# Optomechanics RP

# Manual

## Characterization of a mechanical oscillator using fiber interferometry

(Dated: September 1, 2025)

**Note:**
**1 - when you write the report, you should answer EVERY question in this document. For each question you did not answer 0.5 to 1.0 points will be deducted from the final grade.**
**2 - You are not allowed to use ChatGPT for the production of the report. If we suspect that you used it, we might decide to fail you and report this to the University.**

### I. INTRODUCTION

Micromechanical oscillators are commonly used as sensors in applications ranging from cell phones, computers, in the oil- and gas and aerospace industry, amongst others. They are a very versatile platform as they couple to almost any physical system and are easily read-out through electrical or optical signals. Mechanical oscillators are also found in research settings like atomic force microscopy, as well as in experiments probing the very foundations of physics, like in optomechanics.

The mechanical systems are usually described as damped, driven harmonic oscillators, coupled to a read-out system. This coupling can be sensed as a change in resonance frequency or damping of the mechanical motion and used as a very sensitive probe of changes in for example mass, charge, pressures or temperature.

The oscillators are usually fabricated using processes that are adopted from the microelectronics industry, which makes it possible to integrate them with readout electronics on a single chip. Such devices, known as 'smart sensors' or 'lab-on-a-chip', are promising candidates for applications such as explosives detection, air-quality monitoring and medical diagnostics.

In many applications the mechanical device is driven on or close to one of its resonance frequencies. However, even when no driving force is applied the mechanical motion is not zero, as the oscillator is coupled to the surrounding bath. These thermal fluctuations give rise to random motion at an amplitude that can be relatively large in small devices.
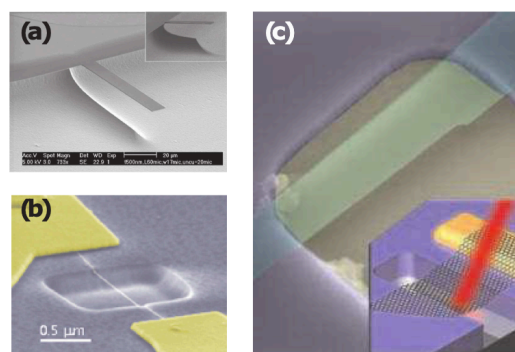


FIG. 1. Shown are three examples of micro- and nanomechanical devices. (a): a miniature silicon nitride 'diving board' clamped on one side to the substrate (singly-clamped devices are usually referred to as *cantilevers*) [1]; (b): A suspended carbon nanotube with a diameter of 2 nm, clamped on both sides [2]. (c) A suspended graphene ribbon [3].

### II. THEORY

In this project you will characterize a fiber-optic interferometer and learn how to use it to detect micromechanical motion. You will start with an analysis of the displacement responsivity of the instrument, and then use it to measure a few characteristic parameters of the mechanical resonator. To actuate the motion, the mechanical devices are mounted on a piezo-ceramic stack, which converts an electrical signal into mechanical motion.

# Q's

# A1

Q: Derive an expression for $L$ in terms of $\lambda$ and give the values for $L$ for which there is constructive and destructive interference. Take $n\prime = 1$ and $\theta\prime = 0 \deg$

A:

$$\delta = \frac{4\pi}{\lambda} L$$

$$P_r = P_i * f$$

$$f = \frac{F \sin^2(\delta/2)}{1 + F \sin^2(\delta/2)}$$

$L$ in terms of $\lambda$:

$$L = \frac{\delta \lambda}{4\pi}$$

Constructive interference when $\sin^2(\delta/2) = 1$ and destructive when $\sin^2(\delta/2) = 0$

$$\sin^2(\delta/2) = 1 \implies \delta/2 = \pi/2 + k\pi \ \text{ for } k \in \mathbb{Z}$$

$$\sin^2(\delta/2) = 0 \implies \delta/2 = k\pi \ \text{ for } k \in \mathbb{Z}$$

Meaning:

$$\text{Constructive int. } L = \frac{\lambda(1 + 2k)}{4}$$
$$\text{Destructive int. } L = \frac{k\lambda}{2}$$

# B1

Beam formula:

$$EI_z \frac{\partial^4 Z(x,t)}{\partial x^4} + \rho A \frac{\partial^2 Z(x,t)}{\partial t^2} = 0$$

$$Z(x,t) = u(x)e^{i\omega t}$$

$$\frac{\partial^4 Z(x,t)}{\partial x^4} = u''''(x)e^{i\omega t}$$

$$\frac{\partial^2 Z(x,t)}{\partial t^2} = u(x) \cdot -\omega^2 e^{i\omega t}$$

$$EI_z u''''(x)e^{i\omega t} - \rho A u(x) \cdot \omega^2 e^{i\omega t} = 0$$

$$EI_z u''''(x)e^{i\omega t} = \rho A u(x) \cdot \omega^2 e^{i\omega t}$$

$$u''''(x) - \frac{\rho A \omega^2}{EI_z} u(x) = 0$$

$$k^4 = \frac{\rho A \omega^2}{EI_z}$$

$$\omega(k) = k^2 \sqrt{\frac{EI_z}{\rho A}}$$