Transmission Valuation Analysis based on Real Options with Price Spikes

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Abstract The presence of optionality in the generation and transmission of power means that valuing physical and financial assets requires using option theory, which in turn requires studying stochastic processes appropriate for the description of power prices. Power prices are much more volatile than other commodity prices and exhibit interesting behavior such as regime switching between normal and spiked states. The probability distributions underlying such stochastic process provide an input for price forecasts, which are based on price history. They also provide an input into valuation of transmission and transmission options in cases where the implied market-based measures of volatilities and correlations are lacking. Combining information from this analysis of stochastic processes for power prices with the Black–Scholes framework for option valuation, specifically using that framework to calculate the value of spread options, yields methods for calculating the value of transmission as well as for calculating the value of financial transmission options, which also depend on spread of power prices. The three main techniques for obtaining the option value include analytical approaches, binomial-type trees (finite difference methods), and Monte Carlo simulations. Each of these techniques presented in the paper has its own advantages and disadvantages and is complementary to the other two, providing independent validation and quality control for transmission valuation algorithms.

Keywords Options · Power · FTRs · Price · Spikes · Transmission · Valuation · Congestion.

1 Introduction

Energy companies often need to move energy from one location to another. These transactions involve the transmission of certain quantity of power from a generator location to a load location. Another example of a transaction is the transportation of

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certain volume of gas via pipeline from a gas storage location to a power plant location. Crude oil often has to be transported to refineries by tankers and then through pipelines to the power plants. Also, energy companies convert one commodity into another. An example of such conversion include burning of fuels, such as natural gas, coal, or oil, to produce electricity at power plants, or the refinery operations refining crude oil into its products.

Common to all aforementioned transactions is the move of a commodity from a place of relative abundance and lower price to a place of scarcity and higher price or conversion of a cheaper commodity into a more expensive one. This makes the operation of transporting or converting the commodity profitable. Moreover, the operation itself is not a zero-sum game and often is worth a lot of money for its potential to continue the delivery or conversion and turning a profit. The value of this operation depends on the costs of delivery or conversion and the difference between the prices at two locations or the prices of two commodities. Once the payoff function is established, one can price the transaction using methods described in this paper, and hence establish a value of the transportation and conversion, whether represented by physical assets or financial instruments.

The financial and energy trading community considers these transactions as spread obligations or spread options. A variety of other names has also been used; these include exchange, transportation, tolling, spark-spread obligations, or options, depending on the commodities exchanged and the nature of transaction. The so-called structured deals can combine a number of simpler transactions into one complex deal.

This paper will focus on the physical transmission of power and associated financial instruments. The examples of such transactions include the following:

- Physical rights to transmit power on transmission lines
- Financial Transmission Rights (FTRs)/Congestion Revenue Rights (CRRs)/ Transmission Congestion Contracts (TCCs)
- Tolling agreements (options)
- FTR options

The paper will focus on the valuation of such transmission-based instruments. We will consider obligations, contrast them with options, and highlight the impact of the optionality on the value of the asset or the financial instrument. We will also examine the impact of the price process model on the valuation of obligations and options.

Post electric deregulation, the transmission assets are owned by Independent Transmission Companies or separated by a "firewall" from the commercial operations to ensure equal access. In the areas where transmission is scheduled by the Independent System Operators (ISOs) or Regional Transmission Organizations (RTOs), market participants, such as generators or Load Serving Entities (LSEs), can enter only into financial transactions on transmission of power within the respective control areas. In regions where the vertically integrated structure still prevails, the market participants may combine financial instruments on transmission with physical reservations of capacity on the transmission lines.

Financial obligations or options on transmission may be available from a centralized market or available only from other market participants. The latter are obtained over-the-counter (OTC). One example of the centralized auction of transmission obligations and options is the FTR/CRR/TCC auction run by an ISO or a RTO with the purpose to auction off the transmission capacity of the network they control to various market participants. The participants use these instruments to hedge their exposure to locational price differential at various locations in the transmission grid when they move power for generation locations or trading hubs, or load aggregation points (LAPs) to load locations. Many OTC transactions in the market place that are outside the domain of an ISO or a RTO are tailored to individual needs of generators and load. Examples of such OTC transactions include tolling agreements (options) to transport power.

The rest of the paper is organized as follows. The next section reviews models of commodity prices in general and power prices in particular. The most appropriate and widely used model is the single factor model. One possible extension is the multifactor family of models, which we briefly describe in the next section as well. The most important step for the purpose of transmission valuation is the introduction of spikes into the commodity prices. This issue is mostly acute in markets where the probability of surges in power prices occasionally changes the dynamics of the marketplace. After the model for the price process is established we proceed to describe the valuation methods, starting with simpler models, which include simplifying approximations and proceeding towards the presentation of models that provide an exact solution. We also discuss the incorporation of spikes into the solution.

Finally, we close with the discussion of the numerical solutions for the methods we have introduced. The simplest approaches have closed form solutions and thus can be evaluated in terms of standard functions. The exact solution requires numerical integration and should be done with proper controls to ensure convergence. One of the most intuitive numerical schemes is a binomial tree approach that builds on a similar approach for valuations of derivatives contingent on one underlying asset or commodity. Finally, we present the Monte Carlo method, which provides a simulation-based approach to the valuation of spread options.

2 Behavior of Commodity Prices

2.1 Models of Spot and Forward Commodity Prices

The single factor model for energy prices is mathematically the simplest approach for modeling energy prices. As the name for this approach suggests, there is only one source of uncertainty in the prices that can be modeled as geometric mean-reverting processes of log type (Schwartz 1997) with time varying mean

$$dS = \eta(m(t) - \ln S)S\,dt + \sigma_0 S\,dz \tag{1}$$

where $S = S(t)$ is the stochastic process of energy spot prices, $m(t)$ is the general trend, η is reversion rate, σ_0 is the standard deviation of the error term (volatility), and dz is the associated white noise process. This single source of uncertainty is the volatility of spot prices. More information about the properties and reasons for the choice of this process can be found in Rosenberg et al. (2002b). It can be shown (Clewlow and Strickland 1999) that this assumption about the spot price, combined with the fact that in a risk-neutral world the futures price of a commodity is equal to its expected spot price, leads to the following equation for the futures commodity price at time T:

$$dF(T) = F(T)\sigma(T)dz \tag{2}$$

where F is the futures price and σ is the corresponding volatility. Equation (2) is the right candidate for the description of future prices. The geometric Brownian motion with zero drift means that there is no preference as to the direction of the change of future prices in the risk-neutral world. In other words, all currently available information was absorbed by the market into the current price of the futures. Again, we must distinguish between the risk-neutral distribution under which valuation takes place and the true/objective/statistical distribution. That true/objective/statistical distribution may have a nonzero drift owing to the presence of a market price of risk (MPR) (detailed general discussions of the MPR can be found in Hull 2002 and Bjork 2004, and a discussion of this in energy markets can be found in Ronn 2002),

$$dF(T) = \mu_1(F,T)dt + F(T)\sigma(T)dz \tag{3}$$

where μ_1 is the nonrandom drift. The variable that determines a particular form of future $F(T)$ in (2) and (3) is the volatility term structure $\sigma(T)$. One of the simplest assumptions consistent with empirical observations for certain period of time is the exponentially decaying volatility term structure:

$$\sigma(T) = \sigma_0 e^{-\eta T} \tag{4}$$

Moreover, the assumption of geometric mean-reverting process (1) requires that the rate of decay is equal to the mean-reverting parameter and, in the limit $T \to 0$, the volatility of future converges to the volatility of spot (Clewlow and Strickland 1999). In practice, the observed nonzero asymptotic value of volatility term structure is the evidence of more complex forward curve dynamics than is allowed by one factor model (Smith and Schwartz 2003). We shall not concern ourselves with these details as they are not important for the purpose of this paper.

2.2 *Stochastic Modeling of Price Spikes*

The time plot on Fig. 1 reveals a rather irregular behavior of daily electricity prices. One can clearly notice sharp and relatively short-term surges of prices – spikes. They are caused by an abrupt increase in the demand of energy, which may be a result of a combination of extreme conditions. Behavior of electricity prices during

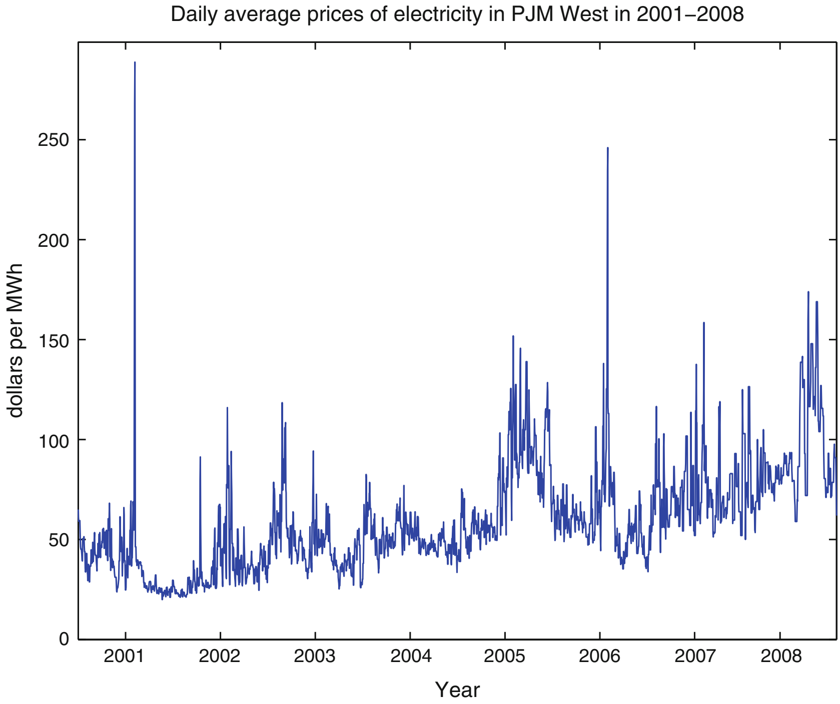

Fig. 1 Daily electricity prices in Pennsylvania–New Jersey–Maryland West region (Data Source: Intercontinental Exchange – theice.com)

spikes is dramatically different from the periods between spikes. Spikes last from several hours to several days, but they can lead to a tremendous price increase (Baron et al. 2001, 2002).

We conclude that, for the process of electricity prices with spikes, (1–3) require different sets of parameters for its two phases, the spike periods and interspike periods. This leads to a *multistate* model (Baron 2004), which represents a nonstationary stochastic process consisting of homogeneous segments (spikes and interspike periods) separated by *change points.* Each change point marks the moment when one segment ends and the next one begins. To model such a process, one has to determine the distribution of prices during each type of segments and the stochastic mechanism of switching between the segments.

2.2.1 Interspike Segments (Regular Mode)

At any time t, (1) yields a *lognormal* distribution of prices with the density

$$f_0(x) = \frac{1}{x\tau_0\sqrt{2\pi}} \exp\{-(In\, x - m(t))^2/2\tau_0^2 \tag{5}$$

with $m(t)$ and τ_0 representing the time dependent mean and volatility. This is a homogeneous (stationary) segment of a stochastic process that is located between two successive change points (Spokoiny 2009). Moreover, since only a finite number of observations can be available, any data satisfying model (1) forms *an autoregressive process of order* 1 or AR(1) (Dixit and Pindyck 1994, Section 3.B). That is, electricity prices $S(t)$ and $S(t-1)$ are connected by the stochastic *difference* equation

$$\ln S(t) - m(t) = \phi\{\ln S(t-1) - m(t-1)\} + \sigma_0 Z(t) \tag{6}$$

for a Gaussian white noise sequence $Z(t)$ and $t = 1, 2, \ldots$. Parameters of this equation are related to (1) and (4) through the formulas $\phi = e^{-\eta}$ and $\sigma_0 = \tau_0\sqrt{1-\phi^2}$. Here ϕ, σ_0, and $\tau_{0,}$ are parameters of the named distributions: ϕ is the first autoregressive coefficient that is related to the reversion rate, τ_0 is the standard deviation of log-price at any time, and σ_0 is the error standard deviation of the AR(1) process of log-prices.

2.2.2 Spikes with Varying Magnitudes (Spike Mode)

According to Fig. 1, electricity prices during one spike can differ very significantly from those during another spike. Different spikes are characterized by different location parameters μ that show the magnitude of a spike. Variable values of μ can be viewed as realizations of a random variable M. This yields a *Bayesian model* (where a parameter has its own distribution, see, e.g., Carlin and Louis 2008) with a lognormal *conditional* density of prices:

$$f_1(x|\mu) = \frac{1}{x\tau_1\sqrt{2\pi}} \exp\left\{-(\ln x - \mu)^2/2{\tau_1}^2\right\}, \tag{7}$$

conditioned on parameter μ, and a normal *prior* distribution of μ,

$$\pi(\mu) = \frac{1}{\omega\sqrt{2\pi}} \exp\{-(\mu-\theta)^2/2\omega^2\}$$

Parameters of this prior distribution have the following meaning. The prior mean, θ, is the average magnitude of spikes, while the prior standard deviation, ω, quantifies the variability of spike magnitudes and how much the spikes can differ from each other.

This Bayesian model reflects the fact that electricity prices are dependent on the same spike (because they are based on the same parameter μ), but independent between different spikes. This reflects the real situation. Indeed, when the price of electricity reaches a rather high mark during a spike, it stays around that level almost until the end of the spike, not affecting, however, the next spike. This Bayesian model can be developed further by letting the prior mean θ of the distribution of

μ to depend on time, $\theta = \mathbb{E}(\mu) = \theta(t)$. This time trend is high during the peak season, gradually reducing during the shoulder months.

2.2.3 Change of Modes

Transitions between the segments are governed by a *Markov chain* with the *transition probability matrix:*

$$P = \begin{pmatrix} 1 - p(t) & p(t) \\ q & 1 - q \end{pmatrix}, \tag{8}$$

where transitions from a regular state to a spike occur with probability $p(t)$, and transitions from a spike to a regular state occur with probability q. Consequently, $\{1 - p(t)\}$ is the probability that a day with no spike is followed by another day with no spike, and $\{1 - q\}$ is the probability for a spike to extend for one more day. Noticeably, $p(t)$ is a time-dependent probability because the chance of a new spike has a seasonal and weekly component. For example, it increases during a peak season and practically vanishes on weekends. On the other hand, the spike-ending probability q is constant, confirming geometric distribution of spike durations.

Estimation of parameters in such a multistate process requires statistical *change-point detection* techniques (Basseville and Nikiforov 1993; Zacks 2009, Chap. 9) to separate the segments as well as standard tools for estimating parameters of segments. Parameters of each state are estimated separately by standard available tools. Monthly and weekly components as well as the general trend are estimated by the method of weighted least squares to reduce the influence of possible spikes on estimates. Then, an autoregressive time series model is fit to detrended log-prices, which are residuals obtained after the estimated trend is subtracted from the observed log-prices. This results in estimates of the autoregressive coefficient and residual variance (volatility). Substituting the estimates of ϕ, η, σ_0, and τ_0 into (4) and (5), we obtain the interspike distribution of electricity prices. This is all we need for the initial estimation of change points between the segments by the sequential cumulative sum algorithm (CUSUM, as in Basseville and Nikiforov 1993; Khan 2009; Zacks 2009, Sect. 9.3). Estimated change points partition the observed time series into spike and interspike segments. Given this, the interspike parameters can be re-estimated based on interspike segments only. Also, we can estimate the spike parameters. Under the assumed Bayesian model, the unconditional distribution of detrended log-prices during spikes is normal with parameters θ and $\omega^2 + \tau_1^2$. Consequently, θ is estimated by the sample mean of all the residuals during spikes and ω^2 and τ_1^2 as the between and within mean square in the one-way analysis of variance during spikes. With the refined parameter estimates, the CUSUM algorithm can be repeated for more accurate estimation of change points. This iterative scheme will be concluded when a new round of CUSUM makes no changes in the obtained change-point estimates. For more details, see Baron (2004).

3 Valuation of Obligations and Options

This section reviews the methodology for valuing obligations and options on transmission. These two classes of transactions require drastically different valuation methods. We begin by looking into valuation of obligations.

3.1 Valuation of Transmission Obligations

Transmission obligation is either a financial gain or loss derived from holding a financial transaction structured to equal the difference between the prices at two locations and may be either positive or negative at any moment in time, depending on which locational price is higher. Examples of such transactions include (a) Financial Transmission Rights (FTRs)/Congestion Revenue Rights (CRRs)/Transmission Congestion Contracts (TCCs) and (b) Basis spread transactions (going long futures or forwards at one location and shorting the same amount of futures or forwards on the same commodity at another location).

If the forwards for each location in the obligation are liquidly traded in the market, the expected value of the difference is currently observed by a holder of the obligation and is equal to the difference of the observed prices. If the forwards/futures are not liquidly traded, such as FTRs and similar nodal contracts, the best estimate of the expected value of the spread can be estimated from forecasting the future spot prices and inferring for the MPR (see the MPR discussion in Ronn 2002). Once both parameters are estimated, the expected value of spread on futures or forwards may be computed. Such value would represent the expected market-transacted price for the purchase or sale of the spread, if such transaction did take place.

The value of such obligations to each market participant depends on his risk aversion, which, unlike MPR, is idiosyncratic to each participant and partly depends on the purpose for which he enters into such transactions. Therefore, one should not confuse the equilibrium or equilibrium-estimated price with the worth of the instrument to individual participant. This is true for obligations, as well as options to which we now turn to.

3.2 Valuation of Spread Options

In this section we discuss methods for valuing options for energy producing and delivery systems. Option valuation is much more complex than the valuation of obligations described in the previous subsection. Given the variety of options and models for energy commodities, such an undertaking could easily become unwieldy. We shall solve this problem by focusing on a specific kind of option, *viz.*, an option on the spread between two futures, and a specific kind of model for the futures prices, *viz.*, single factor geometric Brownian motion. In this subsection we try to illustrate a large number of mathematical techniques that are useful for valuing *any* option on an energy commodity. Given the practical emphasis of this paper, we make no claims of mathematical rigor or completeness, but instead we concentrate on sketching the most basic information and reference sources with more complete information.

We use a simple model of futures prices. Later in the paper we will move beyond the simple model analyzed here to discuss the valuation of spread options when there are spikes in the prices of the underlying asset. A paper that analyzes a spread option with a very simple generalization of single factor geometric Brownian motion in the underlying asset is Alexander and Scourse (2004). The correlation coefficient, which plays a major role in the following analysis, is less well suited for describing non-normal distributions. An influential and lucid discussion of these problems is in Embrechts et al. (1999). A popular alternative to the use of correlation coefficients to describe multivariate probability distributions is the use of copula functions. Cherubini and Luciano (2002) use copulas to analyze a general spread option. Gregoire et al. (2008) describe the use of copulas to model price dependence in energy markets. We hope that the choice of a simple, but very important, option and a simple, but very flexible and useful, model for futures prices will assist the reader clearly understand these useful valuation techniques. Further, we hope that they will provide sufficient foundation to understand and analyze more complex and realistic models, such as the models we describe later in the paper and the models described in the above references.

A versatile and conceptually straightforward method of valuing options consists of deriving and solving a differential equation for the option value. Here we shall display and discuss the differential equation for the value of an option on two futures contracts. As mentioned earlier, we are modeling the behavior of the futures prices by a simple, single factor model. We denote the two futures prices by F_1 and F_2, and we assume that the futures prices have volatilities of σ_1 and σ_2, respectively. With these assumptions, the stochastic differential equations,

$$dF_1(t,T) = \mu_1(F_1, F_2, t)dt + \sigma_1(t,T)F_1(t,T)dz_1(t) \tag{9}$$

$$dF_2(t,T) = \mu_2(F_1, F_2, t)dt + \sigma_2(t,T)F_2(t,T)dz_2(t) \tag{10}$$

where z_1 and z_2 represent standard Wiener processes, describe the behavior of the futures prices. The relation between the two Weiner processes z_1 and z_2 accounts for the correlation between the returns of the futures prices. To describe this correlation more precisely, we define the operator $\mathbb{E}[x]$ to represent the expectation (average) of x; then the correlation coefficient, ρ, of the returns of the two futures is defined so that

$$\mathbb{E}[dz_1(t)dz_2(t)] = \rho(t)dt \tag{11}$$

This correlation coefficient is the usual correlation coefficient between price returns that trading organizations compute. We denote by $V(F_1, F_2, t)$ the value of a financial instrument that depends on the values of the futures prices F_1 and F_2 and the time t. With this model and the assumption of no-arbitrage, the value of the financial instrument must satisfy the differential equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 F_1^2 \frac{\partial^2 V}{\partial F_1^2} + \rho\sigma_1\sigma_2 F_1 F_2 \frac{\partial^2 V}{\partial F_1 \partial F_2} + \frac{1}{2}\sigma_2^2 F_2^2 \frac{\partial^2 V}{\partial F_2^2} - rV = 0 \tag{12}$$

It is important to note that, although here we will focus on spread options, this differential equation (12) is applicable to *any* financial instrument with a value that depends only on the value of two futures prices and time. A derivation of the differential equation (12) based on the methods that would be required to perfectly hedge the options can be found in Rosenberg et al. (2002a), and a more mathematical derivation, based on the Feynman–Kacs formula, can be found in Carmona and Durrleman (2003). (Pedagogical discussions for the Feyman–Kacs formula, which can be used to relate risk-neutral expectations to solutions of partial differential equations, can be found in Bjorck (2004) and Shreve (2004)). A more general version of (12) for n futures or assets is derived in Bjorck (2004), and can also be found in the standard text of Hull (2002).

What differentiates between the many possible financial instruments that have this property and therefore have a value that is described by (12)? To answer this question we note that we have not completely specified the mathematical problem by showing that the financial instrument must satisfy (12); to complete the specification of the mathematical problem we must also specify what happens at the boundaries of the regions where $V(F_1, F_2, t)$ is defined. These boundary conditions differentiate between different financial instruments that depend on the values of two futures prices and time. In the case of a spread option, the relevant boundary is the expiration time, T, of the option and the boundary value is the payoff, $P(F_1, F_2)$. To make these ideas concrete, consider an option to exchange a_1 futures contracts of type 1 for a_2 futures contracts of type 2 plus a fixed sum K, which we will call the strike price, all at an agreed upon future time, the expiration time of the option, which we denote by T. This is a spread option and it satisfies (12) for the value of a general financial instrument, and in addition the value of the option at expiration is known to be

$$V(F_1, F_2, T) = P(F_1, F_2) = \max[a_1 F_1(T) - a_2 F_2(T) - K, 0] \tag{13}$$

Note that K can be positive or negative. For future reference, also notice that the differential equation (12) is time homogeneous, by which we mean that it is not changed by the transformation: $t \to t + \delta t$. The only time that matters in the value of the option is the time until expiration, $\tau \equiv T - t$. In this paper we are concerned with European-type exercise options. For these options the boundary conditions are given by the payoff of the options at expiration. Most energy options are European-type exercise.

3.2.1 Closed-form Solutions

Valuing a spread option becomes much easier if the strike equals to zero, that is, $K = 0$. In that case, as Rosenberg et al. (2002a) prove, a simple dimensional analysis suggests that the option value has the form

$$V(F_1, F_2, t) = F_2 h(z, t) \tag{14}$$

where we have defined as the ratio of the values of the two futures prices

$$z = \frac{F_1}{F_2} \tag{15}$$

In terms of the new variable z, the payoff of the exchange option, given in (13), becomes

$$P(F_1, F_2) = a_1 F_2 \max\left[z - \frac{a_2}{a_1}, 0\right] = F_2 \varphi(z) \tag{16}$$

where we have defined the function $\varphi(z)$ as

$$\varphi(z) = a_1 \max\left[z - \frac{a_2}{a_1}, 0\right] \tag{17}$$

where a_1 and a_2 are the number of futures contracts that are being exchanged. (Good introductions to dimensional analysis include Barenblatt (1996), Barenblatt (2003), Bridgeman (1931), and Wilson (1990) Using the chain rule of calculus, the differential equation (12) for the value of the option becomes a differential equation for $h(z)$,

$$\frac{\partial h}{\partial t} + \frac{1}{2}\tilde{\sigma}^2 z^2 \frac{\partial^2 h}{\partial z^2} - rh = 0 \tag{18}$$

where we have defined the effective volatility

$$\tilde{\sigma} = \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2} \tag{19}$$

where σ_1 and σ_2 are the volatilities of the two futures prices and ρ is the correlation coefficient of the returns of the two futures prices. The boundary condition for $h(z)$ at the expiration time T is given by

$$h(z, T) = \varphi(z) = a_1 \max\left[z - \frac{a_2}{a_1}, 0\right] \tag{20}$$

Now notice that the differential equation (18) for $h(z, t)$ together with the boundary condition (20) describe a_1 vanilla call options on a futures price z, having a strike price of $^{a_2}/_{a_1}$ and a volatility of $\tilde{\sigma}$ (Hull 2002), and so the solution of the differential equation can be obtained directly from the Black-76 formula for an option on a futures contract (derived in Black 1976):

$$h(z,t) = a_1 e^{-r\tau}\left[zN(d_1) - \left(\frac{a_2}{a_1}\right)N(d_2)\right] \tag{21}$$

where we have defined

$$d_1 = \frac{1}{\tilde{\sigma}\sqrt{\tau}}\log\left(\frac{a_1 z}{a_2}\right) + \frac{\tilde{\sigma}\sqrt{\tau}}{2} \tag{22}$$

$$d_2 = \frac{1}{\tilde{\sigma}\sqrt{\tau}}\log\left(\frac{a_1 z}{a_2}\right) - \frac{\tilde{\sigma}\sqrt{\tau}}{2} \tag{23}$$

and $N(x)$ represents the cumulative normal distribution. We remind the reader again that $\tau = T - t$ represents the time until the option expires. Substituting this expression for $h(z,t)$ into (14) for $V(F_1, F_2, t)$ and using the definition (15) of z yields the desired value of an option to exchange two futures contracts,

$$V(F_1, F_2, t) = e^{-r\tau}\left[a_1 F_1 N\left(d_1'\right) - a_2 F_2 N\left(d_2'\right)\right] \tag{24}$$

where

$$d_1' = \frac{1}{\tilde{\sigma}\sqrt{\tau}}\log\left(\frac{a_1 F_1}{a_2 F_2}\right) + \frac{\tilde{\sigma}\sqrt{\tau}}{2} \tag{25}$$

$$d_2' = \frac{1}{\tilde{\sigma}\sqrt{\tau}}\log\left(\frac{a_1 F_1}{a_2 F_2}\right) - \frac{\tilde{\sigma}\sqrt{\tau}}{2} \tag{26}$$

Equations (24)–(26) represent the value of the exchange option and are simply the Margrabe formula (Margrabe 1978) modified so that they apply to futures contracts. The Black-76 formula and the Margrabe formula are both standard results. Clear derivations of both can be found in Hull (2002).

3.2.2 Solutions for Nonzero Strike

The next level of complexity for exchange options is the option to exchange with a fixed strike. The simplification we used to produce the Margrabe formula (24)–(26) does not work for this case. In terms of the dimensional analysis of Rosenberg et al., this simplification occurred because all three of the variables or parameters in the problem that had dimensions of money were explicitly present in the differential equation. These three variables could combine to produce only two dimensionless variables, and so rewriting the differential equation in terms of the two dimensionless variables reduced the number of variables in the equation. In an option with a fixed strike, there exists another parameter with the dimensions of money, *viz.*, the fixed strike. This parameter occurs in the payout, and hence the boundary condition for the differential equation, but not in the differential equation itself, so that dimensional analysis cannot be used to reduce the number of variables in the differential equation. The value of a spread option with a nonzero strike can be solved exactly; however, the expressions are more complex than the Margrabe formula that gives the solution with a zero strike. Specifically, the value of the option can be expressed as a double integral. This solution is derived explicitly in Rosenberg et al. (2002b) and in Carmona and Durrleman (2003). (Mathematically oriented readers can just note that (12) can be turned into a linear second order differential equation with constant coefficients with the substitution $x_i = \log(F_i/K)$. Such equation is amenable to a solution with the method of Fourier transforms.) We just give the results here. The value of the spread option with the payoff of (13) at time T is given by the formula

$$V(F_1, F_2, \tau) = \frac{e^{-r\tau}}{2\pi\sigma_1\sigma_2\tau\sqrt{1-\rho^2}} \int_0^\infty \int_0^\infty \max\left[a_1 F_1' - a_2 F_2' - K, 0\right]$$
$$\times \exp\left[-\frac{y_1^2}{2\sigma_1^2\tau\,(1-\rho^2)} - \frac{\rho y_1 y_2}{\sigma_1\sigma_2\tau\,(1-\rho^2)} - \frac{y_2^2}{2\sigma_2^2\tau\,(1-\rho^2)}\right] \frac{dF_1'}{F_1'}\frac{dF_2'}{F_2'} \tag{27}$$

where we have retained the notation $\tau = T - t$ and used the definitions

$$y_i = \log\left(\frac{F_i}{F_i'}\right) - \frac{\sigma_i^2\tau}{2} \tag{28}$$

of y_1 and y_2 to keep the expression (27) from becoming unwieldy. Equation (27) is the desired *exact* expression for the value of the option to exchange with a fixed strike. It is in the form of a double integral that must be evaluated numerically; however, this numerical evaluation is straightforward, as we discuss below. On a practical note, the double integral in (27) for the option value can be reduced to a set of single integrals of a normal probability density times the cumulative normal distribution of a complicated function. This reduction makes the numerical evaluation of the value of the option much faster, easier, more accurate, and more reliable. The reduction involves completing the square in the exponential function several times.

The ability to express the value of an option as an integral, such as (27), represents a significant step towards valuing and hedging the option. Although integral representations do not have the ease of use of closed form solutions, they are significantly easier to use and work with than binomial trees, finite difference solutions, or Monte Carlo solutions. When closed form solutions cannot be obtained, integral representations, when they can be obtained, have three main advantages over the other methods. First, because an integral representation is still, in essence, an analytical expression for the value of the option, it retains many of the advantages of closed form solutions. For example, the standard procedures of differential calculus can be used to calculate integral representations of the Greeks. Second, there is a large literature on computing integrals numerically and many easy to implement, powerful, and well-understood algorithms are available. Almost every elementary numerical analysis text has a section on numerical integration, such as, to take a very arbitrary and limited selection, the books of Dahlquist and Bjork (1974), Acton (1970), and

Stoer and Bulirsch (1980). The book of Judd (1998) is oriented toward applications to economics, and the truly amazing Press et al. (1992) includes a working computer code. One way to think about numerical integration is that it is the simplest differential equation, and so it is reasonable that the algorithms for solving it should be simpler and more powerful than the algorithms for more complex differential equations. Binomial trees and finite difference methods are essentially methods for solving these more general kinds of differential equations. Third, it is relatively easy to write an algorithm that evaluates an integral to a given, predetermined accuracy. The basic idea is that one evaluates the integral numerically using a certain number of points, checks for numerical convergence, and if the convergence is not reached, one just evaluates the integral with more points. A good, practical discussion of this procedure is given by Press et al. (1992), who give both a clear mathematical explanation and a computer code.

3.2.3 Monte Carlo

One application of Monte Carlo simulation is its use as a mathematical technique for numerically solving stochastic differential equations. In the financial context Monte Carlo simulation is used for pricing options on multiple assets, such as spread options. Below we outline the steps for the use of Monte Carlo simulation to price such options:

1. Generation of large number of scenarios at the expiration date for the prices of two underlying assets. This has to be done in a manner that is consistent with the option volatilities, current futures/forwards prices, and a correlation between the prices (or returns). The joint distribution of asset prices at any time in the future can be fully described by their initial price levels, correlation, and volatilities, under the assumption that they follow geometric Brownian motion processes.
2. Evaluation of option payoff at expiration for every simulated scenario. Each scenario consists of a possible randomly drawn pair of assets' prices.
3. Calculation of the fair value or expected, discounted option payoff by first taking the average of all future payoffs, calculated at the previous step, and then discounting it to the valuation date.

Monte Carlo is a powerful and flexible method for solving problems in option valuation, hedging, and risk management. It is frequently discussed in general books on option modeling, such as Clewlow and Strickland (1998), Hull (2002), and Tavella (2002). Two books devoted solely to Monte Carlo methods in finance are Glasserman (2004), which has thorough and clear discussions of basic issues in using the Monte Carlo method in finance, and Jackel (2002), which provides good insights of more specialized (but useful) topics such as low-discrepancy sequences. Books on numerical solution of stochastic differential equations, such as Kloeden and Platen (1994), also provide information that is useful for financial applications of Monte Carlo simulations.

The spread option is an option on the difference between two commodity prices F_1, F_2, which follow geometric Brownian motion, described by the system of

(9) and (10) with instantaneous correlation ρ in (11). To price this option via Monte Carlo simulation, we need to derive a discrete time version of the system of (9)–(11) under risk-neutral measure, that is, $\mu_1(F_1, F_2, t) = \mu_2(F_1, F_2, t) = 0$:[1]

$$\begin{cases} dF_1(t,T) = \sigma_1(t,T)F_1(t,T)dz_1(t) \\ dF_2(t,T) = \sigma_2(t,T)F_2(t,T)dz_2(t) \end{cases} \tag{29}$$

The following steps detail a discretization scheme for (29).

Application of Ito's Lemma to the logarithms of prices in (29) gives

$$\begin{cases} d \ln\ F_1(t,T) = -\dfrac{\sigma_1^2(t,T)dt}{2} + \sigma_1(t,T)dz_1(t) \\ d \ln\ F_2(t,T) = -\dfrac{\sigma_2^2(t,T)dt}{2} + \sigma_2(t,T)dz_2(t) \end{cases} \tag{30}$$

where $F_1(T)$ and $F_2(T)$ are the assets' futures prices for delivery time T. Random variables z_1 and z_2 come from the standard bivariate normal distribution with correlation ρ.

Simulation of (30) requires the following discretizations:

$$\begin{cases} \ln\ F(t+\Delta t) - \ln\ F(t) = -\dfrac{1}{2}\sigma_1^2\Delta t + \sigma_1 \Delta z_1 \\ \ln F(t+\Delta t) - \ln\ F(t) = -\dfrac{1}{2}\sigma_2^2\Delta t + \sigma_2 \Delta z_2 \end{cases} \tag{31}$$

Integrating (31) from 0 to T and using the definition of implied volatility,

$$\sigma_{imp}^2(0,T) = \frac{1}{T}\int_0^T \sigma^2(\tau, T)d\tau, \tag{32}$$

We finally obtain

$$\begin{cases} F_{1,T} = F_1 e^{-.5\sigma_{imp\,1}^2 T + \sigma_{imp\,1} z_1 \sqrt{T}} \\ F_{2,T} = F_2 e^{-.5\sigma_{imp\,2}^2 T + \sigma_{imp\,2} z_2 \sqrt{T}} \end{cases} \tag{33}$$

Correlated standard normal random variables z_1 and z_2 are derived by simulating and combining independent standard normal variables ε_1 and ε_2 as follows:

$$\begin{aligned} z_1 &= \varepsilon_1 \\ z_2 &= \rho\varepsilon_1 + \sqrt{1-\rho^2}\varepsilon_2 \end{aligned}$$

(This transformation is a special case of the Cholesky decomposition method for obtaining correlated standard normal random variables. This method is discussed

[1] Discussion of various discretizing choices can be found in Kloeden and Platen (1994).

in most of the Monte Carlo references given at the beginning of this section). Note that the system of (33) allows us to simulate the prices at expiry without simulating the entire price path as long as we observe or can compute implied volatilities from now until the exercise. Finally, for each pair of simulated prices we evaluate the option's payoff and discount it to the present, that is, evaluate the following expression: $e^{-rT}\max(F_{2,T} - F_{1,T} - X, 0)$. The average of these numbers will be the fair value of the option V. Specifically,

$$V = e^{-rT}\frac{1}{N}\sum_{i=1}^{N}\max(F_{2,T} - F_{1,T} - X, 0)$$

Note that this solution yields an approximate price. By increasing the number of scenarios N, the accuracy of the result could be improved. The main disadvantage of simple Monte Carlo simulation is that its accuracy increases only as the square root of the number of simulations.

There exist many techniques (like variance reduction and quasi-random techniques) that make it possible to improve accuracy and decrease computation time at the same time. Press et al. (1992) contains a good overview of both methods.

3.2.4 Binomial Spread Options with a Strike

Another method for pricing spread options is the multidimensional binomial method. Kamrad and Ritchken (1991) were the first to suggest it. The presentation in this section applies the key elements of this method and considerations of Clewlow and Strickland (1998) for stocks to futures prices. The binomial model assumes that the asset price follows a binomial process, that is, at any step it can either go up or down with a given probability. Thus it assumes that the asset price has a binomial distribution. On the other hand, many financial models assume that the distribution of asset's returns is normal. The binomial option pricing models make use of the fact that as the number of observations/trials in the binomial distribution increases, the assets' returns approach normal distribution.

There are many ways one can build a price evolution tree that preserves the distribution properties of the assets. Computationally, an addition operation is more efficient than multiplication. In what follows we present a widely used additive binomial tree method. Since the price process represented as an additive process in logarithms is

$$\log\ F_i(t + \Delta t) = \log\ F_i(t) + \xi_t,$$

the tree is built for the logarithms of the assets' prices ($x_i = \log(F_i)$).

Since the risk neutral process for x_i is $dx_i = -\frac{1}{2}\sigma_i^2 dt + \sigma_i dz_i$ (cf. 30), the discrete version of the previous equation will be $\Delta x_i = -\frac{1}{2}\sigma_i^2\Delta t + \sigma_i\sqrt{\Delta t}$ (cf. 31). The discrete time binomial model for x_1 and x_2 is illustrated in Table 1 and Fig. 2. Assuming equal up and down jumps, each of the variables can either go up by Δx_i or down by $-\Delta x_i$. At any time step, the probability that logarithms of both prices go up is p_{uu} and down is p_{dd}; first asset goes up, but second down is p_{ud}, second

Table 1 The joint normal random variable (**X1 (t), X2 (t)**) is approximated by a pair of multinomial discrete normal variables having the following distribution

X1 (t)	X2 (t)	Probability
Δx_1	Δx_2	p_{uu}
Δx_1	$-\Delta x_2$	p_{ud}
$-\Delta x_1$	Δx_2	p_{du}
$-\Delta x_1$	$-\Delta x_2$	p_{dd}

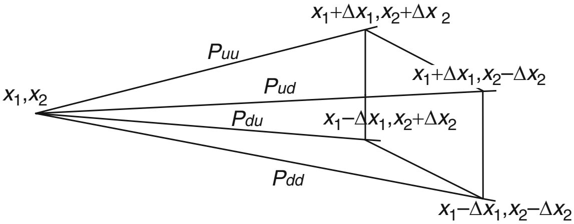

Fig. 2 One branch of the additive two-variable binomial tree for logarithms of assets' prices

goes up, but first goes down is p_{du}. The sum of these probabilities is one, as they contain all four possible outcomes of price movements.

The next step is to establish jump sizes (Δx_1 and Δx_2) and four probabilities so that they match mean, variance, and correlation of the bivariate normal distribution of logarithms of assets' prices. For that we need to solve the system of equations that matches first and second moments of binomial and analytical representations. It is an algebraic system of six equations with six unknowns:

$$\begin{cases} \mathrm{E}(\Delta x_1) = \Delta x_1(p_{uu} - p_{du} + p_{ud} - p_{dd}) = -\frac{1}{2}\sigma_1^2 \Delta t \\ \mathrm{E}(\Delta x_1) = \Delta x_1^2(p_{uu} + p_{du} + p_{ud} + p_{dd}) = \sigma_1^2 \Delta t \\ \mathrm{E}(\Delta x_2) = \Delta x_2(p_{uu} + p_{du} - p_{ud} - p_{dd}) = -\frac{1}{2}\sigma_2^2 \Delta t \\ \mathrm{E}(\Delta x_2) = \Delta x_2^2(p_{uu} + p_{du} + p_{ud} + p_{dd}) = \sigma_2^2 \Delta t \\ \mathrm{E}(\Delta x_1, \Delta x_2) = \Delta x_1 \Delta x_2(p_{uu} - p_{du} - p_{ud} + p_{dd}) = \rho\sigma_1\sigma_2 \Delta t \\ p_{uu} + p_{du} + p_{ud} + p_{dd} = 1 \end{cases} \tag{34}$$

The solution to this system of equations for a given time step and given changes in logarithms of both assets' futures prices is

$$\begin{cases} \Delta x_1 = \sigma_1 \sqrt{\Delta t} \\ \Delta x_2 = \sigma_2 \sqrt{\Delta t} \\ p_{uu} = \dfrac{\Delta x_1 \Delta x_2 - 0.5\Delta x_2 \sigma_1^2 \Delta t - 0.5\Delta x_1 \sigma_2^2 \Delta t + \rho\sigma_1\sigma_2 \Delta t \sigma_2^2}{4\Delta x_1 \Delta x_2} \\ p_{du} = \dfrac{\Delta x_1 \Delta x_2 + 0.5\Delta x_2 \sigma_1^2 \Delta t - 0.5\Delta x_1 \sigma_2^2 \Delta t - \rho\sigma_1\sigma_2 \Delta t \sigma_2^2}{4\Delta x_1 \Delta x_2} \\ p_{ud} = \dfrac{\Delta x_1 \Delta x_2 - 0.5\Delta x_2 \sigma_1^2 \Delta t + 0.5\Delta x_1 \sigma_2^2 \Delta t - \rho\sigma_1\sigma_2 \Delta t \sigma_2^2}{4\Delta x_1 \Delta x_2} \\ p_{dd} = \dfrac{\Delta x_1 \Delta x_2 + 0.5\Delta x_2 \sigma_1^2 \Delta t + 0.5\Delta x_1 \sigma_2^2 \Delta t + \rho\sigma_1\sigma_2 \Delta t \sigma_2^2}{4\Delta x_1 \Delta x_2} \end{cases} \tag{35}$$

Now we have all the information necessary to build a two-dimensional tree and calculate the value of the option. The algorithm proceeds as follows:

A user sets up the granularity of the tree N, that is, the number of steps between valuation and expiration dates of the option. The larger this number is, the more accurate the solution will be. Thus, there will be N branching processes, at every time step $\Delta t = T/N$, where T is the time to expiry for a given option. Triplex (i, j, k) refers to the nodes on the tree, where i is a time step, j is the first asset's price level, k is the second assets price level. Since the tree is built for logarithms of assets' prices (x_1, x_2), we convert them to assets' prices at each node of the tree via these simple transformations:

$$\begin{cases} F_{1,i,j,k} = F_1 e^{(2j-i)\Delta x_1} \\ F_{2,i,j,k} = F_2 e^{(2k-i)\Delta x_2} \end{cases}$$

Separately, each asset in the binomial tree follows a one-dimensional process that can be approximated by the one-dimension tree (see Fig. 3). At the same time, their joint distribution characteristic – instantaneous correlation – reveals itself in their joint probabilities of (35).

We initialize asset prices at expiration time (nodes (N, j, k) of the two-variable binomial tree or (N, j) and (N, k) of the individual assets' one-dimensional trees). At this step we estimate option values at expiration, utilizing asset prices computed at the previous step. For all j and k of the final nodes (N, j, k) of the two-variable binomial tree, the option price $C(N, j, k)$ at expiration is $\max(F_2[k]-F_1[j]-X, 0)$. Next we start to iterate option values $C(i, j, k)$ by stepping back through the tree from the time node (N-1) to 0. Option value at each time step i and price levels (j, k) is a discounted, weighted (by probabilities) average of the four option values at the nodes that are connected to a current node.

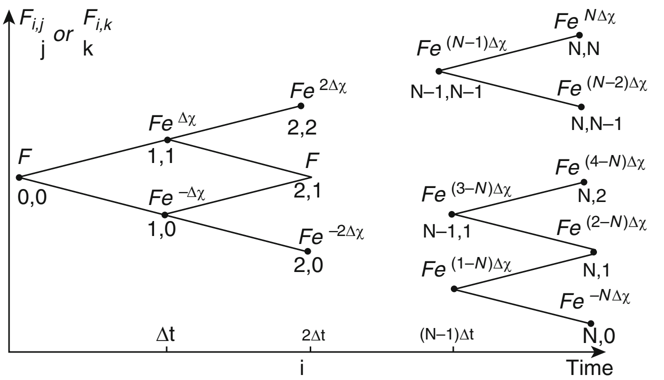

Fig. 3 Individual asset's prices at time node i and price level j for the first asset or k for the second

Specifically,

$$C(i,j,k) = e^{-r\Delta t}\begin{pmatrix} p_{uu}C(i+1,j+1,k+1) + p_{du}C(i+1,j-1,k+1)+ \\ +p_{ud}C(i+1,j+1,k-1) + p_{dd}C(i+1,j-1,k-1) \end{pmatrix}$$

Fair value of the spread option at valuation date is given by the option value at the very first node of the tree, that is, $C(0,0,0)$.

The above-described algorithm can be easily modified to accommodate the American exercise. American options can be exercised at any time during their life. Thus at every time step, a holder of the option has a choice to hold it to expiry (European feature) or to exercise it and receive the option payoff, which is $\max(F_2[k]-F_1[j]-X,0)$ at every time node i (American feature). The last feature can easily be incorporated into our algorithm at the iteration stage 4. Specifically, the American option price at time node i and price levels j and k becomes

$$C(i,j,k) = \max\left\{ e^{-r\Delta t}\begin{pmatrix} p_{uu}C(i+1,j+1,k+1) + p_{du}C(i+1,j-1,k+1)+ \\ +p_{ud}C(i+1,j+1,k-1) + p_{dd}C(i+1,j-1,k-1) \end{pmatrix}, \atop F_2[k]-F_1[j]-X \right\}$$

All other steps remain unaffected. One can also improve the efficiency of the program by premultiplying probabilities by the one step discount factor $e^{-r\Delta t}$.

4 Valuation in the Presence of Spikes

The presence of spikes creates a new complexity in the valuation of obligations and options. In theory, the Black–Scholes analysis assumes continuous dynamic replication of a portfolio. In practice, however, any such rebalancing incurs transaction costs. These costs force hedging to become discrete and hence imperfect. Therefore, in practice all portfolios bear some risk. This situation is exacerbated during the power price spikes when even approximate dynamic replication becomes impossible. The implications of this difficulty are discussed, in the context of jump rather than spike processes, in the original paper of Merton (1976) on valuing options, where the asset returns have jumps, and in the excellent textbook of Shreve (2004). This difficulty further limits the applications of risk-neutrality assumption for option valuation. In this section, we reduce the use of risk-neutral valuation techniques to the net present value (NPV) analysis. The reason for this is twofold: to illustrate the effects of spikes in the simplest possible way and to provide a base from which readers can develop more sophisticated and realistic valuation techniques.

Now, let us account for the possibility of *spikes*. Analysis of commodity prices S_1, S_2 and futures prices F_1, F_2 at both locations relies on a *multistate* model introduced earlier in the paper. In particular, transitions between the states (spikes and interspike segments) are governed by the transition probability matrix $p = p(t)$

specified in (8). The process appears in one state or the other with probabilities computed from this matrix. For a day that is T days away from the current day t, the matrix of transition probabilities is computed as

$$P^{(t,T)} = P(t+T|t) = \prod_{k=0}^{T-1} P(t+k) = P(t)P(t+1)\dots P(t+T-1).$$

Element $P_{i.i}^{(t,T)}$ of this 2-by-2 matrix equals the conditional probability of state j, T days ahead, given the current state i (i,j = 0,1). For example, if there is a spike on day t, then the model predicts a spike on day (t + T) with probability $P_{1.1}^{(t,T)}$ and no spike on that day with probability $P_{1.0}^{(t,T)}$. Consequently, the density of prices on that day is a *mixture*

$$f_{t,T}(x) = P_{i,0}^{(t,T)} f_0(x) + P_{i,1}^{(t,T)} f_1(x), \tag{36}$$

where densities $f_0(x)$ and $f_1(x)$ are given by (5) and (7). Expectation of any function h with respect to this mixture density is the weighted average

$$E\{h(X)\} = \int h(x) f_{t,T}(x)dx = P_{i,0}^{(t,T)} \int h(x) f_0(x)dx + P_{i,1}^{(t,T)} \int h(x) f_1(x)dx. \tag{37}$$

In our context, $X = (F_1, F_2)$, and the role of $E_T\{h(X)\}$ is played by the value $V(F_1, F_2, T) = E_T\{h(F_1, F_2)\}$ of a financial instrument, which is based on both F_1 and F_2 (spread option, transportation option) at the expiry date T. Therefore,

$$V_T(F_1, F_2, t) = P_{i,0}^{(t,T)} V_T^{(0)}(F_1, F_2, t) + P_{i,1}^{(t,T)} V_T^{(1)}(F_1, F_2, t),$$

where $V_T^{(0)}(F_1, F_2, t)$ and $V_T^{(1)}(F_1, F_2, t)$ are values of the financial instruments computed for each mode (regular mode and spikes), respectively.

The joint distribution of commodity prices during spikes at two locations, to be used in (36), is *bivariate lognormal*, with correlation coefficient ρ between the two locations. To evaluate the strength of the spike effect, we fitted the introduced multistate model to a 5-year long sequence of day-ahead electricity prices in ERCOT North and a 3-year long sequence in ERCOT South. ERCOT is the Independent System Operator (ISO) for the State of Texas.

Estimation of the log-price mean value $m(t)$ includes the general linear trend, weekday effects, and seasonal components, showing the long-term growth of prices (at the average rate of 13% per year, according to the ERCOT data), variation during a week, and variation during a year (Fig. 4).

Following the model developed for the interspike distribution in (5), detrended log-prices $X_{j,t} = S_j(t) - m_j(t)$ for $j = 1, 2$ at two locations are correlated AR(1) processes

$$X_{1t} = \phi_1 X_{1,t-1} + Z_{1t} \; and \; X_{2t} = \phi_2 X_{2,t-1} + Z_{2t},$$

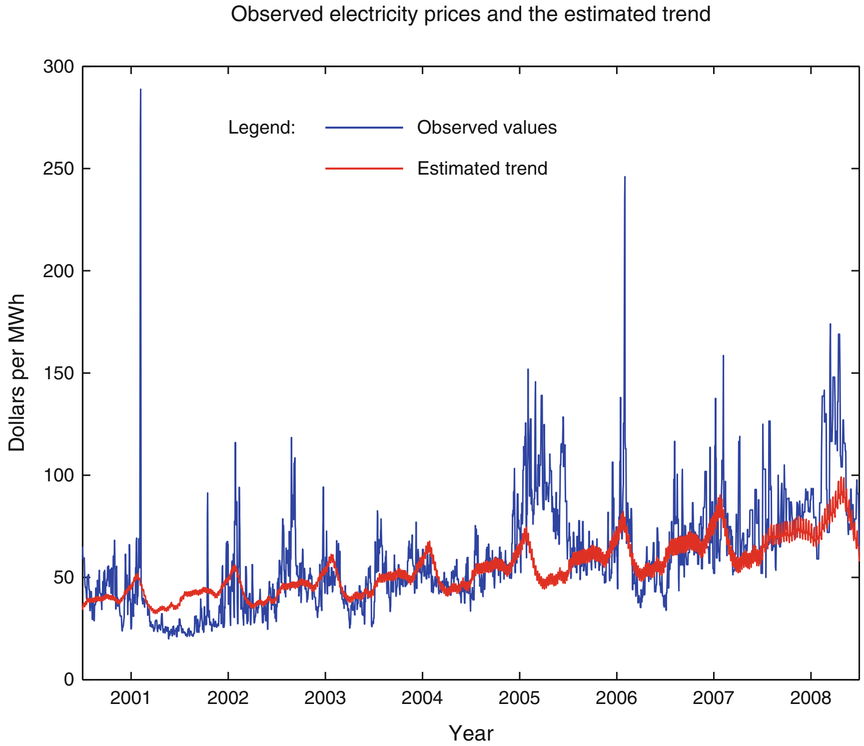

Fig. 4 Electricity prices in ERCOT North and their estimated trend Source: *ICE Trade the World* site (https://www.theice.com)

where (Z_{1t}, Z_{2t}) is a bivariate white noise, $Var(Z_{1t})\sigma_1^2$, $Var(Z_{2t})\sigma_2^2$, and $Cov(Z_{1t}, Z_{2t}) = \rho\sigma_1\sigma_2$. Parameters σ_i, ϕ_i of each process X_{it}, $j = 1, 2$ are estimated by standard methods for time series analysis (see Brockwell and Davis (1991, 2009). To estimate the correlation coefficient ρ, we notice that

$$Cov(X_{1t}, X_{2t}) = Cov\left(\sum_{n=0}^{\infty} \phi_1^n Z_{1t-n}, \sum_{n=0}^{\infty} \phi_2^n Z_{2t-n}\right) = \frac{\rho\sigma_1\sigma_2}{1-\phi_1\phi_2},$$

from where the estimator of ρ based on n days of data is

$$\hat{\rho} = \frac{(1-\hat{\phi}_1)(1-\hat{\phi}_2)\sum_{t=1}^{n}(X_{1t}-\bar{X}_1)(X_{2t}-\bar{X}_2)/n}{\hat{\sigma}_1\hat{\sigma}_2}.$$

For ERCOT North and ERCOT South, the time series parameters are estimated as $\hat{\phi}_1 = 0.9307$, $\hat{\phi}_2 = 0.9592$, $\hat{\sigma}_1 = 0.0742$, $\hat{\sigma}_2 = 0.0729$, and the correlation coefficient is $\hat{\rho} = 0.8211$.

During each spike, the mean vector (μ_1, μ_2) is generated from a bivariate normal distribution with (hyper-) parameters $(\theta_1, \theta_2, \omega_1, \omega_2, \rho_\mu)$, where ρ_μ is the correlation coefficient between the spike means μ_1 and μ_2. Conditioned on (μ_1, μ_2), the vector of detrended log-prices for each spike for the two regions has a bivariate normal distribution with parameters $(\mu_1, \mu_2, \tau_1, \tau_2, \rho^{sp})$. Unconditionally, we have

$$\begin{aligned} Cov(X_{1t}, X_{2t}) &= E\{Cov(X_{1t}, X_{2t}|\mu_1, \mu_2)\} + Cov\{E(X_{1t}|\mu_1), E(X_{2t}|\mu_2)\} \\ &= \rho^{sp}\omega_1\omega_2 + \rho_\mu\tau_1\tau_2. \end{aligned}$$

Then, the spike-mode distribution of detrended log-prices at two locations is bivariate normal with the unconditional correlation coefficient $\frac{\rho^{sp}\omega_1\omega_2+\rho_\mu\tau_1\tau_2}{\sqrt{(\tau_1^2+\omega_1^2)(\tau_2^2+\omega_2^2)}}$.

With all the parameters estimated, we obtain the joint density (36) of electricity prices for the two regions for any given day. This allows us to make forecasts in the form of a predictive joint density as well as the marginal densities at each location. A 3-year forecast of densities of prices for each day of the year 2011 is given on Fig. 5.

For each day of the year, the cross-section of this graph gives a probability density of electricity prices. During the peak season, the probability of low prices reduces, and a heavy tail of high prices appears. It accounts for the possibility of spikes.

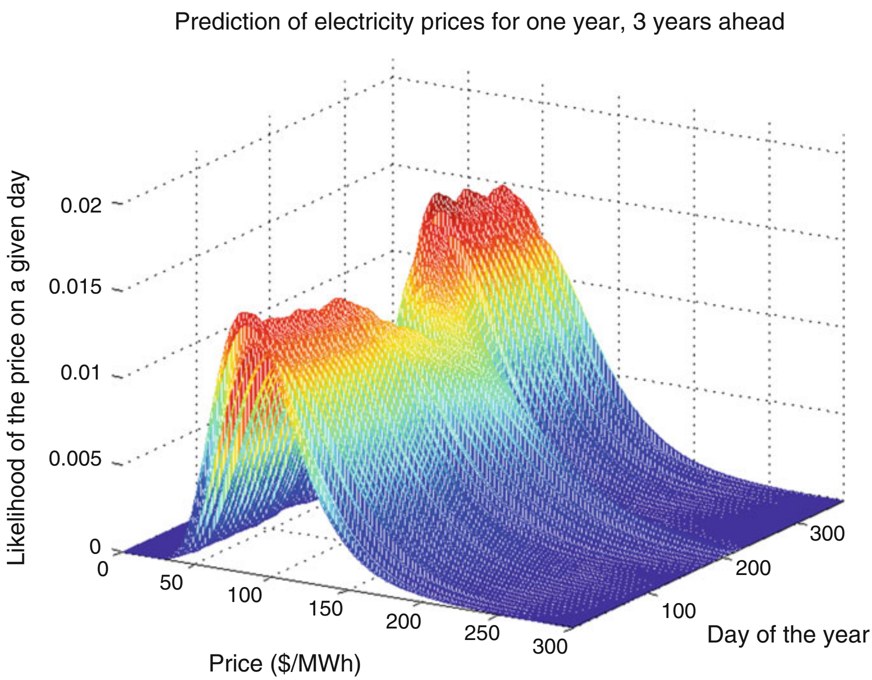

Fig. 5 Predictive densities of electricity prices for each day of the year, 3 years ahead

In Rosenberg et al. (2002a), the authors estimated the effect of accounting for spikes in the estimation of expected value of the spread option payoff. On the example of PJM West–PJM East spread, the use of a multistate model with spikes yielded additional 7% value as compared with the model that did not use spikes and estimated a single lognormal distribution for price returns. This result demonstrated the importance of accounting for spikes in the probability density when computing the values of spread-type instruments.

5 Conclusions

This paper described physical commodity transportation and transmission instruments that have one thing in common: they depend on the spread of locational or cross-commodity prices and thus can be modeled by a family of spread options. As this dependency suggests, the value of the instrument in any future moment depends not only on the level of prices but also on their future distribution function. The inclusion of spikes changes this distribution and hence produces a different value for these assets. The correct calibration and inclusion of these sudden and big deviations from the normal level of prices is very important for correct valuations and mark-to-market procedures. There are a number of methods that may be utilized to solve for the value of assets once the price process is established. They range from approximate solutions under some simplifying assumptions that are quite easy to compute to the exact solution that is reducible to single integrals. Numerical methods include Monte-Carlo and Binomial method and have a different degree of robustness. Monte-Carlo always converges and requires a simple algorithm, but may be slow. Binomial approach requires more analytics and programming, but yields payoffs in the speed of the computation.

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