B.Sc. Engg. SWE 2nd Semester 27 May 2024

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION SUMMER SEMESTER, 2022-2023
DURATION: 3 HOURS FULL MARKS: 150

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer all 6 (six) questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

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1. a) Determine whether the compound propositions $(P \wedge Q) \rightarrow R$ and $(P \rightarrow R) \wedge (Q \rightarrow R)$ are logically equivalent. 8 (CO1) (PO1)

   b) Prove that $\sqrt{3}$ is irrational. 8 (CO1) (PO1)

   c) Let $S(x)$ be the predicate "x is a student," $F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express the following statements: 3×3 (CO1) (PO1)

      i. Every student has asked Professor Gross a question.
      ii. Some students have never been asked a question by a faculty member.
      iii. Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.

2. a) Apply mathematical induction to prove that 6 divides $n^3 - n$ whenever n is a nonnegative integer. 10 (CO2) (PO1)

   b) Suppose that you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly $n \log(n^2 + 1)$ operations and the second algorithm uses exactly $n^{3/2}$ operations. As n grows, which algorithm uses fewer operations? 10 (CO2) (PO1)

   c) List all the ordered pairs in the relation $R = \{\gcd(a, b) = 1\}$ on the set $\{1, 2, 3, 4, 5, 6, 7\}$. Determine whether the relation R is reflexive, symmetric, and/or transitive 10 (CO2) (PO1)

3. a) Use the extended Euclidean algorithm to express $\gcd(252, 356)$ as a linear combination of 252 and 356. 10 (CO3) (PO1)

   b = m×e + b

   b) Show that $a \textbf{ mod } m = b \textbf{ mod } m$, if and only if $a \equiv b \textbf{ mod } m$, where $m \in \mathbb{Z}^+$ and $a, b \in \mathbb{Z}$. 10 (CO3) (PO1)

   c) List all integers between -100 and 100 that are congruent to -1 **mod** 13. 5 (CO3) (PO1)

4. a) If $ac \equiv bc \textbf{ mod } m$ and $\gcd(c, m) = 1$, then show that $a \equiv b \textbf{ mod } m$, where $m \in \mathbb{Z}^+$ and $a, b, c \in \mathbb{Z}$. 10 (CO3) (PO1)

   b) Solve the following congruence using the modular inverses technique: 10 (CO3) (PO1)

$$19x \equiv 4 \textbf{ mod } 141$$

5 = 3×1 + 2
3 = 2×1 + 1
2 = 1×2 + 0
3 = 3×1 + 0

5. a) Draw the ordered rooted tree that represents the expression $(z+3)/(2+y) \times (x-(z+7))$ and answer the following questions about the tree. 4×4 (CO4) (PO1)

   i. Find the height and the leaf nodes of the tree.

   ii. Write the prefix and the postfix forms of the above expression.

   iii. Evaluate the value of the postfix notation when the value of $x = 2, y = 1$, and $z = 6$.

   b) A chain letter starts with a person sending a letter out to 10 others. Each person is asked to send the letter out to 10 others, and each letter contains a list of the previous six people in the chain. Unless there are fewer than six names in the list, each person sends one dollar to the first person in this list, removes the name of this person from the list, moves up each of the other five names one position, and inserts his or her name at the end of this list. If no person breaks the chain and no one receives more than one letter, how much money will a person in the chain ultimately receive? 10 — 9 (CO4) (PO1)

6. a) A sequence $d_1, d_2, \dots, d_n$, where d_i is the degree of the i-th vertex, is called graphic if it is the degree sequence of a simple graph. Determine whether the sequence $6, 5, 4, 3, 2, 1, 2, 2, 2$ is graphic. Draw the graph if it exists. 10 (CO4) (PO1)

   b) Consider a graph G is represented by the following incidence matrix: 5×2 (CO4) (PO1)

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

   i. Draw the graph G represented by the above incidence matrix.

   ii. Define the bipartite graph. Determine weather graph G is a bipartite graph.

   c) Differentiate between Euler circuit and Hamilton circuit with appropriate graph. 5 (CO4) (PO1)