Characterization of a mechanical oscillator using fiber interferometry

(Dated: September 1, 2025)

Note:
1 - when you write the report, you should answer EVERY question in this document. For each question you did not answer 0.5 to 1.0 points will be deducted from the final grade.
2 - You are not allowed to use ChatGPT for the production of the report. If we suspect that you used it, we might decide to fail you and report this to the University.

I. INTRODUCTION

Micromechanical oscillators are commonly used as sensors in applications ranging from cell phones, computers, in the oil- and gas and aerospace industry, amongst others. They are a very versatile platform as they couple to almost any physical system and are easily read-out through electrical or optical signals. Mechanical oscillators are also found in research settings like atomic force microscopy, as well as in experiments probing the very foundations of physics, like in optomechanics.

The mechanical systems are usually described as damped, driven harmonic oscillators, coupled to a read-out system. This coupling can be sensed as a change in resonance frequency or damping of the mechanical motion and used as a very sensitive probe of changes in for example mass, charge, pressures or temperature.

The oscillators are usually fabricated using processes that are adopted from the microelectronics industry, which makes it possible to integrate them with readout electronics on a single chip. Such devices, known as 'smart sensors' or 'lab-on-a-chip', are promising candidates for applications such as explosives detection, air-quality monitoring and medical diagnostics.

In many applications the mechanical device is driven on or close to one of its resonance frequencies. However, even when no driving force is applied the mechanical motion is not zero, as the oscillator is coupled to the surrounding bath. These thermal fluctuations give rise to random motion at an amplitude that can be relatively large in small devices.

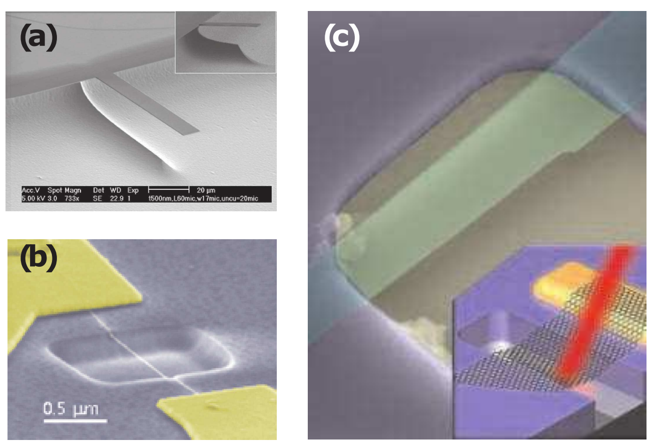

FIG. 1. Shown are three examples of micro- and nanomechanical devices. (a): a miniature silicon nitride 'diving board' clamped on one side to the substrate (singly-clamped devices are usually referred to as *cantilevers*) [1]; (b): A suspended carbon nanotube with a diameter of 2 nm, clamped on both sides [2]. (c) A suspended graphene ribbon [3].

II. THEORY

In this project you will characterize a fiber-optic interferometer and learn how to use it to detect micromechanical motion. You will start with an analysis of the displacement responsivity of the instrument, and then use it to measure a few characteristic parameters of the mechanical resonator. To actuate the motion, the mechanical devices are mounted on a piezo-ceramic stack, which converts an electrical signal into mechanical motion.

A. Fabry-Pérot interferometer

The basic principle of an interferometer is to detect intensity variations that occur when two waves that propagate along different paths, with varying lengths, interfere. Optical interferometers are among the most sensitive displacement detectors, and are frequently used to detect the motion of mechanical devices. Figure 2(a) shows a Fabry-Pérot type interferometer. In the present experiment, the fixed mirror is the tip of an optical fiber, and the moving mirror is the mechanical oscillator device [4]. Multiple reflections between the mirrors are possible and the interferometer is then called a *cavity*. If the two mirrors are placed at a distance L, the optical path difference is given by

$$\Delta L = 2n'L\cos\theta', \tag{1}$$

where n' is the refractive index of the medium between both mirrors and θ' is the angle of refraction. The resulting phase difference for light with wavelength λ can then be written as

$$\delta = \frac{4\pi}{\lambda} n'L\cos\theta'. \tag{2}$$

At each mirror, the light will be partly reflected and transmitted. This results in multiple interference paths between the mirrors. For the total reflected power we can write

$$P_r = P_i \frac{F\sin^2(\delta/2)}{1+F\sin^2(\delta/2)}. \tag{3}$$

Where the parameter F, usually called the finesse, is given by

$$F = \frac{4R}{(1-R)^2}. \tag{4}$$

Here R is the total reflectivity. For a derivation of Eq. 1 see for example Ref. [5].

Question II.A1: Derive an expression for L in terms of λ and give the values for L for which there is constructive and destructive interference. Take $n' = 1$ and $\theta' = 0°$.

B. Beam mechanics

The movement of a cantilever beam can be described by the Euler-Bernoulli equation. The equation consists of a time dependent part and a spatial part:

$$EI_z \frac{\partial^4 Z(x,t)}{\partial x^4} + \rho A \frac{\partial^2 Z(x,t)}{\partial t^2} = 0, \tag{5}$$

where ρ is the density of the material, $A = w \cdot h$ the area of the cross section of the device (with w being the width and h and height), E the Young's modulus and I_z is the area moment of inertia.

Question II.B1: Use separation of variables to split the equation in a time dependent part and a spatial part. Assume that the solution can be written as: $Z(x,t) = u(x)e^{i\omega t}$ and show that the solution for $u(x)$ can be written as $u'''' - k^4 u = 0$. Also give the dispersion relation between k and ω.

Question II.B2: The general solution for $u(x)$ is given by:

$$u(x) = a_1\cosh(kx) + a_2\sinh(kx) + a_3\cos(kx) + a_4\sin(kx). \tag{6}$$

Give the boundary conditions and use them to find a solution for kL (two are hard to find and are given by $u''|_{x=L} = u'''_{x=L} = 0$, which is corresponds to no bending and shear forces at the end of the cantilever). Show that the solution is given by $1 + \cosh(\alpha_n)\cos(\alpha_n) = 0$ and is valid for arbitrary values of $\alpha_n = k_n L$, where L is the length of the cantilever.

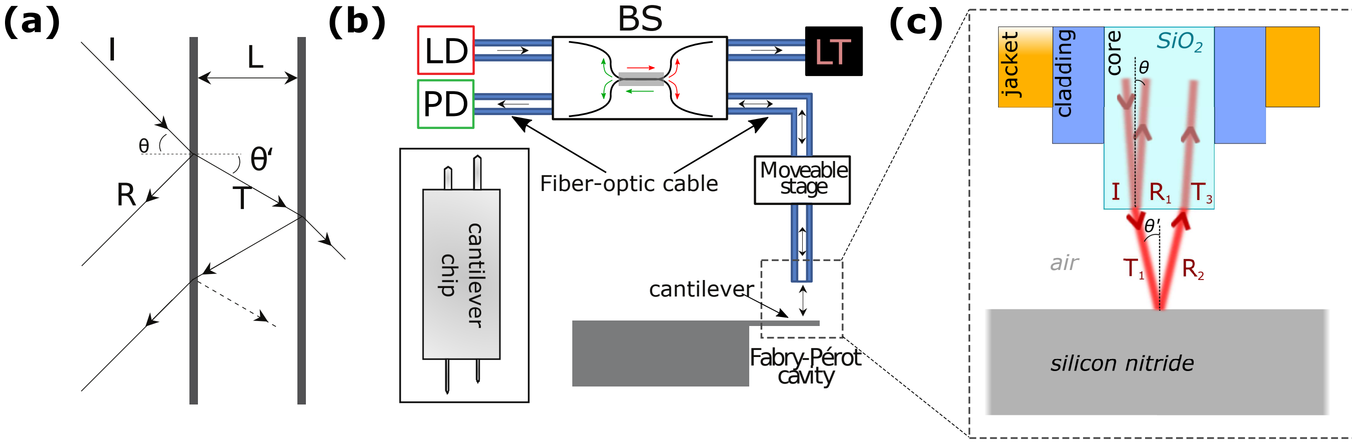

FIG. 2. (a) **Fabry-Pérot cavity.** Reflection and transmission of a plane wave between two mirrors. Upon hitting the mirror, the incoming light will be partly reflected and partly transmitted, resulting in multiple reflected and transmitted beams. Depending on the cavity length L, the interference is constructive or destructive. When the angle of incidence is $\theta' = 0°$, the modulation will be maximized. (b) **Schematic of the measurement setup.** Light is generated by a laser diode (LD), coupled into an optical fiber, and sent through a 50:50 beamsplitter (BS). Half of the incoming light is blocked by a light trap (LT) and the other half is sent to the mechanical device. The position of the spot is manually controlled by a positioning stage. The reflected light travels back and is detected by a photodiode (PD), which converts the intensity of the light into an electrical signal. **Inset:** top-view of the cantilever chip. There are 2 cantilevers on each side. (c) **A close-up of the optical cavity of the setup.** The end of the optical fiber serves as a fixed semi-reflective mirror (due to the difference in the refractive indices of SiO_2 and air). Part of the incident laser light (I) reflects off of the glass/air interface (R1) and a part of it is transmitted (T1). The cantilever chip, which is coated with a very thin layer of metal acts as a movable mirror. The light beam reflected from the chip (R2) travels back to the fiber and it is, again, partly reflected and partly transmitted (T3). Even though multiple such reflections take place, the majority of the interference signal comes from the interference of R1 and T3 (for simplicity T2 and R3 are not shown). In our case $\theta' = 0°$.

Question II.B3: Solve the equation $1 + \cosh(\alpha_n)\cos(\alpha_n) = 0$ numerically (this gives the values for α_n, with mode number n).

Question II.B4: Calculate the area moment of inertia I_z for a beam with a rectangular cross section. Note that $I_z = \int_A z^2 \mathrm{d}A = \int_0^w \mathrm{d}y \int_{-h/2}^{h/2} z^2 \mathrm{d}z$.

Question II.B5: Give an expression for the frequency as function of L, h, E and ρ.

Question II.B6: Using the dimensions of one of the silicon nitride cantilevers that you will be measuring (width $w = 20$ μm, length $L = 100$ μm, height $h = 0.8$ μm), calculate the first 5 resonance frequencies of the cantilever.

III. EXERCISES

A. Characterization of the optical cavity

We start by characterizing the optical cavity formed by the tip of the optical fiber and the sample surface mounted on the piezo stack. A schematic of the setup is shown in Fig2(b). The goal of this exercise is to relate the movement of the mirror (in nanometers [nm]) to the electrical signal that we get as an output from the photodiode (in Volts [V]).

N.B. For instructions on saving data from the oscilloscope or the spectrum analyzer, please refer to **Section IV**.

- The laser diode ($\lambda = \mathbf{635\ nm}$) is connected to a laser controller (the box next to the optical table). In order to turn the laser on, first switch on the main power supply for the controller (the red switch). Press the button that says temperature controller (TC). It should light up yellow once it is on. Then press the laser diode button. After some time it should light up blue and the laser should be on. Please **DO NOT** touch anything else on the laser controller box. It is very sensitive and should be set to the correct values already – in case you cannot turn it on, please contact one of the practicum supervisors.

- Connect the piezostack to the voltage amplifier (PI-E660). **BE CAREFUL** not to touch the output of the amplifier, maximum output voltage is **100 V (!)**. Connect the input of the amplifier to the output of the function generator (TG315). Set the output to a **sawtooth wave** and the frequency in the range of **10 − 50 Hz**. Use a T-piece (a signal splitter) to also connect the output to the oscilloscope.

- Make sure that the fiber is pointed somewhere on the cantilever chip (preferably in the center to ensure maximum reflection) and move the fiber close to the chip using the movable stage. Place the fiber tip at a distance from the chip approximately equal to its diameter.

- Plug the output of the photodiode into the oscilloscope. The photodiode converts intensity to voltage using a built-in amplifier with an adjustable gain (the knob on top). Turn on the power supply for the photodiode (the small black box) and turn on the photodiode using the switch on the side.

- Be sure to turn off the light and keep other vibrating devices, such as laptops, phones, or watches, away from the breadboard where the experiment is conducted, as these devices can introduce unwanted vibrations.

- On the oscilloscope you should be seeing two traces: one from the signal generator (which is your driving signal) and one from the photodiode (which is the signal from the optical cavity).

> **Question III.A1:** Play with the peak-to-peak amplitude of the output signal of the signal generator. Take several traces for different voltages. What do you see? How does changing the driving voltage influence the signal at the output of the photodiode? Why?

> **Question III.A2:** What happens if you increase or decrease the distance between the sample substrate and the fiber tip? You can do this by either changing the distance of the fiber manually, or by applying an offset on the function generator. With this in mind, why does the amplitude of the output signal slowly vary over the course of a full period of the driving signal?

> **Question III.A3:** Using the data obtained in **QIII.A1** and the laser wavelength ($\lambda = 635$ nm) determine the responsivity of the system. Make a proper statistical analysis. Give the mean value and the error of your measurement. The responsivity is a property of the piezo and is defined as the ratio between response (amplitude of oscillation of the piezo in meters) and input (peak-to-peak voltage of signal generated by the signal generator in Volts).

- Now try increasing the frequency of the driving signal while keeping the peak-to-peak amplitude of the driving signal constant. Vary the frequency from **50 Hz − 1.5 kHz** and monitor the output. Take a few traces.

> **Question III.A4:** Above a certain value for the frequency the amplifier can no longer follow the input signal. The frequency at which the output of the amplifier reduces by a factor of 2 (or −3 dB) is called the **cutoff frequency**. Determine the cutoff frequency of the piezo amplifier using the output signal from the optical cavity. Hint: from the calculations for the responsivity of the system, you know the distance that the piezo stack **should** travel per Volt.

B. Detection of resonance motion

Now you will use the interferometer to detect the resonance frequency of a few cantilevers. A spectrum analyzer is used to measure the strength of the photodiode signal in the frequency domain. Connect the output of the photodiode to the input channel of the spectrum analyzer. To achieve optimal results use the averaging function of the spectrum analyzer. Do this by navigating to the "trace" menu and selecting "power average" under "trace type".

This spectrum analyzer also features a tracking generator that allows you to also drive the device. By doing so, you can measure a driven frequency response of a cantilever (for a detailed explanation on spectrum analyzers with a tracking generator please refer to [6]).

- In order to excite the cantilever near its resonance frequency a small dither-piezo is used, which is the wired grey piece on which the cantilever chip is mounted. The dither-piezo performs the same function as the large piezo-stack that we used in part A, but this one can operate at a wider range of frequencies. Connect the output of the function generator (from part A) to the dither piezo. Set the function generator to a **sine wave** between $15 - 45$ kHz. The laser spot is still pointed on the chip itself, not on a cantilever.

> **Question III.B1:** Set the start frequency of the spectrum analyser to 1 kHz and the stop frequency to 50 kHz. What do you see? What happens when you change the frequency/voltage of the signal generator? Why?

- On each side of the cantilever chip there are two rectangular cantilevers. Use the stage to move the fiber tip and shine the laser on top of one of the cantilevers. Now connect the output port of the spectrum analyzer (this is where the tracking generator comes into play). Use the tracking generator to sweep the driving frequency of the dither-piezo. The spectrum analyser is now simultaneously measuring the amplitude and driving the dither-piezo at each frequency. By setting a start and a stop frequency, the so-called **driven freqency response** of the cantilever can be measured.

> **Question III.B2:** Identify at least the two first resonance modes of each cantilever. Compare the measured resonance frequencies to the theoretically calculated ones, and explain any deviations. Make sure that the peaks you identify are coming from the cantilever motion and are not spurious peaks. If you find any spurious peaks, explain what their origins could be.

- Each resonance mode is a standing mechanical wave of the resonator, or in this case the cantilever. This means that along its length (for the longitudinal modes) there are points of no displacement (nodes) and points of maximum displacement (antinodes). By manually moving the laser spot along the length of the cantilever and taking traces using the tracking generator, you can monitor the change of the amplitude of a certain resonance peak. This will give you a "map" of the mode.

> **Question III.B3:** Choose either the short or long cantilever and map out the first two resonance modes of the chosen cantilever.

C. Estimation of absolute cantilever motion

- We can now combine the techniques from part **A** and **B**. From part **A** you know how to determine the cavity length change by looking at the interference pattern. From part **B** you learned how to measure the resonance frequencies of the cantilevers. Point the fiber to the tip of the long cantilever. Connect the signal generator to the dither-piezo and set the output frequency to be the first resonance mode of the long cantilever that you measured with the spectrum analyzer. Look at the signal from the photodiode on the oscilloscope.

> **Question III.C1:** Give an estimate with reasonable errorbar on how much the cantilever moves in absolute terms for a chosen peak to peak amplitude of the driving signal.

IV. OSCILLOSCOPE AND SPECTRUM ANALYZER

To extract the measured data from the oscilloscope and the spectrum analyzer for further processing data should be saved in a correct way.

A. Save data from the oscilloscope

- Put your USB stick in the front panel of the oscilloscope
- To save a waveform press **Run/Stop** so the oscilloscope stops with continuous measuring
- Press **Storage**
- As file format choose .csv (comma separated value)
- Data depth: **Displayed** (only displayed data will be saved)
- Para save ON (Save all settings and parameters)
- Press **external**
- With the small knob next to the display choose Character, Push knob to confirm character
- Press **Save**
- Exit the storage menu and go to the main screen before removing the USB stick

B. Save data from the spectrum analyser

- Put your USB stick in the front panel of the Spectrum analyser
- Press **Storage**
- Select USB drive **Browser** → **Dir** use knob to select mobile disk
- Select **Browser** → **File**
- File type **Trace**
- File format **.csv**
- File source **T1**
- Save (use numerical keys to select name , use right lower button to switch between numbers or character)
- Press **OK** to save file

V. REPORT

General

- The report should contain all the necessary information to repeat the experiment.
- Be concise and be complete.
- Figures should be clear, and their captions self-explanatory. Pay attention to the units - 100 kHz looks much better than 10^5 Hz. **No screenshots. Take raw data and use plotting software (e.g. Matlab, Origin, etc.) to make the figures.**
- Focus on the experiments and results.
- Do not copy and paste text without referencing the source.

- Give full calculations/derivations, rather than only the final answer.
- If you make an estimation, explain under which conditions you expect this estimation to be valid.
- Answer all the questions.
- Clearly write down what you have done (the report must be readable for somebody who did not follow the practical work).

Sections

Each report should contain at least the following sections:

Abstract

- Present in less than 100 words the motivation, the goal, and the main conclusions of the work.

Introduction

- Describe the background and context of the experiment; make it appealing to a broad audience.
- Briefly explain the motivation, goal and the methods to reach it.

Theory

- Here you can include the theory that is necessary to interpret the experimental results.

Experiment

- Description of the experimental procedures.

Results and discussion

- Describe the results with text and figures and explain what you observe.
- If you did not manage to obtain the results you aimed for, explain why and propose alternative experimental routes.
- The figures tell the story. They are worth polishing.

Conclusion

- Briefly present the main result of the work and its main consequences.

References and appendix

- Cite all literature you have used, such as theory textbooks and/or journal publications.

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[5] M. Born & E. Wolf. *Principles of Optics.* Cambridge University Press (1999).

[6] `http://www.radio-electronics.com/info/t_and_m/spectrum_analyser/analyzer-tracking-generator.php`