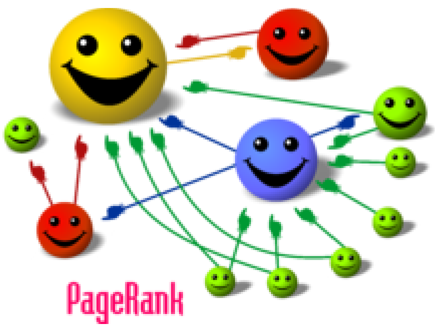
APPLICATIONS

1. Matrix computation for Google's PageRank algorithm

How to solve the matrix equation and obtain Google solution for the textbook example
(Elements of Statistical Learning, pp. 576-578)




```
> x=c(0,1,1,0,0,0,1,0,1,0,0,0,0,0,1,0)
> L = matrix(x,4,4)          # Define a 4x4 matrix L showing linkage among web pages
> L                          # R reads a matrix column by column
     [,1] [,2] [,3] [,4]
[1,]    0    0    1    0
[2,]    1    0    0    0
[3,]    1    1    0    1
[4,]    0    0    0    0

> e = matrix( rep(1,4), 4, 1 )      # Vector of 1s
> e
     [,1]
[1,]    1
[2,]    1
[3,]    1
[4,]    1

> C = diag( colSums(L) )        # Diagonal matrix C showing the number
> C                             # of outgoing links from each web page
     [,1] [,2] [,3] [,4]
[1,]    2    0    0    0        # Matrix multiplication is %*%
[2,]    0    1    0    0        # Inverse matrix is solve(C)
[3,]    0    0    1    0        # Transpose matrix is t(e)
[4,]    0    0    0    1

> A = 0.15*(e%*%t(e))/4 + 0.85*L%*%solve(C)
> A
       [,1]   [,2]   [,3]   [,4]
[1,] 0.0375 0.0375 0.8875 0.0375
[2,] 0.4625 0.0375 0.0375 0.0375
```

```
[3,] 0.4625 0.8875 0.0375 0.8875
[4,] 0.0375 0.0375 0.0375 0.0375

> eigen(A)                          # Computes eigenvalues and eigenvectors of A
$values
[1]  1.000000e+00+0.00e+00i -4.250000e-01+4.25e-01i
[3] -4.250000e-01-4.25e-01i  5.477659e-18+0.00e+00i

$vectors
            [,1]                    [,2]
[1,] -0.64470397+0i  7.071068e-01+0.000000e+00i
[2,] -0.33889759+0i -3.535534e-01-3.535534e-01i
[3,] -0.68212419+0i -3.535534e-01+3.535534e-01i
[4,] -0.06489841+0i -2.175463e-17-4.017163e-18i
                         [,3]           [,4]
[1,]  7.071068e-01+0.000000e+00i -9.941546e-16+0i
[2,] -3.535534e-01+3.535534e-01i -7.071068e-01+0i
[3,] -3.535534e-01-3.535534e-01i -1.968623e-16+0i
[4,] -2.175463e-17+4.017163e-18i  7.071068e-01+0i

>  install.packages("expm")         # This package contains matrix powers
> library(expm)                     # "Exponentiation of a matrix"
> A100 = A%^%100                    # Large power of matrix A
> A100                              # (a simple but not an optimal method)
          [,1]      [,2]      [,3]      [,4]
[1,] 0.3725269 0.3725269 0.3725269 0.3725269
[2,] 0.1958239 0.1958239 0.1958239 0.1958239
[3,] 0.3941492 0.3941492 0.3941492 0.3941492
[4,] 0.0375000 0.0375000 0.0375000 0.0375000
                                    # "Converged"! All columns are the same
> p = A100[,1]                      # Take one column of A^100
> p
[1] 0.3725269 0.1958239 0.3941492 0.0375000
> p=t(t(p))                         # Now R understands it as a column
> p
          [,1]
[1,] 0.3725269
[2,] 0.1958239
[3,] 0.3941492
[4,] 0.0375000
                                    # Verify that it is a solution of p = Ap
> A%*%p
          [,1]
[1,] 0.3725269
[2,] 0.1958239
[3,] 0.3941492
[4,] 0.0375000
                        # p is not properly normalized yet, we need e'p = N = 4
> t(e)%*%p              # but we have the sum of p[i] = 1
```

```
     [,1]
[1,]    1

> p*4                    # This is the solution. Ranked by importance, we have
          [,1]           # web pages ordered as 3, 1, 2, 1
[1,] 1.4901074
[2,] 0.7832956
[3,] 1.5765969
[4,] 0.1500000
```

 # Another way is by iterations (standard method of solving fixed-point equations):

```
> p = matrix(c(1,0,0,0),4,1)     # The initial vector is kind of arbitrary
> p
     [,1]
[1,]    1
[2,]    0
[3,]    0
[4,]    0

> for (n in (1:100)){ p = A%*%p }     # Iterate  p[next] = A*p[current]
> 4*p                                 # Same result
          [,1]
[1,] 1.4901074
[2,] 0.7832956
[3,] 1.5765969
[4,] 0.1500000
```

2. Spam detection

 # The Spam data with description are in https://archive.ics.uci.edu/ml/datasets/Spambase
 # We can also get its formatted version from a package "kernlab"

```
> library(kernlab)
> data(spam)
> dim(spam)
[1] 4601   58
> names(spam)
 [1] "make"              "address"           "all"               "num3d"
 [5] "our"               "over"              "remove"            "internet"
 [9] "order"             "mail"              "receive"           "will"
[13] "people"            "report"            "addresses"         "free"
[17] "business"          "email"             "you"               "credit"
[21] "your"              "font"              "num000"            "money"
[25] "hp"                "hpl"               "george"            "num650"
[29] "lab"               "labs"              "telnet"            "num857"
[33] "data"              "num415"            "num85"             "technology"
[37] "num1999"           "parts"             "pm"                "direct"
[41] "cs"                "meeting"           "original"          "project"
[45] "re"                "edu"               "table"             "conference"
[49] "charSemicolon"     "charRoundbracket"  "charSquarebracket" "charExclamation"
[53] "charDollar"        "charHash"          "capitalAve"        "capitalLong"
```

```
[57] "capitalTotal"      "type"
```

> table(type)
type
nonspam   spam
  2788   1813

 # LOGISTIC REGRESSION

> logreg = glm(type ~ ., data=spam, family = "binomial")
> prob.spam = fitted.values(logreg)
> class.spam = ifelse(prob.spam > 0.5, "spam", "nonspam")
> table(class.spam, type)

```
          type
class.spam nonspam spam
   nonspam    2666  194
   spam        122 1619
```

> mean(type == class.spam)
[1] 0.9313193   **# Correct classification rate = 93% (better be validated by CV)**

 # What are error rates among spam and benign emails?
> sum(type=="nonspam" & class.spam=="spam") / sum(type=="nonspam")
[1] 0.04375897
> sum(type=="spam" & class.spam=="nonspam") / sum(type=="spam")
[1] 0.107005

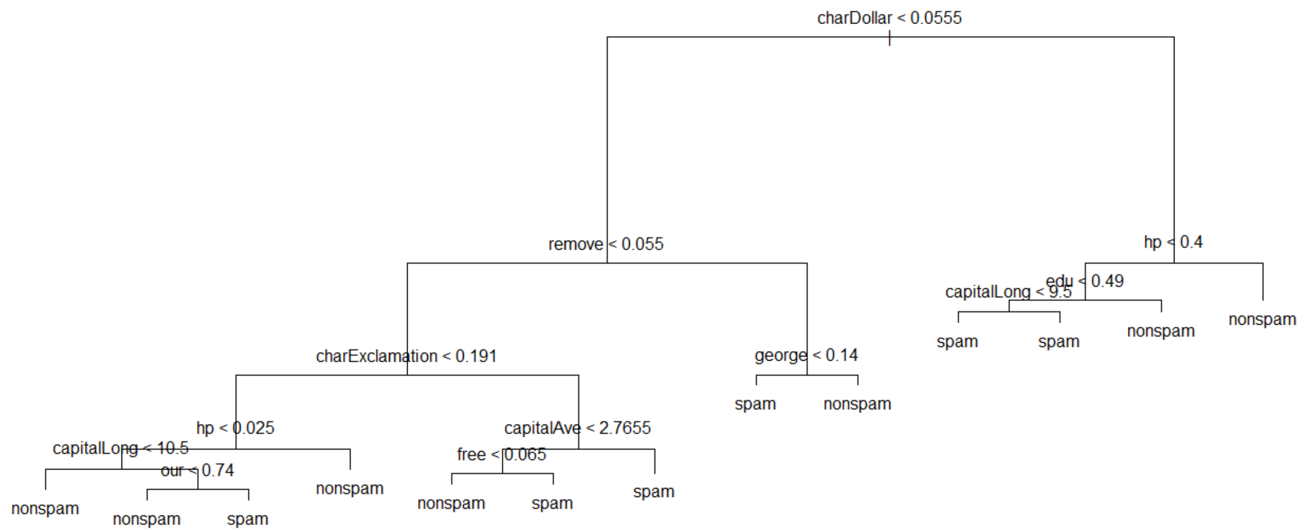
 # Reducing the error rate for nonspam emails, with loss function L(1,0)=1, L(0,1)=10
> class.spam = ifelse(prob.spam > 10/11, "spam", "nonspam")
> table(class.spam, type)

```
          type
class.spam nonspam spam
   nonspam    2759  674
   spam         29 1139
```

> mean(type == class.spam)
[1] 0.8472071
> sum(type=="nonspam" & class.spam=="spam") / sum(type=="nonspam")
[1] 0.01040172
> sum(type=="spam" & class.spam=="nonspam") / sum(type=="spam")
[1] 0.3717595
 # The overall error rate increased, and the error rate of misclassifying spam emails as nonspam increased substantially, but the error rate for benign emails is only 1% now.

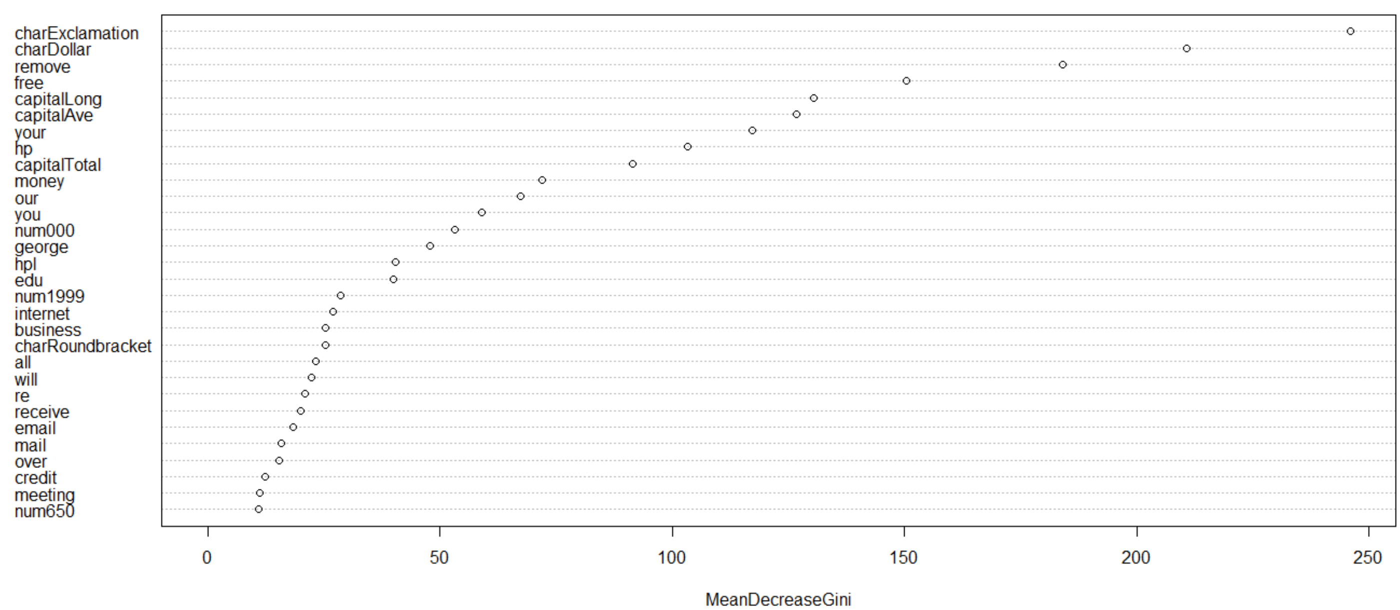
 # DECISION TREES

> tr = tree(type ~ ., data=spam)
> plot(tr); text(tr);



> summary(tr)
Misclassification error rate: 0.08259 = 380 / 4601

 # Optimization – random forests
> library(randomForest)
> rf = randomForest(type ~ ., data=spam)
> varImpPlot(rf) **# To see which variables are most significant in a tree based classification**



> rf
Number of trees: 500
No. of variables tried at each split: 7 **# That's approx. square root of the number of X-variables**

OOB estimate of error rate: 4.52% **# The lowest classification rate we could get so far**

```
Confusion matrix:
        nonspam spam class.error
nonspam    2710   78  0.02797704
spam        130 1683  0.07170436
```

 # DISCRIMINANT ANALYSIS

> library(MASS)
> LDA = lda(type ~ ., data=spam, CV=TRUE)
> table(type, LDA$class)

```
type      nonspam spam
  nonspam    2656  132
  spam        394 1419
```

> mean(type != LDA$class)
[1] 0.114323 **# This error rate is higher**

> QDA = qda(type ~ ., data=spam, CV=TRUE)
> table(type, QDA$class)

```
type      nonspam spam
  nonspam    2090  691
  spam         87 1722
```

> mean(type != QDA$class)
[1] NA
> summary(QDA$class)
nonspam spam NA's
2177 2413 11 **# There are missing values, predictions not computed by QDA**
 # We'll remove these points
> mean(type != QDA$class, na.rm=TRUE)
[1] 0.1694989 **# No improvement with QDA; linear models are ok**

SUPPORT VECTOR MACHINES

> library(e1071)
> SVM = svm(type ~ ., data=spam, kernel="linear")
> summary(SVM)
Number of Support Vectors: 940
(456 484) **# 940 support vectors (not cleanly classified) out of 4601 data points**
> dim(spam)
[1] 4601 58

> class = predict(SVM)
> table(class,type)

```
         type
class     nonspam spam
  nonspam    2663  185
  spam        125 1628
```

> mean(class != type)
[1] 0.06737666 **# Best rate so far?**