B.Sc.Eng. CSE/SWE (2nd Sem.) May 15, 2024

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination — Summer Semester: 2022-2023
Course Number: Math 4241 — Full Marks: 200
Course Title: Integral Calculus and Differential Equations — Time: 3 Hours

Answer all the **6 (Six)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) Sketch the region enclosed by the curves $y = x - 1$ and $y^2 = 2x + 6$. Then find the area of the region by integrating with respect to x. **(11)** (CO3) (PO2)

b) Use cylindrical shell method to find the volume of the solid that is generated when the region enclosed by the curves $y = 2x - 1,\ y = -2x + 3$ and $x = 2$ is revolved about the y-axis. **(12)** (CO3) (PO2)

c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis. **(10)** (CO3) (PO2)

2. a) Find the area of the region that is inside the cardioid $r = 2 + 2\cos\theta$ and outside the circle $r = 3$. **(10)** (CO3) (PO2)

b) Find the nature of singularity of the differential equation **(4)** (CO2) (PO2)

$$2x^2y'' + xy' - (2x + 1)y = 0$$

c) Solve the following differential equation by Fröbenius method: **(20)** (CO3) (PO2)

$$2xy'' + (x + 1)y' + y = 0$$

3. a) A small metal bar, whose initial temperature was 20^0C, is dropped into a large container of boiling water How long will it take the bar to reach 90^0C if it is known that its temperature increases 2^0 in 1 second? How long will it take the bar to reach 98^0C? **(11)** (CO3) (PO2)

b) Find the charge $q(t)$ on the capacitor in an LRC-series circuit when $L = 0.25$ henry (h), $R = 10$ ohms (Ω), $C = 0.001$ farad (f), $E(t) = 0, q(0) = q_0$ coulombs, and $i(0) = 0$. **(11)** (CO3) (PO2)

c) Eliminate arbitrary function ϕ from the equation **(11)** (CO1) (PO1)

$$\phi(\tan x + \sin^{-1} y - \log z\,,\, e^x - \sec y + z^3) = 0\,.$$

4. a) Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(11)** (CO2) (PO2)

b) Prove that $\int_{-1}^{1} x^2 P_{n-1}(x) P_{n+1}(x)\,dx = \frac{2n(n+1)}{(4n^2-1)(2n+3)}$. (12) (CO2) (PO2)

c) Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$. (11) (CO2) (PO2)

5. a) Solve $p\cos(x+y) + q\sin(x+y) = z$, using Lagrange's method (11) (CO1) (PO1)

b) Find the general integral of $p^2x^2 + q^2y^2 = z^2$. (11) (CO1) (PO1)

c) Apply Charpit's method to find the complete integral of (11) (CO2) (PO1)

$$z = px + qy + p^2 + q^2.$$

6. a) Solve $(D_x^2 + 4D_x D_y + 4D_y^2)z = e^{2x+y}$. (13) (CO1) (PO1)

b) Find the solution to the heat conduction problem (20) (CO3) (PO2)

$$T_{xx} = 4T_t;\ 0 < x < 2, t > 0$$

$$T(0,t) = 0, T(2,t) = 0, t > 0$$

$$T(x,0) = 2\sin\tfrac{\pi x}{2} - \sin \pi x + 4\sin 2\pi x,\ 0 \le x \le 2.$$