Titolo del progetto

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Introduction

Objective of the project

Our goal is to investigate whether the salaries earned by the NBA players during the 2023-2024 season are fair in proportion to their performance during the current year's Regular season. To analyse performance, we selected several statistics: from the most common such as points, rebounds, assists to advanced metrics like Usage, Player Impact Estimated and Winning Shares. The idea is to deep dive into the relationship between salaries and performance through different models in order to understand what kind of relationship there is and which model best fits the data. Finally, we will compare actual salaries with those predicted by our models to find out which players (according to the models) are the most overpaid or underpaid.

Steps followed

To perform our analysis we followed these steps:

1. Data collection;
2. Data exploration;
3. Analysis;
4. Interpretation.

We now explain in depth each step.

Data collection

We performed a web scraping operation from the Official NBA Stats website, from which we collected most of the stats. Additionally, we downloaded data about the salaries from Hoopshype and other stats of interest from Basketball reference. All data concerns the 2023-2024 NBA Regular Season.

Why consider only Regular Season data?

Considering only data about Regular Season without considering players performance during playoffs limits a bit the potential of our analysis. On one hand, it's reasonable to infer that player performance during playoffs should have an important weight in determining his salary. On the other hand, considering playoffs in the analysis carries different issues.

There are teams (and consequently players) that go further than others: 14 out of 30 teams can't qualify for the playoffs. For the teams which qualify, playoff stats are calculated on a number of games that could differ greatly between different teams (e.g. if a team loses in the first round, it plays from 4 to 7 games. If a team reaches the finals, it plays from 16 to 28 games). During Regular Season every team plays a fixed number of games, 82.

Additionally, coaches usually rotate players at their disposal in a different way during playoffs: for instance, during regular season approximately 10-12 players for each team take part in the game; during playoffs it is not uncommon to observe only 7-8 players that come into play for each team. Furthermore, usually in a playoff game the best players are more involved compared to Regular season games. It means that, first of all, they play several more minutes. Moreover, they have the ball in their hands for a lot of time and consequently their stats grow a lot; hence, it could happen that few players record a large part of the entire team's statistics. Considering this, including playoffs data in the analysis could lead to an overestimation of performance of 2-3 players and to an underestimation of the performance of the rest of the team.

All in all, it is undeniable that playoffs are a fundamental part of the season. It is also obvious that if a player has more responsibilities in that phase he probably deserves a higher salary. But we think that for the purposes of our analysis, the addition of statistics collected on a small sample of matches, different for practically every team, with highly polarized data between the various players may lead to biases if not handled properly.

We think that considering only the regular season, although leading to a limited analysis, may be sufficient to grasp the main relationships between salaries and performance.

Glossary

- **PLAYER NAME**: name of a player;
- **SALARY**: salary earned by a player for 2023-2024 season (collected from Hoopshype);
- **AGE**: age of a player;
- **POS**: "Position", the playing position of a player.

Traditional stats (collected from the NBA website)

- **GP**: "Games played", the number of games played by a player during the 2023-2024 regular season;
- **FG_PCT**: "Field Goal Percentage", the percentage of field goal attempts that a player makes. Formula: (FGM)/(FGA);
- **FG3_PCT**: "3 Points "Field Goal Percentage", the percentage of 3pt field goal attempts that a player makes;
- **FT_PCT**: "Free throws Percentage", the percentage of free throws attempts that a player makes;
- **OREB**: "Offensive Rebounds", the number of rebounds a player or team has collected while they were on offense;

- **DREB**: “Defensive Rebounds”, the number of rebounds a player or team has collected while they were on defense;
- **REB**: “Rebounds”, a rebound occurs when a player recovers the ball after a missed shot. This statistic is the number of total rebounds a player has collected on either offense or defense;
- **AST**: “Assists”, the number of assists (passes that lead directly to a made basket) by a player;
- **TOV**: “Turnovers”, a turnover occurs when a player on offense loses the ball to the defense;
- **STL**: “Steals”, number of times a defensive player takes the ball from a player on offense, causing a turnover;
- **BLK**: “Blocks”, a block occurs when an offensive player attempts a shot, and the defense player tips the ball, blocking their chance to score;
- **BLKA**: “Blocks Against”, The number of shots attempted by a player or team that are blocked by a defender
- **PF**: “Personal fouls”, the number of personal fouls a player or team committed;
- **PFD**: “Personal fouls drawn”, the number of personal fouls that are drawn by a player or team;
- **PTS**: “Points”, the number of points scored by a player;
- **MIN**: “Minutes played”, number of minutes played by a player during the 2023-2024 Regular season;
- **MIN_G**: “Minutes played per game”.

Advanced stats (collected from the NBA website)

- **OFF_RATING**: “Offensive Rating”, measures a team’s points points scored per 100 possessions while a player is on the court. Formula: 100*((Points)/(POSS);
- **DEF_RATING**: “Defensive Rating”, the number of points per 100 possessions that the team allows while a player is on the court. Formula: 100*((Opp Points)/(Opp POSS));
- **NET_RATING**: “Net Rating”, Measures a team’s point differential per 100 possessions while a player is on the court. Formula: OFFRTG - DEFRTG;
- **AST_TO**: “Assist to Turnover Ratio”, the number of assists for a player compared to the number of turnovers committed;
- **TS_PCT**: “True Shooting Percentage”, a shooting percentage that factors in the value of three-point field goals and free throws in addition to conventional two-point field goals. Formula: Points/ [2*(Field Goals Attempted+0.44*Free Throws Attempted)];
- **USG_PCT**: “Usage Percentage”, the percentage of team plays used by a player when they are on the floor. Formula: (FGA + Possession Ending FTA + TO) / POSS;
- **PIE**: “Player Impact Estimate”, measures a player’s overall statistical contribution against the total statistics in games they play in. PIE yields results which are comparable to other advanced statistics (e.g. PER) using a simple formula. Formula: (PTS + FGM + FTM - FGA - FTA + DREB + (.5 * OREB) + AST + STL + (.5 * BLK) - PF - TO) / (GmPTS + GmFGM + GmFTM - GmFGA - GmFTA + GmDREB + (.5 * GmOREB) + GmAST + GmSTL + (.5 * GmBLK) - GmPF - GmTO).

The stats below are collected from Basketball Reference:

- **WS**: “Win Shares”, attempts to divvy up credit for team success to the individuals on the team. It is calculated using player, team and league-wide statistics and the sum of player win shares on a given team will be roughly equal to that team’s win total for the season (more details on the Basketball Reference page);
- **BPM**: “Box Plus/Minus”, a box score estimate of the points per 100 possessions that a player contributed above a league-average player, translated to an average team;
- **VORP**: “Value Over Replacement Player”, a box score estimate of the points per 100 TEAM possessions that a player contributed above a replacement-level (-2.0) player, translated to an average team and prorated to an 82-game season. Multiply by 2.70 to convert to wins over replacement.

BPM and VORP are calculated per 100 possessions; MIN and WS are calculated over the whole regular season, MIN_G is calculated per game. The other stats are considered per 48 minutes.

Why statistics per 48 minutes?

Considering most statistics projected over 48 minutes avoids overestimating performance for players who play, on average, more minutes in a game. In this way we think that the contribution of each player is fairly evaluated and not distorted by the minutes played.

Data integration and cleaning

Once we had obtained the tables of interest, we selected from each table the statistics useful for analysis (those given in the glossary) and then merged the slices of the various datasets, removing all the players who played less than 480 minutes during the entire regular season.

```
data_traditional_tot <- data_traditional_tot[data_traditional_tot$MIN > 480, ]

final_dataset <- merge(data_salary, data_traditional_per48, by = "PLAYER_NAME", all = TRUE)
final_dataset <- merge(final_dataset, data_advanced, by = "PLAYER_NAME", all = TRUE)
final_dataset <- merge(final_dataset, data_miscellaneous, by = "PLAYER_NAME", all = TRUE)
final_dataset <- merge(final_dataset, data_traditional_tot, by = "PLAYER_NAME", all = TRUE)
final_dataset <- merge(final_dataset, data_vorp, by = "PLAYER_NAME", all = TRUE)
```

The reason why we selected players with at least 480 minutes played is that we wanted to avoid considering stats taken on a too small amount of minutes. After these operation, the final dataset consists of 360 rows and 31 columns.

At this stage, we cleaned the data following these other steps:

- NA removal;
- Matching players' names;
- Transforming the Salary column into a numeric one;
- Putting the players' name as row names for the dataset and thus removing the `PLAYER_NAME` column.

Data exploration

Before studying the data with formal models, we got an overview through an exploratory data analysis. For the first part of our analysis we used only numeric variables, so the categorical parameter `Pos`, which you can see on the table below, was removed from the dataset at this stage.

	Salary	AGE	GP	FG_PCT	FG3_PCT	FT_PCT	OREB	DREB	REB	AST
Aaron Gordon	22266182	28	73	0.556	0.290	0.658	3.6	6.2	9.8	5.4
Aaron Holiday	2346614	27	78	0.446	0.387	0.921	0.9	3.8	4.7	5.3
Aaron Nesmith	5634257	24	72	0.496	0.419	0.781	1.5	5.1	6.6	2.6
Aaron Wiggins	1836096	25	78	0.562	0.492	0.789	2.3	4.9	7.3	3.4
Al Horford	10000000	37	65	0.511	0.419	0.867	2.3	9.1	11.4	4.6

TOV	STL	BLK	BLKA	PF	PTS	OFF_RATING	DEF_RATING	NET_RATING	AST_TO
2.2	1.2	0.9	1.2	3.0	21.2	119.8	111.1	8.7	2.47
2.0	1.6	0.2	0.8	4.7	19.4	110.5	107.6	2.9	2.64
1.5	1.6	1.2	1.2	5.8	21.1	119.3	115.0	4.3	1.69
2.2	2.2	0.7	1.3	3.6	21.2	115.6	110.0	5.7	1.54

TOV	STL	BLK	BLKA	PF	PTS	OFF_RATING	DEF_RATING	NET_RATING	AST_TO
1.3	1.0	1.7	0.3	2.6	15.5	120.9	109.5	11.4	3.50

TS_PCT	USG_PCT	PIE	PFD	MIN	MIN_G	Pos	WS	BPM	VORP
0.607	0.174	0.103	4.7	2296.810	31.46315	PF	7.1	1.3	1.9
0.578	0.158	0.078	2.5	1269.297	16.27303	PG	2.5	-1.5	0.2
0.631	0.158	0.071	3.5	1994.655	27.70354	SF	4.1	-0.5	0.8
0.664	0.163	0.096	2.3	1227.938	15.74280	SG	3.7	0.7	0.8
0.650	0.119	0.105	0.8	1739.797	26.76610	C	6.2	3.6	2.5

Firstly, we perform an analysis, which can be seen in the Figure 1, of the variable Salary, and its logarithmic transformation, that will be the dependent variable in the models.

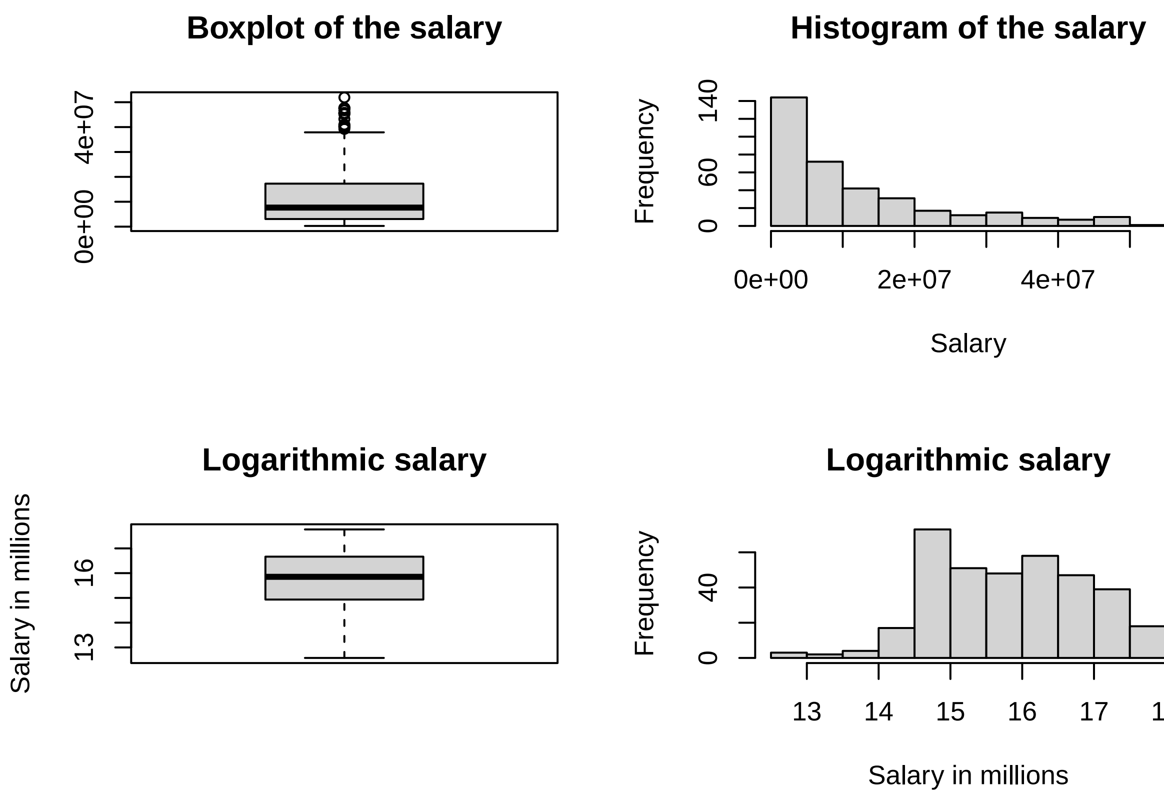

Figure 1: Boxplot and histograms of the dependent variable Salary and of its logarithmic transformation

The boxplot shows that the salary distribution is right skewed, with some outliers in the right side. We expected this kind of distribution, the outliers are the players earning the highest salaries. The histogram also highlights the right skewed distribution. It can be seen that Salary's log transformation reduces the skewness and makes the distribution of the variable closer to normal.

In order to study correlations between the predictors of the model, we used the corrplot function (Figure 2).

```r
library(corrplot)
corrplot(cor(fd_numeric), method = 'color')
```

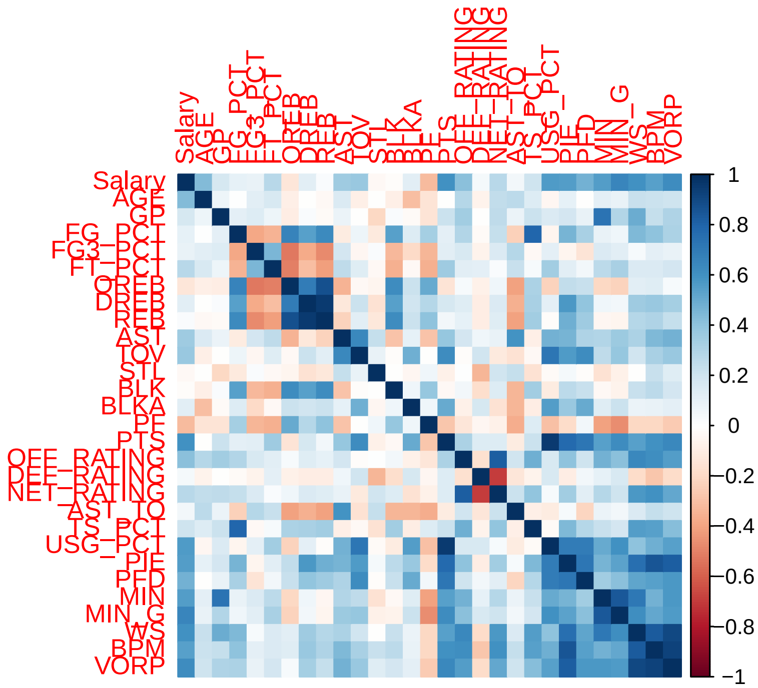

Figure 2: Correlation plot of the independent numeric variables

Different correlations between the variables emerge from the `corrplot`. With regard to the variable Salary, it is interesting to notice that Salary is positively correlated with PTS and advanced stats like USG_PCT, BPM and VORP: all of these variables are related to players' shots and point contribution. For what concerns the other variables, there are some obvious correlations: for instance, between variables MIN (total minutes played during the regular season) and MIN_G (minutes played per game) and between variables REB, OREB and DREB (all related to rebounds, with the relation REB = OREB + DREB). Additionally, we expected the positive correlation between BPM and VORP because are both related to players point estimation. A strong positive correlation emerges between PTS and USG_PCT. The usage percentage is "The percentage of team plays used by a player when they are on the floor. Formula: (FGA + Possession Ending FTA + TO) / POSS". Thus, players with a high USG_PCT often make the last play in an offensive possession (a shot, a free throw or a turnover): it is straightforward that if a player often ends the offensive possession of his team, he has more opportunities to score points. For what concerns the negative correlations, the most interesting are the ones between rebounds variables (OREB, DREB, REB), FT_PCT and FG3_PCT. Players that grab a lot of rebounds are usually the tallest ones and these players are not great free throws shooters or 3 point shooters (on average).

Models

We started creating a linear regression model in order to predict salaries (Figure 3).

```
##
## Call:
## lm(formula = Salary ~ +., data = fd_numeric)
##
```

```
## Residuals:
##       Min        1Q    Median        3Q       Max
## -18450260  -4028989    276645   4003025  20712902
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -12550803   28966256  -0.433   0.6651
## AGE            1057586      93293  11.336   <2e-16 ***
## GP              -21024     118116  -0.178   0.8588
## FG_PCT        35836089   19086013   1.878   0.0613 .
## FG3_PCT         -56045    4195524  -0.013   0.9893
## FT_PCT          975535    6135667   0.159   0.8738
## OREB           4054377    6865112   0.591   0.5552
## DREB           4473997    6846450   0.653   0.5139
## REB           -4315225    6838752  -0.631   0.5285
## AST             -98527     667680  -0.148   0.8828
## TOV            2003183    1516145   1.321   0.1873
## STL             -69046     985541  -0.070   0.9442
## BLK             601287     664109   0.905   0.3659
## BLKA          -2253383    1230646  -1.831   0.0680 .
## PF             -616626     640191  -0.963   0.3362
## PTS            1117890     623630   1.793   0.0740 .
## OFF_RATING    16646653    6955245   2.393   0.0172 *
## DEF_RATING   -16681974    6953008  -2.399   0.0170 *
## NET_RATING   -16610236    6957987  -2.387   0.0175 *
## AST_TO         -252115     978605  -0.258   0.7969
## TS_PCT       -63710004   30332753  -2.100   0.0365 *
## USG_PCT      -40539821   73391644  -0.552   0.5811
## PIE         -131534170  115933751  -1.135   0.2574
## PFD             101829     397847   0.256   0.7981
## MIN              -4774       4689  -1.018   0.3094
## MIN_G           696311     299872   2.322   0.0208 *
## WS             1845668     740418   2.493   0.0132 *
## BPM            -391702     764769  -0.512   0.6089
## VORP            663105    1436430   0.462   0.6446
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6712000 on 331 degrees of freedom
## Multiple R-squared:  0.7081, Adjusted R-squared:  0.6834
## F-statistic: 28.67 on 28 and 331 DF,  p-value: < 2.2e-16
```

```
## [1] "MSE of the complete linear model =  4.142347e+13"
```

The complete model has a good adjusted R-squared of 0.68 and a MSE of 4.14e+13. It emerges that many variables are not significative in determining the response. Through the residual analysis it is noticeable that the relationship between fitted values and residuals is not exactly linear (1st graph). Additionally, in the third graph the points are not are included in a band of constant amplitude parallel to the x-axis, hence the omoschedasticity assumption can be doubted.

```
##
## Call:
## lm(formula = log(Salary) ~ +., data = fd_numeric)
```

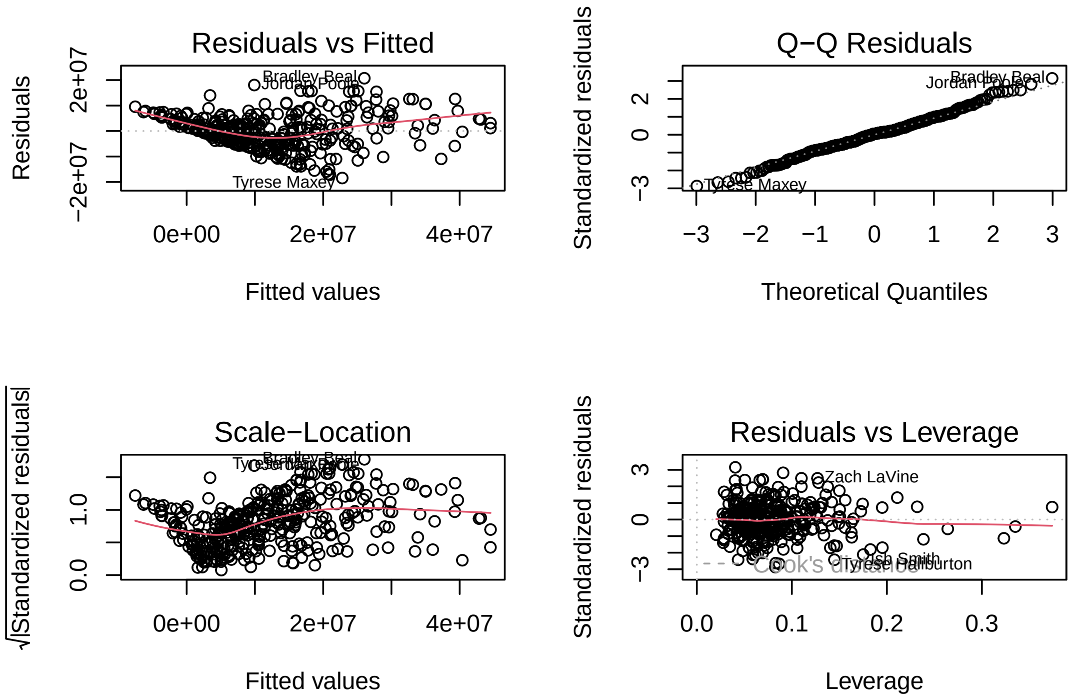

Figure 3: Residuals plot of the complete linear model

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.75872 -0.32207  0.02064  0.42623  1.62364
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.258e+01  2.744e+00   4.583 6.49e-06 ***
## AGE          9.669e-02  8.838e-03  10.940  < 2e-16 ***
## GP          -5.087e-04  1.119e-02  -0.045   0.9638
## FG_PCT       3.669e+00  1.808e+00   2.029   0.0432 *
## FG3_PCT     -7.625e-02  3.975e-01  -0.192   0.8480
## FT_PCT      -1.574e-01  5.813e-01  -0.271   0.7867
## OREB         3.180e-01  6.504e-01   0.489   0.6252
## DREB         4.105e-01  6.486e-01   0.633   0.5273
## REB         -3.435e-01  6.479e-01  -0.530   0.5963
## AST          3.954e-02  6.325e-02   0.625   0.5323
## TOV          1.536e-03  1.436e-01   0.011   0.9915
## STL          5.077e-02  9.337e-02   0.544   0.5870
## BLK          7.242e-02  6.291e-02   1.151   0.2505
## BLKA        -1.802e-01  1.166e-01  -1.545   0.1232
## PF          -5.294e-02  6.065e-02  -0.873   0.3834
## PTS          9.972e-02  5.908e-02   1.688   0.0924 .
## OFF_RATING   1.508e+00  6.589e-01   2.289   0.0227 *
## DEF_RATING  -1.499e+00  6.587e-01  -2.276   0.0235 *
## NET_RATING  -1.504e+00  6.592e-01  -2.282   0.0231 *
```

```
## AST_TO       -4.552e-02  9.271e-02  -0.491   0.6237
## TS_PCT       -6.498e+00  2.874e+00  -2.261   0.0244 *
## USG_PCT      -2.881e+00  6.953e+00  -0.414   0.6789
## PIE          -1.715e+01  1.098e+01  -1.562   0.1193
## PFD           3.123e-02  3.769e-02   0.829   0.4079
## MIN          -5.447e-05  4.442e-04  -0.123   0.9025
## MIN_G         5.720e-02  2.841e-02   2.014   0.0449 *
## WS            1.246e-01  7.014e-02   1.777   0.0765 .
## BPM           5.637e-02  7.245e-02   0.778   0.4371
## VORP         -1.639e-01  1.361e-01  -1.204   0.2293
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6359 on 331 degrees of freedom
## Multiple R-squared:  0.6516, Adjusted R-squared:  0.6222
## F-statistic: 22.11 on 28 and 331 DF,  p-value: < 2.2e-16
```

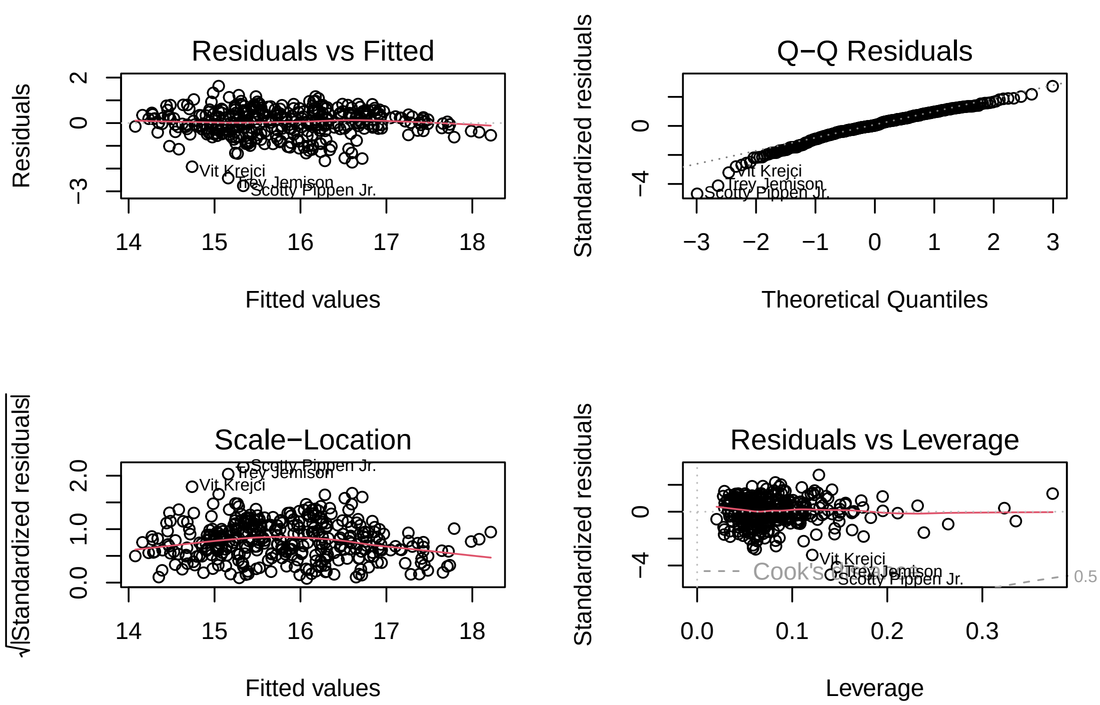

Figure 4: Residuals plot of the complete linear model with logarithmic Salary

```
## [1] "MSE of the complete linear model with logarithmic Salary =  4.703165e+13"
```

With a logarithmic transformation of the dependent variable, the model shows a slightly lower adjusted R-squared (0.62) and a slightly higher MSE (4.70e+13). This can be seen in the Figure 4. Applying a logarithmic transformation to the dependent variable `Salary`, the first graph shows a more linear relationship and the third graph allows to infer a more constant variance in the error terms. In both models many variables are not significative in determining the response: for this reason, to avoid a model that is unnecessary complex, we performed a variable selection. A logarithmic transformation of the dependent variable Salary will be applied because, although it slightly worsens the performance of the model, it makes the salaries distribution closer to normal, it improves the linearity of the model and it reduces residuals eteroschedasticity.

Variable selection

We selected a subset of relevant features starting from the predictors used in the complete model in order to have a simpler model that is easier to interpret, without redundant variables and less prone to overfitting. To do so, we used the `regsubsets` function which performs best subset selection by identifying the best model that contains a given number of predictors, according to the RSS metric. We set the function to return results up to the best 28-variables model.

To find the best balance between model simplicity and precision, we evaluated the number of parameters to be included in the model through Mallow's Cp, BIC and Adjusted R-squared.

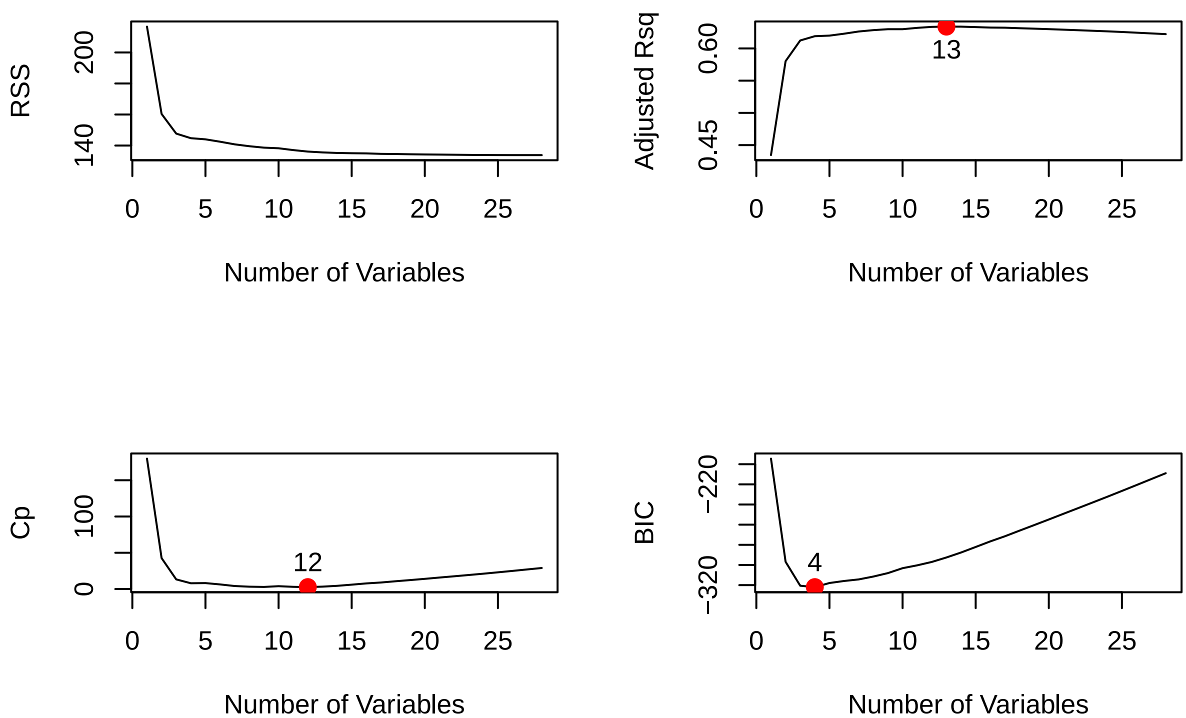

Figure 5: Evaluation of the number of parameters through RSS, Adjusted R-squared, Mallow's Cp and BIC

Considering Mallow's Cp, the best number of parameters for our model is 12. This result can be seen in the Figure 5. We obtained the list of parameters from the `regsubset` function to get the best model with 12 parameters.

```
##
## Call:
## lm(formula = selected.formula, data = fd_numeric)
##
## Residuals:
```

```
##      Min      1Q  Median      3Q     Max
## -2.6476 -0.3193  0.0482  0.4315  1.5814
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.852025   1.626347   6.673 9.95e-11 ***
## AGE          0.095227   0.008503  11.199  < 2e-16 ***
## FG_PCT       3.445692   1.155704   2.981  0.00307 **
## BLK          0.083159   0.049934   1.665  0.09674 .
## BLKA        -0.167470   0.106759  -1.569  0.11764
## PTS          0.058816   0.011460   5.132 4.78e-07 ***
## OFF_RATING   1.531106   0.629095   2.434  0.01544 *
## DEF_RATING  -1.513509   0.629188  -2.405  0.01667 *
## NET_RATING  -1.520988   0.629121  -2.418  0.01614 *
## TS_PCT      -5.839664   1.438599  -4.059 6.09e-05 ***
## PIE         -6.258145   2.750650  -2.275  0.02351 *
## MIN_G        0.059152   0.006890   8.585 3.10e-16 ***
## WS           0.053790   0.026074   2.063  0.03986 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6263 on 347 degrees of freedom
## Multiple R-squared:  0.6457, Adjusted R-squared:  0.6334
## F-statistic:  52.7 on 12 and 347 DF,  p-value: < 2.2e-16
```

The reduced model shows a slightly higher adjusted R-squared, 0.63, compared to the complete logarithmic model (0.62). It means that, despite the lower number of variables, this model fits better the data. Different variables are strongly significant:

- **AGE**: the positive coefficient associated to the variable shows that older players earn, on average, more than youngsters. This makes sense since the youngest players in the league, rookies (first year in NBA) and sophomores (second year in NBA), usually earn less in the first years due to particular specifications in their contracts;
- **PTS**: this is quite straightforward. Players who score more points, on average, have higher salaries;
- **TS_PCT**: for what concerns true shooting percentage, the situation is peculiar. TS_PCT weights a player's shooting percentages based on the shot type (3-pointer, 2 pointer or free throw). The negative coefficient seems counterintuitive: a better TS_PCT reflects, on average, a lower salary. A possible explanation is that this metric is high for two players categories. The first one is composed by tall players who take most of their shots near the basket, thus getting a high percentage. The second category is composed by 3-point shooting specialists, because the weight for a 3 point shoot is higher for the metric. These players are crucial into a team, but we can say that they often have a limited role: the former have to score mostly near the basket, the latter from behind the 3-point line. Consequently, it makes sense if the model assigns a lower salary for players with a limited role. Additionally, shooting percentages are also high for players that shoot only few shots in a game; it is reasonable to think that scoring only few shots it's not enough to earn a high salary.
- **MIN_G**: players that play on average more minutes in a game earn, on average, a higher salary.

The variable FG_PCT is less significative than TS_PCT, but the coefficient here is positive. Both the stats measure shooting percentages, but FG_PCT does not weight shots and does not consider free throws. In this way, the previous mentioned effect on 3 point shooting specialists is reduced. It is possible to infer that FG_PCT represents better, within this model, the positive impact of good shooting percentages on wages.

The variables OFF_RATING, DEF_RATING, NET_RATING, PIE and WS have a level of significance between 0.01 and 0.05. The positive sign of OFF_RATING and WS coefficients and the negative sign of DEF_RATING coefficient are in line with what we expected. OFF_RATING (DEF_RATING) represents the points scored (conceded) by the team when the player is playing, WS measures the player contribution to the team wins. We didn't expected a negative signs for NET_RATING (OFF_RATING - DEF_RATING) and PIE, that measures the player impact in the game.

For what concerns PIE, the negative sign has different possible explanations: projecting PIE per 48 minutes inflates the metric for players who have a high impact on the game but few minutes played. It considers a lot of stats, even stats that seem to be not significative in determining salary; PIE difference between high salary players and low salary ones is not proportional to the differences in salaries. It is always difficult consider defensive contribution with this kind of metric and it is reasonable to think that defensive contribution plays an important role in determining a players salary. Furthermore, PIE does not consider aspects like leadership and IQ that, as defensive contribution, will certainly have an impact on the salaries. Anyway, beyond all the possible explanations, these unexpected negative signs likely depend from other variables not included in the model.

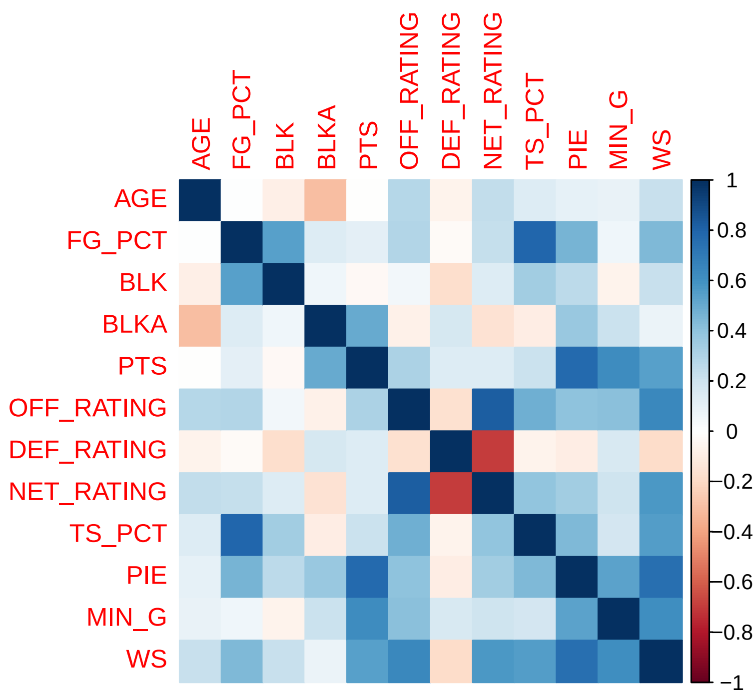

Figure 6: Correlation between dependent variables of the reduced model

Correlation between dependent variables It can be seen in the Figure 6 that there are, also in this case, different correlations between the dependent variables.

Residual analysis From what can be seen in the Figure 7, the assumptions of the linear model seem to be fulfilled. There are some players who are outliers in each graph: they probably have special contracts (two-way contracts). This means that they usually play in the team's second team (in a so called development league) and occasionally in the first team, so they have really low salaries compared to the league average.

MSE

```
## [1] "MSE of the full linear model with logarithmic Salary =  4.775116e+13"
```

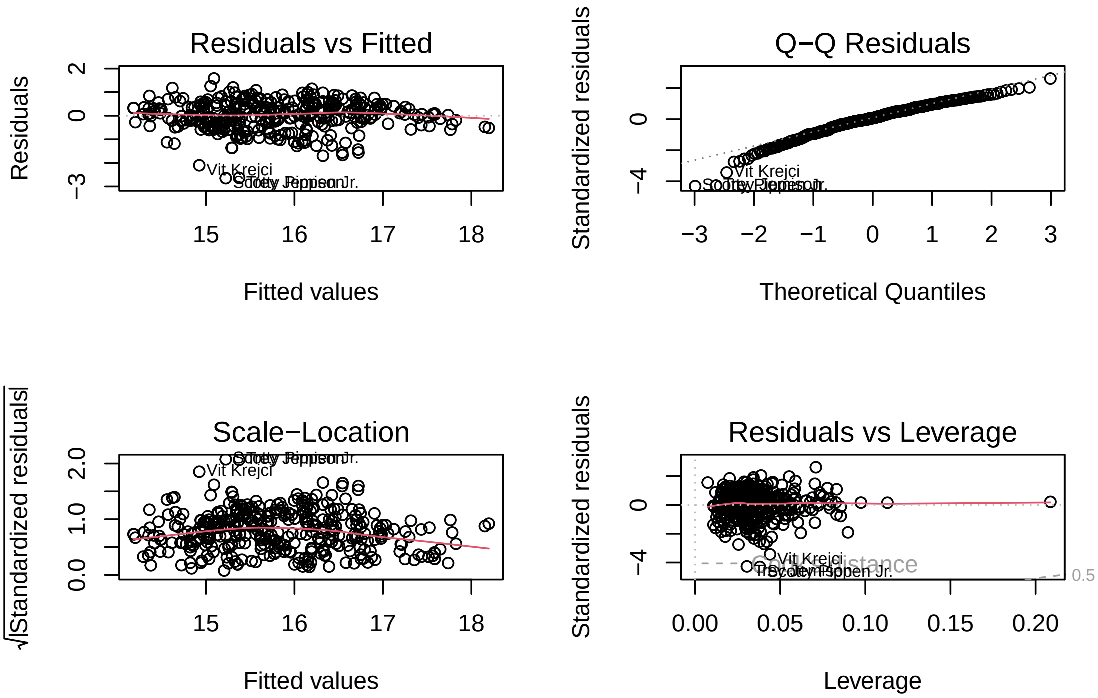

Figure 7: Residual plot of the reduced model with 12 covariates

The Mean Squared Error of the reduced model is really close to the one of the complete linear model with the logarithmic transformation of the Salary, 4.77e+13 against 4.70e+13. Considering that the complete model has 28 variables and the reduced one 12, the latter model represents quite an improvement.

Real salaries vs salaries prediction

Table 4: Ten most overpaid players according to the reduced model

	Salary	Predicted salary	Difference
Bradley Beal	46741590	21945158	24796432
Darius Garland	34005250	10821355	23183895
Zach LaVine	40064220	17030435	23033785
Trae Young	40064220	18771385	21292835
Deandre Ayton	32459438	12251585	20207853
Michael Porter Jr.	33386850	13910889	19475961
Zion Williamson	34005250	14543954	19461296
Karl-Anthony Towns	36016200	17724464	18291736
Jordan Poole	27955357	10348315	17607042
Gordon Hayward	31500000	15311898	16188102

Table 5: Ten most underpaid players according to the reduced model

	Salary	Predicted salary	Difference
LeBron James	47607350	79971554	32364204
Kevin Durant	47649433	76482185	28832752
DeMar DeRozan	28600000	51810315	23210315
Kyrie Irving	37037037	52868441	15831404
Nikola Vucevic	18518519	33136916	14618397
Jalen Brunson	26346666	40868798	14522132
Russell Westbrook	3835738	18318987	14483249
Tyrese Maxey	4343920	18257013	13913093
Kelly Oubre Jr.	2891467	15308000	12416533
Brook Lopez	25000000	37263780	12263780

Here we have a comparison between real salaries and predicted ones. The tables contain, respectively, the 10 most overpaid players and the 10 most underpaid players according to the model. The aim of this comparison is to analyse the major differences between predictions and actual salaries to understand whether, despite a big difference, the model's predictions seem reasonable.

MOST OVERPAID PLAYERS

The most overpaid player results to be Bradley Beal. After some brilliant seasons with Washington Wizards in which he was the league top scorer, he signed in 2022 a maximum contract (251 million $ in 5-years). In Washington he was the best player by far, his statlines in the past years justify the huge contract. In 23-24 he was traded to Phoenix (keeping the same contract) to play with Durant and Booker (two superstars) in a team that was, on the paper, a contender for the title. Beal, being no longer the first offensive option, had a quite different statline compared to the previous years. Additionally, the whole Phoenix Suns team disappointed the expectations. These facts are enough to explain that Beal 23-24 performance is not in line with his salary.

Darius Garland signed a big contract (near to the maximum) starting from 23-24 season. After showing superstar potential in 22-23, Cleveland Cavaliers renewed his contract with an important salary increase but Garland's performance decreased in 23-24. He is only 24, the team bet heavily on him taking a weighted risk in order to keep with them a high potential player. This bet didn't paid in 23-24 season.

Trae Young and Zach Lavine have superstar contracts respectively in Atlanta and Chicago, but they are not carrying their teams as expected. Both players could be traded during this summer.

Regarding Deandre Ayton, he was an amazing prospect but he repeatedly failed to meet expectations at the most important moments. He signed a big contract in 2022 but his performance were not at the same level as the salary. He was traded to Washington (keeping the same contract) but also this year in a different team he did not fulfil expectations.

Zion Williamson and Michael Porter Jr. (especially the former) are young players that in their still short careers have not shown their full potential due to injuries. Their contracts, let's say, consider their potential performance at the top of their form. Jordan Poole had an exploit in the previous seasons playing with a top team, Golden State Warriors, that somehow justifies his salary. He seemed to be ready to carry a team on his own, he was traded to Washington but his first season was a failure.

MOST UNDERPAID PLAYERS

Lebron James and Kevin Durant are two of the best players in the league for many years now. Even though, according to our model they should earn much more than the maximum wage. For sure their careers and their performance motivate a high salary, but equally surely they are not underpaid. We think that this overestimation depends in part on the fact that the variable AGE in the model is strongly significant, Lebron

James is 39 and Kevin Durant is 35. The same reasoning could apply to Kyrie Irving (32) and especially Demar Derozan (34).

For what concerns Nikola Vucevic, his stats are always more than respectable. His salary is lower than the expected probably because he seems to lack characteristics not included in the model or generally difficult to quantify such as defense, leadership and consistency at key moments of the season.

Jalen Brunson has shown this year that he is one of the best players in the NBA after being somewhat underrated in the years past. We expected the difference between his predicted and actual salary. Very similar the situation of Tyrese Maxey, in the last year of his rookie contract. He has shown by his performances that he is worth much more than his salary says.

Russell Westbrook is in the waning phase of his career. On the expiry of his last superstar contract, no team in the league offered him a comparable salary (he earned 47 millions in 2022). Consequently, he accepted a 3.8 millions salary (veteran minimum contract) to play with Los Angeles Clippers. For sure he is no longer a player worth 47 millions, but he is not worth 3.8 millions either. Our model interprets pretty well the situation, stating that Westbrook should earn a 18.3 millions salary: not a superstar one, but not a minimum wage either.

Given the presence of correlations between the independent variables, the presence of multicollinearity is likely. For this reason, we decided to implement models that perform well when the variables are collinear such as Ridge regression and Lasso regression. In the next paragraphs we want to see if the performances of these models are better than that of the models seen so far.

Ridge regression

The subset selection method uses least squares to fit a linear model with a subset of the predictors. On the other hand, ridge regression does not select a subset of the coefficients β_j of the model, but it fits a model with all p predictors adding a term $\lambda \sum_{j=1}^{p} \beta_j^2$. This term is called shrinkage penalty, since it has the effect to push the coefficient estimates towards zero. Lambda (λ) is a tuning parameter that controls the impact of the penalty on the estimates. In order to determine a good value for λ, we used cross-validation.

```
## [1] "The best lambda is =  755675"
## [1] "The estimated test MSE with the best lambda is =  5.446854e+13"
```

In the plot of the Figure 8 the red dotted line represents the cross-validation curve along with upper and lower standard deviation curves along the λ sequence (error bars). We chose the value of λ (755675.4) that gives minimum mean cross-validated error. The mean squared error on the test set is 5.45e+13.

The final model was fitted with the best λ on all data. The trace plot, in the Figure 9, shows how the coefficients change if λ increases.

```
## [1] "R-squared =  0.693844944021397"
```

```
## [1] "MSE =  4.344009e+13"
```

Once fitted the model on all data with the best lambda, we evaluated the performance. The 0.69 R-squared highlights a better fit compared to the previous model (0.64) obtained after a subset selection. Also the MSE improves: we get 4.34e+13 instead of the previous 4.77e+13.

The final step is the comparison between real salaries and predicted ones.

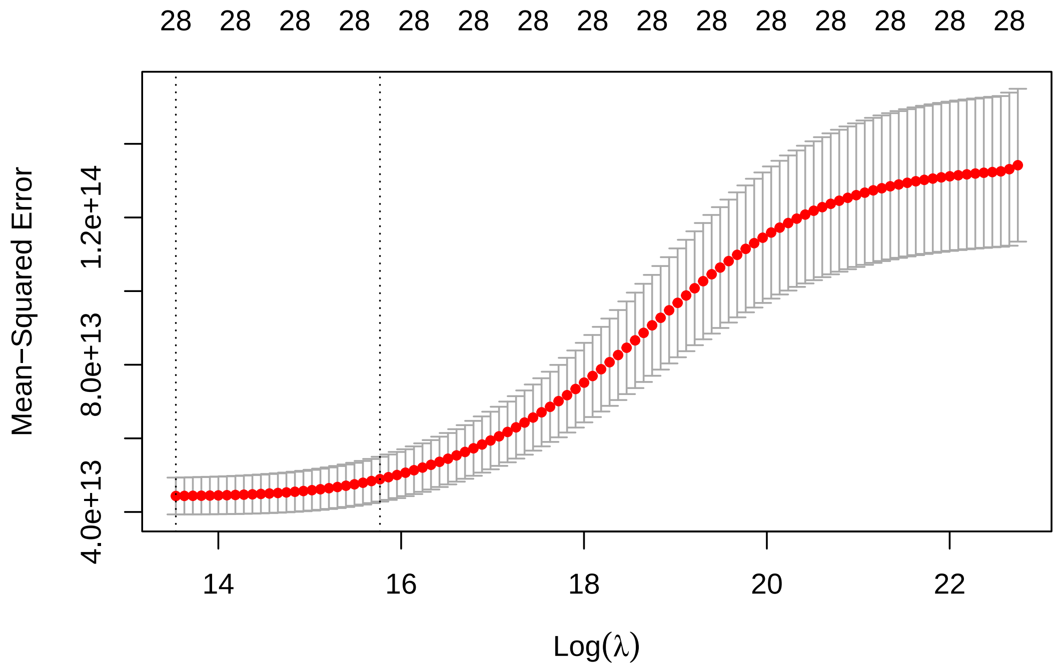

Figure 8: Plot of the MSE with respect to the value of λ in the Ridge regression model

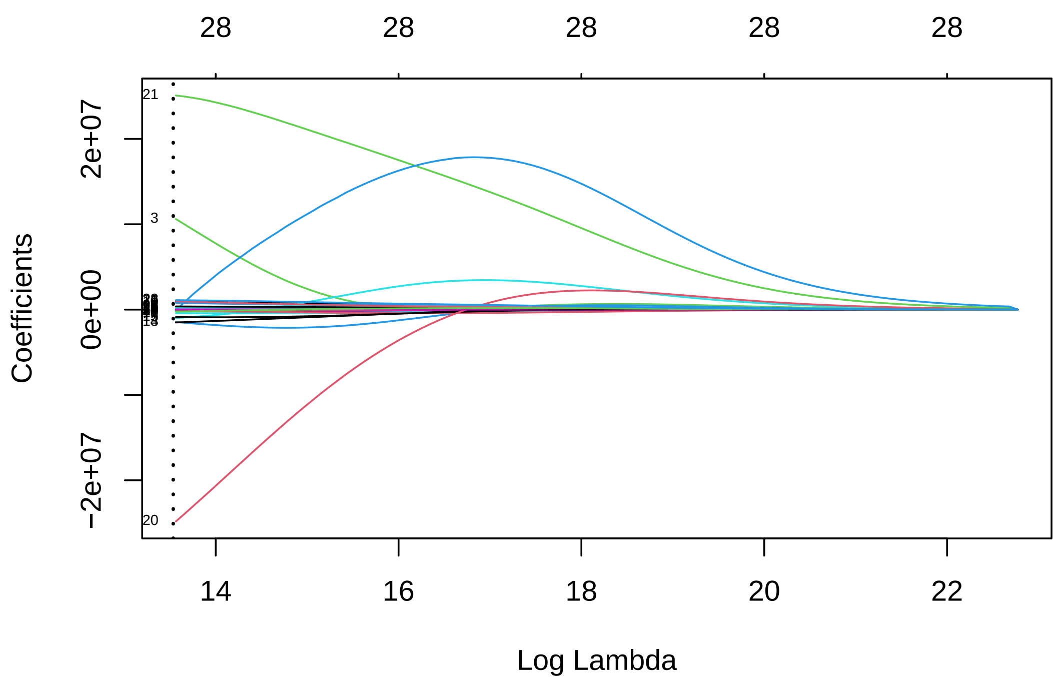

Figure 9: Plot of the values of the parameters with respect to λ in the Ridge regression model

Table 6: Ten most overpaid players according to the Ridge regression model

	Salary	Predicted salary	Difference
Bradley Beal	46741590	23835819	22905771
Jordan Poole	27955357	10348317	17607040
Klay Thompson	43219440	25696260	17523180
Zach LaVine	40064220	22726915	17337305
Deandre Ayton	32459438	15961704	16497734
Darius Garland	34005250	17803526	16201724
Michael Porter Jr.	33386850	17289756	16097094
Tobias Harris	39270150	23308324	15961826
Fred VanVleet	40806300	25497363	15308937
Rudy Gobert	41000000	27551539	13448461

Table 7: Ten most underpaid players according to the Ridge regression model

	Salary	Predicted salary	Difference
Tyrese Maxey	4343920	24414677	20070757
Russell Westbrook	3835738	20700675	16864937
Eric Gordon	3196448	19918750	16722302
Desmond Bane	3845083	20436019	16590936
Tyrese Haliburton	5808435	22145870	16337435
Alperen Sengun	3536280	19253555	15717275
Cam Thomas	2240160	17089707	14849547
Jalen Williams	4558680	18820694	14262014
Kelly Oubre Jr.	2891467	16543402	13651935
Anthony Edwards	13534817	26285734	12750917

MOST OVERPAID PLAYERS

Here there are a different similarities between the previous model: Beal, Poole, LaVine, Ayton, Garland and Porter Jr. still result in the most overpaid players.

Klay Thompson, after being a key piece in the Golden State Warriors dinasty, suffered a serious injury few years ago. After that, he was no longer the same player and the salary was, let's say, no longer adequate to his performance. His contract with Golden State ended after the 23-24 season and he recently signed with Dallas Mavericks for 50 millions in 3 years, thus he will earn a salary closer (even lower) to the predicted one.

Regarding Tobias Harris, this was the last contract year with Philadelphia 76ers. He signed this contract in 2019, team's situation was really different, Harris seemed to be the missing piece to build a contender for the title. After 5 years and a lot of changes, his situation is similar to Thompson's: salary not in line with performance. In fact, he also signed recently with another team (Detroit Pistons) for 52 millions in 2 years, really close to the prediction.

Fred Vanvleet signed a big contract with Houston Rockets last year. The team has a young core, they are in a rebuilding phase so for the moment they don't have ambitions for the title. Without being a contender, teams are less attractive for the superstars. For this reason, they signed a really good player paying him like a superstar: the fact that he results as really overpaid was quite predictable.

Discussions about Rudy Gobert's value are always controversial. He doesn't shine for his technique, he is a so called hustle player: a great defender (three times best defender of the year) who gives a great contribution in terms of intangibles aspects that are really difficult to grasp with stats. It is really difficult to assess his value, especially with this type of model.

MOST UNDERPAID PLAYERS

At first glance we can see that Ridge regression does not classify players with very high salaries (such as Lebron James and Kevin Durant) as the most underpaid. We believe that in this respect the prediction is better than the previous model one.

Again, we find Westbrook, Maxey and Oubre Jr. in this tier. Oubre's last contract was 30 millions in 2 years with Phoenix Suns, so in line with the predicted one. Last year he signed a small 2.89 million one-year contract with Philadelphia 76ers for several reasons: injury history, lack of performance consistency, market dynamics. Probably he will sign a new contract soon.

Eric Gordon is a veteran, he signed for a very small salary with Phoenix Suns in order to play with a contender. This move is not uncommon for good players in the final part of their career, especially if they never won a NBA title like Gordon. In the previous contract with Houston Rockets Gordon earned 75.6 millions in 4 years, perfectly in line with the prediction.

Bane, Haliburton, Sengun, Thomas and Williams have a Maxey-like situation: they are young players which are still in their rookie contracts but they are clearly overperforming considering how much they earn. Maybe the model over evaluates a bit Cam Thomas, because he produces really good offensive numbers (the stats and the models capture the offensive contribution really well, much less the defensive one) when called on but his performance decrease when it comes to defense. Additionally, he could improve in leadership and understanding of the game.

Anthony Edwards had an amazing season, he carried his team to playoffs conference finals. 23-24 season was the last one in his rookie contract (he perceives a higher salaries than the previous mentioned players in their rookie contracts because he was a better prospect when drafted), for this reason he perceived a lower salary compared to the model's expectation. It is really likely that he will receive a big offer in the near future.

All in all, Ridge regression has shown better results compared to the previous model: better R-squared, lower mean squared error and in some cases more meaningful predictions.

Lasso regression

A disadvantage of ridge regression is that, unlike subset selection, it includes all p predictors in the final model. Also lasso regression shrinks the coefficients estimates towards zero but it has an absolute value shrinkage penalty instead of a quadratic one: $\lambda \sum_{j=1}^{p} |\beta_j|$. When λ is sufficiently large, some coefficient estimates become exactly equal to zero. Hence, like best subset selection, lasso performs a variable selection.

```
## [1] "The best lambda is =  86884"
## [1] "The estimated test MSE with the best lambda is =  5.419323e+13"
```

We again used the cross-validation method and we chose the λ value that guarantees the lower mean cross-validated error, as it can be seen in the Figure 10. Once chosen the best λ, the final model was fitted with that λ on all data. The trace plot, in the Figure 11, shows how the coefficient estimates change with increasing λ. Observing the coefficients, 6 are shrunk to zero. Among them, the coefficients of PIE and NET_RATING that were significant in the best subset selection model.

```
## [1] "MSE =  0.69421191533625"
```

```
## [1] "MSE =  4.338802e+13"
```

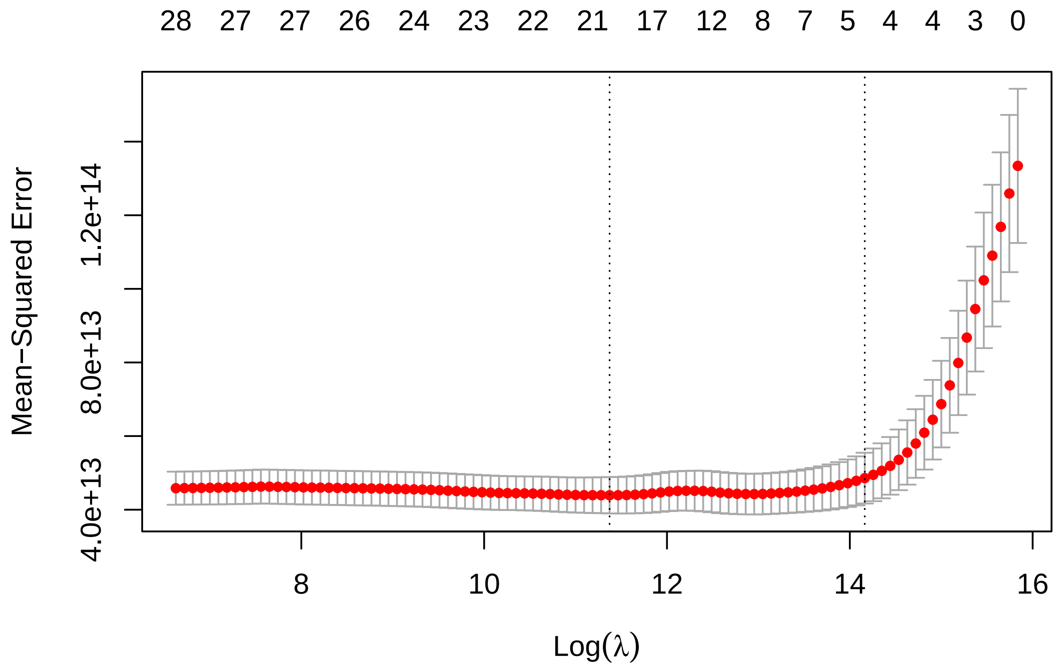

Figure 10: Plot of the MSE with respect to the value of λ in the Lasso regression model

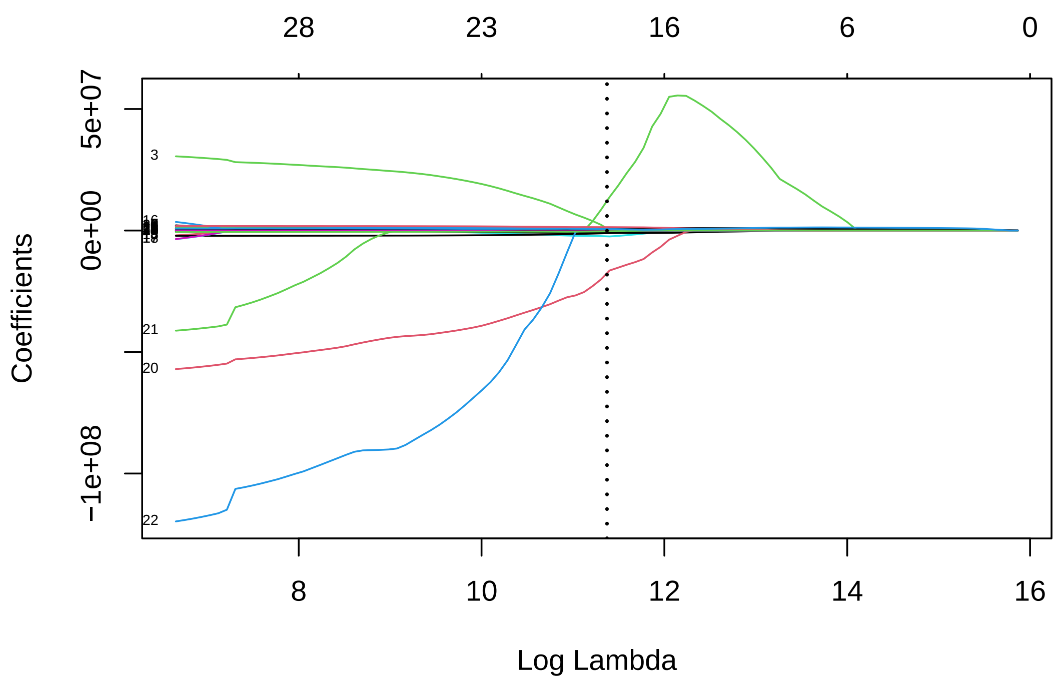

Figure 11: Plot of the values of the parameters with respect to λ in the Ridge regression model

Once fitted the model on all data with the best λ, we evaluated the performances The R-squared is marginally better than that of the ridge, 0.6942 against 0.6938. The Mean squared error is slightly lower compared to the ridge one, 4.338802e+13 against 4.344009e+13. The performance of these two models is very similar and both outperform the model obtained with the best subset selection.

Table 8: Ten most overpaid players according to the Lasso regression model

	Salary	Predicted salary	Difference
Bradley Beal	46741590	24401038	22340552
Jordan Poole	27955357	10171469	17783888
Klay Thompson	43219440	25865270	17354170
Zach LaVine	40064220	23367732	16696488
Darius Garland	34005250	17735816	16269434
Michael Porter Jr.	33386850	17520241	15866609
Deandre Ayton	32459438	16626254	15833184
Tobias Harris	39270150	23634443	15635707
Fred VanVleet	40806300	26335599	14470701
Trae Young	40064220	26643866	13420354

Table 9: Ten most underpaid players according to the Lasso regression model

	Salary	Predicted salary	Difference
Tyrese Maxey	4343920	24351859	20007939
Desmond Bane	3845083	21627917	17782834
Eric Gordon	3196448	20056953	16860505
Russell Westbrook	3835738	20610467	16774729
Tyrese Haliburton	5808435	22042488	16234053
Alperen Sengun	3536280	18956778	15420498
Jalen Williams	4558680	19044255	14485575
Kelly Oubre Jr.	2891467	17305075	14413608
Cam Thomas	2240160	16357848	14117688
Kevin Love	3835738	16608865	12773127

MOST OVERPAID PLAYERS

It is interesting to note that 9 out of 10 players in this table are the same as in the corresponding table for ridge. Also the difference between real salaries and predicted ones is very similar to that of the previous model. The only change is the presence of Trae Young here (he was one the most overpaid players in the best subset selection model) instead of Rudy Gobert in the ridge.

MOST UNDERPAID PLAYERS

Also in this case, 9 out of 10 players are the same as in ridge and the differences are really small. The only change is the presence of Kevin Love. As Eric Gordon, he is a veteran and he signed for a small salary with Miami Heat.

After analyzing these three models, it emerges that ridge and lasso regression outperform the model obtained through the best subset selection. Additionally, the predictions for the most underpaid players seem to be more reasonable in ridge and lasso regression. In conclusion, lasso regression results (even only slightly compared to the ridge regression) the model that best fits our data.

Models per position