The long Cahn

The *Allen–Cahn equation* is used as a model for systems that prefer to be in one of two stable states. The governing PDE is

$$u_t = u(1-u^2) + \epsilon u_{xx}. \tag{1}$$

Aside from the diffusion, at each x there is an ODE that has stable steady states at $u = \pm 1$. The boundary conditions and diffusion are the only factors that drive any other behavior.

For this lab you will solve the PDE on $-1 \le x \le 1$, with boundary conditions $u(\pm 1, t) = -1$ and initial condition

$$u_0(x) = -1 + \beta e^{-20x^2}, \tag{2}$$

where β is a parameter. (The initial condition doesn't satisfy the boundary conditions exactly, but numerically it is very close.)

Goals

You will use the method of lines to investigate the long-term behavior of solutions of this PDE.

Preparation

Read section 11.2.

Procedure

Download the template. In this case it's a function and not a script, so the whole file must be executed each time. The function contains code for making an animation; the animations and final function will be your submission material.

1. Define the functions `extend` and `chop` that map between a complete vector of u values at a given time and the interior values. Extension to the boundary is defined via the boundary conditions.
2. Write a function `timederiv` that maps interior values of u to interior values of u_t, using $\epsilon = 10^{-3}$. Use a Chebyshev discretization in space.
3. Define the initial condition using $\beta = 1.6$ and solve the PDE over $0 \le t \le 5$. You should see the bump grow to touch $u = 1$. Save this animation as `anim_grow.gif`.
4. Repeat step 3 using $\beta = 1$. This time the bump dies away down to nothing, leaving the constant solution $u \equiv -1$. Save this animation as `anim_vanish.gif`.
5. Now modify the function to solve for the boundary conditions $u(-1, t) = -1$, $u(1, t) = -\cos(t)$, using $\beta = 1$. Save this animation as `anim_moving.gif`.