We didn't start the fire

The initial-value problem

$$\frac{dr}{dt} = r^2(1-r), \qquad t > 0, \quad r(0) = r_0, \tag{1}$$

is a simple model for the radius of a spherical flame ball in zero gravity. If r is initially small, it grows slowly for a while before rapidly increasing to a value close to 1, which it approaches asymptotically.

Although the solution is quite simple, it proves to be surprisingly challenging for some IVP solvers, including Euler's method.

Preparation

Read sections 6.1 and 6.2.

Goals

You will solve the flame ball equation numerically using Euler's method, observing its convergence.

Procedure

Download the template code and edit it to perform the following steps.

1. Follow the script to create a high-accuracy reference solution of the problem with $r_0 = 0.01$. Plot the reference solution for $0 \le t \le 500$.

2. For $n = 100, 200, 400, 800, 1600, 6400, 25600$, use the `eulerivp` function to solve the IVP with $r_0 = 0.01$ up to time $t = 100$. Find the error in the Euler solution by taking the difference between its value of r at and the reference solution at $t = 100$. Make a log-log plot of the errors and verify that the method is first-order convergent.

3. Repeat part 2 but at time $t = 200$. This time, the convergence behavior is a lot less smooth than first-order accuracy might seem to imply.

4. Plot the solution $r(t)$ over $0 \le t \le 500$ using `eulerivp` with n = 150, 200, 250, and 300. Even though the errors are getting smaller, these could hardly be called "good" solutions qualitatively!