ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION SUMMER SEMESTER, 2022-2023
DURATION: 1 HOUR 30 MINUTES FULL MARKS: 75

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer all 3 (three) questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

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1. a) Let $V = \{1, 2, \ldots, 30\}$, and $A[1..30]$ be an array such that for each index i between 1 and 30, $A[i] = i \times i - 1$. For the elements $A[1], A[2], \ldots, A[30]$, write a predicate of the following statements: — 3 × 3 (CO1) (PO1)

   i. Every entry in the array is nonnegative.

   ii. The value $A[30]$ is the largest value.

   iii. Every element of array A is nonzero.

  b) Negate the following statements and write the equivalent formulas in negation normal form: — 3 + 4 (CO1) (PO1)

   i. $\forall x \in \mathbb{N} \, (x \text{ is prime} \rightarrow x^2 + 1 \text{ is even})$

   ii. $\forall x \, (\forall y \, (x < y \rightarrow (\exists z \, (x < z \land z < y))))$

  c) Determine whether the compound proposition $(\neg P \land (P \rightarrow Q)) \rightarrow \neg Q$ is a tautology using the logical equivalence rules. Write explicitly the logical rules you used in each stage. — 9 (CO1) (PO1)

2. a) Let $A, B,$ and C be three sets. Answer the following question: — 4 + 4 (CO2) (PO1)

   i. Draw the Venn diagrams for combinations $A \cap (B - C)$.

   ii. How many elements does $\mathscr{P}(\mathscr{P}(\varnothing))$ set have where, $\mathscr{P}$ and $\varnothing$ are power and empty sets respectively. Write the all elements that belong to the set $\mathscr{P}(\mathscr{P}(\varnothing))$

  b) Find the solution of following recurrence relation, where the initial condition is $a_0 = 1$. — 9 (CO2) (PO1)
$$a_n = 2n a_{n-1}$$

  c) Suppose the number of Corona viruses triples per hour. Construct a recurrence relation to monitor the number of Corona virus cases after n hours. Calculate the number of Corona viruses inhabiting after 12 hours, beginning with an initial count of 99. — 4 + 4 (CO2) (PO1)

3. a) Prove that if x and y are real numbers, then $max(x, y) + min(x, y) = x + y$. — 8 (CO1) (PO1)

  b) Consider the following argument: — 10 (CO1) (PO1)
If either wages or prices are raised, there will be inflation. If there is inflation, then either Government must regulate it or the people will suffer. If the people suffer, Government will be unpopular. Government will not regulate inflation, and Government will not be unpopular.

   Determine whether the conclusion "Wages will not rise" derived from the above arguments is valid or invalid using the rules of inference.

  c) Determine whether following function $f(x)$ is a bijection from R to R. In case of your answer is no, then determine the necessary condition for $f(x)$ to be a bijection. — 7 (CO2) (PO1)
$$f(x) = \frac{x + 2}{2x - 1}$$