A fitting challenge

Some phenomena, like the growth of tumors, can be predicted by theory to follow a **Gompertz function**,

$$g(t) = Ae^{-be^{-ct}}, \tag{1}$$

where A, b, c are positive parameters. As t varies over the real line, g increases from 0 to A.

Suppose you have data (t_i, z_i), $i = 1, \ldots, m$, that you believe follow a Gompertz curve with unknown parameters. Taking the log, we get

$$\log(z_i) \approx (\log A) - be^{-ct_i}. \tag{2}$$

We let $y_i = \log z_i$ and let $a = \log A$. Define a function $\boldsymbol{f}$ whose components are

$$f_i(a, b, c) = a - be^{-ct_i} - y_i. \tag{3}$$

In order to fit the data, we seek to minimize $\|\boldsymbol{f}\|_2$ as a function of a, b, and c, which together we call the vector $\boldsymbol{x}$. This is a nonlinear least squares problem that can be solved by the Levenberg iteration.

Preparation

Read section 4.7.

Goals

You will find a nonlinear least squares fit to given data and use the result to predict the asymptotic value $g(\infty) = A$.

Procedure

Download the script template and the file `gompertz_data`.

1. Load the data file, which has two vectors `t` and `z`. Make a plot of z versus t.
2. In a separate file, write a function

   ```
   function f = gomp(x,t,z)
   ```

   that returns the m-vector defined by (3), given a value of $\boldsymbol{x} = [a; b; c]$ and the vectors `t` and `z`.
3. In the script file, use `levenberg` to find best fitting values for a, b, c.
4. Using the result of the fit, calculate the value of $\lim_{t\to\infty} g(t)$.
5. On top of the data points, make a plot of the fitting function (1) over $0 \le t \le 40$. (Remember that the function fits the log of the values, not the values themselves.)