Find the maximum and minimum value of the function
$x^3 - 3x^2 - 9x + 12$

$f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$= 3x^2 - 9x + 3x - 9$

$= 3x^2 + 3x - 9x - 9$

$= 3x(x+1) - 9(x+1)$

$= (x+1)(3\cancel{x} - 9)$

$f'(x) = 0$ $\quad x+1 = 0$ $\quad 3x - 9 = 0$

$x = -1$ $\quad 3x = 9$

$x = 3$

consider critical point $x = -1$

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 6x - 6$

$f''(x=-1) = 6(-1)-6 = -12 < 0$

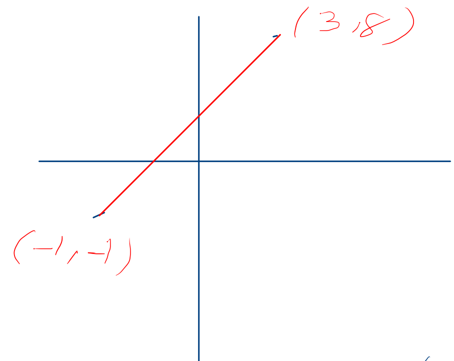
$x=-1$ is local maxima point

$f''(x=3) = 6(3) - 6 = 18-6 = 12 > 0$

$x=3$ is local minima point

Q-2 Calculate slope and equation of line passes through $(-1,-1)$ & $(3,8)$

$$\frac{y_2-y_1}{x_2-x_1} = \frac{8+1}{3+1} = \frac{9}{4} = m$$



$y = mx + c$

consider point $(-1,-1)$

$-1 = 9/4(-1) + c$

$c = -1 + 9/4 = \frac{-4+9}{4} = 5/4$

$y = 9/4x + 5/4$

Q.3 Solve for $w'(z)$

$$w(z) = \frac{4z-5}{2-z}$$

$u = 4z - 5$

$v = 2 - z$

$$\frac{d}{dz}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dz} - u\frac{dv}{dz}}{v^2}$$

$$= \frac{(2-z) \times \frac{d}{dz}(4z-5) - (4z-5)\frac{d}{dz}(2-z)}{(2-z)^2}$$

$$= \frac{(2-z)\times 4 - (4z-5)\times 1}{(2-z)^2}$$

$$= \frac{-4z+8-4z+5}{(2-z)^2}$$

$$w'(z) = \frac{-8z+13}{(2-z)^2}$$

$f(y) = 2x^3+6x^2+3$ identify the critical values and varify local maxima & minima

$$f(x) = 2x^3 + 6x^2 + 3x$$

$$f'(x) = 6x^2 + 12x + 3$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$6x^2 + 12x + 3$$

$a = 6 \quad b = 12 \quad c = 3$

$$= \frac{-12 \pm \sqrt{12^2 - 4 \times 6 \times 3}}{2 \times 6}$$

$$= \frac{-12 \pm \sqrt{144 - 72}}{12}$$

$$= \frac{-12 \pm \sqrt{72}}{12} = \frac{-12 \pm \sqrt{36 \times 2}}{12}$$

$$= \frac{-12 \pm 6\sqrt{2}}{12}$$

$$x = \frac{-2+\sqrt{2}}{2} \qquad x = \frac{-2-\sqrt{2}}{2}$$

$$x = -0.2928 \qquad x = -1.7071$$

consider critical points

$$f''(x) = 12x + 12$$

$$f'(x = -0.2928) = 12(-0.2928) + 12$$

$$= -3.51 + 12$$

$$= 8.486 > 0$$

x is local minima

$$f''(x = -1.707) = 12(-1.707) + 12$$

$$= -20.48 + 12$$

$$= -8.484 < 0$$

$x = -1.7071$ is local maxima

Determine the critical point & identify local minima & maxima

$$y = 2x_1^2 + 2x_1 x_2 + 2x_2 + 6x_1$$

Let us assume $x_1 = x$

$x_2 = y$

Hence the equation becomes

$$2x^2 + 2xy + 2y^2 + 6x$$

Let us first find the critical points

$$f_x = 4x + 2y + 6$$

$$f_y = 4y + 2x$$

The critical points are the points where

$f_x = 0$ & $f_y = 0$

$$4x + 2y + 6 = 0$$

$$2(2x + y + 3) = 0$$

$$2x + y = -3$$

$$y = -3 - 2x$$

substitute in f_y

$$f_y = 4(-3 - 2x) + 2x$$

$$= -12 - 8x + 2x$$

$$= -12 - 6x$$

$$f_y = 0$$

$$-12 - 6x = 0$$

$$-12 = 6x$$

$$x = -12/6 = -2$$

similarly $y = 1$

$$\frac{\partial}{\partial x} f = 4x + 2y + 6 \qquad \frac{\partial f}{\partial y} = 2x + 4y$$

$$A \quad \frac{\partial^2 f}{dx^2} = 4 \qquad B \qquad C \quad \frac{\partial^2 f}{\partial y^2} = 4$$

$$B \quad \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$AC - B^2 > 0$$

$$4 \times 4 - 4 = 12$$

This value is coming positive hence local minima at $x = -2$ & $y = 1$