Division Algorithm in $\mathbb{Z}$

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1. State and prove the Division Algorithm in $\mathbb{Z}$.

2. Prove that the quotient q and remainder r in the Division Algorithm are unique.

3. Find q and r when $a = 93$, $b = 8$ such that $a = bq + r$, where $0 \leq r < |b|$.

4. If $a = -45$ and $b = 7$, find q and r satisfying the Division Algorithm.

5. Verify the Division Algorithm for $a = 127$ and $b = 11$.

6. Find the remainder when $a = 500$ is divided by $b = 13$.

7. Determine all integers a for which the remainder $r = 0$ when divided by $b = 5$.

8. Show that for all integers a, b $(b \neq 0)$, there exist unique integers q, r such that $a = bq + r$ with $0 \leq r < |b|$.

9. Use the Division Algorithm to compute $\gcd(252, 105)$.

10. Apply the Euclidean Algorithm to find $\gcd(270, 192)$ and express it as a linear combination of a and b.

11. If $a = bq + r$ and $r = 0$, show that b divides a.

12. Show by example that the Division Algorithm does not hold when $b = 0$.

13. Let $a = -25$, $b = -4$. Find integers q, r satisfying $a = bq + r$ with $0 \leq r < |b|$.

14. For $a = 10n + 3$, find the remainder when a^2 is divided by 10.

15. If a leaves remainder r when divided by m, show that a^2 leaves remainder r^2 when divided by m if and only if m divides $r(r - 1)$.

16. Find the remainder when $7^2, 7^3$, and 7^4 are divided by 5.

17. Compute the remainder when 9^3 and 9^4 are divided by 7.

18. Prove that for any integer a, $a^2 \equiv 0$ or $1 \pmod 4$.

19. Show that the square of any odd integer is congruent to $1 \pmod 8$.

20. Explain the role of the Division Algorithm in finding patterns of integer powers (square, cube, and fourth power) modulo n.
