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Aggregated Markov-based reliability analysis of multi-state systems under combined dynamic environments

Xujie Jia[1] | Liudong Xing[2] | Xueying Song[1]

[1]School of Science, Minzu University of China, Beijing, China

[2]Department of Electrical and Computer Engineering, University of Massachusetts Dartmouth, Dartmouth, Massachusetts

Correspondence
Xujie Jia, School of Science, Minzu University of China, Beijing, China.
Email: jiaxujie@126.com

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Abstract

Reliability of a system may differ greatly when operating under different environments. However, the existing works have either neglected the environment factor in system reliability analysis or considered this factor for binary systems or systems subject to a single environment (parameter). In this paper, we make contributions by modeling a multi-state system operating under hybrid dynamic environments affected by multiple environmental parameters. Different Markov chains with finite states are used to represent the random system behavior and dynamic environments, leading to an aggregated Markov process that models the overall system behavior. An effective approach based on state partitions and aggregations is suggested for assessing the system reliability indexes, including reliability, availability, multi-point availability, and environment-based reliability. A high-pressure homogenizer system is analyzed to demonstrate the proposed model and show the comparison of the reliability of system under fixed and dynamic environment.

KEYWORDS

aggregated Markov process, availability, combined dynamic environment, multi-state system, reliability

1 | INTRODUCTION

Modern manufacturing organizations employ very complex, flexible machines. It is well known that many factors can influence the rate at which machine tools degrade. Among them, the environment of the manufacturing equipment can significantly impact the system operation.[1] For instance, significant fluctuations in the ambient temperature or humidity can cause workpiece expansion and contraction and even premature corrosion. Motors can fail under vibrations of high intensity or other environments. Machines that operate in days and nights under different temperatures can show different performances. Particularly, the reliability of a system may differ greatly when operating under different environments. So, it is necessary to consider the influence of environments when studying the reliability of systems subject to complex dynamic environments.

Considerable efforts have been expended in the reliability study of systems operating under dynamic environments. For example, Ni et al[2] proposed an algebraic solution to the fault tree reliability analysis of static and dynamic systems. Eryilmaz and Bozbulut[3] studied reliability of nonrepairable multi-state weighted k-out-of-n: G system based on an algorithmic approach. Verlinden et al[4] built a hybrid reliability model based on simulations for a nuclear reactor safety system under different environments. In addition, stochastic failure models that attempt to capture the impact of a randomly evolving environment have been examined extensively in the reliability field.[5] For example, Esary et al[6] provided physically motivated models and analyzed wear and shock accumulates based on Poisson process and renewal theory. Cinlar[7] demonstrated that the joint process of a unit's wear level and the state of its ambient environment may be considered as a Markov-additive process. Li and Luo[8] suggested that the shock inter-arrival times and the random shock damage are both governed by a Markov chain. Kiessler et al[9] investigated the average availability of a system whose time-varying wear rates are governed by a continuous-time Markov chain (CTMC); Kharoufeh et al[5] extended the model of Kiessler et al[9] by including damage-inducing shocks.

Among the rich works on stochastic models, considerable research efforts have been devoted to the Markov process–based switching model. This model was first applied in the financial fields.[10-13] In 1994, Kim and Park[14] applied the Markov process–based switching model, particularly, a CTMC to determine the reliability of a phased-mission system with random phase durations following general distributions. Based on the research in Kim and Park,[14] Lim[15] further considered the switching between regimes (environments) and described the failure process. Hawkes et al[16] introduced a new model where the switching between regimes is governed by an alternating renewal process, and the observed processes are Markovian. Based on the model of Hawkes et al,[16] Wang et al[17] studied a Markov repairable system with stochastic switching between different environments. Li et al[18,19] analyzed nonrepairable systems with cyclic-mission switching and multimode failure components. Cui et al[20] obtained the closed-form solution about reliability and other sojourn time distributions for discrete-state systems with cyclic mission periods. Kharoufeh[21] derived the failure time distribution for a single-unit system that operates in a Markovian environment. Based on the research of Kharoufeh,[21] Liu et al[22] built a Markovian environment process and obtained some interesting reliability indexes. In addition, semi-Markov processes have been explored for systems that cannot be modeled by Markov processes (see, eg, previous studies[1,23,24]).Although there is a large literature on the study of systems with environmental impacts, the existing works mainly focus on binary system models. For example, Liu et al[22] considered the case where the environment evolves as a homogeneous CTMC, and derived indexes of the system performance including availability and system uptime distributions. Shen and Cui[25] considered a dynamic reliability system operating under a cycle of K regimes, which is modeled as a CTMC with K different transition rate matrices; the system availability and probability distributions of the first uptime were derived.The binary system models are inadequate for modeling and analyzing some practical systems exhibiting multiple states, for example, systems operating under different environments. However, the multi-state model has received little attention in the study of systems with environmental impacts. Particularly, based on the ion-channel modeling theory, Li et al[19] studied the reliability and availability of a multi-state system operating under a single random environment, which is governed by a Markov chain. To the best of our knowledge, no works have been done on modeling multi-state systems subject to multiple different random environments. In this work, we make new contributions by modeling multi-state systems operating under a hybrid dynamic environment based on aggregated Markov processes. For the research of the multi-state systems, there are rich research results.[26-31] Zhao et al[32] analyzed multi-state balanced systems in a shock environment. Iscioglu[33] analyzed the dynamic performance evaluation of multi-state systems under nonhomogeneous continuous time Markov process degradation. An efficient method based on state partitions and aggregations[34-39] is further suggested for deriving reliability indexes of the considered multi-state system.

The rest of the paper is organized as follows. Section 2 presents the multi-state system model under a combination of two random environments. State partitions and aggregations are presented in Section 3. In Section 4, reliability indexes such as system reliability, availability, multi-point availability, and environment-based reliability are derived based on the aggregated Markov process. Section 5 is the generalization to multiple environments. In Section 6, numerical examples are presented to demonstrate the proposed model. Finally, conclusions and future possible continuations of the present research are given in Section 7.

2 | SYSTEM MODEL

The system states can be classified into three categories/subsets: perfect functioning, degradation, and failure. The system behavior follows a homogeneous, irreducible, discrete-time Markov chain $\{Y_n, n = 1,2,\ldots,\infty\}$ under specific environment on the finite state space $S_Y = \{1,2,\ldots,M\}$. There are two different kinds of environments, respectively, modeled by homogeneous, irreducible, discrete-time Markov chains $\{X_n, n = 1,2,\ldots,\infty\}$ and $\{U_n, n = 1,2,\ldots,\infty\}$ with state space $S_X = \{1,2,\ldots,N\}$ and $S_U = \{1,2,\ldots,V\}$. At any time, the system operates under a dynamic environment that is a combination of those two random environments.

Note that the Markov chains $\{X_n, n = 1,2,\ldots,\infty\}$ and $\{U_n, n = 1,2,\ldots,\infty\}$ for modeling the environments are independent; both of them have effects on the Markov chain $\{Y_n, n = 1,2,\ldots,\infty\}$ modeling the system behavior. The one-step transition matrices of the Markov chains $\{X_n, n = 1,2,\ldots,\infty\}$ and $\{U_n, n = 1,2,\ldots,\infty\}$ are given as follows:

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NN} \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1V} \\ q_{21} & q_{22} & \cdots & q_{2V} \\ \vdots & \vdots & \ddots & \vdots \\ q_{V1} & q_{V2} & \cdots & q_{VV} \end{pmatrix},$$

where $h_{ab} = P\{X_{n+1} = b|X_n = a\}, q_{\alpha\beta} = P\{U_{n+1} = \beta|U_n = \alpha\}, a,b \in S_X, \alpha,\beta \in S_U$.

Under a specific combined environment (a, α) $(a \in S_X,\ \alpha \in S_U)$, the Markov chain $\{Y_n, n = 1,2,\ldots,\infty\}$ has one-step transition matrix as follows:

$$\mathbf{P}^{(a,\alpha)} = \begin{pmatrix} p_{11}^{a\alpha} & p_{12}^{a\alpha} & \cdots & p_{1M}^{a\alpha} \\ p_{21}^{a\alpha} & p_{22}^{a\alpha} & \cdots & p_{2M}^{a\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1}^{a\alpha} & p_{M2}^{a\alpha} & \cdots & p_{MM}^{a\alpha} \end{pmatrix}, \tag{1}$$

where $p_{ij}^{a,\alpha} = P(Y_{n+1} = j|Y_n = i, X_{n+1} = a, U_{n+1} = \alpha), i,j \in S_Y$.

For further investigation on the system, a new Markov chain $\{Z_n, n = 1,2,\ldots,\infty\}$, where $Z_n = (X_n, U_n, Y_n)$ is defined. Its state space is denoted as $\Omega = S_X \times S_U \times S_Y = \{(a, \alpha, m) : a \in S_X, \alpha \in S_U, m \in S_Y\}$, where state (a, α, m) means the system is in state m under combined environment (a, α). The one-step transition matrix of $\{Z_n, n = 1,2,\ldots,\infty\}$ is denoted as $\mathbf{G} = (G_{ia\alpha,jb\beta})$ whose elements $G_{ia\alpha,jb\beta}$ $(i,j \in S_Y, a,b \in S_X, \alpha,\beta \in S_U)$ are given as follows:

$$\begin{aligned}
&G_{ia\alpha,jb\beta} := P(Y_{n+1} = j, X_{n+1} = b, U_{n+1} = \beta|Y_n = i, X_n = a, U_n = \alpha) \\
&= P(Y_{n+1} = j, X_{n+1} = b|Y_n = i, X_n = a, U_n = \alpha, U_{n+1} = \beta) \\
&\quad \times P(U_{n+1} = \beta|Y_n = i, X_n = a, U_n = \alpha) \\
&= P(Y_{n+1} = j|Y_n = i, X_n = a, X_{n+1} = b, U_n = \alpha, U_{n+1} = \beta) \\
&\quad \times P(X_{n+1} = b|Y_n = i, X_n = a, U_n = \alpha, U_{n+1} = \beta) \times P(U_{n+1} = \beta|U_n = \alpha) \\
&= P(Y_{n+1} = j|Y_n = i, X_{n+1} = b, U_{n+1} = \beta) \times P(X_{n+1} = b|X_n = a) \\
&\quad \times P(U_{n+1} = \beta|U_n = \alpha) \\
&= h_{ab} q_{\alpha\beta} p_{ij}^{b,\beta}.
\end{aligned}$$

3 | STATE PARTITIONS AND AGGREGATIONS

The system state space contains three subsets $S_Y = A \cup B \cup D$, where A,B,D denote perfect functioning state subset, degradation state subset, and failure state subset, respectively. Denote $A_{a\alpha}, B_{a\alpha}, D_{a\alpha}$ as subsets of A,B,D when the system is in combined environment (a, α) $(a \in S_X,\ \alpha \in S_U)$, respectively. Thus, $A = \bigcup_{\alpha=1}^{V} \bigcup_{a=1}^{N} A_{a,\alpha}$, $B = \bigcup_{\alpha=1}^{V} \bigcup_{a=1}^{N} B_{a,\alpha}$, and $D = \bigcup_{\alpha=1}^{V} \bigcup_{a=1}^{N} D_{a,\alpha}$. Define $W = \bigcup_{\alpha=1}^{V} \bigcup_{a=1}^{N} W_{a,\alpha} = \bigcup_{\alpha=1}^{V} \bigcup_{a=1}^{N} (A_{a,\alpha} \cup B_{a,\alpha})$, which represents the entire set of system working states including perfect functioning states and degradation states.

For Markov chain $\{Z_n, n = 1,2,\ldots,\infty\}$, the one-step transition matrix $\mathbf{G} = (G_{ia\alpha,jb\beta})$ can be divided into sub-matrix, $(a, b \in S_X,\ a,b = 1,\ldots,N;\ \ \alpha,\beta \in S_U, \alpha,\beta = 1,\ldots,V)$.

$$\mathbf{G}_{(a,\alpha)(b,\beta)} = \begin{pmatrix} \mathbf{G}^{A_{a,\alpha}A_{b,\beta}} & \mathbf{G}^{A_{a,\alpha}B_{b,\beta}} & \mathbf{G}^{A_{a,\alpha}D_{b,\beta}} \\ \mathbf{G}^{B_{a,\alpha}A_{b,\beta}} & \mathbf{G}^{B_{a,\alpha}B_{b,\beta}} & \mathbf{G}^{B_{a,\alpha}D_{b,\beta}} \\ \mathbf{G}^{D_{a,\alpha}A_{b,\beta}} & \mathbf{G}^{D_{a,\alpha}B_{b,\beta}} & \mathbf{G}^{D_{a,\alpha}D_{b,\beta}} \end{pmatrix}, \tag{2}$$

where $\mathbf{G}^{A_{a,\alpha}B_{b,\beta}}$ is the transition probability matrices of the state transfer from A to B when the environments transfer from (a,α) to (b,β). Equation (3) gives the transition matrix when the environments transfer from (a,α) to any other environment states.

$$\mathbf{G}_{(a,\alpha)(\ )} = \begin{pmatrix} \mathbf{G}_{(a,\alpha)(1,1)} & \mathbf{G}_{(a,\alpha)(1,2)} & \cdots & \mathbf{G}_{(a,\alpha)(1,V)} \\ \mathbf{G}_{(a,\alpha)(2,1)} & \mathbf{G}_{(a,\alpha)(2,2)} & \cdots & \mathbf{G}_{(a,\alpha)(2,V)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}_{(a,\alpha)(N,1)} & \mathbf{G}_{(a,\alpha)(N,2)} & \cdots & \mathbf{G}_{(a,\alpha)(N,V)} \end{pmatrix}. \tag{3}$$

Thus, the one-step transition matrix $\mathbf{G} = (G_{ia\alpha,jb\beta})$ can be expressed as

$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_{(1,1)(\ )} & \mathbf{G}_{(2,1)(\ )} & \cdots & \mathbf{G}_{(N,1)(\ )} \\ \mathbf{G}_{(1,2)(\ )} & \mathbf{G}_{(2,2)(\ )} & \cdots & \mathbf{G}_{(N,2)(\ )} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}_{(1,V)(\ )} & \mathbf{G}_{(2,V)(\ )} & \cdots & \mathbf{G}_{(N,V)(\ )} \end{pmatrix}. \tag{4}$$

Different environments have different effects on system performance. The influence of environment on the system is reflected in the change of transition probability between states. So the one-step transition matrix $\mathbf{G} = (G_{ia\alpha,jb\beta})$ is the most important influencing factors in the analysis of the system reliability indexes.

4 | RELIABILITY ANALYSIS OF THE AGGREGATED MARKOV SYSTEM

In this section, reliability indexes (including system reliability, availability, multi-point availability, and environment-based reliability) are analyzed based on the aggregated Markov process defined in Section 2 and state partitions and aggregations in Section 3.

For the traditional Markov process method, the states of the system under the process $S_Y = \{1,2,\ldots,M\}$, and $S_X = \{1,2,\ldots,N\}$, $S_U = \{1,2,\ldots,V\}$ are $M \times N \times V$. The transition probability matrix is an $(M \times N \times V) \times (M \times N \times V)$ matrix. While based on the method of state partitions and aggregations, as there are three subsets A,B,D, the final aggregated matrix is a 3×3 matrix. Based on it, we just need to deal with the sub-matrices of this matrix.

4.1 | System reliability

Define $T_D := \inf\{t \in N^+; (X_t, U_t, Y_t) \in (a, \alpha, j), a \in S_X, \alpha \in S_U, j \in D\}$ as the first time to system state D under the combined dynamic environment (a,α). To get the system reliability, we assume system state D is an absorbing state. Then Markov chain Z_n changes to $\{\tilde{Z}_n, n = 1,2,\ldots,\infty\}$. The difference between the Markov chain Z_n and $\tilde{Z}_n$ is that the state D in $\tilde{Z}_n$ is changed to an absorbing state. The state space of $\tilde{Z}_n$ is $\Omega = S_X \times S_U \times S_Y = \{(a,\alpha,m) : a \in S_X, \alpha \in S_U, m \in S_Y\}$ with the absorbing state D. The one-step transition matrix can be denoted as $\tilde{\mathbf{G}}_{NVM \times NVM}$. Let $\tilde{\mathbf{G}}^{W_{a,\alpha}W_{b,\beta}}$ denote the transition matrix of system in the working state when the environments transfer from (a,α) to (b,β). Thus,

$$\tilde{\mathbf{G}}^{W_{a,\alpha}W_{b,\beta}} = \begin{pmatrix} \mathbf{G}^{A_{a,\alpha}A_{b,\beta}} & \mathbf{G}^{A_{a,\alpha}B_{b,\beta}} \\ \mathbf{0} & \mathbf{G}^{B_{a,\alpha}B_{b,\beta}} \end{pmatrix}, \tag{5}$$

where $\mathbf{0}$ denotes zero matrix with appropriate dimensions.

Then

$$\tilde{\mathbf{G}}^{W_{,\alpha}W_{,\beta}} = \begin{pmatrix} \tilde{\mathbf{G}}^{W_{1,\alpha}W_{1,\beta}} & \tilde{\mathbf{G}}^{W_{1,\alpha}W_{2,\beta}} & \cdots & \tilde{\mathbf{G}}^{W_{1,\alpha}W_{N,\beta}} \\ \tilde{\mathbf{G}}^{W_{2,\alpha}W_{1,\beta}} & \tilde{\mathbf{G}}^{W_{2,\alpha}W_{2,\beta}} & \cdots & \tilde{\mathbf{G}}^{W_{2,\alpha}W_{N,\beta}} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{G}}^{W_{N,\alpha}W_{1,\beta}} & \tilde{\mathbf{G}}^{W_{N,\alpha}W_{2,\beta}} & \cdots & \tilde{\mathbf{G}}^{W_{N,\alpha}W_{N,\beta}} \end{pmatrix}, (\alpha,\beta \in S_U, \alpha,\beta = 1,\ldots,V), \tag{6}$$

$$\tilde{\mathbf{G}}^{WW} = \begin{pmatrix} \tilde{\mathbf{G}}^{W_{,1}W_{,1}} & \tilde{\mathbf{G}}^{W_{,1}W_{,2}} & \cdots & \tilde{\mathbf{G}}^{W_{,1}W_{,V}} \\ \tilde{\mathbf{G}}^{W_{,2}W_{,1}} & \tilde{\mathbf{G}}^{W_{,2}W_{,2}} & \cdots & \tilde{\mathbf{G}}^{W_{,2}W_{,V}} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{G}}^{W_{,V}W_{,1}} & \tilde{\mathbf{G}}^{W_{,V}W_{,2}} & \cdots & \tilde{\mathbf{G}}^{W_{,V}W_{,V}} \end{pmatrix}, \tag{7}$$

$\tilde{\mathbf{G}}^{WW}$ is the transition matrix of system in the working state when the environments transfer between any states. Similarly,

$$\tilde{\mathbf{G}}^{WD} = \begin{pmatrix} \tilde{\mathbf{G}}^{W_{,1}D_{,1}} & \tilde{\mathbf{G}}^{W_{,1}D_{,2}} & \cdots & \tilde{\mathbf{G}}^{W_{,1}D_{,V}} \\ \tilde{\mathbf{G}}^{W_{,2}D_{,1}} & \tilde{\mathbf{G}}^{W_{,2}D_{,2}} & \cdots & \tilde{\mathbf{G}}^{W_{,2}D_{,V}} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{G}}^{W_{,V}D_{,1}} & \tilde{\mathbf{G}}^{W_{,V}D_{,2}} & \cdots & \tilde{\mathbf{G}}^{W_{,V}D_{,V}} \end{pmatrix}, \tilde{\mathbf{G}}^{DW} = \begin{pmatrix} \tilde{\mathbf{G}}^{D_{,1}W_{,1}} & \tilde{\mathbf{G}}^{D_{,1}W_{,2}} & \cdots & \tilde{\mathbf{G}}^{D_{,1}W_{,V}} \\ \tilde{\mathbf{G}}^{D_{,2}W_{,1}} & \tilde{\mathbf{G}}^{D_{,2}W_{,2}} & \cdots & \tilde{\mathbf{G}}^{D_{,2}W_{,V}} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{G}}^{D_{,V}W_{,1}} & \tilde{\mathbf{G}}^{D_{,V}W_{,2}} & \cdots & \tilde{\mathbf{G}}^{D_{,V}W_{,V}} \end{pmatrix}.$$

Then

$$\tilde{\mathbf{G}} = \begin{pmatrix} \tilde{\mathbf{G}}^{WW} & \tilde{\mathbf{G}}^{WD} \\ \tilde{\mathbf{G}}^{DW} & \tilde{\mathbf{G}}^{DD} \end{pmatrix}.$$

Based on the above discussions, assume the system starts operation at time 0, the system reliability can be evaluated as follow.

$$\begin{aligned} R_{sys}(t) &= P(T_D > t) = \sum_{a \in S_X} \sum_{\alpha \in S_U} \sum_{j \in W} P(\forall n \in \{0,\ldots,t-1\}, Z_n \in S_X \times S_U \times W, Z_t = (a,\alpha,j)) \\ &= \boldsymbol{\pi}_W \left(\tilde{\mathbf{G}}^{WW}\right)^t \boldsymbol{\Delta}_1, \end{aligned} \tag{8}$$

where $t \in N^+$, $\boldsymbol{\pi}_W = (\boldsymbol{\pi}_A, \boldsymbol{\pi}_B) = (1,0,\ldots,0)_{|W|}$, $\boldsymbol{\Delta}_1 = (1,1,\ldots,1)^T{}_{|W|}$, $|W|$ is the number of the working states.

4.2 | System availability

Availability is the probability that the system is working at time instant or point k. The traditional point availability, denoted as $A(t)$, is given by

$$A(t) = P(Y_t \in W) = P\left(Z_t \in \bigcup_{\alpha=1}^{V} \bigcup_{a=1}^{N} (\{a\} \times \{\alpha\} \times W_{a,\alpha})\right) = \boldsymbol{\gamma} \mathbf{G}^t \boldsymbol{\Delta}_2, \tag{9}$$

where $t \in N^+$, $\boldsymbol{\gamma}$ is the initial probability vector of the system with $t \times |W|$ dimensions.

$$\mathbf{\Delta}_2 = \left(\mathbf{1}^T_{|W_{1,1}|}, \mathbf{0}^T_{M-|W_{1,1}|}, \mathbf{1}^T_{|W_{1,2}|}, \mathbf{0}^T_{M-|W_{1,2}|}, \ldots, \mathbf{1}^T_{|W_{1,V}|}, \mathbf{0}^T_{M-|W_{1,V}|}, \mathbf{1}^T_{|W_{2,1}|}, \mathbf{0}^T_{M-|W_{2,1}|}, \ldots, \mathbf{1}^T_{|W_{N,V}|}, \mathbf{0}^T_{M-|W_{N,V}|}\right)^T,$$

$\mathbf{1}$ and $\mathbf{0}$ are column vectors with all elements being 1 and 0, respectively. $|W_{i,j}|$ is the element number in set $W_{i,j}$.

We also evaluate another type of point availability, which is point availability under the given environment, denoted as $A^{a,\alpha}(t)$. Specifically, $A^{a,\alpha}(t)$ is the probability of the system being at a working state under the combined dynamic environment $(a, \alpha), a \in S_X, \alpha \in S_U$. It can be evaluated as

$$A^{a,\alpha}(t) = P(Y_t \in W_{a,\alpha}) = P(Z_t \in \{a\} \times \{\alpha\} \times W_{a,\alpha}) = \boldsymbol{\gamma}\mathbf{G}^t\mathbf{\Delta}_{a,\alpha}, \tag{10}$$

where $\mathbf{\Delta}_{a,\alpha} = \left(\mathbf{0}^T_{|W_{1,1}|}, \mathbf{0}^T_{M-|W_{1,1}|}, \ldots, \mathbf{0}^T_{|W_{1,V}|}, \mathbf{0}^T_{M-|W_{1,V}|}, \ldots, \mathbf{1}^T_{|W_{a,\alpha}|}, \mathbf{0}^T_{M-|W_{a,\alpha}|}, \ldots, \mathbf{0}^T_{|W_{N,V}|}, \mathbf{0}^T_{M-|W_{N,V}|}\right)^T$. $\mathbf{1}$ and $\mathbf{0}$ are column vectors with all elements being 1 and 0, respectively. $\boldsymbol{\gamma}\mathbf{G}^t$ is a probability vector and shows the probability of being each state at time t. $\mathbf{\Delta}_{a,\alpha}$ sums up the probabilities of all the working states at environment (a, α). Also, when all the vector except $\mathbf{1}_{|W_{a,\alpha}|}$ in $\mathbf{\Delta}_2$ are $\mathbf{0}$, $A(t) = A^{a,\alpha}(t)$.

4.3 | Multi-point availability of the system

Due to the influence of environments, multi-point availability under different environments is also important. First, we consider the multi-point availability under environment $(a, \alpha), a \in S_X, \alpha \in S_U$. Let $A^{a,\alpha}(t_1, t_2, \ldots, t_d)$ denote the availability at all the time points $t_1, t_2, \ldots, t_d (d \in N^+)$, that is, the probability of the system being in state $W_{a,\alpha}$ at all the time points $t_1, t_2, \ldots, t_d$ under environment $(a, \alpha), a \in S_X, \alpha \in S_U$. Then,

$$\begin{aligned} A^{a,\alpha}(t_1, t_2, \ldots, t_d) &= P\{(X_t, U_t, Y_t) \in \{a\} \times \{\alpha\} \times W_{a,\alpha}, \forall t \in \{t_1, t_2, \ldots, t_d\}\} \\ &= \boldsymbol{\gamma}\mathbf{G}^{t_1}\mathbf{E}_{a,\alpha}\mathbf{G}^{(t_2-t_1)}\mathbf{E}_{a,\alpha}\cdots\mathbf{G}^{(t_d-t_{d-1})}\mathbf{E}^*_{a,\alpha}, \end{aligned} \tag{11}$$

where

$$\mathbf{E}_{a,\alpha} = \begin{pmatrix} \mathbf{0}_{((a-1)(\alpha-1)M)\times((a-1)(\alpha-1)M)} & \mathbf{0}_{((a-1)(\alpha-1)M)\times|W_{a,\alpha}|} & \mathbf{0}_{((a-1)(\alpha-1)M)\times(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)} \\ \mathbf{0}_{|W_{a,\alpha}|\times((a-1)(\alpha-1)M)} & \mathbf{I}_{|W_{a,\alpha}|\times|W_{a,\alpha}|} & \mathbf{0}_{|W_{a,\alpha}|\times(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)} \\ \mathbf{0}_{(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)\times((a-1)(\alpha-1)M)} & \mathbf{0}_{(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)\times|W_{a,\alpha}|} & \mathbf{0}_{(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)\times(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)} \end{pmatrix}_{NVM\times NVM},$$

$$\mathbf{E}^*_{a,\alpha} = \begin{pmatrix} \mathbf{0}_{(a-1)(\alpha-1)M\times 1} \\ \mathbf{I}_{|W_{a,\alpha}|\times 1} \\ \mathbf{0}_{(NV-(a-1)(\alpha-1)M-|W_{a,\alpha}|)\times 1} \end{pmatrix}.$$

$\boldsymbol{\gamma}$ is the initial probability vector of the system. $\boldsymbol{\gamma}\mathbf{G}^{t_1}$ is a probability vector and shows the probability of being at each state at time t_1. $\mathbf{G}^{(t_i-t_{i-1})}(i = 1, 2, \ldots, d)$ is the transition probability matrix after time $t_i - t_{i-1}$. Matrix $\mathbf{E}_{a,\alpha}$ keeps the system starting from the working states $\{a\} \times \{\alpha\} \times W_{a,\alpha}$, and $\mathbf{E}^*_{a,\alpha}$ sums up the probabilities of the system being in the working states. When $d = 1$, the multi-point availability is reduced to the point availability in Equation (10).

We also consider the multi-point availability under an arbitrarily environment (i,j), $(i = 1,2,\ldots,N, j = 1,2,\ldots,V)$, denoted as $A^{(i,j)}(t_{1,1}, \ldots, t_{1,V}, t_{2,1}, \ldots, t_{2,V}, \ldots, t_{i,j}, \ldots, t_{N,V})$ with $t_{i,j}$ representing the time when the system is under environment (i,j). Without losing generality, assume $t_{1,1} < t_{1,2} < \cdots < t_{1,V} < t_{2,1} < \cdots < t_{N,V-1} < t_{N,V}$. Thus,

$$\begin{aligned} &A^{(i,j)}\left(t_{1,1}, \ldots, t_{1,V}, t_{2,1}, \ldots, t_{2,V}, \ldots, t_{i,j}, \ldots, t_{N,V}\right) \\ &= P\left\{\left(X_{t_{i,j}}, U_{t_{i,j}}, Y_{t_{i,j}}\right) \in \{i\} \times \{j\} \times W_{i,j},\ i = 1, 2, \ldots, N, j = 1, 2, \ldots, V\right\} \\ &= \boldsymbol{\gamma}\mathbf{G}^{t_{1,1}}\mathbf{E}_{1,1}\mathbf{G}^{(t_{1,2}-t_{1,1})}\mathbf{E}_{2,1}\cdots\mathbf{G}^{(t_{N,V}-t_{N,V-1})}\mathbf{E}^*_{N,V}, \end{aligned} \tag{12}$$

where

$$\mathbf{E}_{1,1}=\begin{pmatrix}\mathbf{I}_{|W_{1,1}|\times|W_{1,1}|} & \mathbf{0}_{|W_{1,1}|\times(NVM-|W_{1,1}|)}\\ \mathbf{0}_{(NVM-|W_{1,1}|)\times|W_{1,1}|} & \mathbf{0}_{(NVM-|W_{1,1}|)\times(NVM-|W_{1,1}|)}\end{pmatrix},\mathbf{E}^{*}_{N,V}=\begin{pmatrix}\mathbf{0}_{(NV-1)M\times 1}\\ \mathbf{I}_{|W_{N,V}|\times 1}\\ \mathbf{0}_{(M-|W_{N,V}|)\times 1}\end{pmatrix},$$

$$\mathbf{E}_{i,j}=\begin{pmatrix}\mathbf{0}_{((i-1)(j-1)M)\times((i-1)(j-1)M)} & \mathbf{0}_{((i-1)(j-1)M)\times|W_{i,j}|} & \mathbf{0}_{((i-1)(j-1)M)\times\left(NV-(i-1)(j-1)M-|W_{i,j}|\right)}\\ \mathbf{0}_{|W_{i,j}|\times((i-1)(j-1)M)} & \mathbf{I}_{|W_{i,j}|\times|W_{i,j}|} & \mathbf{0}_{|W_{i,j}|\times\left(NV-(i-1)(j-1)M-|W_{i,j}|\right)}\\ \mathbf{0}_{\left(NV-(i-1)(j-1)M-|W_{i,j}|\right)\times((i-1)(j-1)M)} & \mathbf{0}_{\left(NV-(i-1)(j-1)M-|W_{i,j}|\right)\times|W_{i,j}|} & \mathbf{0}_{\left(NV-(i-1)(j-1)M-|W_{i,j}|\right)\times\left(NV-(i-1)(j-1)M-|W_{i,j}|\right)}\end{pmatrix}_{NVM\times NVM}.$$

4.4 | Environment-based reliability

The system has limited tolerance to the environment change; when going beyond a certain state range, the environment can have a fatal impact on the system performance. Suppose the environment states $\{(1,1),(1,2),\ldots,(r,s)\},(r<N,$ $s<V)$ are safe environment states. States in the set $NA=\{(r+1,s+1),\ldots,(N,V)\}$ are dangerous environment states (the system will be in a failure state under the environment from set NA). We aggregate the following transition probability matrices,

$$\underset{(i\le s,j\le s)}{\tilde{\mathbf{G}}_1^{W_{,i}W_{,j}}}=\begin{pmatrix}\tilde{\mathbf{G}}^{W_{1,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{1,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{1,i}W_{N,j}}\\ \tilde{\mathbf{G}}^{W_{2,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{2,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{2,i}W_{N,j}}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\mathbf{G}}^{W_{N,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{N,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{N,i}W_{N,j}}\end{pmatrix},\underset{(i\le s,j>s)}{\tilde{\mathbf{G}}_1^{W_{,i}W_{,j}}}=\begin{pmatrix}\tilde{\mathbf{G}}^{W_{1,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{1,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{1,i}W_{r,j}}\\ \tilde{\mathbf{G}}^{W_{2,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{2,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{2,i}W_{r,j}}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\mathbf{G}}^{W_{N,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{N,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{N,i}W_{r,j}}\end{pmatrix},$$

$$\underset{(i>s,j\le s)}{\tilde{\mathbf{G}}_1^{W_{,i}W_{,j}}}=\begin{pmatrix}\tilde{\mathbf{G}}^{W_{1,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{1,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{1,i}W_{N,j}}\\ \tilde{\mathbf{G}}^{W_{2,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{2,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{2,i}W_{N,j}}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\mathbf{G}}^{W_{r,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{r,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{r,i}W_{N,j}}\end{pmatrix},\quad\underset{(i>s,j>s)}{\tilde{\mathbf{G}}_1^{W_{,i}W_{,j}}}=\begin{pmatrix}\tilde{\mathbf{G}}^{W_{1,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{1,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{1,i}W_{r,j}}\\ \tilde{\mathbf{G}}^{W_{2,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{2,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{2,i}W_{r,j}}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\mathbf{G}}^{W_{r,i}W_{1,j}} & \tilde{\mathbf{G}}^{W_{r,i}W_{2,j}} & \cdots & \tilde{\mathbf{G}}^{W_{r,i}W_{r,j}}\end{pmatrix}.$$

$\tilde{\mathbf{G}}_1^{WW}$ can be expressed as

$$\tilde{\mathbf{G}}_1^{WW}=\begin{pmatrix}\tilde{\mathbf{G}}_1^{W_{,1}W_{,1}} & \tilde{\mathbf{G}}_1^{W_{,1}W_{,2}} & \cdots & \tilde{\mathbf{G}}_1^{W_{,1}W_{,V}}\\ \tilde{\mathbf{G}}_1^{W_{,2}W_{,1}} & \tilde{\mathbf{G}}_1^{W_{,2}W_{,2}} & \cdots & \tilde{\mathbf{G}}_1^{W_{,2}W_{,V}}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{\mathbf{G}}_1^{W_{,N}W_{,1}} & \tilde{\mathbf{G}}_1^{W_{,N}W_{,2}} & \cdots & \tilde{\mathbf{G}}_1^{W_{,N}W_{,V}}\end{pmatrix}(r<N,s<V). \tag{13}$$

$\tilde{\mathbf{G}}_1^{WW}$ is the transition matrix of system in the working state when the environments transfer between any states under the environment-based state aggregation. Then the environment-based system reliability $R_e(t)$ is

$$R_e(t)=\boldsymbol{\gamma}_2\left(\tilde{\mathbf{G}}_1^{WW}\right)^t\mathbf{U}, \tag{14}$$

where $\mathbf{U}=(1,1,\ldots,1)^T_{|W|-\sum_{i=r+1}^{N}\sum_{j=s+1}^{V}|W_{i,j}|}$, $\boldsymbol{\gamma}_2$ is an initial probability vector with $t\times|W|$ dimensions.

5 | GENERALIZATION TO MULTIPLE ENVIRONMENTS

While the evaluation method proposed in Section 4 is presented for systems undergoing a combination of two random environments, the methodology is applicable to system subject to multiple environments. Consider the system that operates under k kinds of environments, respectively, modeled by homogeneous, irreducible, discrete-time Markov chains $X_1=(X_n^1)_{n\in N^+}, X_2=(X_n^2)_{n\in N^+}, \ldots, X_k=(X_n^k)_{n\in N^+}$ with state space $S_{X_1}=\{1,2,\ldots,N_1\}$, $S_{X_2}=\{1,2,\ldots,N_2\},\ldots,S_{X_k}=\{1,2,\ldots,N_k\}$, where N_i is the number of states of environment i ($i=1,\ldots,k$). The one-step transition matrices of these Markov chains are $\mathbf{H}_1=\left(h_{ij}^1\right)_{N_1\times N_1}$, $\mathbf{H}_2=\left(h_{ij}^2\right)_{N_2\times N_2}, \ldots, \mathbf{H}_k=\left(h_{ij}^k\right)_{N_k\times N_k}$, respectively. The Markov chains $X_1=(X_n^1)_{n\in N^+}, X_2=(X_n^2)_{n\in N^+}, \ldots, X_k=(X_n^k)_{n\in N^+}$ are independent, and all have effects on the Markov chain $\{Y_n, n=1,2,\ldots,\infty\}$ modeling the system behavior. Then the one-step transition matrix of the system under the k different environments $(\alpha_1,\ldots,\alpha_k)(\alpha_i\in S_{X_i}=\{1,\ldots,N_i\}, i=1,\ldots,k)$ is $\mathbf{P}^{(\alpha_1,\ldots,\alpha_k)}=\left(p_{ij}^{\alpha_1,\ldots,\alpha_k}; i,j\in S_Y\right)_{M\times M}$, where

$$p_{ij}^{\alpha_1,\ldots,\alpha_k}=P\left(Y_{n+1}=j|Y_n=i, X_{n+1}^1=\alpha_1,\ldots,X_{n+1}^k=\alpha_k\right).$$

To analyze the reliability of the system, an aggregated Markov process $Z^*=(Z_n^*)_{n\in N^+}=(X_n^1,\ldots,X_n^k,Y_n)_{n\in N^+}$ is defined, and the state space is $\Omega^*=S_{X_1}\times\ldots\times S_{X_k}\times S_Y$. Then the one-step transition matrix changes to $\mathbf{G}^*=(G^*(i,\alpha_1,\ldots,\alpha_k;j,\beta_1,\ldots,\beta_k))$, where

$$G^*(i,\alpha_1,\ldots,\alpha_k;j,\beta_1,\ldots,\beta_k)=h_{\alpha_1\beta_1}^1 h_{\alpha_2\beta_2}^2\cdots h_{\alpha_k\beta_k}^k p_{ij}^{\beta_1,\ldots,\beta_k}.$$

As shown in Section 3, the system state space contains three subsets $S_Y=A\cup B\cup D$. When the environments transfer from $(\alpha_1,\ldots,\alpha_k)$ to $(\beta_1,\ldots,\beta_k)(\alpha_i,\beta_i\in S_{X_i}=\{1,\ldots,N_i\},\ i=1,\ldots,k)$, the one-step transition matrix $\mathbf{G}^*=(G^*(i,\alpha_1,\ldots,\alpha_k;j,\beta_1,\ldots,\beta_k))$ can be divided into sub-matrices,

$$\mathbf{G}^*_{(\alpha_1,\ldots,\alpha_k)(\beta_1,\ldots,\beta_k)}=\begin{pmatrix}\mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}A_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}B_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}D_{\beta_1,\ldots,\beta_k}}\\ \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}A_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}B_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}D_{\beta_1,\ldots,\beta_k}}\\ \mathbf{G}^{D_{\alpha_1,\ldots,\alpha_k}A_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{D_{\alpha_1,\ldots,\alpha_k}B_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{D_{\alpha_1,\ldots,\alpha_k}D_{\beta_1,\ldots,\beta_k}}\end{pmatrix},$$

where $\mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}B_{\beta_1,\ldots,\beta_k}}$ is the transition probability matrices of the state transfer from A to B when the environments transfer from $(\alpha_1,\ldots,\alpha_k)$ to $(\beta_1,\ldots,\beta_k)$. $\mathbf{G}^*_{(\alpha_1,\ldots,\alpha_k)(,\ldots,)}$ gives the transition matrix when the environments transfer from $(\alpha_1,\ldots,\alpha_k)$ to any other environment states.

$$\mathbf{G}^*_{(\alpha_1,\ldots,\alpha_k)(,\ldots,)}=\begin{pmatrix}\mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}A_{,\ldots,}} & \mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}B_{,\ldots,}} & \mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}D_{,\ldots,}}\\ \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}A_{,\ldots,}} & \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}B_{,\ldots,}} & \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}D_{,\ldots,}}\\ \mathbf{G}^{D_{\alpha_1,\ldots,\alpha_k}A_{,\ldots,}} & \mathbf{G}^{D_{\alpha_1,\ldots,\alpha_k}B_{,\ldots,}} & \mathbf{G}^{D_{\alpha_1,\ldots,\alpha_k}D_{,\ldots,}}\end{pmatrix}.$$

5.1 | System reliability

Expression (5) gives the transition matrix of system in the working state when the combination of two environments transfers from (a,α) to (b,β). The matrix of (5) can be generalized to (15) for systems undergoing k different random environments.

$$\tilde{\mathbf{G}}^{*W_{\alpha_1,\ldots,\alpha_k}W_{\beta_1,\ldots,\beta_k}}=\begin{pmatrix}\mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}A_{\beta_1,\ldots,\beta_k}} & \mathbf{G}^{A_{\alpha_1,\ldots,\alpha_k}B_{\beta_1,\ldots,\beta_k}}\\ \mathbf{0} & \mathbf{G}^{B_{\alpha_1,\ldots,\alpha_k}B_{\beta_1,\ldots,\beta_k}}\end{pmatrix}. \tag{15}$$

Similarly, $\tilde{\mathbf{G}}^{*W_{,\ldots,\alpha_i,\ldots}W_{,\ldots,\beta_i,\ldots}}$ consists of $\tilde{\mathbf{G}}^{*W_{\alpha_1,\ldots,\alpha_k}W_{\beta_1,\ldots,\beta_k}}$.

$$\tilde{\mathbf{G}}^{*W,\ldots,\alpha_i,\ldots W,\ldots,\beta_i,\ldots} = \begin{pmatrix} \mathbf{G}^{A,\ldots,\alpha_i,\ldots A,\ldots,\beta_i,\ldots} & \mathbf{G}^{A,\ldots,\alpha_i,\ldots B,\ldots,\beta_i,\ldots} \\ \mathbf{0} & \mathbf{G}^{B,\ldots,\alpha_i,\ldots B,\ldots,\beta_i,\ldots} \end{pmatrix}.$$

$\tilde{\mathbf{G}}^{*WW}$ consists of $\tilde{\mathbf{G}}^{*W,\ldots,\alpha_i,\ldots W,\ldots,\beta_i,\ldots}$. Then the system reliability is $\boldsymbol{\pi}^*{}_W\left(\tilde{\mathbf{G}}^{*WW}\right)^t\boldsymbol{\Delta}^*{}_1$, where $\boldsymbol{\pi}_W = (1,0,\ldots,0)_{|W|} = (\boldsymbol{\pi}_A, \boldsymbol{\pi}_B)$, $\boldsymbol{\Delta}_1 = (1,1,\ldots,1)^T{}_{|W|}$, $|W|$ is the number of the working states.

5.2 | System availability

For the system operating under k kinds of environments, the traditional point availability becomes $A(t) = \boldsymbol{\gamma}^*(\mathbf{G}^*)^t\boldsymbol{\Delta}_2$ where $\boldsymbol{\gamma}^*$ is the initial probability vector and $\tilde{\mathbf{G}}^* = \begin{pmatrix} \tilde{\mathbf{G}}^{*WW} & \tilde{\mathbf{G}}^{*WD} \\ \tilde{\mathbf{G}}^{*DW} & \tilde{\mathbf{G}}^{*DD} \end{pmatrix}$. Similarly, for multi-point availability of the system under environments $(\alpha_1, \ldots, \alpha_k)$, the transition matrix changes from $\tilde{\mathbf{G}}^{(\cdots)}$ to $\tilde{\mathbf{G}}^{*(\cdots)}$ to reflect the generalization to multiple environments.

5.3 | Environment-based reliability

Suppose the environment states $NA^* = \{(r_1+1, \ldots, r_k+1), \ldots, (N - r_1, \ldots, N - r_k)\}(r_i < N_i, i = 1, \ldots, k)$ are dangerous environment states when the system operates under k kinds of environments. Thus, the environment-based reliability is $R_e{}^*(t) = \boldsymbol{\gamma}^*{}_2\left(\tilde{\mathbf{G}}^*{}_1{}^{WW}\right)^t\mathbf{U}$, where $\tilde{\mathbf{G}}_1^{*WW}$ consists of $\tilde{\mathbf{G}}_1^{*W,\ldots,\alpha_i,\ldots W,\ldots,\beta_i,\ldots}$.

6 | NUMERICAL EXAMPLES

To illustrate the proposed model, we apply it to analyze a high-pressure homogenizer. The homogenizer is a system that uses high-pressure reciprocating pump as power to convey materials. It is a key equipment for homogenizing and refining liquid materials. It can make the suspension material flow through the high-pressure homogeneous chamber with special internal structure at high speed under the action of high pressure (up to 60 000 psi) and make a series of changes in physical, chemical, and structural properties of the material, so as to achieve the effect of homogenization. For high-pressure homogenizer, multiple (eg, three) plunger reciprocating pumps are common. The basic structure of a high-pressure homogenizer with three plungers is shown in Figure 1.

The three-plunger reciprocating pump is composed of three identical working parts, each including the studio, plunger, one-way feed valve, and one-way discharge valve. The three studios are not connected with each other, but the feeding and discharging pipes are connected so that the discharge flow is balanced.

The reciprocating pump system consists of three pumps, which are all influenced by two important environment parameters, temperature (environment I) and pressure (environment II). There are two states for each pump: working and failed. For the three-plunger reciprocating pump system, the state is $S_Y = \{0,1,2,3\}$, with $i = 0,1,2,3$ denoting i failed pumps in the system. The three subsets A,B,D for this example are $A = \{0,1\}$, $B = \{2\}$, and $D = \{3\}$. For temperature (environment I), there are three states including high, medium, and low, denoted by $S_X = \{1,2,3\}$. For pressure (environment II), there are two states including high and low pressure, denoted by $S_U = \{1,2\}$. Suppose the system follows a time-homogeneous Markov chain $\{Y_n, n = 1,2,\ldots,\infty\}$ under specific environment on the finite state space $S_Y = \{0,1,2,3\}$. The combined environment of the system can be described by two time-homogeneous, irreducible CTMCs $\{X_n, n = 1,2,\ldots,\infty\}$ and $\{U_n, n = 1,2,\ldots,\infty\}$ with state spaces $S_X = \{1,2,3\}$ and $S_U = \{1,2\}$, respectively.

Take a month as one step, the one-step transition matrix of the Markov chain $\{Y_n, n = 1,2,\ldots,\infty\}$ under combined environment $(a, \alpha)(a \in S_X, \alpha \in S_U)$ is given as follows:

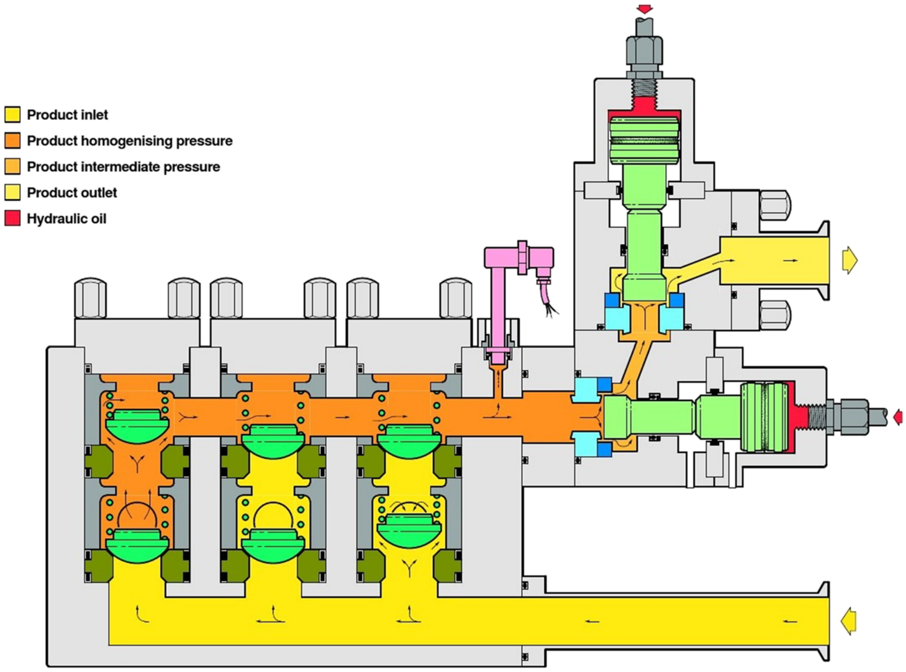

FIGURE 1 Sketch of a high-pressure homogenizer[40] [Colour figure can be viewed at wileyonlinelibrary.com]

$$\mathbf{P}^{(a,\alpha)}=\begin{pmatrix} p_{(a,\alpha)}^{n} & np_{(a,\alpha)}^{n-1}\left(1-p_{(a,\alpha)}\right) & C_n^2 p_{(a,\alpha)}^{n-2}\left(1-p_{(a,\alpha)}\right)^2 & 1-p_{(a,\alpha)}^{n}-np_{(a,\alpha)}^{n-1}\left(1-p_{(a,\alpha)}\right)-C_n^2 p_{(a,\alpha)}^{n-2}\left(1-p_{(a,\alpha)}\right)^2 \\ 0 & p_{(a,\alpha)}^{n-1} & (n-1)p_{(a,\alpha)}^{n-2}\left(1-p_{(a,\alpha)}\right) & 1-p_{(a,\alpha)}^{n-1}-(n-1)p_{(a,\alpha)}^{n-2}\left(1-p_{(a,\alpha)}\right) \\ 0 & 0 & p_{(a,\alpha)}^{n-2} & 1-p_{(a,\alpha)}^{n-2} \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $n=3$, $p_{(1,1)}=0.95$, $p_{(1,2)}=0.93$, $p_{(2,1)}=0.98$, $p_{(2,2)}=0.96$, $p_{(3,1)}=0.9$, $p_{(3,2)}=0.91$. The one-step transition matrices of the Markov chains $\{X_n, n=1,2,\ldots,\infty\}$ and $\{U_n, n=1,2,\ldots,\infty\}$ are given as follows:

$$\mathbf{H}=\begin{pmatrix} 0.2 & 0.5 & 0.3 \\ 0.3 & 0.1 & 0.6 \\ 0.4 & 0.4 & 0.2 \end{pmatrix}, \mathbf{Q}=\begin{pmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{pmatrix}.$$

For comparison, we first analyze the system reliability when the impact of the environment is neglected. Assume the system works in a fixed environment (high temperature and low pressure) ($a=1$, $\alpha=2$). Under this environment, the one-step transition matrix is

$$\mathbf{P}^{(1,2)}=\begin{pmatrix} 0.8044 & 0.1816 & 0.0137 & 0.0003 \\ 0 & 0.8649 & 0.1302 & 0.0049 \\ 0 & 0 & 0.9300 & 0.0700 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

As the working state is $W=A\cup B=\{0,1,2\}$, and the failing state is $D=\{3\}$, the blocking matrix is

$$\mathrm{P}^{(1,2)}=\begin{pmatrix} 0.8044 & 0.1816 & 0.0137 & 0.0003 \\ 0 & 0.8649 & 0.1302 & 0.0049 \\ 0 & 0 & 0.9300 & 0.0700 \\ 0 & 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} \mathrm{P}_{WW}^{(1,2)} & \mathrm{P}_{WD}^{(1,2)} \\ 0 & \mathrm{P}_{DD}^{(1,2)} \end{pmatrix}$$

Then the system reliability is

$$R'_{sys}(t) = P(T > t) = \boldsymbol{\pi}'_W \left(\mathbf{P}^{(1,2)}_{WW}\right)^t \boldsymbol{\Delta}'_1,$$

where $\boldsymbol{\pi}'_W = (1,0,0)$, $\boldsymbol{\Delta}'_1 = (1,1,1)^T$. Figure 2 presents the reliability of the system under the fixed environment ($a = 1, \alpha = 2$).

Next we evaluate the reliability of the system considering impacts of the dynamic operating environment. When the environment changes dynamically, according to Equation (2), there are 36 transition matrices $\mathbf{G}_{(1,1)(1,1)}$, $\mathbf{G}_{(1,1)(1,2)}$, … , $\mathbf{G}_{(3,2)(3,2)}$. $\mathbf{G}_{(a,\ \alpha)(b,\ \beta)}$ $(a,b \in S_X,\ a,b = 1,2,3;\quad \alpha,\beta \in S_U, \alpha,\beta = 1,2)$ is the transition probability matrices of the system state B when the environments transfer from (a, α) to (b, β). We partition the matrices according to the system state. For example, for $\mathbf{G}_{(2,2)(3,1)}$,

$$\mathrm{G}_{(2,2)(3,1)} = \begin{pmatrix} 0.2624 & 0.0875 & 0.0097 & 0.0004 \\ 0 & 0 & 0.0648 & 0.0036 \\ 0 & 0 & 0.3240 & 0.0360 \\ 0 & 0 & 0 & 0.3600 \end{pmatrix} = \begin{pmatrix} \mathrm{G}^{A_{2,2}A_{3,3}} & \mathrm{G}^{A_{2,2}B_{3,3}} & \mathrm{G}^{A_{2,2}D_{3,3}} \\ \mathrm{G}^{B_{2,2}A_{3,3}} & \mathrm{G}^{B_{2,2}B_{3,3}} & \mathrm{G}^{A_{2,2}A_{3,3}} \\ \mathrm{G}^{D_{2,2}A_{3,3}} & \mathrm{G}^{D_{2,2}B_{3,3}} & \mathrm{G}^{D_{2,2}D_{3,3}} \end{pmatrix}$$

where $\mathbf{G}^{A_{a,\alpha}B_{b,\beta}}$ is the transition probability matrices of the system state transfer from A to B when the environments transfer from (a, α) to (b, β).

Based on Equations ((5)) to ((7)), the transition matrix of system in the working state when the environments transfer $\tilde{\mathbf{G}}^{WW}$ is as follow:

$$\tilde{\mathbf{G}}^{WW} = \begin{pmatrix}
0.051 & 0.008 & 0 & 0.141 & 0.009 & 0.066 & 0.022 & 0.002 & 0.097 & 0.022 & 0.002 & 0.310 & 0.039 & 0.002 & 0.158 & 0.047 \\
0 & 0.054 & 0.006 & 0 & 0.144 & 0 & 0 & 0.016 & 0 & 0 & 0.016 & 0 & 0.323 & 0.027 & 0 & 0.174 \\
0 & 0 & 0.057 & 0 & 0 & 0 & 0 & 0.081 & 0 & 0 & 0.112 & 0 & 0 & 0.336 & 0 & 0 \\
0.077 & 0.012 & 0.001 & 0.028 & 0.002 & 0.131 & 0.044 & 0.005 & 0.169 & 0.038 & 0.003 & 0.062 & 0.008 & 0 & 0.316 & 0.094 \\
0 & 0.081 & 0.009 & 0 & 0.029 & 0 & 0 & 0.032 & 0 & 0 & 0.027 & 0 & 0.065 & 0.005 & 0 & 0.348 \\
0.103 & 0.016 & 0.001 & 0.113 & 0.007 & 0.044 & 0.015 & 0.002 & 0.225 & 0.051 & 0.004 & 0.248 & 0.031 & 0.001 & 0.105 & 0.031 \\
0 & 0.108 & 0.011 & 0 & 0.115 & 0 & 0.049 & 0.011 & 0 & 0.242 & 0.036 & 0 & 0.258 & 0.022 & 0 & 0.116 \\
0 & 0 & 0.114 & 0 & 0 & 0 & 0 & 0.054 & 0 & 0 & 0.260 & 0 & 0 & 0.269 & 0 & 0 \\
0.103 & 0.016 & 0.001 & 0.282 & 0.017 & 0.131 & 0.044 & 0.005 & 0.064 & 0.015 & 0.001 & 0.177 & 0.022 & 0 & 0.090 & 0.027 \\
0 & 0.108 & 0.011 & 0 & 0.288 & 0 & 0.149 & 0.032 & 0 & 0.069 & 0.010 & 0 & 0.184 & 0.015 & 0 & 0.099 \\
0 & 0 & 0.114 & 0 & 0 & 0 & 0 & 0.162 & 0 & 0 & 0.074 & 0 & 0 & 0.192 & 0 & 0 \\
0.154 & 0.024 & 0.001 & 0.056 & 0.003 & 0.262 & 0.087 & 0.010 & 0.097 & 0.022 & 0.002 & 0.035 & 0.004 & 0 & 0.181 & 0.054 \\
0 & 0.162 & 0.017 & 0 & 0.058 & 0 & 0 & 0.065 & 0 & 0 & 0.016 & 0 & 0.037 & 0.003 & 0 & 0.199 \\
0 & 0 & 0.171 & 0 & 0 & 0 & 0 & 0.324 & 0 & 0 & 0.112 & 0 & 0 & 0.038 & 0 & 0 \\
0.206 & 0.032 & 0.002 & 0.226 & 0.014 & 0.087 & 0.029 & 0.003 & 0.129 & 0.029 & 0.002 & 0.142 & 0.018 & 0 & 0.060 & 0.018 \\
0 & 0.217 & 0.023 & 0 & 0.230 & 0 & 0 & 0.022 & 0 & 0 & 0.021 & 0 & 0.147 & 0.012 & 0 & 0.066
\end{pmatrix}_{16\times 16}.$$

According Equation (8), the system reliability under dynamic environments is illustrated in Figure 2.

Figure 2 shows the changes of the reliability over time. As shown in the figure, the reliability under an unchanging environment is higher than that under the dynamic environment. Moreover, the reduction speed of the system reliability under the fixed environment is lower than that under the dynamic environment. The comparison result shows the significant impact of the dynamic environment on the system reliability. Therefore, when analyzing a system impacted by its operating environment, if the environment factors are ignored, then the reliability result obtained will be distorted.

According to Equation (9), the system point availability is calculated. The comparison of the system availability under fixed and dynamic environment is shown in Figure 3. According to Equation (14), the environment-based reliability is calculated as shown in Figure 4. Also, the comparison is presented. As shown in Figures 3 and 4, the environment-based reliability declines more rapidly over time.

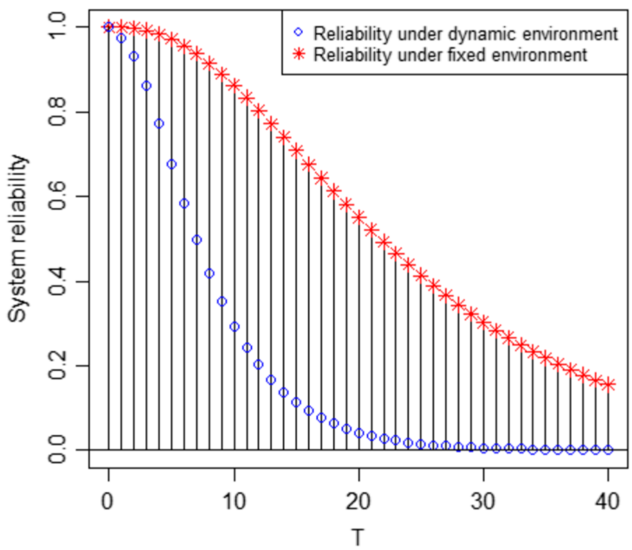

FIGURE 2 Comparison of the system reliability: fixed vs. dynamic environment [Colour figure can be viewed at wileyonlinelibrary.com]

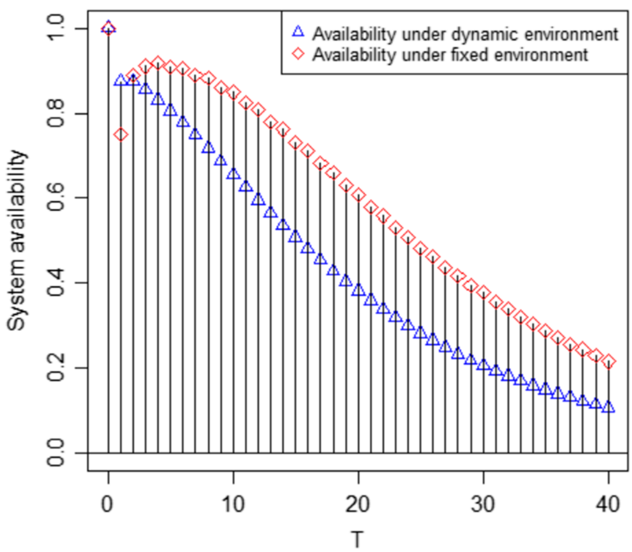

FIGURE 3 Comparison of the system availability: fixed vs. dynamic environment, where fixed environment refers to high temperature and dry condition ($a = 1$, $\alpha = 1$) [Colour figure can be viewed at wileyonlinelibrary.com]

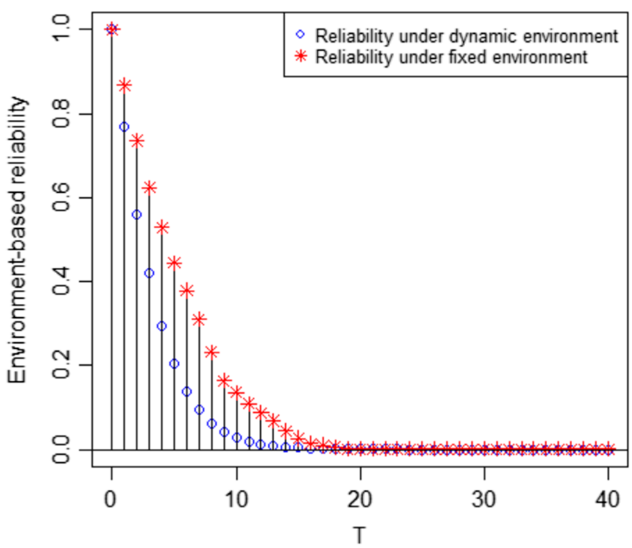

FIGURE 4 Comparison of the environment-based reliability: fixed vs. dynamic environment, where fixed environment refers to high temperature and dry condition ($a = 1$, $\alpha = 1$) [Colour figure can be viewed at wileyonlinelibrary.com]

7 | CONCLUSION

The working environment of systems or products influences their performance. Thus, it is crucial to consider the environment factor when evaluating the system reliability performance. In this article, multi-state systems under combined dynamic environments are modeled using aggregated Markov processes. The complexity of reliability analysis using the Markov processes constructed increases exponentially with the increase in the component states. To solve this problem, we propose an effective approach based on the state partitions and aggregations to evaluate the system reliability indexes, including system reliability, availability, multi-point availability, and environment-based reliability for systems under two random environments. The generalization of the proposed method to multiple environments is also addressed. Examples are provided to demonstrate applications of the proposed model. Numerical results and comparisons demonstrate the significance of studying the combined dynamic environment for assessing the system reliability metrics.The model can have many applications, not only in the system reliability presented in this work but also in other fields such as marketing, risk analysis, and supply chain management under dynamic environments. Future studies such as maintenance and reliability optimization under dynamic environments can be pursued. In addition, the different environments are often dependent, for example, when the temperature rises, the pressure tends to increase. Thus, in the future, we are also interested in investigating systems subject to dependent multiple environments.

NOMENCLATURES

a,b	Elements of S_X
α,β	Elements of S_U
A,B,D	Perfect functioning, degradation, failure states subset
$A(t)$	System availability
$A^{a,\alpha}(t_1, t_2, \cdots, t_d)$	Multi-point availability, ie, the probability of the system in state $W_{a,\alpha}$ at all the time points t_1,t_2,..., t_d
$\mathbf{G}$	Transition probability matrix of Z_n and with elements $G_{ia\alpha,jb\beta}$ (i,$j \in S_Y$,a,$b \in S_X$,α,$\beta \in S_U$)
$\mathbf{G}^{A_{a,\alpha}B_{b,\beta}}$	Transition probability matrices of the state transfer from A to B when the environments transfer from (a, α) to (b, β)
$\mathbf{G}_{(a,\alpha)(\ ,\ )}$	Transition probability matrix when the environments transfer from (a, α) to any other environment states.
$\tilde{\mathbf{G}}_{NVM \times NVM}$	Transition probability matrix of $\tilde{Z}_n$
$\tilde{\mathbf{G}}^{W_{a,\alpha}W_{b,\beta}}$	Transition probability matrix of system in the working state when the environments transfer from (a, α) to (b, β)
$\tilde{\mathbf{G}}^{WW}$	Transition probability matrix of system in the working state when the environments transfer between any states
$\mathbf{H}$,$\mathbf{Q}$,$\mathbf{P}$	Transition probability matrices of X, U, Y
NA	Dangerous environment state set,$NA = \{(r+1,s+1),...,(N,V)\}$
N_i	The number of states of environment i ($i = 1, ..., k$)
$R_e(t)$	Environment-based reliability
$R_{sys}(t)$	System reliability
S_U	State space of U, $S_U = \{1,2,...,V\}$
S_X	State space of X, $S_X = \{1,2,...,N\}$
S_Y	State space of Y, $S_Y = \{1,2,...,M\}$
T_D	First time of the system to state D
W	Working state subset consist of perfect functioning states and degradation states of the system: $W = A \cup B$
$\lvert W \rvert$	Number of the working states
$\lvert W_{i,j} \rvert$	Elements number in set $W_{i,j}$.
$X = (X_n)$,$U = (U_n)$	Markov chains of modeling environments
$Y = (Y_n)$	Markov chain of modeling the system
$Z = (Z_n)$	Markov chain $Z_n = (X_n, U_n, Y_n)$ with state space is $\Omega = S_X \times S_U \times S_Y$, $n = 1,2,...,\infty$
$\{\tilde{Z}_n, n = 1, 2, ..., \infty\}$	Aggregated Markov process of $Z = (Z_n)$ with state space $\Omega = S_X \times S_U \times S_Y = \{(a, \alpha, m) : a \in S_X, \alpha \in S_U, m \in S_Y\}$ and an absorbing failure state.

$\boldsymbol{\gamma}$ Initial probability vector of the system
$\boldsymbol{\gamma}^*$ Initial probability vector

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ORCID

Xujie Jia https://orcid.org/0000-0002-7623-6509
Liudong Xing https://orcid.org/0000-0003-1606-1644

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AUTHOR BIOGRAPHIES

Xujie Jia is an associate professor in College of Science at Minzu University of China. She received her PhD degree of Management Science and Engineering from Beijing Institute of Technology in 2009. She was a visiting scholar in the Department of Electrical and Computer Engineering, University of Massachusetts (UMass), Dartmouth, USA from October 2016 to October 2017. She is a director of the Reliability Branch of Operations Research Society of China. Her main research interests are in quality and reliability engineering, stochastic modeling, and operations research.

Liudong Xing received her PhD degree in Electrical Engineering from the University of Virginia, USA, in 2002. She is currently a professor at the Department of Electrical and Computer Engineering, University of Massachusetts (UMass) Dartmouth, USA. She was the recipient of several "only one per year" awards of UMass Dartmouth, including Leo M. Sullivan Teacher of the Year Award (2014), Scholar of the Year Award (2010), and Outstanding Women Award (2011). She was the recipient of the Changjiang Scholar Award from the Ministry of Education of China in 2015 and IEEE Region 1 Technological Innovation (Academic) Award in 2007. She is a co-recipient of 2018 Premium Award for Best Paper in IET Wireless Sensor Systems and the Best (Student) Paper Award at several international conferences. Prof Xing is or was an associate editor or editorial board member of multiple journals including Reliability Engineering & System Safety, International Journal of Systems Science, International Journal of Systems Science: Operations & Logistics, and IEEE Transactions on Reliability. She has served as program committee chair, co-chair, vice-chair for multiple different international conferences, and technical program committee member for numerous international workshops and conferences. Her research focuses on reliability modeling and analysis of complex systems and network.

Xueying Song is a master in College of Science at Minzu University of China. She is a member of the Reliability Branch of Operations Research Society of China. Her research interest includes stochastic modeling and reliability engineering.

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