Factor Models

Barcelona GSE. Empirical Time Series Methods for Macroeconomic Analysis.
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1 Introduction:

There are two important limitation to VARs:
1) They can usually include only a limited amount of variables (6-8 at most).
2) Agents usually use a larger information set to make decision, rather than the one included in the VAR.
Think, for instance, of monetary authorities. These for sure use more than 6-8 variables when deciding to set the interest rate. What is the problem related to the two points above? Identified shocks are combinations of innovations with respect to a **wrong** information set. This leads to non-fundamentalness and misleading impulse response analysis. An example: the price puzzle. The solution to overcome these problems would be to enlarge the information set used to identify the economic shocks of interest. How can we do this? One alternative are Structural Factor Models (henceforth, SFM). In factor models, the possibility of using large information sets (a lot of variables) helps in aligning the information set of the economic agents and that of the econometrician.

2 The Factor Model:

Consider the following model, introduced by Forni, Giannone, Lippi and Reichlin (Econometric Theory 2009):

$$x_t = Af_t + \xi_t \tag{1}$$

$$D(L)f_t = \epsilon_t \tag{2}$$

$$\epsilon_t = Ru_t \tag{3}$$

where:

- x_t is a vector containing the n variables of the panel.
- Af_t is the common component.
- f_t is a vector containing $r < n$.
- u_t is a vector containing $q < r$ structural macro shocks.
- R is a rxq matrix of coefficients.
- $D(L)$ is a rxr matrix of polynomials in the lag operator.
- ξ_t is a vector of n idiosyncratic components, orthogonal to the common one.

Notice that, combining equations (1) and (2), we can write the dynamic representation of the model, in term of structural shocks:

$$\begin{aligned} x_t &= AD(L)^{-1}\epsilon_t + \xi_t \\ &= AD(L)^{-1}Ru_t + \xi_t \\ &= B(L)u_t + \xi_t \end{aligned} \tag{4}$$

where $B(L) = AD(L)^{-1}R$ is a nxq matrix of impulse responses to structural shocks.

Suppose the we have data on 112 US monthly macroeconomic series and we want to use this data to study the effects of monetary policy shocks on the macroeconomy. In what follows, we aim at replicating Forni and Gambetti (JME, 2010), using a structural factor model with $r = 16$ static factors, $q = 4$ dynamic factors and a recursive identification scheme for the monetary policy shock, in which the impact effects on both industrial production and prices are zero. How do we consistently estimate the model described in (4)?

1. First of all, we can estimate the principal component f_t. Let $\hat{\Gamma}^x$ be the sample variance-covariance matrix of the data at hand. Let loadings $\hat{A} = (\hat{a}_1', \hat{a}_2', ..., \hat{a}_n')'$ be the nxr matrix having on the columns the normalized eigenvectors corresponding to the first largest $\hat{r}$ eigenvalues of $\hat{\Gamma}^x$. Then, the factors (principal components) are given by $\hat{f}_t = \hat{A}'(x_{1t}, x_{2t}, ..., x_{nt})'$. The function PC performs exactly this.

```
function [pc,A,L]=PC(data,npc)

% Function that computes the principal components:

Sigma = cov(data);
opts.disp = 0;
[ A, L ] = eigs(Sigma,npc,'LM',opts);
pc = data*A;

end
```

2. To estimate $D(L)^{-1}$ and ϵ_t , we can estimate a $VAR(p)$ for $\hat{f}_t$ and obtain $\hat{D}(L)^{-1}$ and $\hat{\epsilon}_t$.

```
% Steps 1:

[pc,A,¬]=PC(finaldata,npc); % principal components pc and estimated ...
    loading matrix A

% Step 2:

[pi_hat,Y,X,Y_initial,Yfit,err]=VAR(pc,p,1);
BigA=[pi_hat(2:end,:)'; eye(npc*p-npc) zeros(npc*p-npc,npc)]; % BigA ...
    companion form, npxnp matrix

Dinv=zeros(npc,npc,20);

for j=1:20
    BigDinv=BigA^(j-1);
    Dinv(:,:,j)=BigDinv(1:npc,1:npc); % Impulse response functions of ...
        the Wold representation
end
```

3. Now, let $\hat{\Gamma}^{\epsilon}$ be the sample variance-covariance matrix of $\hat{\epsilon}_t$. As the third step, having an estimate $\hat{q}$ of the number of dynamic factors (shocks), an estimate of a non-structural representation of the common components is obtained by using the spectral decomposition of $\hat{\Gamma}^{\epsilon}$. Precisely, let μ_j^{ϵ} , $j = 1, ..., \hat{q}$, be the $j - th$ eigenvalue of $\hat{\Gamma}^{\epsilon}$, in decreasing order, $\hat{M}$ the $q \times q$ diagonal matrix with μ_j^{ϵ} as its (j, j) entry, $\hat{K}$ the $r \times q$ matrix with the corresponding normalized eigenvectors on the columns. Setting $\hat{S} = \hat{K}\hat{M}$, the estimated matrix of non-structural impulse response functions is given by:

$$\hat{C}(L) = \hat{A}\hat{D}(L)^{-1}\hat{S} \tag{5}$$

```
% Step 3:

Nonstrucinv=zeros(size(finaldata,2),q,20);

opt.disp=0;

Sigma = cov(err);
[ K, MM ] = eigs(Sigma, q, 'LM',opt);
M = diag(sqrt(diag(MM)));

for j=1:20
    Nonstrucinv(:,:,j)=A*(Dinv(:,:,j)^(-1))*K*M;
end
```

4. Finally, $\hat{H}$ and $\hat{b}_i(L) = \hat{c}_i(L)\hat{H}$, where $i = 1, 2, ..., n$, are obtained by imposing the identification restrictions on:

$$\hat{B}(L) = \hat{C}(L)\hat{H} \tag{6}$$

To replicate Forni and Gambetti (2010), we use as restriction that federal funds rate doesn't affect contemporaneously industrial production and CPI, whereas it does affect the exchange rate contemporaneously.

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% Step 5 - Cholesky identification:

intv = [5;96;75;106];
C_hat=zeros(length(intv),q);

for i=1:length(intv)

    C_hat(i,:)=Nonstrucinv(intv(i),:,1);

end

G=chol(C_hat*C_hat','lower');

H=(C_hat^(-1))*G;

% Step 6 - Structural IRF:

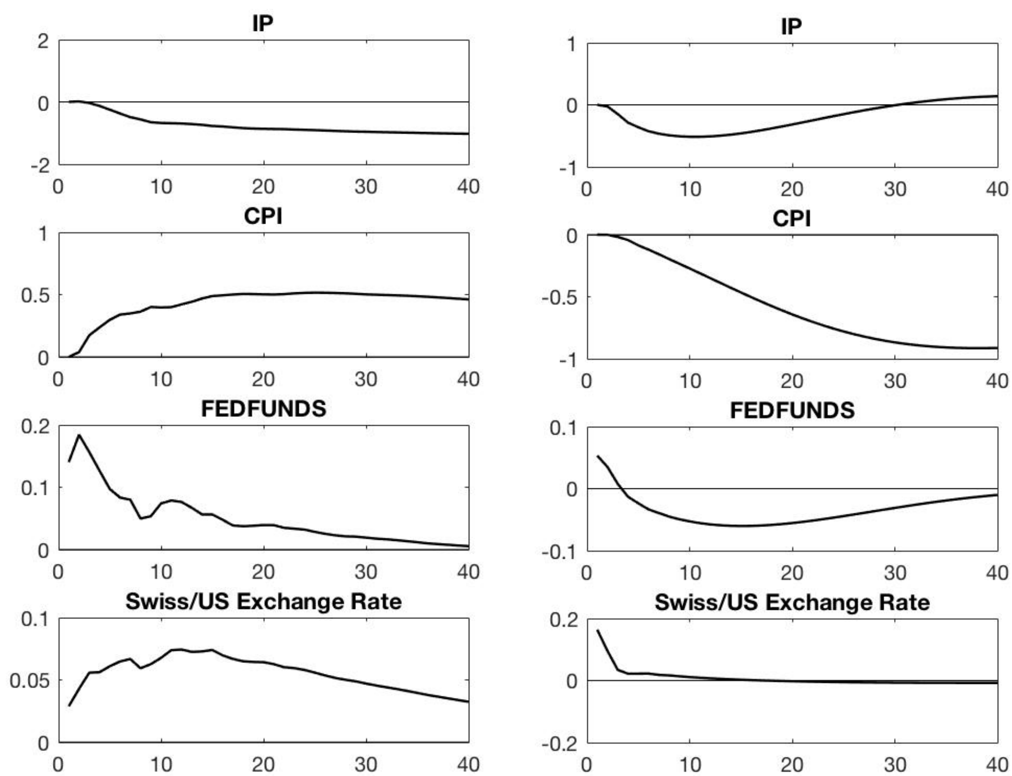
strucirf=zeros(size(finaldata,2),q,20);

for j=1:40
    strucirf(:,:,j)=Nonstrucinv(:,:,j)*H;
end
```

This is achieved using a Cholesky factorization in which we order first and second IP and CPI, third the federal funds rate and last the Swiss/US exchange rate.
How is this restriction performed? Let $\hat{B}_m(L)$ be the matrix of impulse response functions of industrial production, CPI, federal funds rate and Swiss/US real exchange rate (in this order), and $\hat{B}_m(0)$ is restricted to be lower triangular. Let G_m be the (lower triangular) Cholesky factor of $\hat{C}_m(0)\hat{C}_m(0)'$, where $\hat{C}_m(L)$ is estimated in step 3. The restriction is then obtained by setting $\hat{H} = \hat{C}_m(0)^{-1}G_m$. The monetary policy shock is the third one. Figure 1 compares the impulse responses of the variables of interest in the case of a simple VAR (estimated with $p = 9$ lags) with Cholesky and the structural factor model presented above. Two striking results:

1. Price puzzle solved

2. Delayed overshooting disappears



(a) Cholesky VAR

(b) Structural Factor Model

Figure 1: Impulse Response Functions - Cholesky VAR vs. Structural factor Model