

STAT-S520: Homework #3

Due: Tuesday, February 18, 2020 by 11:59 PM

Submit homework as a pdf on Canvas.

1. Suppose that X is a continuous random variable with probability density function (pdf) f defined as follows:

$$f(x) = \begin{cases} 0 & x < 20 \\ \frac{1}{20} & 20 \leq x \leq 40 \\ 0 & x > 40 \end{cases}$$

(2 points each)

- (a) Graph f .
 - (b) Verify that f is a pdf.
 - (c) Compute $P(0 < X < 35)$.
 - (d) Find the CDF of X , $F(y)$, for all y .
2. Trosset chapter 5.6 exercise 2.

Suppose that X is a continuous random variable with probability density function (pdf) f defined as follows:

$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 2(x-1) & \text{if } 1 \leq x \leq 2 \\ 0 & \text{if } x > 2 \end{cases}$$

(2 points each)

- (a) Graph f .
 - (b) Verify that f is a pdf.
 - (c) Compute $P(1.5 < X < 1.75)$.
3. (3 points) Let X be a random variable with PDF

$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x < 20 \\ \frac{1}{60} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

Find the expected value of X .

4. Trosset chapter 5.6 exercise 7. (Use R and give code.)

Let X be a normal random variable with mean $\mu = -5$ and standard deviation $\sigma = 10$. Compute the following:

(2 points each)

- (a) $P(X < 0)$
- (b) $P(X > 5)$
- (c) $P(-3 < X < 7)$
- (d) $P(|X + 5| < 10)$
- (e) $P(|X - 3| > 2)$

5. Trosset chapter 5.6 exercise 8.

Suppose that $X_1 \sim \text{Normal}(1, 9)$ and $X_2 \sim \text{Normal}(3, 16)$ are independent. Determine the mean and variance of each of the following normal random variables:

(2 points each)

- (a) $X_1 + X_2$
- (b) $-X_2$
- (c) $X_1 - X_2$
- (d) $2X_1$
- (e) $2X_1 - 2X_2$

6. (4 points) A local gas station waits 4 days to receive a delivery of regular gasoline to replenish its inventory. The wait period to receive inventory is known as the lead time. The demand during the lead-time period for regular gasoline, as measured in gallons, follows the normal distribution with a mean of 930 gallons and standard deviation of 140 gallons. The station manager places the next order for regular gas when the inventory is 1200 gals (known as the reorder point). What's the probability that the station will run out of gas before the order arrives? (Use R and give code.)

7. Let X be a standard normal random variable. Let $Y = X^2$. (Use R and give code.)

(2 points each)

- (a) Find $P(-1.5 < X < 2.5)$.
- (b) Find $P(Y > 1)$.