Eye See You

We have used least-squares fitting to create functions of a single variable, $y = f(t)$, which can be plotted as a curve to represent data. Some curves, however, cannot be represented as a single function. A more flexible representation is a parametric curve:

$$x = f(t), \qquad y = g(t). \tag{1}$$

Given points in the plane as (x_i, y_i), we can separately fit them as functions of a third parametric variable t and use the curve $(f(t), g(t))$ to pass near the points.

Preparation

Read section 3.1.

Goals

You will capture an image of an eye and find points along the top and bottom eyelids, then do two least-squares fits to represent the eyelids as curves. Because both x and y are periodic as you go around the eye once, you will use periodic functions for the least-squares fitting implied in equation (1):

$$f(t) = b_1 + b_2\cos(2\pi t) + b_3\cos(4\pi t) + b_4\cos(6\pi t) + b_5\sin(2\pi t) + b_6\sin(4\pi t) + b_7\sin(6\pi t), \tag{2}$$
$$g(t) = c_1 + c_2\cos(2\pi t) + c_3\cos(4\pi t) + c_4\cos(6\pi t) + c_5\sin(2\pi t) + c_6\sin(4\pi t) + c_7\sin(6\pi t). \tag{3}$$

Procedure

Download the template script and edit it to perform the following steps.

1. Using a phone, take a picture of an open eye (your own or someone else's). Load the image into MATLAB using `imread` and display it using `image`.

2. Enter the command

   ```
   [xup,yup] = ginput(10);
   ```

   This will create a crosshair in the image window. Click at ten roughly evenly spaced points along the upper eyelid **from right to left**. Get close to the corners of the eye, but don't put points on the corners. Afterward both `xup` and `yup` will be 10×1 vectors representing the selected points.

3. Repeat step 2 using `[xlo,ylo] = ginput(10)` and clicking along the lower eyelid **from left to right**.

4. Stack `xup` and `xlo` into a vector `x`, and stack `yup` and `ylo` into a vector `y`. Both of these should be 20×1. On top of your eye image, plot the points (x_i, y_i) using `'o'` markers. (If the points don't lie close to the eyelids, you have done something wrong.)

5. Now let `t` be a 20×1 vector where $t_i = (i-1)/20$ for $i = 1, \ldots, 20$. Referring back to equations (2) and (3), create a 20×7 matrix `A` whose columns are the values of the functions 1, $\cos(2\pi t)$, and so on, through $\sin(6\pi t)$.

6. Apply linear least squares (using backslash) to solve for the coefficients b_j in (2) using the `x` data, and to solve for the coefficients c_j in (3) using the `y` data.

7. Evaluate the functions in (2) and (3) at 500 equally spaced values of t between 0 and 1. On top of the axes showing the eye image and the selected points, and using the coefficients from the previous step, plot the curve defined by equation (1).