Computational Macroeconomics
Exercises

Week 5

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Contents

1. Deterministic vs stochastic simulations in Dynare

What is the difference between deterministic and stochastic simulations in Dynare? In other words, what is the difference between using `perfect_foresight_solver` or `stoch_simul`?

2. New Keynesian Model: Simulations in Dynare

Consider the basic New Keynesian (NK) model without capital and a linear production function which is given in the following Dynare file called `new_keynesian_common.inc`:

progs/dynare/new_keynesian_common.inc

```
%----------------------------------------------------------------
% Copyright Notice
%----------------------------------------------------------------
% This file implements the baseline New Keynesian model of
% Jordi Gali (2015): Monetary Policy, Inflation, and the Business Cycle,
% Princeton University Press, Second Edition, Chapter 3
%
% THIS MOD-FILE REQUIRES DYNARE 4.5 OR HIGHER
%
% This implementation was written originally by Johannes Pfeifer and
% adapted by Willi Mutschler (willi@mutschler.eu).
%
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%
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%----------------------------------------------------------------
% Declaration of endogenous variables
%----------------------------------------------------------------
var
c           ${c}$                       (long_name='consumption')
w           ${w}$                       (long_name='real wage')
pie         ${\pi}$                     (long_name='gross inflation')
n           ${n}$                       (long_name='hours worked')
R           ${R}$                       (long_name='nominal interest rate')
r           ${r}$                       (long_name='real interest rate')
y           ${y}$                       (long_name='output')
div         ${div}$                     (long_name='real profits')
Q           ${Q}$                       (long_name='bond price')
mc          ${mc}$                      (long_name='real marginal costs')
pstar       ${p^*}$                     (long_name='price inefficiency distortion')
ptilde      ${\widetilde{p}}$           (long_name='Optimal reset price')
s1          ${s_1}$                     (long_name='aux. sum 1 recursive price setting')
s2          ${s_2}$                     (long_name='aux. sum 2 recursive price setting')
a           ${a}$                       (long_name='technology level')
z           ${z}$                       (long_name='discount factor shifter')
nu          ${\nu}$                     (long_name='monetary policy shifter')
ahat        ${\widehat{a}}$             (long_name='technology level (log dev from ss)')
zhat        ${\widehat{z}}$             (long_name='preference shifter (log dev from ss)')
yhat        ${\widehat{y}}$             (long_name='output (log dev from ss)')
what        ${\widehat{w}}$             (long_name='real wage (log dev from ss)')
nhat        ${\widehat{n}}$             (long_name='hours worked (log dev from ss)')
piehat_an   ${\widehat{\pi}^{ann}}$     (long_name='annualized inflation rate (log dev from ss)')
Rhat_an     ${\widehat{R}^{ann}}$       (long_name='annualized nominal interest rate (log dev from ss)')
rhat_an     ${\widehat{r}^{ann}}$       (long_name='annualized real interest rate (log dev from ss)')
mchat       ${\widehat{mc}}$            (long_name='real marginal costs (log dev from ss)')
;
%----------------------------------------------------------------
% Declaration of exogenous variables (i.e. shocks)
%----------------------------------------------------------------
varexo
eps_a    ${\varepsilon_a}$      (long_name='technology shock')
eps_z    ${\varepsilon_z}$      (long_name='preference shock')
eps_nu   ${\varepsilon_\nu}$    (long_name='monetary policy shock')
;
%----------------------------------------------------------------
% Declaration of parameters
%----------------------------------------------------------------
parameters
BETA      ${\beta}$        (long_name='discount factor')
RHO_A     ${\rho_a}$       (long_name='autocorrelation technology process')
RHO_NU    ${\rho_{\nu}}$   (long_name='autocorrelation monetary policy process')
RHO_Z     ${\rho_{z}}$     (long_name='autocorrelation preference shock')
SIGMA     ${\sigma}$       (long_name='inverse elasticity of intertemporal substitution')
VARPHI    ${\varphi}$      (long_name='inverse Frisch elasticity')
```

```
PHI_PIE    ${\phi_{\pi}}$    (long_name='inflation feedback Taylor Rule')
PHI_Y      ${\phi_{y}}$      (long_name='output feedback Taylor Rule')
EPSILON    ${\epsilon}$      (long_name='demand elasticity')
THETA      ${\theta}$        (long_name='Calvo parameter')
PIESTAR    ${\pi^*}$         (long_name='inflation target')
;

%----------------------------------------------------------------
% Model Equations
%----------------------------------------------------------------
%model(block,bytecode); % you can also use this
model;
%marginal utilities
#Un = -z*n^VARPHI;
#Uc = z*c^(-SIGMA);
#Ucp = z(+1)*c(+1)^(-SIGMA);

[name='labor supply']
w = -Un/Uc;

[name='Euler equation']
Uc = BETA*Ucp*r;

[name='optimal price setting']
ptilde*s1 = EPSILON/(EPSILON-1)*s2;

[name='optimal price setting auxiliary recursion 1']
s1 = y*Uc + BETA*THETA*pie(+1)^(EPSILON-1)*s1(+1);

[name='optimal price setting auxiliary recursion 2']
s2 = mc*y*Uc + BETA*THETA*pie(+1)^EPSILON*s2(+1);

[name='law of motion for optimal reset price']
1=THETA*pie^(EPSILON-1)+(1-THETA)*ptilde^(1-EPSILON);

[name='marginal costs / labor demand']
mc=w/a;

[name='real profits']
div=y-w*n;

[name='aggregate demand']
c=y;

[name='aggregate supply']
pstar*y = a*n;

[name='law of motion for price inefficiency distortion']
pstar = (1-THETA)*ptilde^(-EPSILON) + THETA*pie^EPSILON*pstar(-1);

[name='price of a zero-coupon bond']
Q=1/R;

[name='Fisher equation']
R = r*pie(+1);

[name='monetary policy rule']
R=steady_state(R)*(pie/PIESTAR)^PHI_PIE*(y/steady_state(y))^PHI_Y*exp(nu);

[name='preference shifter']
log(z) = RHO_Z*log(z(-1)) + eps_z;

[name='technology process']
log(a) = RHO_A*log(a(-1)) + eps_a;

[name='monetary policy shock process']
nu = RHO_NU*nu(-1) + eps_nu;

[name='definition log output (dev. from ss)']
yhat = log(y) - log(steady_state(y));

[name='definition log real wage (dev. from ss)']
what = log(w) - log(steady_state(w));

[name='definition log hours worked (dev. from ss)']
nhat = log(n) - log(steady_state(n));

[name='definition log annualized inflation (dev. from ss)']
piehat_an = 4*(log(pie)-log(steady_state(pie)));

[name='definition log annualized nominal interest rate (dev. from ss)']
Rhat_an = 4*(log(R)-log(steady_state(R)));

[name='definition log annualized real interest rate (dev. from ss)']
rhat_an = 4*(log(r)-log(steady_state(r)));

[name='definition log technology (dev. from ss)']
ahat = log(a) - log(steady_state(a));

[name='definition log discount factor shifter (dev. from ss)']
zhat = log(z) - log(steady_state(z));

[name='definition log real marginal costs (dev. from ss)']
mchat = log(mc) - log(steady_state(mc));
end;

%----------------------------------------------------------------
%  Steady state model
%----------------------------------------------------------------
```

```
steady_state_model;
z   = 1;
a   = 1;
nu  = 0;
pie = PIESTAR;
ptilde = ( (1-THETA*pie^(EPSILON-1))/(1-THETA) )^(1/(1-EPSILON));
mc = (EPSILON-1)/EPSILON * ptilde * (1-BETA*THETA*pie^EPSILON) / (1-BETA*THETA*pie^(EPSILON-1)) ;
pstar = (1-THETA)/(1-THETA*pie^EPSILON) * ptilde^(-EPSILON);
Q = BETA/pie;
R = 1/Q;
r = R/pie;
w = mc*a;
n = (w/(a/pstar)^SIGMA)^(1/(VARPHI+SIGMA));
y = a*n/pstar;
c = y;
div = y-w*n;
s1 = c^(-SIGMA)*y/(1-BETA*THETA*pie^(EPSILON-1));
s2 = c^(-SIGMA)*y*mc/(1-BETA*THETA*pie^EPSILON);

yhat=0;what=0;nhat=0;piehat_an=0;Rhat_an=0;rhat_an=0;ahat=0;zhat=0;mchat=0;
end;
```

Assume that the parameters have the following values:

Table 1: Parameter Values

Parameter	**Value**
β	0.990
ρ_a	0.900
ρ_z	0.500
ρ_ν	0.400
σ	1.000
φ	5.000
θ	0.750
ϵ	9.000
ϕ_π	1.500
ϕ_y	0.125
Π^*	1.000

1. Use stochastic simulations (i.e. the `stoch_simul(order=1,irf=30)` command) for the *hat* variables to study the effects of a
   a) unit shock to technology $\varepsilon_{a,t}$.
   b) contractionary monetary policy shock of an increase of 25 basis points in $\varepsilon_{\nu,t}$
   c) discount factor shock by setting the size of the initial shock $\varepsilon_{z,t}$ to -0.5 percentage points.

   Try to explain the responses of the agents and model variables in your own words using the economic mechanisms of this model.

2. Redo the previous exercise using deterministic simulations, i.e. using `perfect_foresight_setup` and `perfect_foresight_solver`.

3. Fiscal Policy in General Equilibrium: Simulations in Dynare

Consider a version of the Baxter and King (1993) model which is given in the following Dynare file called `Baxter_King_common.inc`:

progs/dynare/Baxter_King_common.inc

```
%-------------------------------%
% declare variables and parameters %
%-------------------------------%
var
    y    % output
    c    % consumption
    iv   % private investment
    g    % government spending
    ivg  % government investment
    lam  % marginal utility of consumption
    k    % private capital stock
    kg   % public capital stock
    r    % real interest rate
    w    % real wage
    tau  % net tax rate
    tr   % fiscal transfers
    z    % productivity process
    n    % labor
    y_obs     (long_name='output')
    c_obs     (long_name='consumption')
    iv_obs    (long_name='investment')
    n_obs     (long_name='labor')
    w_obs     (long_name='wages')
    r_obs     (long_name='interest')
    tr_obs    (long_name='transfers')
    g_obs     (long_name='gov.spending')
    ivg_obs   (long_name='gov.investment')
;

varexo
    e_z    % productivity shock
    e_g    % government spending shock
    e_ivg  % government investment shock
    e_tau  % tax rate shock
;

parameters
    BETA     % discount factor
    ETA      % public capital productivity
    ALPHA    % private capital productivity
    THETA_L  % utility weight for labor
    DELTA    % capital depreciation rate
    RHO_Z    % persistence parameter technology process
    RHO_G    % persistence parameter government spending process
    RHO_IVG  % persistence parameter government investment process
    RHO_TAU  % persistence parameter tax process
    Z_BAR    % target value of technology level
    G_BAR    % target value of government spending
    IVG_BAR  % target value of government investment
    TAU_BAR  % target value of net tax rate
;

%----------------------%
% calibrate parameters %
%----------------------%
%% parameter calibration using targeted steady-state values
Y_BAR   = 1;
G_BAR   = 0.2*Y_BAR;
IVG_BAR = 0.02*Y_BAR;
TR_BAR  = 0;
W_BAR   = 2;
N_BAR   = 1/3;
Z_BAR   = 1; %normalize
% public capital productivity ETA should be lower than capital share in production ALPHA
ALPHA = 1-W_BAR*N_BAR/Y_BAR; % from labor demand in steady-state
ETA = 0.3*ALPHA; % ETA is lower
% exogenous processes
RHO_Z   = 0.75;
RHO_G   = 0.75;
RHO_IVG = 0.75;
RHO_TAU = 0.75;
% Set some reasonable values for DELTA and BETA (see e.g. RBC model)
DELTA = 0.025;
KG_BAR = IVG_BAR/DELTA; % from public capital law of motion in steady-state
```

```
K_BAR = (Y_BAR/(Z_BAR*KG_BAR^ETA*N_BAR^(1-ALPHA)))^(1/ALPHA); % from production function
IV_BAR = K_BAR*DELTA;
R_BAR = ALPHA*Y_BAR/K_BAR; % from capital demand
TAU_BAR = (G_BAR + IVG_BAR + TR_BAR)/(W_BAR*N_BAR+R_BAR*K_BAR); % fiscal budget ins steady-state
BETA = 1/(1-DELTA+(1-TAU_BAR)*R_BAR); % from savings decision in steady-state
C_BAR = Y_BAR - IV_BAR - G_BAR - IVG_BAR; % from ressource constraint in steady-state
% labor utility weight from labor supply decision
THETA_L = (1-TAU_BAR)*W_BAR*(1-N_BAR)/C_BAR;

%-----------------%
% model equations %
%-----------------%
model;
[name='labor-leisure decision']
(1-tau)*w = THETA_L*c/(1-n);
[name='savings decision']
lam = BETA*lam(+1)*(1-DELTA + (1-tau(+1))*r(+1));
[name='marginal utility of consumption']
lam = 1/c;
[name='law of motion private capital stock']
k  = (1-DELTA)*k(-1)  + iv;
[name='law of motion public capital stock']
kg = (1-DELTA)*kg(-1) + ivg;
[name='production function']
y = z*kg(-1)^ETA*k(-1)^ALPHA*n^(1-ALPHA);
[name='labor demand']
w*n = (1-ALPHA)*y;
[name='private capital demand']
r*k(-1) = ALPHA*y;
[name='productivity process']
log(z/Z_BAR) = RHO_Z*log(z(-1)/Z_BAR) + e_z;
[name='government budget constraint']
g + ivg + tr = tau*(w*n + r*k(-1));
[name='fiscal rule: government spending']
g - G_BAR = RHO_G*(g(-1)-G_BAR) + e_g;
[name='fiscal rule: government investment']
ivg - IVG_BAR = RHO_IVG*(ivg(-1)-IVG_BAR) + e_ivg;
[name='fiscal rule: tax rate']
log(tau/TAU_BAR) = RHO_TAU*log(tau(-1)/TAU_BAR) + e_tau;
[name='market clearing']
y = c + iv + g + ivg;
y_obs = 100*(y-steady_state(y))/steady_state(y);
c_obs = 100*(c-steady_state(c))/steady_state(c);
iv_obs = 100*(iv-steady_state(iv))/steady_state(y);
n_obs = 100*(n-steady_state(n))/steady_state(n);
w_obs = 100*(w-steady_state(w))/steady_state(w);
r_obs = 100*(r-steady_state(r))/steady_state(r);
tr_obs = 100*(tr-steady_state(tr));
g_obs = 100*(g-steady_state(g)/steady_state(g));
ivg_obs = 100*(ivg-steady_state(ivg)/steady_state(ivg));
end;

%--------------%
% computations %
%--------------%
initval;
y   = Y_BAR;
c   = C_BAR;
iv  = IV_BAR;
g   = G_BAR;
ivg = IVG_BAR;
lam = 1/C_BAR;
k   = K_BAR;
kg  = KG_BAR;
r   = R_BAR;
w   = W_BAR;
tau = TAU_BAR;
tr  = TR_BAR;
z   = Z_BAR;
n   = N_BAR;
end;
steady; % compute steady-state
```

1. Use stochastic simulations (i.e. the `stoch_simul(order=1,irf=30)` command) for the *obs* variables to study the effects of a 1% shock to (i) public consumption and (ii) public investment. Try to explain the responses of the agents and model variables in your own words using the economic mechanisms of this model.

2. What is the productivity effect of public capital on the private sector? To answer this, redo the previous exercise but with a lower value for η (e.g. 0.05) and compare results.

References

Baxter, Marianne and Robert G. King (1993). “Fiscal Policy in General Equilibrium”. In: *The American Economic Review* 83.3, pp. 315–334. ISSN: 00028282.

Rupert, Peter and Roman Šustek (Apr. 2019). “On the Mechanics of New-Keynesian Models”. In: *Journal of Monetary Economics* 102, pp. 53–69. ISSN: 03043932. DOI: `10.1016/j.jmoneco.2019.01.024`.

A. Solutions

1 Solution to Deterministic vs. stochastic simulations in Dynare The key difference between stochastic and deterministic models stems from the role of uncertainty. In a deterministic world we have perfect knowledge about all future events including policy actions. Given some initial data we can derive optimal trajectories leading to a steady state which generates the highest outcome, e.g. total utility flows. Contrary to that, in a stochastic setting there is always some randomness involved. That is, the agents do not know if or when a shock will hit the economy. They do, however, build (mathematical) expectations because the probability distribution of those shocks is known to the agents.

- Consequentially deterministic models can be used if an occurrence of an innovation in the future is completely certain and predictable, independent of the duration the shock takes place. The corresponding so-called *deterministic* simulation provides us with a good impression about the propagation of this shock. The *unknowns* that we search for are the trajectories of the variables given the model equations, not a recursive decision role aka policy function. We typically use deterministic simulations to study changes in taxes or the introduction of a new currency, etc. Also, when studying transition dynamics from one steady-state to another one deterministic simulations are typically used. Particularly, when we want to take all non-linearities of the model into account, as we do not approximate a policy function or a decision rule. Now given the nature of perfect foresight, there is no uncertainty in the model except when a shock hits on impact (in period 0). This is important to keep in mind when studying the effects of news shocks or anticipated shocks, i.e. shocks that are announced on impact, but materialize later. Only the announcement is a surprise, everything else is known with certainty.

- Stochastic simulations rely on the probability distribution of the shocks, i.e. the exact value and timing of shocks is not known, but that they *might* happen and agents do form expectations about that knowing only the distribution of the shocks. Such models are in a sense more realistic as the future is uncertain and agents make probability statements about their decision. Stochastic simulations are useful to study transmission mechanisms of stochastic shocks (impulse-response analysis), how important is the variability of shocks for the variance of the variables (variance decomposition), and also to estimate model parameters with data. The unknowns that we search for are the so-called *policy functions or decision rules* of the agents, that describe optimal recursive behavior given the current state of the economy and the current realization of shocks. There is a downside to use stochastic simulations as we need to find the policy function or approximate it numerically.

2 Solution to Basic New Keynesian Model: Simulations in Dynare

1. First of all, we put a minus sign in front of the preference shifter `eps_z` to answer c):

```
[name='preference shifter']
log(z) = RHO_Z*log(z(-1)) - eps_z;
```

Then we can include the model equations in the following mod file called `new_keynesian_irfs_stoch.mod`:

progs/dynare/new_keynesian_irfs_stoch.mod

```
@#include "new_keynesian_common_with_minus_eps_z.inc"

%----------------------------------------------------------------
%  Calibration
%----------------------------------------------------------------
BETA    = 0.99;
RHO_A   = 0.9;
RHO_Z   = 0.5;
RHO_NU  = 0.4;
SIGMA   = 1;
VARPHI  = 5;
THETA   = 0.75;
EPSILON = 9;
PHI_PIE = 1.5;
PHI_Y   = 0.125;
PIESTAR = 1;

%----------------------------------------------------------------
%  Shock Variances
%----------------------------------------------------------------
shocks;
var eps_a  = 1^2;     % unit shock to technology
var eps_nu = 0.25^2; % 25 basis points
var eps_z  = 0.5^2;  % initial shock is set to -0.5 percentage points
                     % note: there needs to be a minus before eps_z!
end;

%----------------------------------------------------------------
% generate IRFs
%----------------------------------------------------------------

stoch_simul(order=1,irf=30) nu ahat zhat
                            yhat what nhat
                            piehat_an Rhat_an rhat_an mchat;
```

Before we dive in, keep in mind that all variables are simultaneously determined; any description of the transmission channels can only be a cursory way to provide intuition, rather than an accurate characterization of the model's dynamics. The magic clearly happens through the equations, macroeconomists then try to tell an economic story behind the dynamics (depending on the focus of their analysis).

General remarks for intuition:

- Keep in mind that we are in an environment of monopolistic competition, constant elasticity demand curves and randomly arriving opportunities to adjust prices.
- When firms set prices, they are concerned about future inflation, because there is a chance that they won't be able to adjust prices for several periods.
- The inflation process is forward-looking, current inflation is a function of expected future inflation. More specifically, inflation is the present discounted value of current and future real marginal costs.

- We have a linear production function.
- Real marginal cost is equal to real wage divided by marginal product of labor.
- Real wage is equal to marginal rate of substitution between leisure and consumption.
- *Taylor principle*: When inflation rises, the central bank rises the nominal interest rate more than one-to-one. In other words, it guarantees that the real interest rate eventually rises with inflation. The increase in the real interest rate creates a counter-effect to inflation, since a higher real interest rate causes a fall in the output gap and in deviations of the marginal cost from the steady state. This is the underlying economic principle behind the ability of monetary policy to anchor inflation expectations.

Note that in the mod file, we focus on the hat variables which are defined as **log deviations from steady-state**. This is useful for interpreting the Impulse Response Functions as Dynare plots these in **deviation from steady-state**. So for hat variables this corresponds (at `order=1`) to **percentage deviations** from the original level variables from steady state.

Technology shock

The positive unit shock on total factor productivity has on impact a positive effect on output

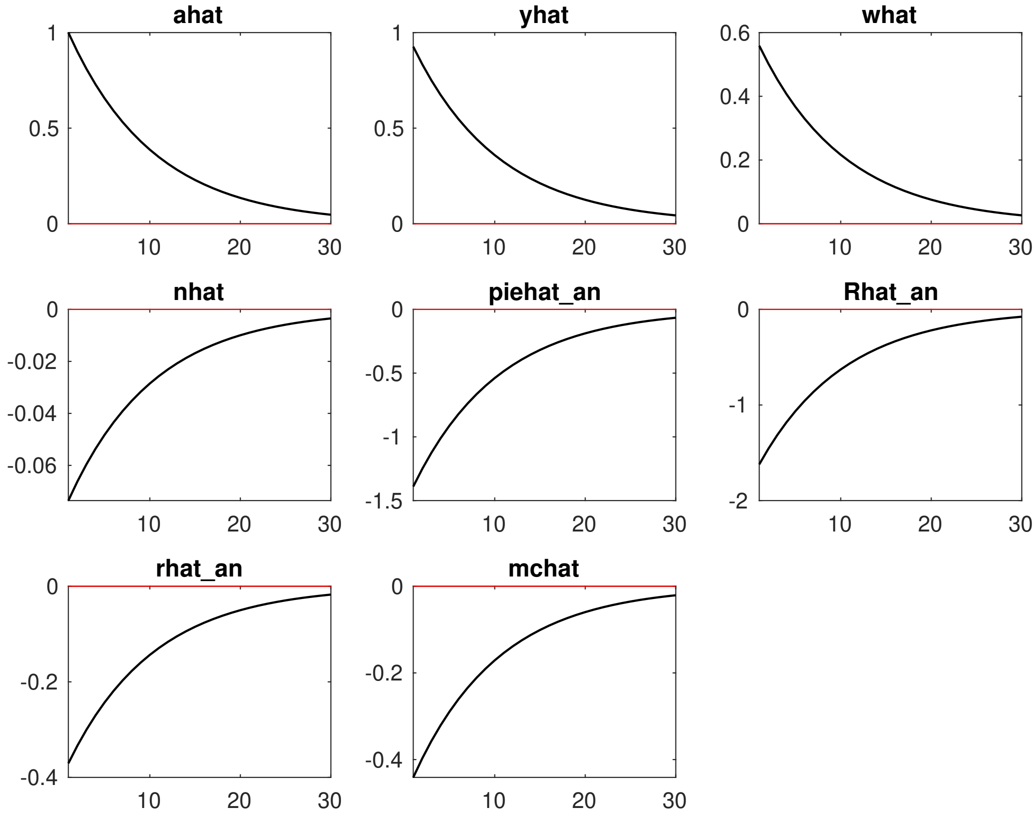

Figure 1: Impulse response functions (orthogonalized shock to ε_a).

(and also consumption), which then follows a downward sloping path due to the persistence of the technological process. On the other hand, we see on impact deflation, higher wages, lower hours worked, and lower interest rates (both the nominal interest rate and the real interest rate), which then all follow an upward-sloping path back to steady-state.

The intuition is that the boost in productivity increases the marginal productivity of labor, which affects not only the consumption-saving and labor-leisure decision, but also the price-setting decision of firms. In more detail, marginal cost of firms falls and this creates incentives for firms to cut their prices. The firms that can reset their price are concerned about not being able to reset it again during the future productivity boost period, so they lower their prices more than they would under flexible prices, implying a larger drop in inflation (compared to the flex-price case) and a large drop in real marginal costs. Likewise, the increase in output is not as large as with flexible prices as the firms that cannot lower their price lower output. According to the Taylor rule, the central bank reacts to the deflationary pressure by lowering the nominal interest rate more than one-to-one. In accordance, the real interest rate falls on impact (but not as much as with flexible-prices) and then follows an upward sloping path. This reflects the wish of the households to smooth consumption via the Euler equation, as consumption in the future becomes less attractive and households prefer to consume more during the periods of productivity boost.

The effect on wages and hours worked depends on the production function (in our case linear) and the calibration of the model; particularly, the Calvo probability, the utility elasticity parameters and the feedback parameters in the Taylor rule. In the chosen calibration, we see an increase in wages, because the increase in the marginal productivity of labor is higher than the reduction in marginal costs. A different Calvo probability might flip this around. Similarly, the effect on hours worked, is in principle ambiguous. A sufficient condition to cause a decline in hours worked is $\sigma \geq 1$ or a sufficiently large feedback coefficient ϕ_π, which in the chosen calibration is both fulfilled.

Preference shifter shock

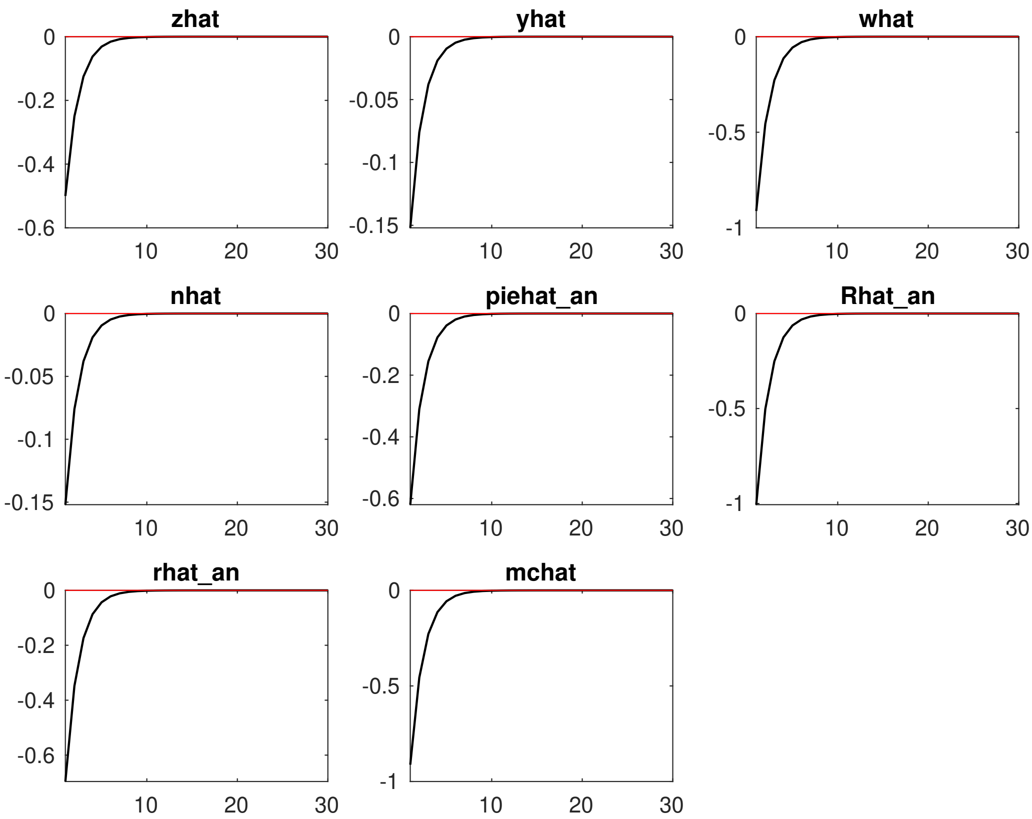

Figure 2: Impulse response functions (orthogonalized shock to ε_z).

The decrease in the preference shifter means that the effective discount factor ($z_t\beta^t$ for $t = 0, 1, 2...$) becomes lower; in other words, households are temporarily becoming more patient and prefer to postpone their consumption, output drops. This decreased demand creates incentives for firms to decrease their price and inflation falls. Note though, that not all firms can reset their price. Nevertheless, lower prices imply lower marginal costs and lower wages. According to the Taylor rule, the central bank reacts to the deflationary pressure and decline in output by decreasing the nominal interest rate more-than-one-for-one ($\phi_\pi > 1$). In accordance to the Fisherian equation, the real interest rate decreases on impact and then follows an upward sloping path. This reverses the effect on the consumption-savings decision of the households via the Euler equation, as consumption in the present becomes again more attractive. However, the decline in the real interest rate is not sufficient to prevent the overall contraction in economic activity (as in the flex-price model). Again the effect on hours worked, is in principle ambiguous dependent on the production function and the values of σ and ϕ_π.

Monetary policy shock

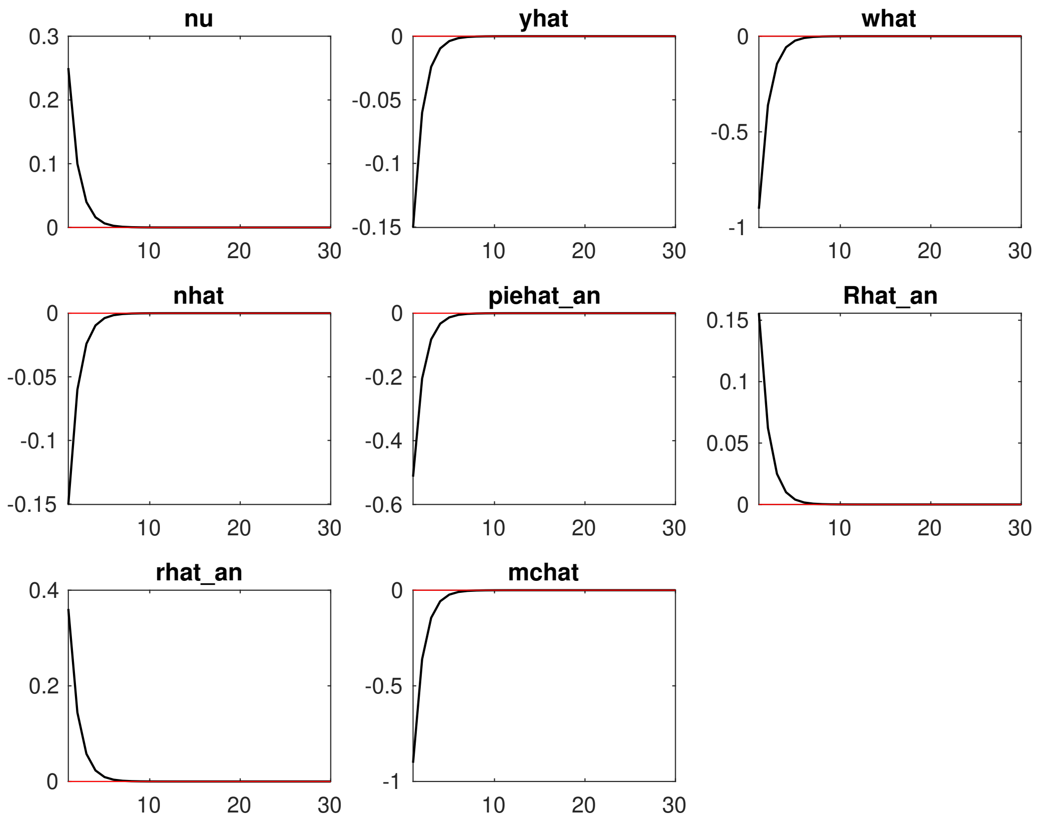

Figure 3: Impulse response functions (orthogonalized shock to ε_ν).

From the empirical VAR literature we can motivate the *real rate channel of monetary policy*: the central bank controls the short-term nominal interest rate and has leverage over the ex-ante real interest rate, because nominal prices are sticky.

Now let's look at this evidence through the lens of our Baseline New-Keynesian model. An exogenous tightening of monetary policy, i.e. a positive realization of v_t, indeed replicates all these facts, i.e. output and inflation decline, whereas the nominal and real interest rates increase. In other words, a monetary tightening, in the form of a positive shock to the Taylor rule that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered price setting. This rise in the real interest rate then causes households to cut back on their current consumption spending. Finally, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock.

Some notes of caution with this interpretation:

- One can show, that for higher values of ρ_ν the nominal interest rate can decline in response to a contractionary monetary policy shock. The intuition is the following: Assume the initial monetary policy shock increases the nominal and thus the real interest rate. The central bank then reacts to lower output and inflation endogenously by lowering the nominal interest rate. If the response is strong enough, it overcompensates the initial increase due to the shock. The ex-ante real rate, however, always increases, irrespective of the shock persistence in the baseline New Keynesian model, as it is inversely related to monetary policy shocks.

- A challenge to this *real rate channel of monetary policy* is given by e.g. Rupert and Šustek (2019) who show that similar to the flexible-price case, inflation is determined by current and expected future monetary policy shocks. According to the New-Keynesian Phillips-curve output temporarily drops when inflation temporarily declines. The reason is that firms that cannot adjust prices reduce output. The real interest rate only reflects the desire and ability of households to keep consumption smooth in face of such temporary changes, but it is not the actual driving force of the dynamics. Particularly, they show that when introducing capital into the model, monetary policy shocks can generate a decline in output and inflation, while the reaction of the real interest rate depends on the calibration: it can increase, decline or stay constant.
- To sum up, don't look solely at interest rates to see whether monetary policy is expansionary or contractionary.

2. Consider the following mod file called `new_keynesian_irfs_det.mod`:

progs/dynare/new_keynesian_irfs_det.mod

```
@#include "new_keynesian_common_with_minus_eps_z.inc"

%-----------------------------------------------------------------
%  Calibration
%----------------------------------------------------------------
BETA     = 0.99;
RHO_A    = 0.9;
RHO_Z    = 0.5;
RHO_NU   = 0.4;
SIGMA    = 1;
VARPHI   = 5;
THETA    = 0.75;
EPSILON  = 9;
PHI_PIE  = 1.5;
PHI_Y    = 0.125;
PIESTAR  = 1;

%-----------------------------------------------------------------
%  unit shock to technology
%----------------------------------------------------------------
steady;% start at steady-state
shocks;
var eps_a; periods 1; values 1;
var eps_z; periods 1; values 0;
var eps_nu; periods 1; values 0;
end;
perfect_foresight_setup(periods=50);
perfect_foresight_solver(maxit=100,tolf=1e-5,tolx=1e-5,stack_solve_algo=0);
tit = 'eps_a';
new_keynesian_irfs_do_plots;
sgtitle('unit shock to technology');

%-----------------------------------------------------------------
%  initial preference shifter shock is set to -0.5 percentage points
%----------------------------------------------------------------
steady;% start at steady-state
shocks;
var eps_a; periods 1; values 0;
var eps_z; periods 1; values 0.5; % important to change the sign of eps z to a
    minus
var eps_nu; periods 1; values 0;
```

```
end;
perfect_foresight_setup(periods=50);
perfect_foresight_solver(maxit=100,tolf=1e-5,tolx=1e-5,stack_solve_algo=0);
tit = 'eps_z';
new_keynesian_irfs_do_plots;
sgtitle('initial preference shifter shock is set to -0.5 percentage points');

%-----------------------------------------------------------------
% contractionary monetary policy shock of 25 basis points
%-----------------------------------------------------------------
steady;% start at steady-state
shocks;
var eps_a; periods 1; values 0;
var eps_z; periods 1; values 0;
var eps_nu; periods 1; values 0.25;
end;
perfect_foresight_setup(periods=50);
perfect_foresight_solver(maxit=100,tolf=1e-5,tolx=1e-5,stack_solve_algo=0);
tit = 'eps_nu';
new_keynesian_irfs_do_plots;
sgtitle('contractionary monetary policy shock of 25 basis points')
```

The helper plotting function is given by `new_keynesian_irfs_do_plots.mod:`

progs/dynare/new_keynesian_irfs_do_plots.m

```
%% PLOTS
options_.irf = 30;
PLOTVARS = ["nu" "ahat" "zhat" "yhat" "what" "nhat" "piehat_an" "Rhat_an" "rhat_an
    " "mchat"];
hh = dyn_figure(options_.nodisplay,'name',['Shock to ' tit]);
x = 0:options_.irf;
plt_nbr = 1;
for j = 1:length(PLOTVARS)
    subplot(4,3,plt_nbr);
    hold on;
    y = oo_.endo_simul(ismember(M_.endo_names,PLOTVARS(j)),3:(options_.irf+3));
    plot(x,y,'linewidth',2);
    title(PLOTVARS(j));
    grid on;
    hold off;
    plt_nbr = plt_nbr+1;
end
dyn_saveas(hh,[M_.dname, '/graphs/' M_.fname '_IRF_' tit],options_.nodisplay,
    options_.graph_format);
```

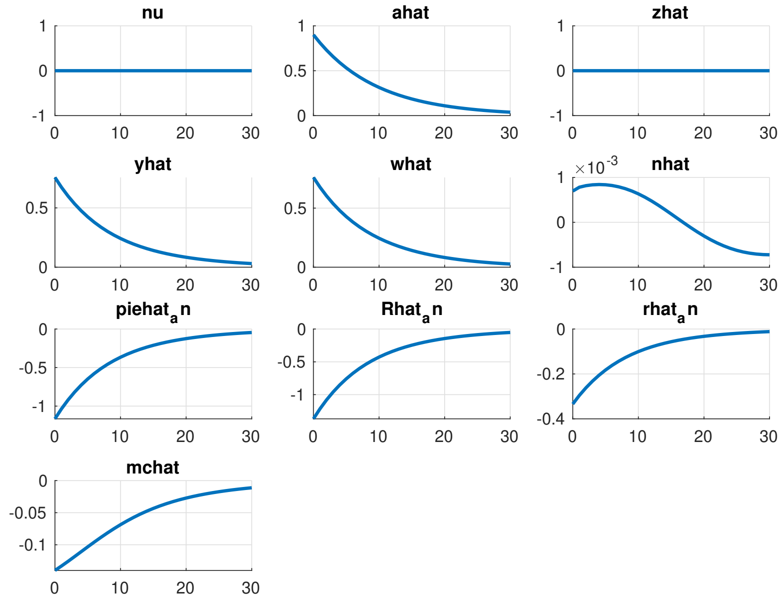

Figure 4: Deterministic Simulations: Impulse response functions (shock to ε_a).

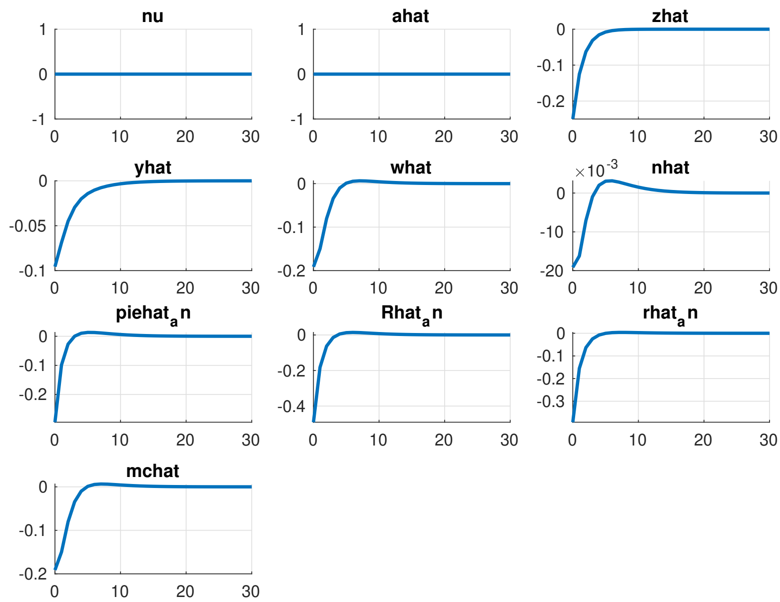

Figure 5: Deterministic Simulations: Impulse response functions (shock to ε_z).

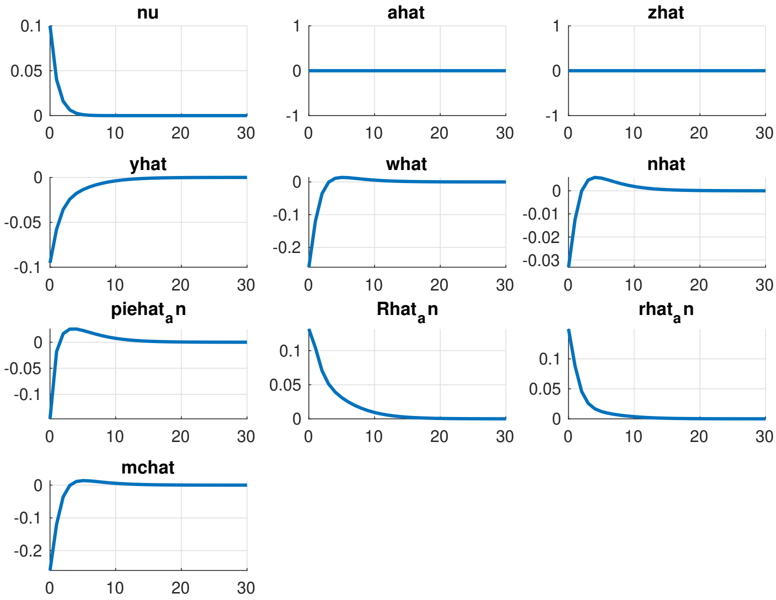

Figure 6: Deterministic Simulations: Impulse response functions (shock to ε_ν).

The impulse response functions of the deterministic simulation are given in figures 4, 5, and 6. Overall, the dynamics are very similar in the sign of the reactions, but not so much in the size of the deviation from steady-state, as the deterministic simulation tends to be somewhat more muted compared to the stochastic impulse-response functions. Also for some variables there is some short overshooting in the adjustment dynamics, and the reaction of wages and employment slightly differs even though the magnitude is very small. To sum up, there are slight differences between whether one is more concerned about non-linearities or stochastics.

3 Solution to Fiscal Policy in General Equilibrium: Simulations in Dynare

1. The following code can be used to compare the impulse response functions with respect to government spending and investment shocks.
   `Baxter_King_irfs.mod`:

progs/dynare/Baxter_King_irfs.mod

```
@#include "Baxter_King_common.inc"

shocks;
var e_g = 0.01^2;
var e_ivg = 0.01^2;
end;

stoch_simul(order=1,irf=200) y_obs c_obs iv_obs n_obs w_obs r_obs tr_obs
                             g_obs ivg_obs;

verbatim; % verbatim tells the preprocessor to ignore the following comands but run them verbatim in
    MATLAB

PLOTVARS = ["y_obs" "c_obs" "iv_obs" "n_obs" "w_obs" "r_obs" "tr_obs"...
                             "g_obs" "ivg_obs"];
hh = dyn_figure(options_.nodisplay,'name','Stochastic Simulations: Gov. spending vs. investment');
sgtitle('Impulse Response Function');
x = 0:(options_.irf-1);

for j = 1:length(PLOTVARS)
    subplot(3,3,j);
    y1 = oo_.irfs.(strcat(PLOTVARS(j),'_e_g'));
    y2 = oo_.irfs.(strcat(PLOTVARS(j),'_e_ivg'));
    plot(x,[y1;y2],'linewidth',2);
    hold on;
    yline(0,'--','linewidth',2);
    title(M_.endo_names_long(ismember(M_.endo_names,PLOTVARS(j))));
    hold off;
    if j==length(PLOTVARS)
        legend({'Gov.spending','Gov.Investment'});
    end
end
dyn_saveas(hh,[M_.dname, '/graphs/' M_.fname '_IRF_comparison' ],options_.nodisplay,options_.
    graph_format);

end; %verbatim end
```

Comparison government spending vs government investment shock
Figure 7 shows the IRFs for both government spending and government investment shocks.

Let us focus first on government spending. We see that there is on impact a positive effect on output, labor and the interest rate; whereas consumption, investment and wages decrease. The return to steady-state happens gradually but rather quickly. In more detail, when government spending goes up, the household feels poorer, because it has to pay more taxes either now or in the future (see the transfers irf). This makes it want to consume less and work more. Working more raises output. So the mechanism through which government spending impacts output in this model is not by stimulating demand, but rather through a wealth effect channel wherein people feel poorer and supply more labor. As the drop in consumption is persistent, we see from the aggregate resource constraint that private investment must fall due to the increase in government spending. Therefore, a shock to government spending will crowd out private investment. By how much depends largely on the the persistence parameter of the government spending process. Because of perfectly competitive factor prices, the decrease in investment and increase in labor supply is reflected in a decreasing marginal productivity of labor (wage goes down) and increasing marginal productivity of capital (interest rate goes up).

Now turning to government investment, we see several differences both in the magnitude and direction of the responses. Keep in mind that government investment spills over to the private sector by increasing overall productivity and the marginal products of labor and capital rise and so do wages and interest rates. This is very similar to a TFP shock in the basic RBC model, as the increase in government investment has a direct positive effect in the public capital stock which is an input in the production function. Therefore, even though the effect on output is on impact slightly negative, the following periods we actually see a high and hump-shaped increase in output. On the other hand, as mentioned above, there is a negative wealth effect, because these increases have to be paid by more taxes either now or in the future. However, the

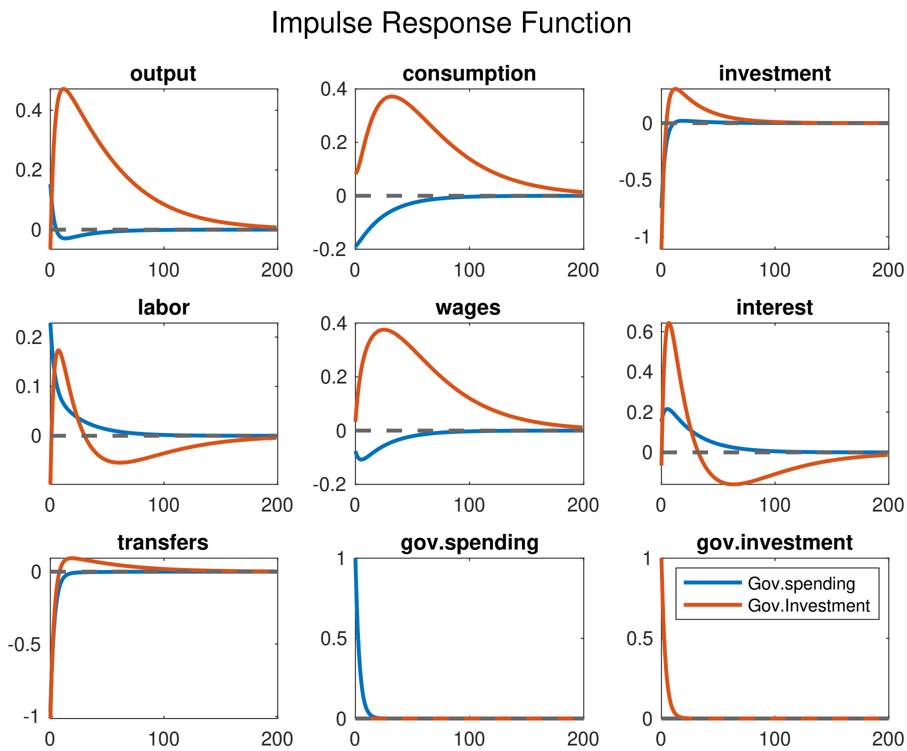

Figure 7: Impulse response functions of unit government shocks.

productivity boost outweighs in this parametrization the negative wealth effect. Even though the stock of public capital accumulate over time (and hence directly and persistently increases output), the schedules for private labor supply and investment shift over time the sign of their reaction as a result of the rising stock of public capital. Note that public capital competes with private capital. With an increase of the former, the interest rate increases on impact and there is an incentive for private agents to slowdown investment. However, in this case, the negative substitution effect is dominated by a positive wealth effect and investment stays positive.

2. The following code can be used to compare the impulse response functions with respect to investment shocks for different values of η.
`Baxter_King_irfs_eta.mod:`

progs/dynare/Baxter_King_irfs_eta.mod

```
@#include "Baxter_King_common.inc"

shocks;
var e_ivg = 0.01^2;
end;

ETA = 0.1;
stoch_simul(order=1,irf=30,nograph) y_obs c_obs iv_obs n_obs w_obs r_obs tr_obs
                                    g_obs ivg_obs;
irfs_high_eta = oo_.irfs;

ETA = 0.05;
stoch_simul(order=1,irf=30,nograph) y_obs c_obs iv_obs n_obs w_obs r_obs tr_obs
                                    g_obs ivg_obs;
irfs_low_eta = oo_.irfs;

verbatim;

PLOTVARS = ["y_obs" "c_obs" "iv_obs" "n_obs" "w_obs" "r_obs" "tr_obs"...
                                    "g_obs" "ivg_obs"];
hh = dyn_figure(options_.nodisplay,'name','Productivity effect of government investment');

sgtitle('Impulse Response Function');
x = 0:(options_.irf-1);

for j = 1:length(PLOTVARS)
    subplot(3,3,j);
    y1 = irfs_high_eta.(strcat(PLOTVARS(j),'_e_ivg'));
    y2 = irfs_low_eta.(strcat(PLOTVARS(j),'_e_ivg'));
    plot(x,[y1;y2],'linewidth',2);
```

```
    hold on;
    yline(0,'--','linewidth',2);
    title(M_.endo_names_long(ismember(M_.endo_names,PLOTVARS(j))));
    hold off;
    if j==length(PLOTVARS)
        legend({'$\eta=0.1$','$\eta=0.05$'},'interpreter','latex');
    end
end

dyn_saveas(hh,[M_.dname, '/graphs/' M_.fname '_IRF_comparison' ],options_.nodisplay,options_.
    graph_format);


end;
```

Comparison productivity of government investment

Figure 8 shows the IRFs of a government investment shock with two different productivity

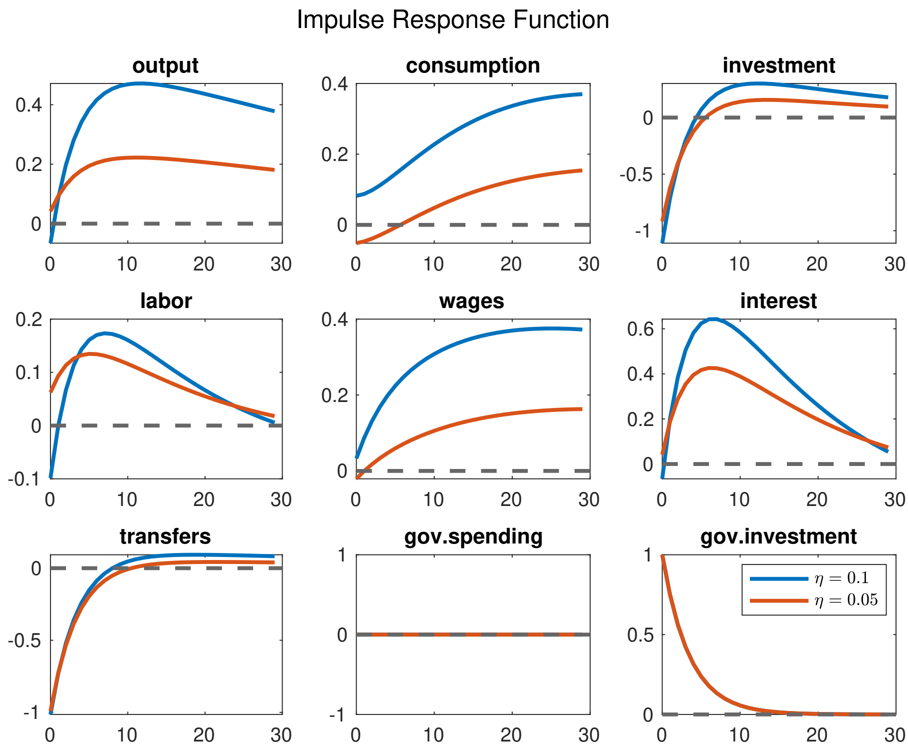

Figure 8: Productivity of government investment

values. Not surprisingly, the higher the productivity parameter, the more clearly government investment acts like a TFP process.