ERRATUM AND CLARIFICATION: MINIMUM VERTEX COVERS AND THE SPECTRUM OF THE NORMALIZED LAPLACIAN ON TREES

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In [1], we relate the spectrum of the normalized Laplacian of trees to minimum vertex covers. Using the interlacing technique suggested by Haemers [4], we find an upperbound for the distance between eigenvalue 1 and other eigenvalues (see [1, Theorem 4]).

We thank Dr. Li Jianxi for pointing out that a word is missing when we cite Haemers interlacing techniques in [1, Theorem 3]. The theorem is only valid for symmetric matrix. So the beginning of Theorem 3 should be "suppose that the rows and the columns of the *symmetric* matrix ...", as in its original formulation in [4].

It is then necessary to explain why Haemers interlacing techniques still apply.

As the referee and many readers have noticed, the normalized Laplacian $\mathcal{L}$ that we use is indeed not a symmetric matrix. However, it is diagonally similar to the symmetric normalized Laplacian matrix (see for example [2]), whose (u, v)-entry is $1/\sqrt{\deg u \deg v}$ if u is adjacent to v.

Furthermore, it is a symmetric operator in the sense that

$$\langle \mathcal{L}f, g\rangle = \sum_{v\in V} (\mathcal{L}f)(v)g(v)\mu_v = \langle f, \mathcal{L}g\rangle,$$

see for example [3, Lemma 2.2] or the calculation in the proof of [1, Theorem 2]. Note that $\langle \cdot, \cdot\rangle$ is an inner product with respect to the measure $\mu_v = \deg(v)$ (see the context of [1, Theorem 2]). In matrix form, $\mathcal{L}$ is symmetric in the sense that $D\mathcal{L} = \mathcal{L}^\intercal D$.

Haemers' interlacing techniques then naturally apply to our normalized Laplacian. We invite curious readers to verify from the original proofs in [4].

References

[1] H. Chen and J. Jost. Minimum vertex covers and the spectrum of the normalized Laplacian on trees. *Linear Algebra and its Applications* 437 (4):1089–1101, 2012.

[2] Fan R. K. Chung. *Spectral graph theory.* CBMS Regional Conference Series in Mathematics, vol. 92. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 1997.

[3] A. Grigoryan. Analysis on Graphs. *Lecture Notes at University of Bielefeld*, 2009. available at http://www.math.uni-bielefeld.de/~grigor/aglect.pdf.

[4] Willem H. Haemers. Interlacing eigenvalues and graphs. *Linear Algebra Appl.* 226/228:593–616, 1995.

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