Formuleblad

te gebruiken bij

Voortgezette Analyse voor TN

Curvilineaire coördinaten

1. $\boldsymbol{\nabla} t = \frac{1}{f}\frac{\partial t}{\partial u}\hat{\mathbf{u}} + \frac{1}{g}\frac{\partial t}{\partial v}\hat{\mathbf{v}} + \frac{1}{h}\frac{\partial t}{\partial w}\hat{\mathbf{w}}$
2. $\boldsymbol{\nabla} \bullet \mathbf{A} = \frac{1}{fgh}\left(\frac{\partial}{\partial u}(ghA_u) + \frac{\partial}{\partial v}(fhA_v) + \frac{\partial}{\partial w}(fgA_w)\right)$
3. $\boldsymbol{\nabla} \times \mathbf{A} = \frac{1}{gh}\left(\frac{\partial}{\partial v}(hA_w) - \frac{\partial}{\partial w}(gA_v)\right)\hat{\mathbf{u}} + \frac{1}{fh}\left(\frac{\partial}{\partial w}(fA_u) - \frac{\partial}{\partial u}(hA_w)\right)\hat{\mathbf{v}} + + \frac{1}{fg}\left(\frac{\partial}{\partial u}(gA_v) - \frac{\partial}{\partial v}(fA_u)\right)\hat{\mathbf{w}}$
4. $\boldsymbol{\nabla}^2 t = \frac{1}{fgh}\left(\frac{\partial}{\partial u}\left(\frac{gh}{f}\frac{\partial t}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{fh}{g}\frac{\partial t}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{fg}{h}\frac{\partial t}{\partial w}\right)\right)$

Enkele goniometrische formules

5. $\sin(\alpha)\sin(\beta) = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$
6. $\cos(\alpha)\cos(\beta) = \frac{1}{2}(\cos(\alpha-\beta) + \cos(\alpha+\beta))$
7. $\sin(\alpha)\cos(\beta) = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

Formules voor Fourierreeksen

8. Ongelijkheid van Bessel: $\sum_{k=1}^{\infty} |c_k|^2 \|\phi_k\|^2 \leq \|f\|^2$
9. Formule van Parseval: $\sum_{k=1}^{\infty} |c_k|^2 \|\phi_k\|^2 = \|f\|^2$

Fouriertransformatie

Laat $F(\omega) = \mathcal{F}\{f(t)\}(\omega)$ en $G(\omega) = \mathcal{F}\{g(t)\}(\omega)$ en laat $a \in \mathbb{R}$.

10. $\mathcal{F}\{f(at)\}(\omega) = \frac{1}{a}F\left(\frac{\omega}{a}\right), \quad a > 0$
11. $\mathcal{F}\{f(t-a)\}(\omega) = e^{-ia\omega}F(\omega)$
12. $\mathcal{F}\{e^{iat}f(t)\}(\omega) = F(\omega - a)$
13. $\mathcal{F}\left\{\frac{d}{dt}f(t)\right\}(\omega) = i\omega F(\omega)$
14. $\mathcal{F}\{tf(t)\}(\omega) = i\frac{d}{d\omega}F(\omega)$
15. Convolutiestelling: $\mathcal{F}\{(f*g)(t)\}(\omega) = F(\omega)G(\omega)$
16. Formule van Parseval: $\int_{-\infty}^{\infty} |f(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$

Laplacetransformatie

Laat $F(s) = \mathcal{L}\{f(t)\}(s)$ en $G(s) = \mathcal{L}\{g(t)\}(s)$ en laat $a, b, c \in \mathbb{R}$ en $n = 0, 1, 2, \cdots$.

17. $\mathcal{L}\{1\}(s) = \dfrac{1}{s}, \quad s > 0$
18. $\mathcal{L}\{e^{at}\}(s) = \dfrac{1}{s-a}, \quad s > a$
19. $\mathcal{L}\{t^n\}(s) = \dfrac{n!}{s^{n+1}}, \quad s > 0$
20. $\mathcal{L}\{t^p\}(s) = \dfrac{\Gamma(p+1)}{s^{p+1}}, \quad p > -1 \text{ en } s > 0$
21. $\mathcal{L}\{\sin at\}(s) = \dfrac{a}{s^2+a^2}, \quad s > 0$
22. $\mathcal{L}\{\cos at\}(s) = \dfrac{s}{s^2+a^2}, \quad s > 0$
23. $\mathcal{L}\{\sinh at\}(s) = \dfrac{a}{s^2-a^2}, \quad s > |a|$
24. $\mathcal{L}\{\cosh at\}(s) = \dfrac{s}{s^2-a^2}, \quad s > |a|$
25. $\mathcal{L}\{e^{at}\sin bt\}(s) = \dfrac{b}{(s-a)^2+b^2}, \quad s > a$
26. $\mathcal{L}\{e^{at}\cos bt\}(s) = \dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
27. $\mathcal{L}\{t^n e^{at}\}(s) = \dfrac{n!}{(s-a)^{n+1}}, \quad s > a$
28. $\mathcal{L}\{u_c(t)\}(s) = \dfrac{e^{-cs}}{s}, \quad c > 0 \text{ en } s > 0$
29. $\mathcal{L}\{u_c(t)f(t-c)\}(s) = e^{-cs}F(s), \quad c > 0$
30. $\mathcal{L}\{e^{ct}f(t)\}(s) = F(s-c)$
31. $\mathcal{L}\{f(ct)\}(s) = \dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
32. $\mathcal{L}\left\{\displaystyle\int_0^t f(t-\tau)g(\tau)\,d\tau\right\}(s) = F(s)G(s)$
33. $\mathcal{L}\{\delta(t-c)\}(s) = e^{-cs}, \quad c > 0$
34. $\mathcal{L}\{f^{(n)}(t)\}(s) = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
35. $\mathcal{L}\{(-t)^n f(t)\}(s) = F^{(n)}(s)$