Jermann (1998)

Motivation

Facts

- cyclical variations in securities returns and risk premia
- asset returns are leading economic indicators

Question:

What does my RBC model need to explain both:

- business cycles
- asset market facts

Underlying: Equity premium puzzle

Equity risk premium

= Equity returns

− Gov. Bonds returns

$\approx$ 5% to 8% (historically)

Risk premium is a compensation for taking higher risk, but this value is not in line with feasible values of risk aversion.

<u>Model:</u> One-sector RBC model with
- capital adjustment costs
- consumption habit formation

<u>Findings:</u> We need both features to generate equity premium

Technology growth:

$$\frac{X_{t+1}}{X_t} = \gamma$$

Firm problem

$$E_t \sum_{s=0}^{\infty} (\beta^*)^s \frac{\Lambda_{t+s}}{\Lambda_t} \Big(A_{t+s} F(K_{t+s}, X_{t+s} N_{t+s}) - W_{t+s} N_{t+s} - I_{t+s} \Big)$$

subject

$$K_{t+1} = (1-\delta) K_t + \underbrace{\phi\left(\frac{I_t}{K_t}\right)}_{\text{cap. adjustment costs}} \cdot K_t$$

$$(\beta^*)^s \frac{\Lambda_{t+s}}{\Lambda_t}$$: Stochastic Discount factor

$$\phi\left(\frac{I_t}{K_t}\right) = \frac{b}{1-a} \left(\frac{I_t}{K_t}\right)^{1-a} + c$$

a, b, c parameters

Detrend

$$\frac{D_t}{X_t} = A_t \cdot \frac{K_t^{\alpha}}{X_t^{\alpha}} \frac{(X_t \cdot N_t)^{1-\alpha}}{X_t^{1-\alpha}} - \frac{W_t \cdot N_t}{X_t} - \frac{I_t}{X_t}$$

$$d_t = A_t \cdot k_t^{a} \cdot N_t^{1-a} - w_t \cdot N_t - i_t$$

$$\frac{K_{t+1}}{X_t} \cdot \frac{X_{t+1}}{X_{t+1}} = (1-\delta) \cdot \frac{K_t}{X_t} + \left(\frac{b}{1-a} \left(\frac{I_t / X_t}{K_t / X_t} \right)^{1-a} + c \right) \cdot \frac{K_t}{X_t}$$

$$\gamma \cdot k_{t+1} = (1-\delta) \cdot k_t + \left(\frac{b}{1-a} \left(\frac{i_t}{k_t} \right)^{1-a} + c \right) \cdot k_t$$

$$L = E_t \sum_{s=0}^{\infty} (\beta^*)^s \cdot \frac{\Lambda_{t+s}}{\Lambda_t} \cdot d_{t+s} - (\beta^*)^s \frac{\Lambda_{t+s}}{\Lambda_t} \cdot q_{t+s} \left((1-\delta) k_t + \ldots \right)$$

$$\frac{\partial L}{\partial k_{t+1}} = 0$$

$$\Lambda_t \cdot q_t \cdot \gamma = (\beta^*) \cdot \Lambda_{t+1} \left(a \cdot A_{t+1} \cdot k_{t+1}^{a-1} \cdot N_t^{1-a} + q_{t+1} \left(1 - \delta + c + \frac{ab}{1-a} \left(\frac{i_{t+1}}{k_{t+1}} \right)^{1-a} \right) \right)$$

$$\Leftrightarrow 1 = (\beta^*) \frac{\Lambda_{t+1}}{\Lambda_t} \cdot \underbrace{\frac{1}{q_t \cdot \gamma} \left(\quad \right)}_{R_t^K}$$

Pricing Equation:

Investment FOC:

$$\frac{\partial L}{\partial i_t} = 0$$

$$\Leftrightarrow \quad 1 = q \cdot b \left(\frac{i_t}{k_t} \right)^{-a}$$

<u>Labor FOC:</u>

$$\frac{\partial L}{\partial N_t} = 0$$

$$\Leftrightarrow \quad w_t = (1-\alpha) A_t \cdot k_t^{\alpha} \cdot N_t^{-\alpha}$$

Household:

$$U_t = \max E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C_{t+s} - h \cdot C_{t+s-1})^{1-\tau}}{1-\tau} \right.$$

$$(1-\tau) \cdot \frac{U_t}{X_t^{1-\tau}} = \beta^0 \left(\frac{C_t}{X_t} - h \cdot \frac{C_{t-1}}{X_t} \right)^{1-\tau} + \beta^1 \left(\frac{C_{t+1}}{X_t} - h \cdot \frac{C_t}{X_t} \right)^{1-\tau}$$
$$+ \beta^2 \left(\frac{C_{t+2}}{X_t} - h \cdot \frac{C_{t+1}}{X_t} \right)^{1-\tau} + \ldots$$

$$= (\gamma^{1-\tau})^0 \beta^0 \left(c_t - \frac{h}{\gamma} c_{t-1} \right)^{1-\tau}$$
$$+ (\gamma^{1-\tau})^1 \beta^1 \left(c_{t+1} - \frac{h}{\gamma} \cdot c_t \right)^{1-\tau}$$

$$+ \left(\gamma^{1-\tau}\right)^2 \beta^2 \left(c_{t+2} - \frac{h}{\gamma} \cdot c_{t+1}\right)^{1-\tau}$$

$$+ \dots$$

$$\max E_t \sum_{s=0}^{\infty} [\underbrace{\beta (\gamma^{1-\tau})}_{\beta^*}]^s \frac{\left(c_{t+s} - \frac{h}{\gamma} \cdot c_{t+s-1}\right)^{1-\tau}}{1-\tau}$$

subject to:

$$\frac{W_t}{X_t} \cdot N_t^s + a_t' \left(\frac{V_t^a}{X_t} + \frac{D_t^a}{X_t}\right) = \frac{C_t}{X_t} + a_{t+1}' \frac{V_t^a}{X_t}$$

a_t: vector of financial assets held at t chosen at $t-1$

V_t^a: vector of asset prices (value of asset)

D_t^a: vector of current payouts (dividends)

$$L = \dots$$

$$\frac{\partial L}{\partial c_t} = 0 \iff \Lambda_t = \left(c_t - \frac{h}{\gamma} \cdot c_{t-1}\right)^{-\tau} - (\beta^*) \cdot \frac{h}{\gamma} \left(c_{t+1} - \frac{h}{\gamma} \cdot c_t\right)^{-\tau}$$

$$\frac{\partial L}{\partial a_{t+1}} = 0$$

$$\iff \Lambda_t \cdot V_t^a = (\beta^*) \cdot \Lambda_{t+1} \cdot \left(V_{t+1}^a + d_{t+1}^a\right)$$

What about N_t^S?

Normaliz time:

$$N_t^s + L_t = 1$$

since L_t is not in the utility function $\Rightarrow N_t^S = 1$

$$\Rightarrow N_t = 1$$

Rare Disaster Risk DSGE Models

Define a rare disaster:
event that occurs with a very small probability, but has extreme effects on economy

effects of actual disaster
- locally: large negative effects
- macro: might be positive effects due to recovery
- local vs. global disasters

effects of fear of a disaster
- Rietz (1988) and Barro (2006)

$\Rightarrow$ important determinant of the risk premium

$\Rightarrow$ fear $\uparrow$ $\Rightarrow$ triggers a recession without actual disaster happening

Time-varying fear of disaster
→ Disaster risk shocks has
measurable and real effects
on economy

RBC models: Gabaix (2011, 2012)
Gourio (2012)

Current paper: What happens on the
macroeconomic side
→ consumption

What features do we need to
get "precautionary savings",
so contraction consumption

Finding: - Price Stickiness
- Certain value of EIS
- Epstein-Zin preferences

(#) Epstein-Zin preferences

- disentangle EIS from risk aversion
- recursive formulation

(#) Introduce a disaster

$$K_{t+1} = \left[(1-\delta)\cdot K_t + S\left(\frac{I_t}{K_t}\right)\cdot K_t\right]\cdot e^{x_{t+1}\cdot \ln(1-\Delta)}$$

x : binary, with small probability Θ_t a large share Δ is destroyed $(x_{t+1} = 1)$

time-varying

$$\ln(\Theta_t) = (1-\rho_\Theta)\cdot \bar{\Theta} + \rho_\Theta \cdot \ln(\Theta_{t-1}) + \varepsilon_{\Theta,t}$$

Disaster Risk shock

Firms:

labor-enhancing productivity growth

$$\frac{z_{t+1}}{z_t} = e^{\mu + \varepsilon_{z,t+1} + x_{t+1} \cdot \ln(1-D)}$$

Problem:

x is a binary random variable with potential large effects such that we deviate far away from the Balanced-Growth-Path

$\Rightarrow$ Perturbation methods are not valid

"Stationarization Trick" (Gourio, 2012)

detrend by z_t, $k_t = \frac{K_t}{z_t}$, $i_t = \frac{I_t}{z_t}$

$$k_{t+1} = \frac{(1-\delta) \cdot k_t + S\left(\frac{i_t}{k_t}\right) \cdot k_t}{e^{\mu + \varepsilon_{z,t+1}}}$$

$\rightarrow$ large disaster event x vanishes from DETRENDED system

- → Small shock $\varepsilon_{\theta,t}$ to a small probability θ_t remains
- → Perturbation methods are valid
- → at least 3rd order to get time-variation in risk premia