STAT-S520: Homework #2

Due: Tuesday, February 11, 2020 by 11:59 PM

Submit homework as a pdf on Canvas.

1. Trosset chapter 3.7 exercise 7.
   Suppose that $P(A) = 0.7$, $P(B) = 0.6$, and $P(A^c \cap B) = 0.2$. (2 points each)

   (a) Draw a Venn diagram that describes this experiment.

   (b) Is it possible for A and B to be disjoint events? Why or why not?

   (c) What is the probability of $A \cup B^c$?

   (d) Is it possible for A and B to be independent events? Why or why not?

   (e) What is the conditional probability of A given B?

   (Note: You can draw the Venn diagram by hand, scan it and include in your homework document.)

2. A statistics class contains 35 students: 11 undergrads and 24 grad students. Of the undergraduates, 4 are female and 7 and male. Of the grad students, 5 are female and 19 are male. (2 points each)

   (a) I randomly select a student from the class. Given that the student I select is a male, what is the conditional probability that they are an undergraduate?

   (b) I randomly select *two* students from the class, without replacement, in order. Given that the first student I select is a grad student, what is the conditional probability the second student I select is an undergraduate?

3. Trosset chapter 3.7 exercise 12, parts (e) to (g).
   For each of the following pairs of events, explain why A and B are dependent or independent. (2 points each)

   (e) Consider the population of Hollywood feature films produced during the 20th century. A movie is selected at random from this population. Let A denote the event that the movie was filmed in color and let B denote the event that the movie is a western.

   (f) Consider the population of U.S. college freshmen, from which a student is randomly selected. Let A denote the event that the student attends the College of William & Mary, and let B denote the event that the student graduated from high school in Virginia.

   (g) Consider the population of all persons (living or dead) who have earned a Ph.D. from an University in the United States, from which one is randomly selected. Let A denote the event that the person's Ph.D. was earned before 1950 and let B denote the event that the person is female.

(Note: You might need make some reasonable assumptions or use intuition, search for necessary background information online, or try to find data/numbers to do certain calculations when you work on this problem. There is one definite answer but there could be various approaches. Please provide sufficient justification.)

4. Let X be a random variable with CDF

$$F(y) = \begin{cases} 0 & y < -2 \\ (y+2)/4 & -2 \leq y < 2 \\ 1 & y \geq 2 \end{cases}.$$

Graph F and compute the following probabilities: (2 points each)

(a) Provide a graph of the CDF.

(b) What's $P(X \leq 1)$?

(c) What's $P(X > 1)$?

(d) What's $P(X \geq 1)$?

(e) What's $P(-1.5 < X < 0.5)$?

(f) What's $P(|X| > 1)$?

(Note: It is required to insert the graph of the CDF in your submission.)

5. Trosset chapter 4.5 exercise 3.

Consider an urn that contains 10 tickets, labelled

$$\{1, 1, 1, 1, 2, 5, 5, 10, 10, 10\}.$$

From this urn, I propose to draw a ticket. Let X denote the value of the ticket I draw. Determine each of the following: (2 points each)

(a) The probability mass function of X.

(b) The cumulative distribution function of X.

(c) The expected values of X.

(d) The variance of X.

(e) The standard deviation of X.

6. Trosset chapter 4.5 exercise 14.

The Association for Research and Enlightenment (ARE) in Virginia Beach, VA, offers daily demonstrations of a standard technique for testing extrasensory perception (ESP). A "sender" is seated before a box on which one of five symbols (plus, square, star, circle, wave) can be illuminated. A random mechanism selects symbols in such a way that each symbol is equally likely to be illuminated. When a symbol is illuminated, the sender concentrates on it and a "receiver" attempts to identify which symbol has been selected. The receiver indicates a symbol on the receiver's box, which sends a signal to the sender's box that cues it to select and illuminate another symbol. This process of illuminating, sending, and receiving a symbol is repeated 25 times. Each selection of a symbol to be illuminated is independent of the others. The receiver's score (for a set of 25 trials) is the number of symbols that s/he correctly identifies. For the purpose of this exercise, please suppose that ESP does not exist. (2 points each)

(a) How many symbols should we expect the receiver to identify correctly?

(b) The ARE considers a score of more than 7 matches to be indicative of ESP. What is the probability that the receiver will provide such an indication?

(c) The ARE provides all audience members with scoring sheets and invites them to act as receivers. Suppose that, as on August 31 2002, there are 21 people in attendance: 1 volunteer sender, 1 volunteer receiver, and 19 additional receivers in the audience. What is the probability that at least one of the 20 receivers will attain a score indicative of ESP?