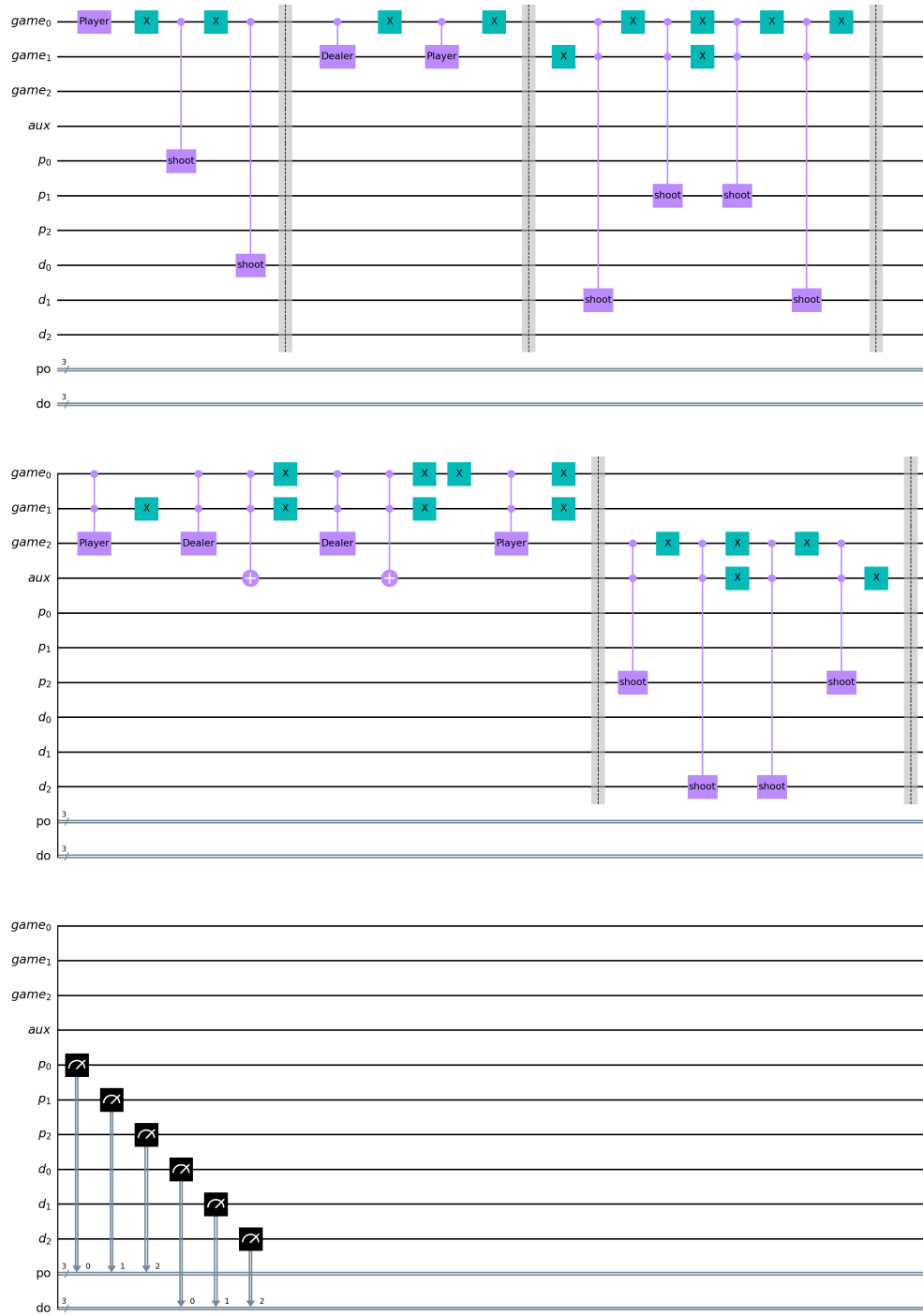
Figure 22: Circuito para o Buckshot Roulette

Além disso, para encontrar a estratégia, foram inseridos dois parâmetros dentro do Oracle do player, sendo possível configurar qualquer valor θ e ϕ para modificar a rotação na Bloch Sphere.

Após verificar os possíveis valores, a rotação que entregou o melhor resultado foi $\theta \approx 3.0853981633974477$, $\phi \approx 3.7853981633974474$ radianos. Usando essa estratégia, os resultados foram semelhantes a versão clássica.
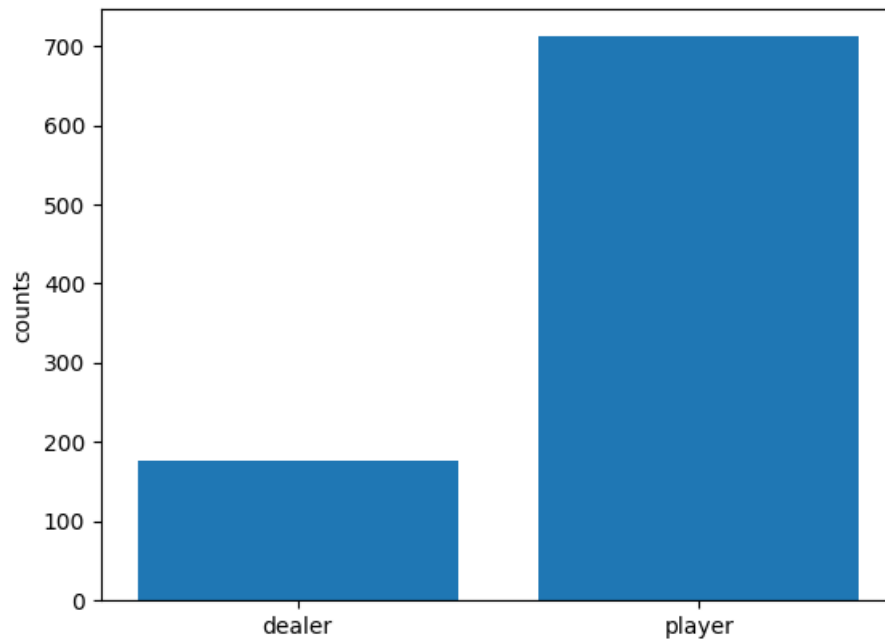
Figure 23: Resultado Buckshot Roulette quântico - Qiskit AER

Observando a Bloch Sphere do estado gerado por essa rotação, é possível ver também que a estratégia se aproxima da versão clássica, com o player preferindo atirar no dealer a maior parte do tempo (o valor 1 representa atirar no outro jogador e 0 em si mesmo).
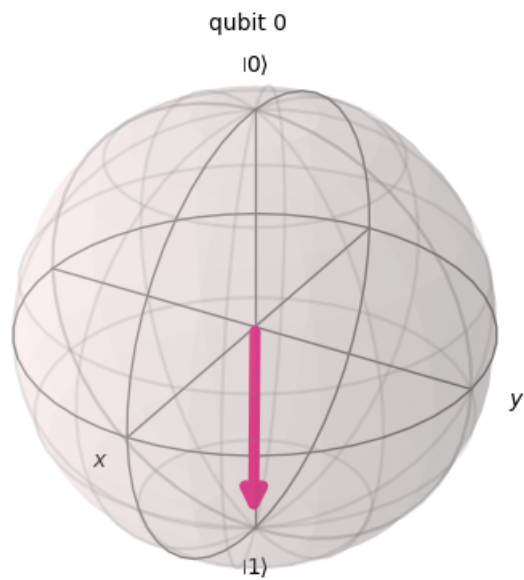
Figure 24: Melhor estratégia Buckshot Roulette quântico - Bloch Sphere

Como uma última nota sobre o circuito, no exemplo 23, o total de partidas ganhas por cada jogador não chega ao total jogado (nesse caso 1000 partidas foram simuladas). Isso acontece devido ao design do circuito, o qual não é possível verificar a jogada do player anterior, acarretando na continuação do jogo mesmo que um dos players já tenha perdido, o que cria a necessidade do uso de pós processamento para limpar os resultados inválidos.

3.4.4 Conclusões

Para esse problema, não há uma competição certa entre as duas versões, já que uma é diretamente inspirada na outra. Contudo, a versão quântica possui ainda a possibilidade de explorar mais valores do que a versão clássica, deixando o player mais aberto a escolha de novas estratégias, o que pode ser visto como um ponto a favor da versão quântica. Em suma, ambos as simulações atingiram o mesmo resultado e foi demonstrado que é possível usar o quantum Oracle como uma representação de um player dentro do circuito.

3.5 QRAM

Por fim, o último projeto realizado foi o de uma *QRAM* utilizando os Oracles. Nessa versão, foi testado a criação de *QROMs* (com dados estáticos dentro), e uma possível maneira de utilizar uma QRAM hábil para escrita.
Neste projeto, foi tido como objetivo o armazenamento de estados quânticos (superposições), e não apenas de bit-strings clássicas. Isso pois, para garantir a real eficiência da computação quântica, a superposição é indispensável, e seu armazenamento pode ser um ponto chave para algoritmos melhores.

3.5.1 QROM

Para a QROM, são utilizados n qubits para o barramento de endereços e m qubits para a o barramento de dados, sem a necessidade desses valores estarem correlacionados, podendo assim ser utilizado, por exemplo, $n = 3; m = 10$. Nessa estrutura, podemos mapear diversas superposições diferentes e aplicá-las quando certo endereço for chamado. Sendo assim, o algoritmo armazena os valores a partir da configuração de gates controlados interiores ao Oracle, criando uma superposição apenas quando certo valor de entrada é inserido, seguindo o formato: $|0\rangle^{\otimes m}|a_{n-1}a_{n-2}...a_0\rangle$.
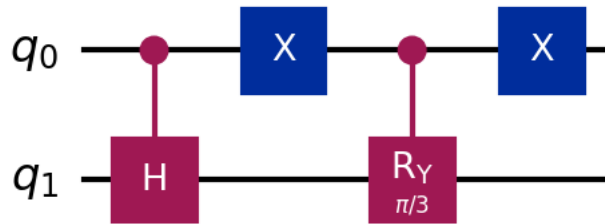

Figure 25: Exemplo circuito - QROM

Em 25, q_0 age como o barramento de endereços, enquanto q_1 como o barramento de dados. Aqui configuramos para mapear o endereço $0 \rightarrow RY(\frac{\pi}{3})$ e $1 \rightarrow H$. Sendo assim, para n qubits no barramento de endereços é possível mapear para 2^n estados, e com os m qubits é possível criar estados mais complexos aumentando sua quantidade e utilizando outros gates acionados para um mesmo endereço.
A partir da abstração desse circuito para um Oracle,é possível utilizar a QROM em um circuito maior, chamando-o novamente sempre que for necessário um certo estado. Além disso, no formato de Oracle, há a possibilidade de colocar os endereços em superposição e ter uma mistura de estados na saída.
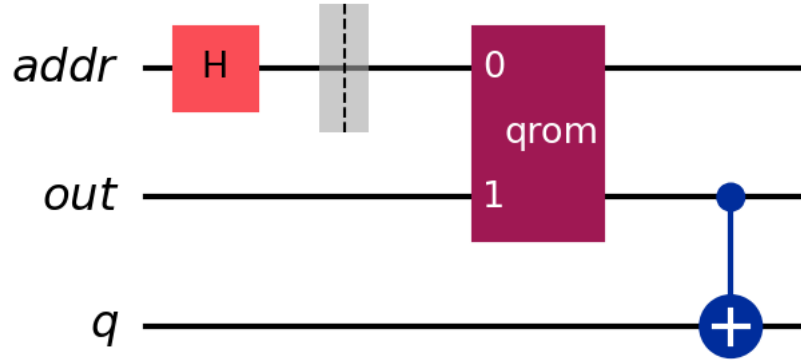
Figure 26: Exemplo circuito usando a QROM com endereços em superposição

Nesse exemplo 26, os endereços são colocados em superposição no qubit *addr* e assim os estados em internos do Oracle são colocados em uma sobreposição de $50 - 50$ no qubit *out*. Com isso, pode-se aproveitar do resultado de *out* em outros qubits, como nesse caso o qubit *q*.

Contudo, devido ao no-cloning-theorem, não é possível copiar esse estado para outro qubit alvo. Sendo assim, não é possível ter dois qubits com o mesmo estado a partir daquele armazenado, podemos apenas pegar o resultado de uma superposição e utilizar o valor binário como trigger para outra operação.

Uma opção para solucionar isso, é utilizar o teleporte quântico, destruindo assim o estado interno do Oracle e movendo-o para outro qubit desejado.

3.5.2 QRAM

Para criar uma QRAM com a possibilidade de escrita, o teleporte quântico, já citado anteriormente, é um caminho para isso. Com ele, podemos ter n qubits, sendo cada qubit um endereço único, e utilizar do teleporte para mover um estado que estava no circuito, para o domínio da QRAM.
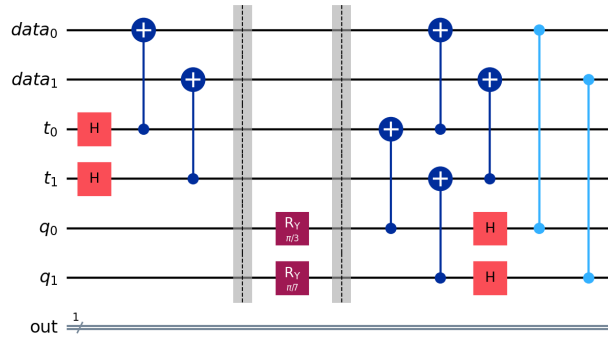

Figure 27: Exemplo circuito - QRAM

Aqui, os n qubits agem tanto como endereços quanto dados (qubits *data*). Além disso, são necessários mais n qubits para o teleporte (qubits *t*).

Com isso, é possível ver que o circuito cresce de forma linear a medida que mais endereços são requisitados, sendo assim $O(2 * n)$ em relação à quantidade de qubits total.

Na configuração acima 27, é possível sobre-escrever valores, assim como interferir com outras superposições apenas teleportando novos valores para o qubit i. Dessa forma, podemos criar uma memória menor e, conforme necessário, remover e adicionar outros valores.

3.5.3 Conclusões

Com esse projeto, e com a literatura usada [13][14], é possível entender que criar versões quânticas de memória é uma tarefa desafiadora, e ainda não é possível tomar proveito de todo o seu potencial usando as superposições e estados de outras bases a não ser a base computacional $(0, 1)$. Fatores como, complexidade de mapear dados, complexidade de utilizar a memória (já que é necessário reaplica-lá toda vez que for requisitado seu uso), no-cloning-theorem, decorrência, etc. Influenciam diretamente na possibilidade de sua criação.Mesmo sendo possível implementar pequenos circuitos que agem como memória, como os mostrados aqui, ainda não é usual e muito menos universal para qualquer tipo de máquina quântica.

Além disso, por esses fatores, a QRAM, pode dificultar a execução de múltiplas tarefas, uma vez que o valor presente nela não pode ser copiado, e ao move-lo para outro qubit, o valor anterior da QRAM é completamente destruído.

Como mostrado na literatura, para resolver esses problemas, o melhor approach para a sua implementação, é a utilização de um hardware especifico para essa finalidade, sem a intervenção de circuitos quânticos.

Em suma, mesmo sendo possível criar pequenos circuitos para implementar uma memória, seu uso está longe de se comparar as versões clássicas.

3.6 Conclusão

Perante o exposto, foi evidenciado que a computação quântica ainda tem muito potencial. No entanto, é possível ver que certos fatores, e a falta de alguns recursos, prejudicam o seu uso no momento.

Como já mostrado pelas inúmeras pesquisas em áreas como, química, machine learning, criptografia, otimização, etc. A computação quântica pode, num futuro próximo, ser um ponto crucial para conseguir resultados mais precisos e, em certos casos, em menor tempo.

No entanto, na era NISQ, para conseguir utilizar todo seu potencial, é necessário ter em conjunto máquinas clássicas para pré e/ou pós processamento, seja para executar alguma tarefa computacionalmente custosa para um computador quântico, ou para o uso de algoritmos de detecção e correção de erros. Como demonstrado aqui, ao utilizar esse conjunto, é possível ter o melhor dos dois mundos, mesmo que na maioria do casos, esse formato de implementação não se sobressaí as versões já utilizadas classicamente, com o tempo e o aperfeiçoamento das técnicas e do hardware darão uma abrangência maior aos usos da computação quântica.

Em resumo, é possível tirar proveito da computação quântica para problemas que conhecemos classicamente. No entanto, é necessário averiguar se há algum fator quântico que pode ser explorado para conseguir alguma vantagem perante a sua versão clássica, se houver, é necessário verificar também se todas as tarefas são mais vantajosas ao serem implementadas usando o algoritmo quântico, ou se ao explorar uma abordagem híbrida os ganhos podem ser maiores.

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