NETWORK MODELING, VISUALITZATION AND ANALYSIS II

The igraph library for R and Python

Master en Bioinformática
Universitat de València (UV)
October 10th , 2013

Marta Bleda Latorre
PhD at the Computational Genomics Institute
Centro de Investigación Príncipe Felipe (CIPF)
Valencia, Spain

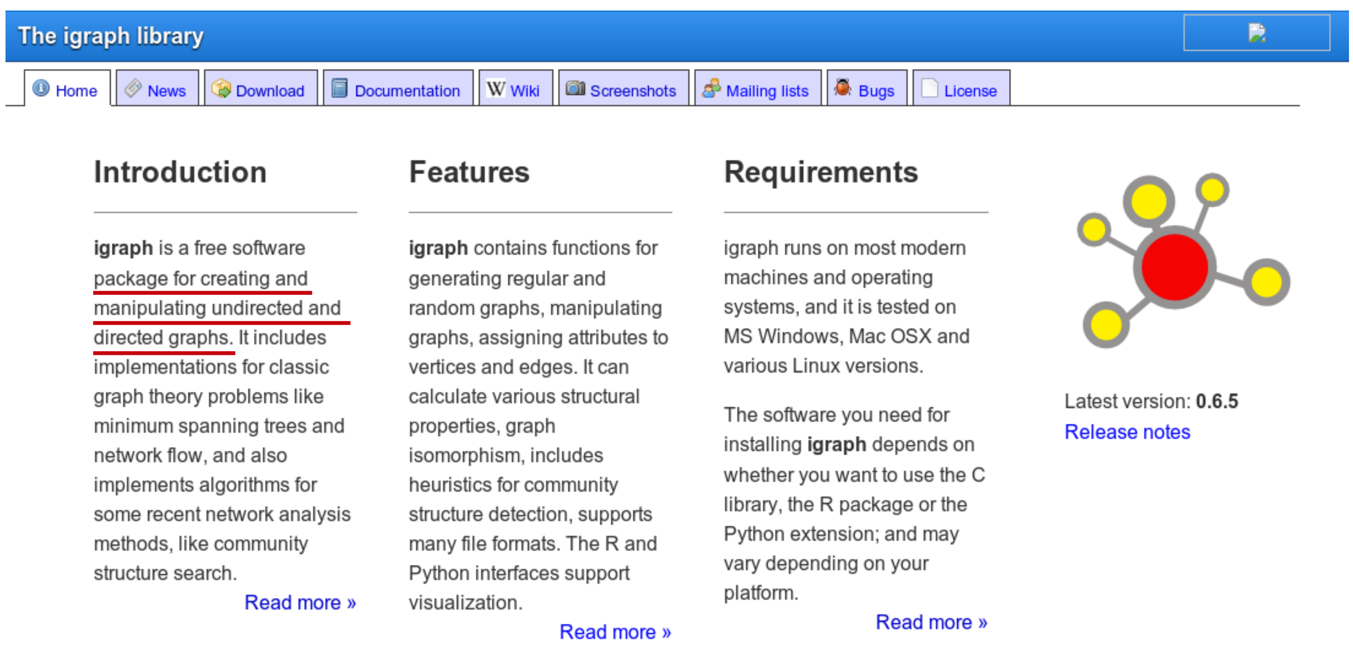
Overview

- The igraph library
- Installation in R
- igraph basics
  - Exercise
- Network models and robustness
  - Exercise
- igraph for Python

- Homework :)

The igraph library

- For complex network analysis
- Lots of graph algorithms implemented
- Core functionality is implemented as a C library
- High level interfaces for **R** and **Python**
- Open source
- **http://igraph.sourceforge.net/**



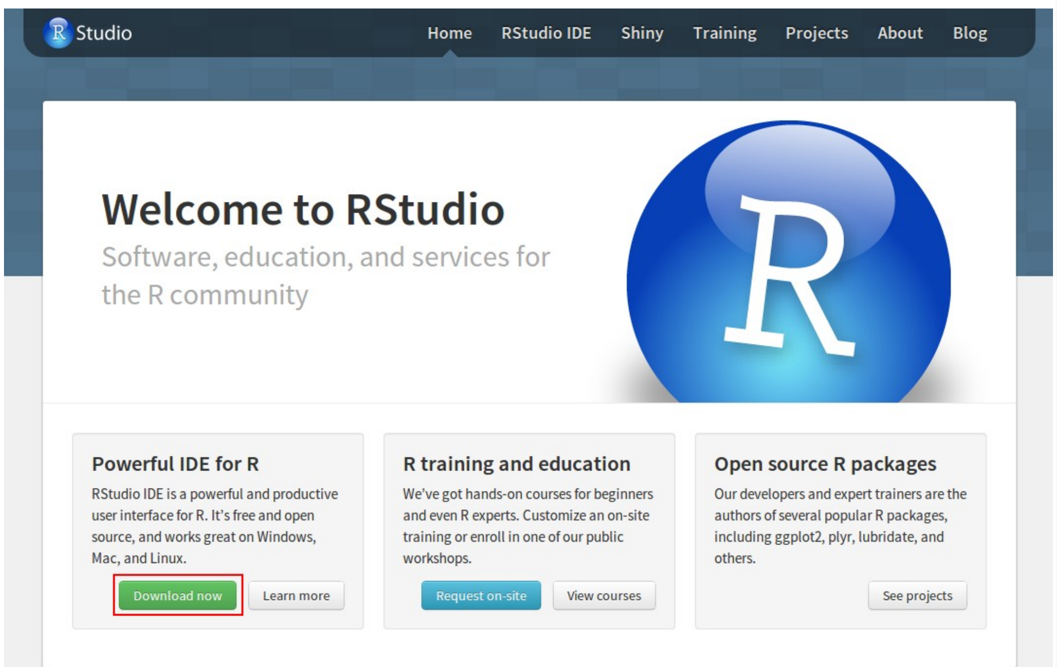
What do we need?

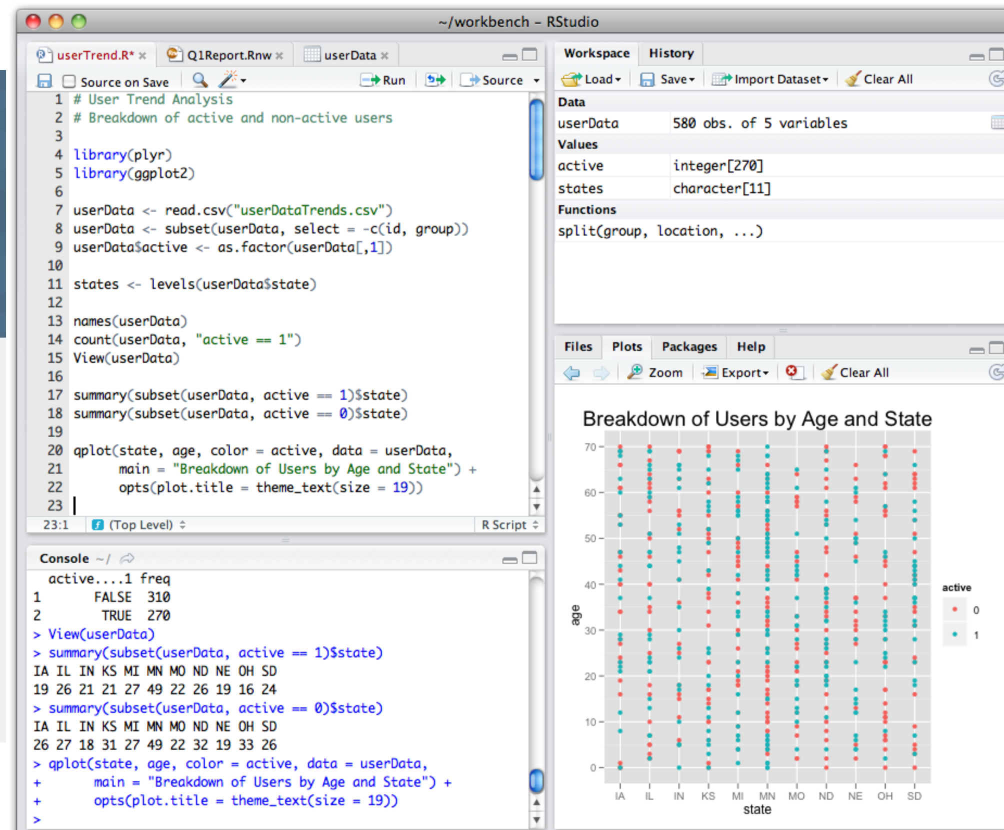
R & RStudio

- If you don't have R installed just type (in Ubuntu shell):

```
$ sudo apt-get update
$ sudo apt-get install r-base
```

- And now Rstudio…





http://www.rstudio.com/

igraph installation in R

- Very easy!
- Enter the R shell and type:

```
> install.packages("igraph")
```

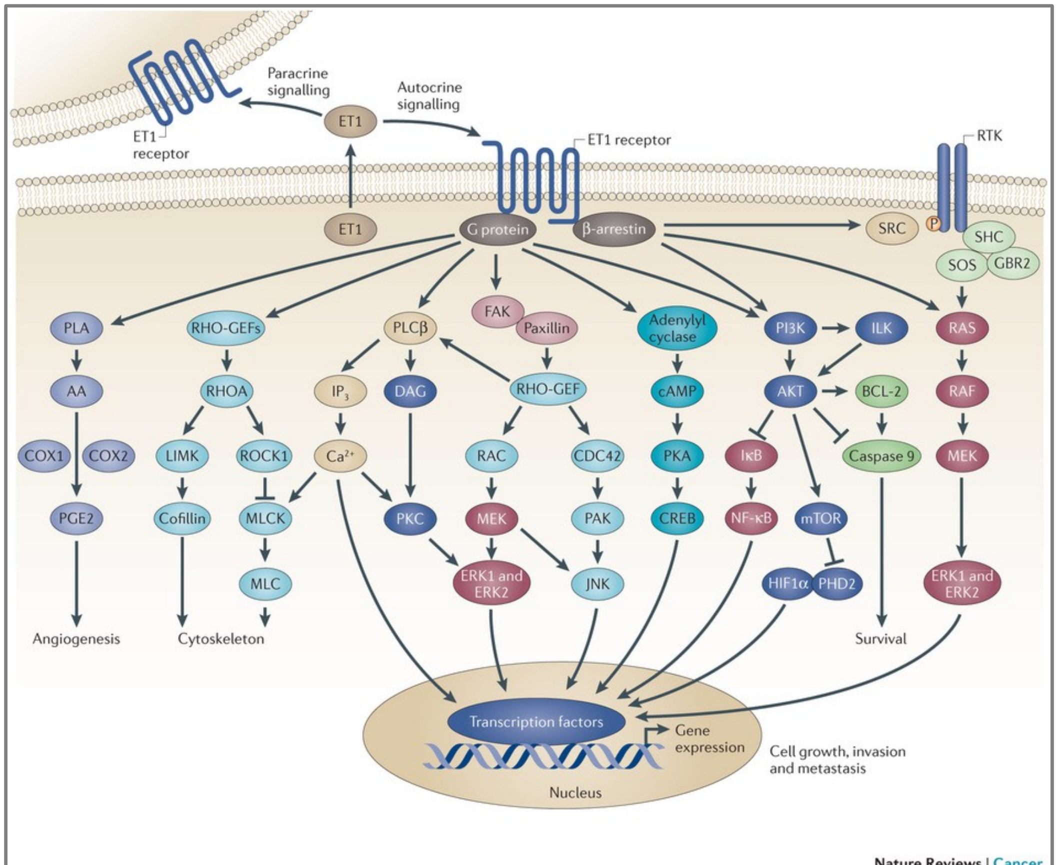
- Load it!

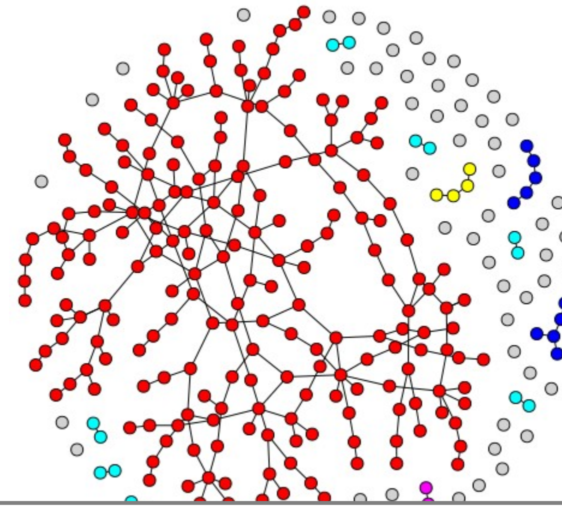
```
> library(igraph)
```

The igraph library

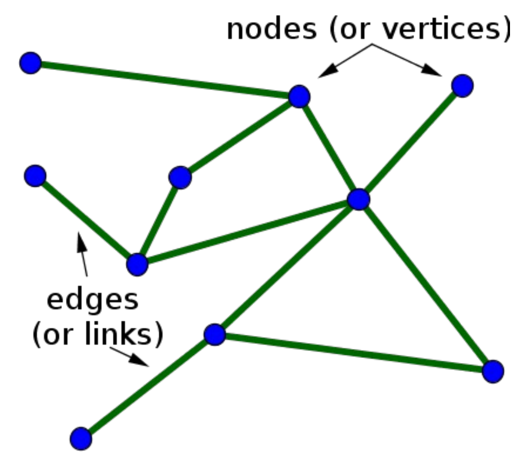
Basic concepts

Network (informal concept)
Graph (formal mathematical object)





vertices = {A, B, C, D, E}
edges = ({A,B}, {A,C}, {B,C}, {C,E})



The igraph library in R

Creating a graph

- Vertices and edges have numerical vertex ids in igraph. Vertex ids are always consecutive and they are **numbered from 1** (from igraph 0.6.5 onwards)
- To create a graph whose nodes are letters we have to ***translate*** vertex names to ids:

  V = {A, B, C, D, E}

  E = ((A, B), (A, C), (B, C), (C, E))

  A = 1, B = 2, C = 3, D = 4, E = 5

```
> g <- graph( c(1,2, 1,3, 2,3, 3,5), n=5 )
```

- By default, igraph creates a **directed** graph, but we can change it:

```
> g <- graph( c(1,2, 1,3, 2,3, 3,5), n=5, directed=FALSE )
```

- Let's play a little bit around...

  Open the R script **`igraph_basic_concepts.R`**

The igraph library in R

Accessing graph information

Description	Function
Vertex and edge counts	`vcount(g)`
	`ecount(g)`
Vertex and edge lists	`V(g)`
	`E(g)`
Vertex and edge attributes	`V(g)$name`
	`E(g)$weight`
Vertex and edge attribute lists	`list.vertex.attributes(g)`
	`list.edge.attributes(g)`

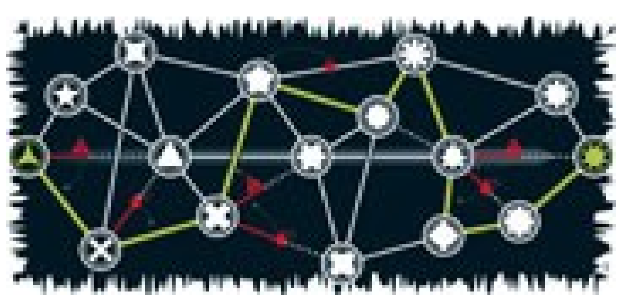
The igraph library in R

Accessing topological information

Description	Function
Node degree	`degree(g)`
Degree distribution	`degree.distribution(g)`
Betweenness	`betweenness(g)`
Closeness	`closeness(g)`
Clustering coefficient	`transitivity(g)`
Shortest paths	`shortest.paths(g)`
	`get.shortest.paths(g)`
Diameter	`diameter(g)`
	`get.diameter(g)`

Error and attack tolerance of complex networks

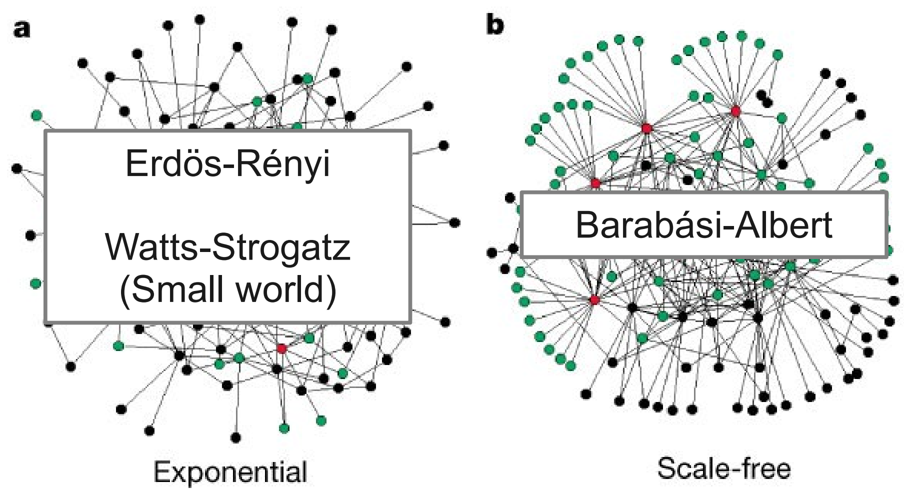
- Components regularly **malfunction**
- However, this failure rarely perturbs the ability of the network → Many complex systems display a surprising degree of **tolerance**
- Stability attributed to the **redundant wiring** of the functional web
- **Robustness**: There must be at least two alternative paths between any two points on the network, so that removal of any one component leaves the network fully operational.
- But… what happens with metabolic networks?
- However, extremely **vulnerable** to the attack of a few nodes

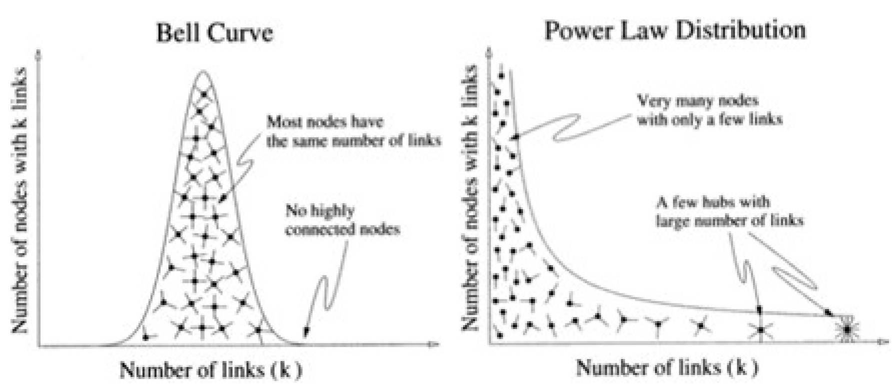


Network models

Types of networks

Complex networks can be divided into **two major classes** depending on the degree distribution





- 130 nodes and 215 links
- RED: 5 nodes with the highest number of links
- GREEN: The first neighbours
- Exponential, only 27% of the nodes are reached
- Scale-free, > 60% of the nodes are reached

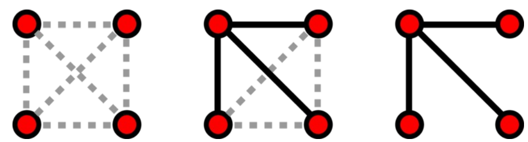
- Mechanism:
  - Growth
  - Preferential attachment

Network models

Types of networks

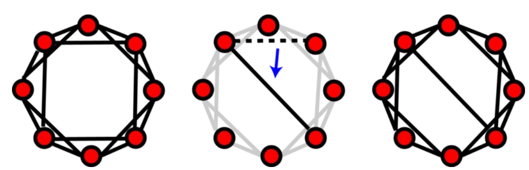
- **Random Networks (Erdös-Rényi** (ER)**)**

  Every node is connected to a randomly selected number of other nodes with **equal probability**. Two parameters needed: the number of nodes and the probability that a link should be formed between any two nodes.



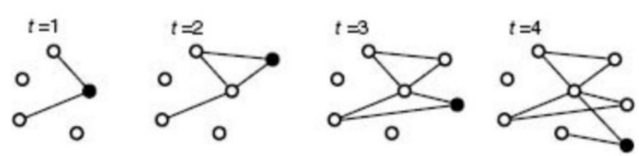
- **Small-World Networks (Watts-Strogatz** (WS)**):**

  Produces graphs with small-world properties, including short average path lengths and high clustering.



- **Scale-free Networks (Barabási-Albert** (BA)**)**

  A scale free network is a network that its degree distribution obeys a power law: $P(k) \sim k^{-\gamma}$. Model used to demonstrate a preferential attachment or a "**rich-get-richer**" effect.



Vulnerability and robustness

- A **perturbation** in a network can be defined as a set of elementary changes:
  - Failure of a node
  - Failure of a link
  - Instability of a network due to removal of a stabilizing link

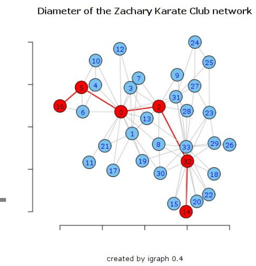
- 2 types of attacks
  - **Random attack**: Removal of nodes randomly (% of network)
  - **Targeted attack**: Removal of nodes with particular attributes values (highest degree nodes)

- Article:

  Error and attack tolerance of complex networks. Albert R, Jeong H, Barabasi AL. *Nature*. 2000 Jul 27;406(6794):378-82.

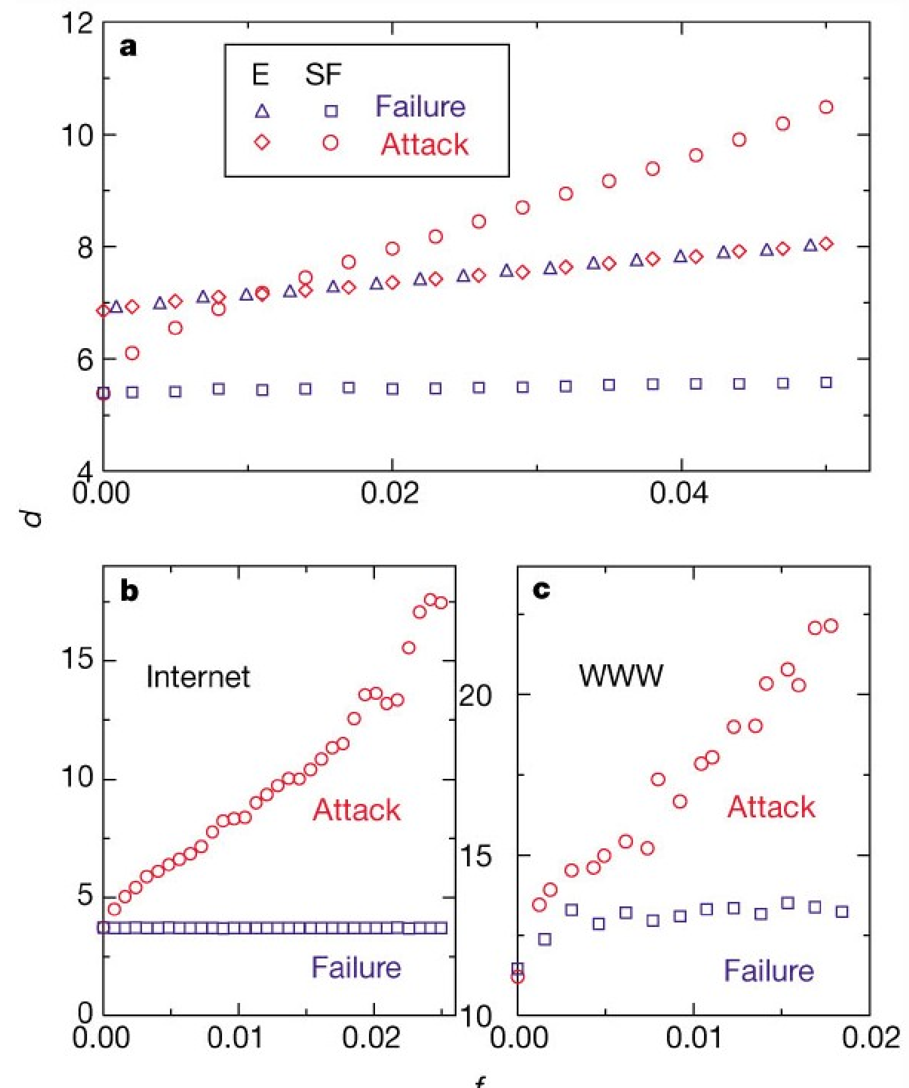
Vulnerability and robustness

Network diameter



- **2 networks**: a random network (ER) and a scale-free network (BA)
  - 10,000 nodes and 20,000 links
- ***d*** = diameter, ***f*** = % nodes removed randomly
- Attack over the **most connected nodes**

	Random network	Scale-free network
Failure	Diameter **increases monotonically** with *f*. All nodes have ~ same number of links → **contribute equally**.	Diameter remains **unchanged**. Lots of nodes with low connectivity → Removal of “small nodes” does **not alter the path structure**.
Attack	Due to the **homogeneity** of the network there is **no substantial difference** with failure attack.	Drastically different behavior → **diameter increases rapidly**. Vulnerability due to **inhomogeneity** → connectivity maintained by a few highly connected nodes.

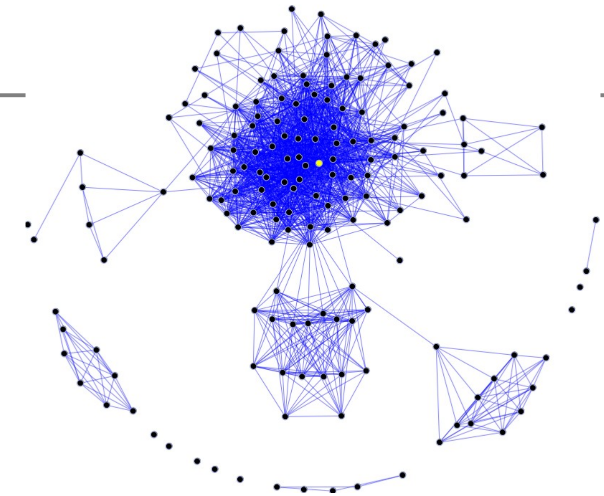


6,209 nodes
12,200 links

325,729 nodes
1,498,353 links

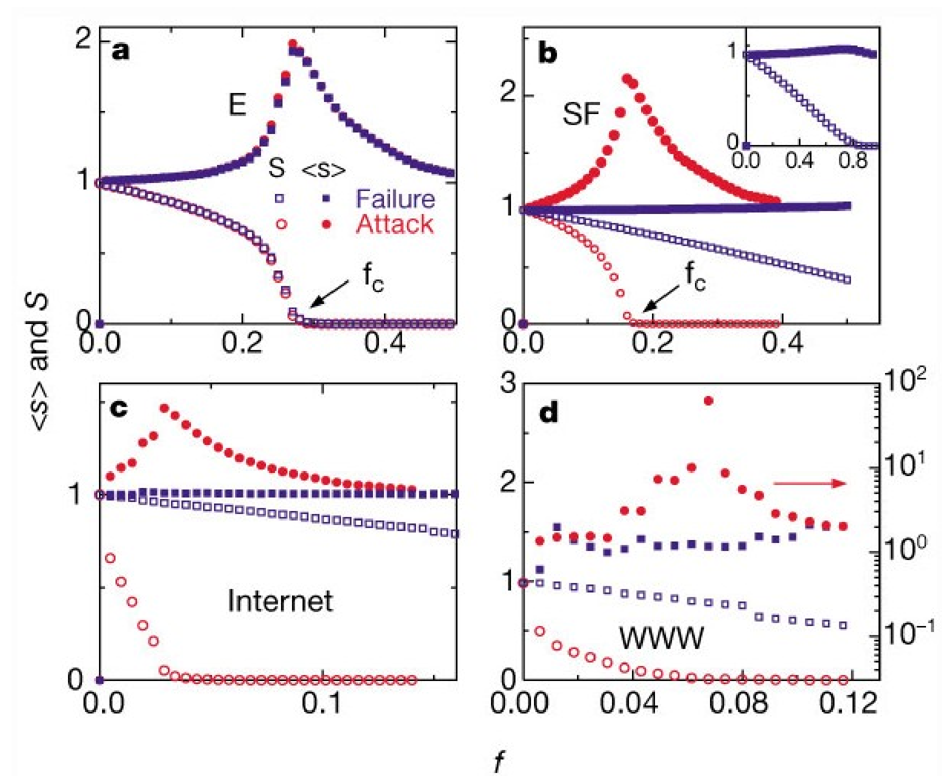
Vulnerability and robustness

Network fragmentation

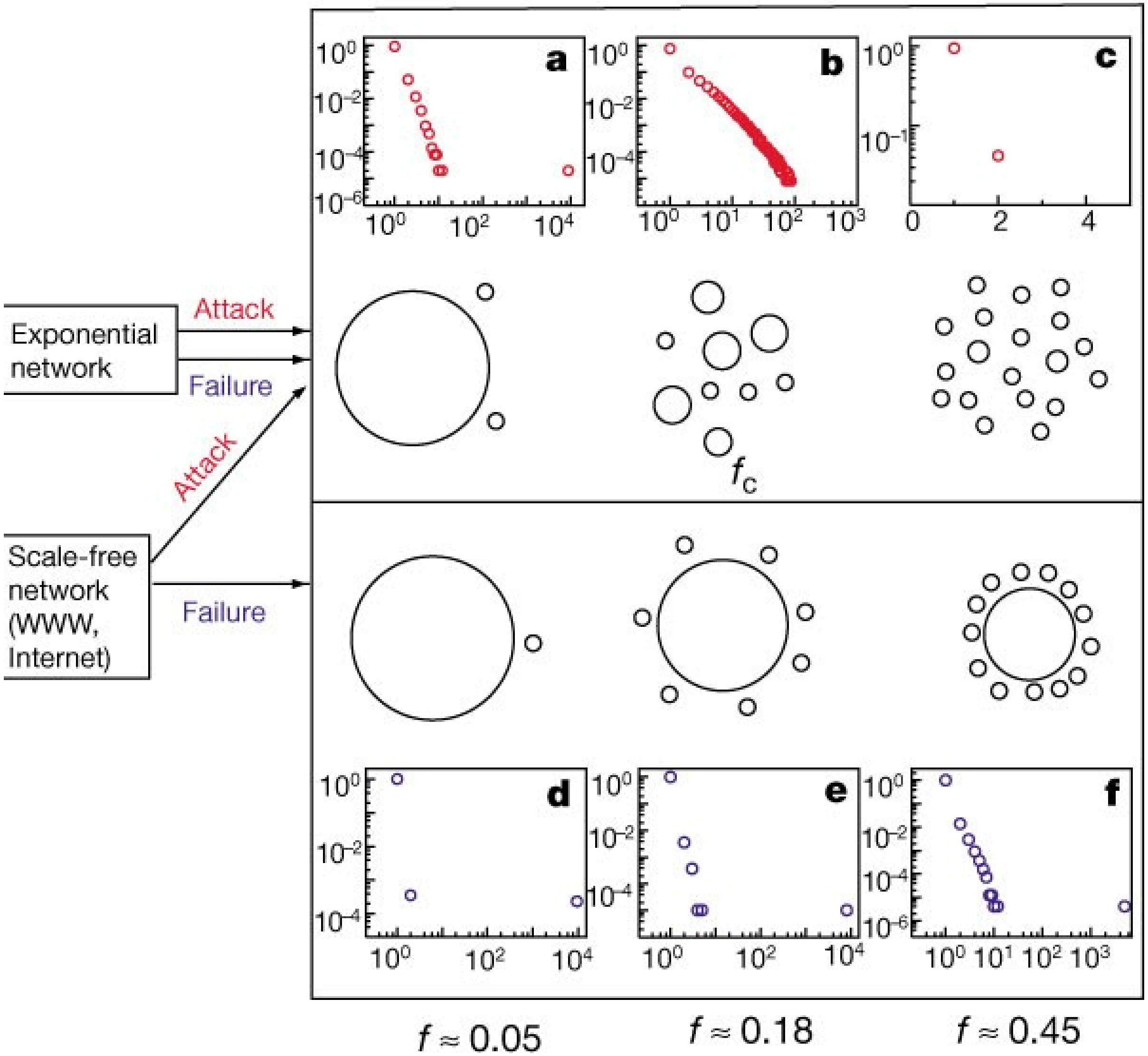


- When nodes are removed, clusters of **nodes** whose links to the system disappear may be cut off (**fragmented**) **from the main cluster**
- S = size (number of nodes) of the largest component (cluster)
- $\langle s \rangle$ = average size of the isolated components, except the largest one

	Random network	Scale-free network
Failure	No difference between failure and attack. S **decreases rapidly** until the main cluster is completely fragmented → S ~ 0. $\langle s \rangle$ **increases rapidly** until $\langle s \rangle$ = 2, after which it decreases to $\langle s \rangle$ = 1.	S **decreases slowly** without observed thershold. $\langle s \rangle$ ~ 1, network deflated by nodes breaking off one by one. Largest component stays together for very high f → **stability**
Attack		**Similar to a random network** but falling apart at a smaller value of f.



Vulnerability and robustness
Conclusions



- **Random graphs** are more **sensitive** than scale-free when nodes are removed randomly.
- **Scale-free networks** display a surprising high degree of **tolerance** against random failure when compared to random networks.
- Under a **targeted attack,** scale-free networks increase their diameter rapidly and break into many **isolated fragments**.

Homework

Simulate and attack

1. Simulate 3 networks
   - Erdös-Rényi (n=1000, p.or.m=2/1000, type="gnp")
   1. Watts-Strogatz (dim=1, size=1000, nei=4, p=0.5)
   2. Barabási-Albert (n=1000)

2. For these 3 networks, perform a random and a targeted attack damaging progressively the 0%, 0.1%, 0.2%, 0.3%, 0.4%, 0.5%, 0.6%, 0.7%, 0.8%, 0.9%, 1%, 1.5%, 2% and 2.5% of the nodes.

3. Study the variability of the following parameters:
   1. Average degree
   2. Diameter
   3. Size of the giant component
   4. Betweenness

4. Summarize this information in a plot and comment your results briefly.

THANK YOU.