Estimate value → average

$$\hat{x}_k = \frac{1}{k}(z_1 + z_2 + \cdots z_k)$$

$$= \frac{1}{k}\cdot\frac{k-1}{k-1}\cdot(z_1+z_2+\cdots+z_{k-1}) + \frac{1}{k}\cdot z_k \quad (\hat{x}_{k-1})$$

$$= \frac{k-1}{k}\cdot\hat{x}_{k-1} + \frac{1}{k}z_k$$

$$= \hat{x}_{k-1} - \frac{1}{k}\hat{x}_{k-1} + \frac{1}{k}z_k$$

$$\therefore \hat{x}_k = \hat{x}_{k-1} + \frac{1}{k}(z_k - \hat{x}_{k-1})$$

$k\uparrow$, $\frac{1}{k}\to 0$

$\therefore \hat{x}_k \to \hat{x}_{k-1}$

令 $\frac{1}{k} = K_k$ 系数

∴ 当前估计值 = 上一次估计值 + 系数 × (当前测量值 − 上一次估计值)

K_k : Kalman Gain

Estimate error : e_{Est}

Measurement error : e_{Mea}

$$K_k = \frac{e_{Est_{k-1}}}{e_{Est_{k-1}} + e_{Mea_k}}$$

Step 1 : 计算 K_k

Step 2 : 计算 $\hat{x}_k$

Step 3 : 更新 $e_{Est_k} = (1-K_k)e_{Est_{k-1}}$

两称：测同一个物品　　根准差

例　$z_1 = 30g$　　$\sigma_1 = 2g$

$z_2 = 32g$　　$\sigma_2 = 4g$

估计的真实值

$$\hat{z} = z_1 + K(z_2 - z_1)$$

$K \in [0,1]$

求 K 使 $\sigma_{\hat{z}}$ 最小 ⇒ 方差最小 $\sigma^2_{\hat{z}}$

$$\sigma^2_{\hat{z}} = Var(z_1 + K(z_2 - z_1)) = Var[(1-K)z_1 + Kz_2]$$

∵ z_1 与 z_2 相互独立

$$\therefore \sigma^2_{\hat{z}} = Var[(1-K)z_1] + Var(Kz_2)$$

∵ $(1-K)$, K 为常数

$$\therefore = (1-K)^2 \cdot Var(z_1) + K^2 Var(z_2)$$

$$= (1-K)^2\sigma_1^2 + K^2\sigma_2^2$$

$$\frac{d\sigma^2_{\hat{z}}}{dK} = 0 \qquad K = \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} = 0.2$$

$$\hat{z} = 30 + 0.2\times(32-30)$$

$$= 30.4g$$

根据两个称的结果预测 30.4g

State Space Representation