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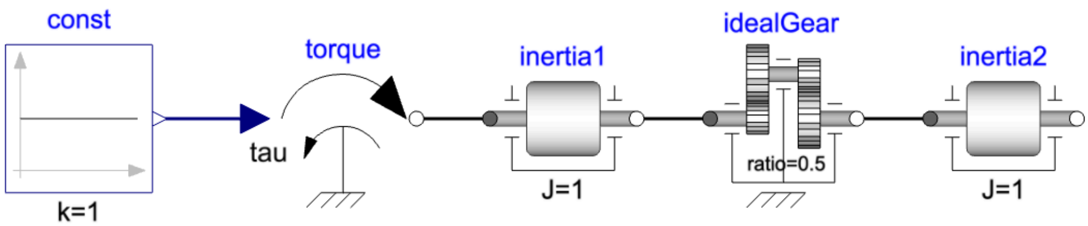
Equivalent Inertia

While this is a basic engineering concept, it is often unclear for many how intertias can be lumped together. Inertia directly coupled can be added. Yet if these are separated by some power transmission elements - such as a nut-screw - things get slightly more complicated.

This short note aims at giving a rationale for a simple example - explaining thus the logic to be reproduced for any system.

The system under study

Let's focus on the case of two inertias (`inertia1` and `inertia2`) separated with a reducer (`idealGear`) as the image below illustrates.



Quick analysis of the system

It is relevant to note that no elasticity is placed between the inertias - this is an infinitely stiff junction with a reducer.

Both `inertia1` and `inertia2` are set to $1kg.m^2$ and the `idealGear` is set to a ratio of $1/2$.

At first sight, it is however unclear what such a gear ratio means. Looking at the documentation of idealGear does not clarify how the ratio is applied. Luckily, the code of Modelica components is visible and a quick look at it allows us to see that the `ratio` is defined as "Transmission ratio (flange_a.phi/flange_b.phi)" - with `flange_a` connected to `inertia_1` and `flange_b` to `inertia_b` - and the following equations apply:

$$\phi_a = ratio * \phi_b;$$

$$0 = ratio * flange_a.\tau + flange_b.\tau;$$

Where:

- ϕ_a and ϕ_b are respectively the angular position of the flange a and b of the ideal gear and
- τ is the variable for the torque.

Hence a `ratio` of $1/2$ means that `flange_a` is rotating at half the speed of `flange_b`, while the torque is respectively the double (power conservation).

Bringing `inertia_2` on the other side of the idealGear

What should be the value of the equivalent inertia if both would be brought to the left side (`flange_a`) of the `idealGear` ?

One way to see this is to bring up the second Newton's law - involving a constant inertia:

$$\sum \tau = J.\ddot{\phi}$$

Let's now apply this equation to `inertia_2` and progressively substitute the variables of the `flange_b` by the relations of `flange_a` .

$$\tau_b = J_b.\ddot{\phi_b}$$

And it is known from the Modelica code that $0 = ratio * flange_a.\tau + flange_b.\tau$. This can be rewritten as $\tau_b = -ratio * \tau_a$.

Hence:

$$\tau_b = -ratio * \tau_a = J_b.\ddot{\phi_b}$$

The relation between the angular positions can be used. $\phi_a = ratio * \phi_b$ gives $\phi_b = \phi_a/ratio$. Hence:

$$-ratio * \tau_a = J_b.\ddot{\phi_a}/ratio$$

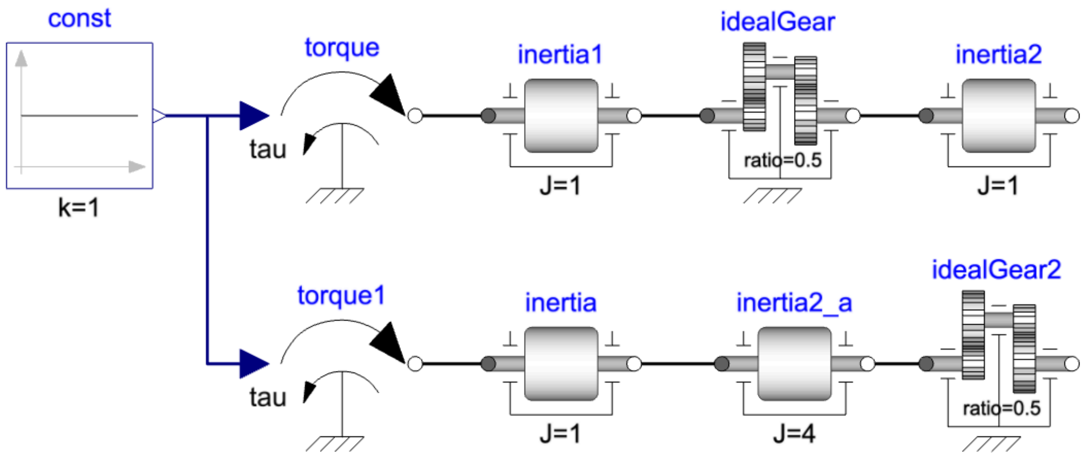
This can be brought back into the form of the second Newton's law on `flange_a` as:

$$\tau_a = -J_b/ratio^2.\ddot{\phi_a}$$

So `inertia_2` brought back on the other side of the `idealGear` shall take a value of $J_b/ratio^2 = 1/(1/2)^2 = 4$.

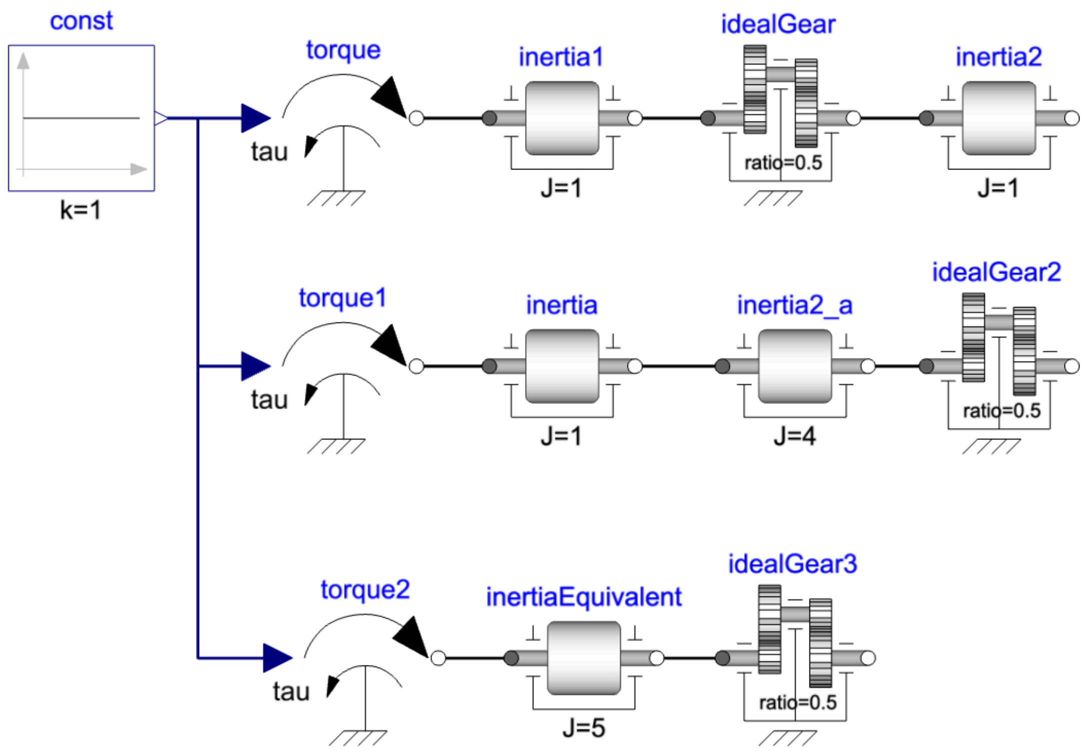
> Note: what about the negative sign?
>
> This is only due to the Modelica sign convention. Note how the torque equation gives opposite signs for τ_a and τ_b.

The following image shows the same initial system with its equivalent once the `inertia_2` is brought back on the other side of the gear.

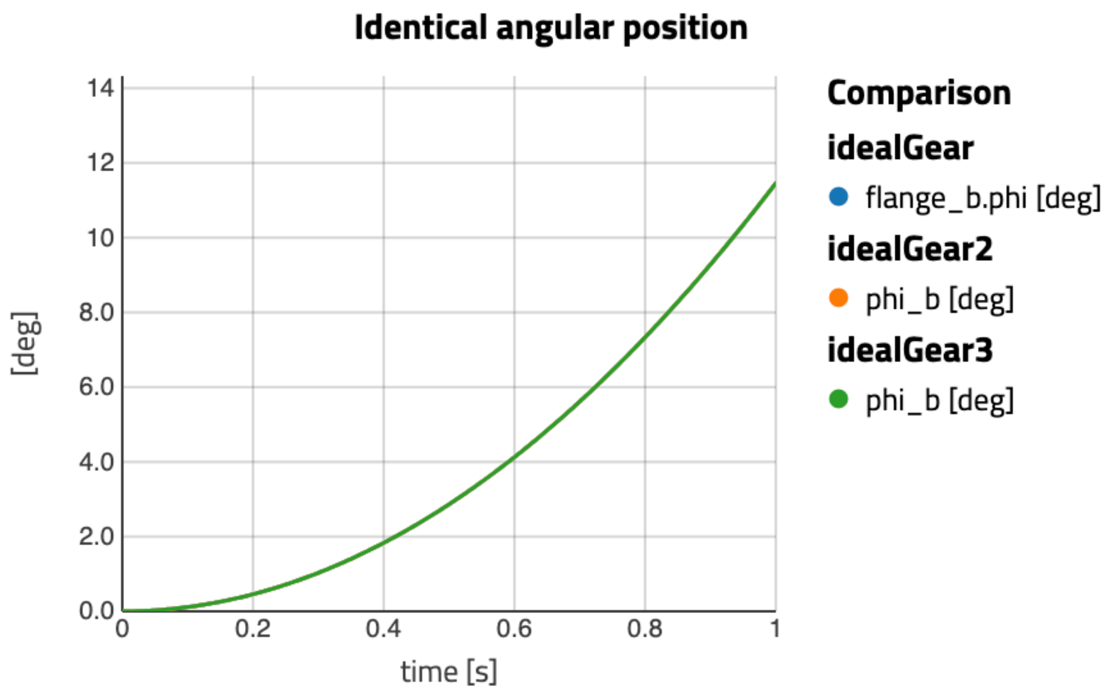


Computing the equivalent inertia

Now we have both inertia directly coupled and thus these can be simply added - leading to an equivalent inertia of value $1 + 4 = 5$.



The following graph shows three curves overlapping plotting the angular position `phi` of `flange_b` (the right side) of the `idealGear` on the three different stages of our system: its initial stage, after bringing `inertia_2` on the left side of the `idealGear` and after lumped both inertias together.



By the way, the inertia in rotation is of unit $kg.m^2$ - which corresponds to a mass in kg on a mechanical lever of a length in m (linear to rotation transformation). Noticing the square in the unit is a good way to remember that the conversion will require a square.

Another method?

As often (if not always), there are different ways to get the same result. These are often equivalent in some sense and yet showing the solution from a different angle can help understanding a topic better.

Another way to see this is to start from power conservation on both sides of the `idealGear`:

$$\tau_a * \dot{\phi}_a = \tau_b * \dot{\phi}_b$$

Now it is possible to bring back the relations applied in the previous method ($\tau_a = J_a * \ddot{\phi}_a$, $\tau_b = J_b * \ddot{\phi}_b$ and $\phi_b = \phi_a / ratio$), leading to:

$$\tau_a * \dot{\phi}_a = (J_b * \ddot{\phi}_b) * \dot{\phi}_b$$

$$\Rightarrow \tau_a * \dot{\phi}_a = (J_b * \ddot{\phi}_a / ratio) * \dot{\phi}_a / ratio$$

$$\Rightarrow \tau_a * \dot{\phi}_a = J_b / ratio^2 * \ddot{\phi}_a * \dot{\phi}_a$$

$$\Rightarrow \tau_a = J_b / ratio^2 * \ddot{\phi}_a$$

$$\Rightarrow J_a * \ddot{\phi}_a = J_b / ratio^2 * \ddot{\phi}_a$$

$$\Rightarrow J_a = J_b / ratio^2$$

This leads to the same result, as expected.

Why does this matter?

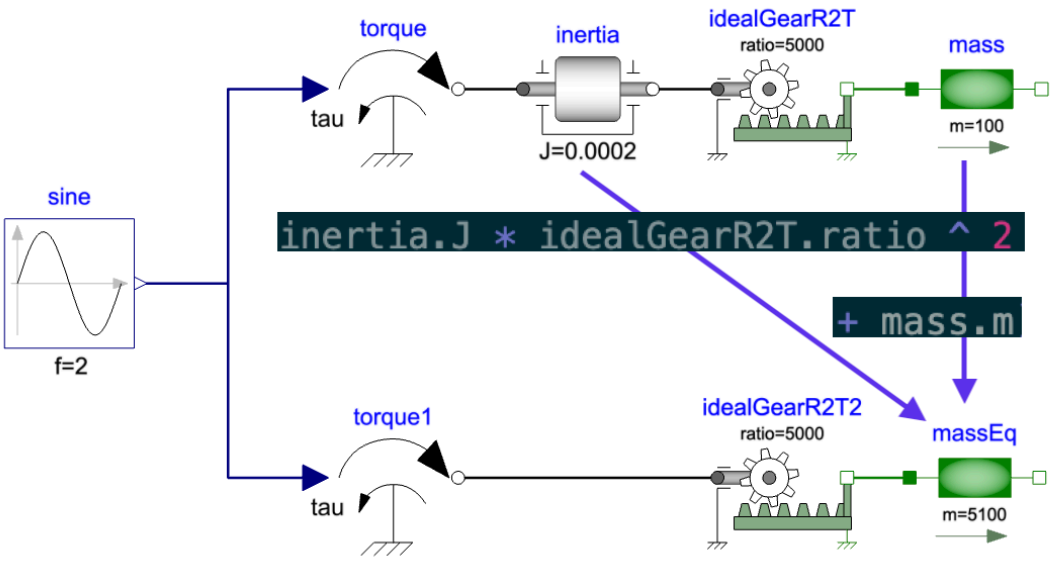
Consider a simple electro-mechanical actuator (EMA) that consists of

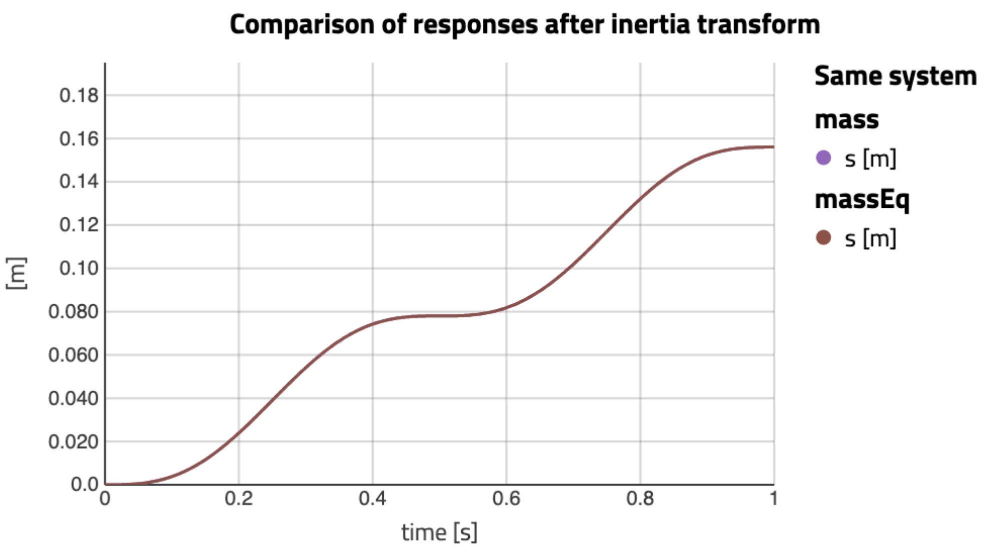
- a motor with a maximum speed of $10000 rev/min$ ($\approx 1050 rad/s$) and an inertia of $2e-4 kg.m^2$,
- an epicycloidal reducer and a ball-screw that jointly converts converts the power to linear motion with a maximum speed of $0.210 m/s$ - which means a conversion ratio of 5000 between the fast and slow axes, and
- an external load of $100 kg$.

The motor inertia appears to be very small compared to the external load.

However, the motor inertia is on a fast axis! Reflected on the slow axis, the same inertia is equivalent to a mass of 5 000 kg!

Indeed, the mechanical power transmission having a combined conversion ratio of 5000, this means that the rotor inertia will reflect on the load side (translational) with a factor 25000000 times higher! The small rotating inertia is not negligible in case of fast control or in case of failure when the EMA reaches its end-stops at full speed.





This is a typical explanation for the comments in section 3.2.2 of this paper for example, that states that the motor inertia can reflect "20 times greater than the load itself" for electro-mechanical actuators.

Summary in few points

- Directly connected inertias can be added.
- Inertia can be brought back through power transmission elements by applying simple laws of physics carefully. The conversion will most likely be at the (inverse of the) square of the speed conversion ratio. This is consistent with the rotational inertia units.
- Small inertia on one end of a power transmission element might not be neglible as seen from the other end.

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I am sharing insights about MBSE and System Simulation.

All my LinkedIn (Dr. Clément Coïc) content is made available on GitHub here. These are technology bits and pieces that I will reuse for the open-source book Model Driven Engineering.

Feel free to reach out if you find something unclear or if you want to leverage some of this content.