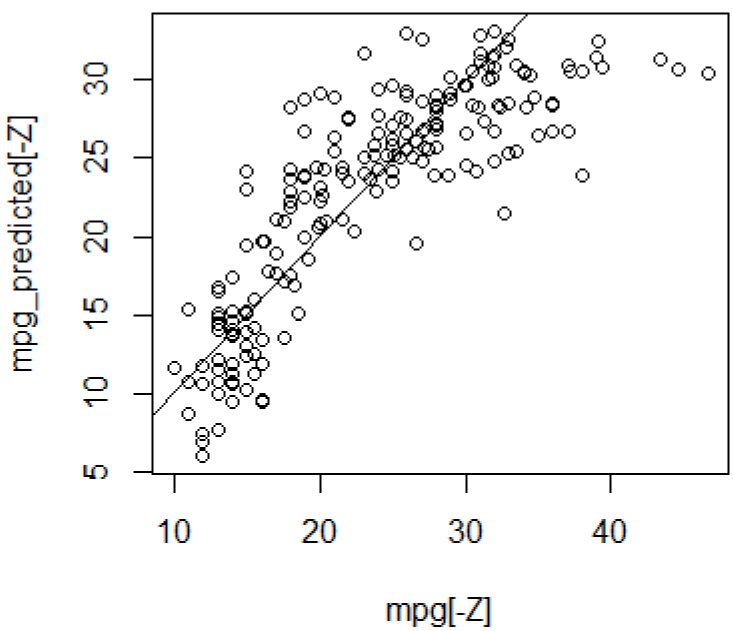
CROSS-VALIDATION

1. Validation Set Approach

> library(ISLR2)
> attach(Auto); n = length(mpg);
> n
[1] 392
> Z = sample(n, 250) **# Random subsample of size 250 (does not have to be n/2)**
 # Works similarly to generating a Bernoulli variable
> reg.fit = lm(mpg ~ weight + horsepower + acceleration, subset=Z) **# Fit using training data**
> mpg_predicted = predict(reg.fit, newdata=Auto[-Z ,])
 # Use this model to predict the testing data [-Z]
> plot(mpg[-Z],mpg_predicted) **# We can see a nonlinear component**
> abline(0,1) **# Compare with the line y=x.**



> mean((mpg[-Z] - mpg_predicted)^2) **# Estimate the mean-squared error MSE**
[1] 18.58128

2. Jackknife (Leave-One-Out Cross-Validation, LOOCV)

"Manually":

> Yhat = numeric(n)
> for (i in 1:n){
+ reg = lm(mpg ~ weight + horsepower + acceleration, data=Auto[-i,])
+ Yhat[i] = predict(reg, newdata=Auto[i,])
+ }
> plot(mpg,Yhat)

```
> mean((mpg-Yhat)^2)
[1] 18.25595
```

Using package "boot":

```
> install.packages("boot")
> library(boot)
> glm.fit = glm(mpg ~ weight + horsepower + acceleration)  # Default "family" is Normal, so this GLM model
> cv.error = cv.glm( Auto, glm.fit )                        # is the same as LM, standard linear regression.
> names(cv.error)                                           # This cross-validation tool has several outputs
[1] "call"  "K"     "delta" "seed"                         # We are interested in "delta"

> error$delta
[1] 18.25595 18.25542
```

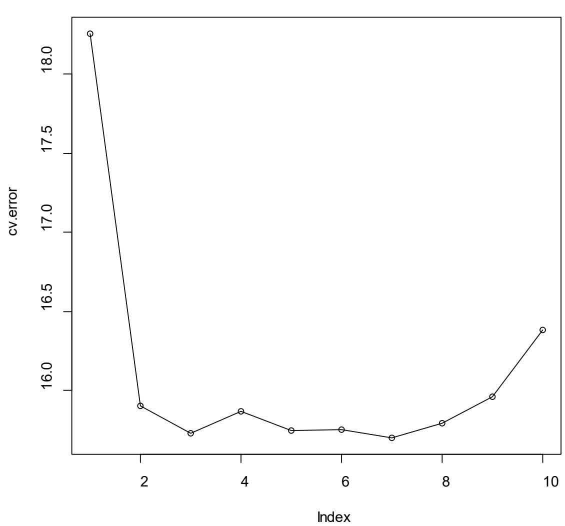
 # Delta consists of 2 numbers – estimated prediction error and its version adjusted for the lost sample size due to cross-validation.

 # Example – what power of "horsepower" is optimal for this prediction?

```
> cv.error = rep(0,10)                                               # Initiate a vector of estimated errors
> for (p in 1:10){                                                   # Fit polynomial regression models
+ glm.fit = glm( mpg ~ weight + poly(horsepower,p) + acceleration )  # with power p=1..10
+ cv.error[p] = cv.glm( Auto, glm.fit )$delta[1]  }                  # Save prediction errors
> cv.error                                                           # Look at the results
 [1] 18.25595 15.90163 15.72995 15.86879 15.74517 15.74989 15.70073 15.79314 15.95933 16.38301
```

 # Although p=7 yields the lowest estimated prediction error, after p=2, the improvement is very little.

```
> plot(cv.error)
> lines(cv.error)
```



3. K-Fold Cross-Validation

 # We can specify K within cv.glm (Omitted K is K=1 by default, which is LOOCV).

```
> cv.error = rep(0,10)
> for (p in 1:10){ glm.fit = glm( mpg ~ weight + poly(horsepower,p) + acceleration )
+ cv.error[p] = cv.glm( Auto, glm.fit, K=30 )$delta[1]    }
> cv.error
 [1] 18.22699 15.87030 15.73565 15.85911 15.80686 15.71585 16.09625 15.67797 15.82097 16.48196
> which.min(cv.error)
[1] 6
```

4. Cross-validation in classification problems. Loss function.

 # Command cv.glm, as we used it above, calculates MSE, the mean squared error, and calls them "delta".
 # In classification problems, MSE can be used to measure the distance between the response variable Y
 # and the predicted probability p. For this, Y has to be 0 or 1, and p should be the probability of Y=1.
 # However, the correct classification rate and the error classification rate are more standard measures
# of classification accuracy. We can force cv.glm to return these measures by introducing a suitable loss
 # function. For example, we'll be predicting whether a student is depressed or not.

 # We define a loss function L(Y,p), a function of true response Y and predicted probability p. The loss = 1 if
 # the predicted response is different from the actual response and 0 otherwise. Suppose the threshold is 0.5

```
> loss = function(Y,p){ return( mean( (Y==1 & p < 0.5) | (Y==0 & p >= 0.5) ) ) }
> loss(1,0.3)
[1] 1
> loss(c(1,1),c(0.3,0.7))
[1] 0.5
```

 # Now we attach the Depression data, skip missing values, fit logistic regression model, and estimate
 # the error classification rate by LOOCV.

```
> Depr = read.csv(url("http://fs2.american.edu/~baron/627/R/depression_data.csv"))
> D = na.omit(Depr)
> attach(D)
> library(boot)
> lreg = glm(Diagnosis ~ Gender + Guardian_status + Cohesion_score, family="binomial")
> cv = cv.glm( D, lreg, loss )
> cv$delta[1]
  [1] 0.1615721
```

 # Is Guardian_status **significant? Let's compare the error rates with and without variable "**Guardian_status**".**

```
> lreg = glm(Diagnosis ~ Gender + Cohesion_score, family="binomial")
> cv.glm( D, lreg, loss )$delta[1]
[1] 0.1572052
```

 # The error classification rate is lower without the "Guardian_status".