Computational Macroeconomics Exercises

Week 6

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Contents

1. Brock and Mirman (1972) Model

Consider a simple RBC model with log-utility and full depreciation. The objective is to maximize

$$\max \; E_t \sum_{j=0}^{\infty} \beta^j \log(c_{t+j})$$

subject to the law of motion for capital k_t at the end of period t

$$k_t = a_t k_{t-1}^{\alpha} - c_t \tag{1}$$

with $\beta < 1$ denoting the discount factor and E_t is expectation given information at time t. Productivity a_t is the driving force of the economy and evolves according to

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t} \tag{2}$$

where ρ_a denotes the persistence parameter and $\varepsilon_{a,t}$ is assumed to be normally distributed with mean zero and variance σ_a. Finally, we assume that the transversality condition and the following non-negativity constraints are fullfilled: $k_t \geq 0$ and $c_t \geq 0$.

1. Show that the first-order condition is given by

$$1 = \alpha\beta E_t \frac{c_t}{c_{t+1}} a_{t+1} k_t^{\alpha-1} \tag{3}$$

2. Compute the steady-state of the model with pen and paper, in the sense that there is a set of values for the endogenous variables that in equilibrium remain constant over time.

3. Write MATLAB scripts that:

   a) Pre-process the model and save both the static and dynamic model equations as well as Jacobians to script files that can be evaluated for any value of parameters, dynamic endogenous variables and exogenous variables.

   b) Compute the steady-state for the following parametrization: $\alpha = 0.3$, $\beta = 0.96$, $\rho_a = 0.5$, and $\sigma_a = 0.2$.

   c) Compute the first-order approximated policy function (i.e. g_x and g_u in the notation used in the lecture) for the following parametrization: $\alpha = 0.3$, $\beta = 0.96$, $\rho_a = 0.5$, and $\sigma_a = 0.2$.

   d) Draw $T = 100$ shocks from the standard normal distribution and simulate data for consumption c_t for $t = 1, ...T$ by using the first-order approximated policy function.

4. Professor Mutschler taught you the concept of a policy function,

$$\begin{pmatrix} c_t \\ k_t \\ a_t \end{pmatrix} = g(a_{t-1}, k_{t-1}, \varepsilon_{a,t})$$

which maps the current state of the economy (k_{t-1} and a_{t-1}) and current shocks $\varepsilon_{a,t}$ to current decisions of the agents. You remember him stating, that there is no way to derive the function in closed-form, so that's why we use perturbation approximation techniques. Willi, a fellow student, disagrees with this assessment and claims that he is able to compute the policy function in closed-form for this model. He claims that the policy function is linear in $a_t k_{t-1}^{\alpha}$, i.e. it has the form

$$c_t = g^c a_t k_{t-1}^{\alpha} \tag{4}$$

$$k_t = g^k a_t k_{t-1}^{\alpha} \tag{5}$$

where g^c and g^k are some scalars which are a function of model parameters α and β only. Professor Mutschler is surprised and therefore asks you on the exam to derive the scalar values g^c and g^k. Do so by inserting the guessed policy functions (4) and (5) into the model equations (1) and (3).

5. Compare simulated data for the endogenous variables as well as impulse response functions of a one-standard-deviation technology shock based on the true solution with the first-order approximated one.

Readings

- Brock and Mirman (1972)
- Hansen and Sargent (2013, Ch.5)

2. An and Schorfheide (2007)

Consider the New Keynesian model of An and Schorfheide (2007). The model equations are given by

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right] && (6) \\
1 &= \phi \left(\pi_t - \pi \right) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right] - \phi \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{y_{t+1}}{y_t} \left(\pi_{t+1} - \pi \right) \pi_{t+1} \right] + \frac{1}{\nu} \left[1 - c_t^{\tau} \right] && (7) \\
y_t &= c_t + \frac{g_t - 1}{g_t} y_t + \frac{\phi}{2} \left(\pi_t - \pi \right)^2 y_t && (8) \\
R_t &= R_t^{*1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_r \epsilon_{R,t}} && (9) \\
\ln(z_t) &= \rho_z \ln(z_{t-1}) + \sigma_z \epsilon_{z,t} && (10) \\
\ln(g_t) &= (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_{t-1}) + \sigma_g \epsilon_{g,t} && (11) \\
y_t^* &= (1 - \nu)^{\frac{1}{\tau}} g_t && (12)
\end{aligned}
$$

where $\epsilon_{R,t}$, $\epsilon_{g,t}$ and $\epsilon_{z,t}$ are iid normally distributed with a standard error of 1. Moreover, we have the following auxiliary parameter relationships:

$$
\gamma = 1 + \frac{\gamma^Q}{100}, \qquad \beta = \frac{1}{1 + r^A/400}, \qquad \bar{\pi} = 1 + \frac{\pi^A}{400}, \qquad \phi = \tau \frac{1 - \nu}{\nu \bar{\pi}^2 \kappa}
$$

and one auxiliary variable R_t^*:

$$
R_t^* = R \left(\frac{\pi_t}{\bar{\pi}} \right)^{\psi_1} \left(\frac{y_t}{y_t^*} \right)^{\psi_2}
$$

Note that y_t^* is an endogenous model variable, whereas $\bar{\pi}$ and $\bar{g}$ are parameters and R denotes the steady-state of R_t.

Parametrization If not otherwise stated, use the following parameter values for the exercises:

$$
\begin{aligned}
&\tau = 2, \quad \kappa = 0.33, \quad \nu = 0.1, \quad \rho_g = 0.95, \quad \rho_z = 0.9, \quad r^A = 1, \\
&\pi^A = 3.2, \quad \gamma^Q = 0.5, \quad \bar{g} = 1/0.85, \quad \sigma_z = \sqrt{0.9}, \quad \sigma_g = \sqrt{0.36}
\end{aligned}
$$

We will consider two different parametrizations of the monetary policy rule $(\psi_1, \psi_2, \rho_R, \sigma_r)$, one is called **hawkish**:

$$
\psi_1 = 1.5, \qquad \psi_2 = 0.125, \qquad \rho_R = 0.75, \qquad \sigma_r = \sqrt{0.4}
$$

and the other one **dovish**:

$$
\psi_1 = 1.043195, \qquad \psi_2 = 0.918417, \qquad \rho_R = 0.792647, \qquad \sigma_r = \sqrt{0.446783}
$$

Exercises

1. Provide intuition behind each equation of the model. How are nominal price rigidities modelled? *Hint: Have a look at the relevant parts of the underlying paper.*

2. Write a Dynare mod file for this model.

3. Derive the steady state of all model variables analytically and include these in a `steady_state_model` block. If you struggle to do that, use an `initval` block.

4. Calibrate the parameters such that the ratio of steady-state consumption to steady-state output is equal to 90%. For the following exercises, though, revert to the original parametrization.

5. Investigate the steady-state properties and theoretical moments of the two different parametrizations (dovish vs. hawkish) by running the `steady` and `stoch_simul(order=1)` commands. *Hint: Look at Dynare's output in the command window for the Steady-State values and Theoretical Moments.*

6. Now compare the impulse-response functions of the two different parametrizations (dovish vs. hawkish) for output and inflation to a one standard deviation TFP shock and a one standard deviation monetary policy shock. Provide intuition for the impulse response functions.

7. The determinacy region is defined as the parameter space for which the Blanchard & Khan (1980, Econometrica) order and rank conditions are full-filled. Dynare's sensitivity toolbox provides means to get a graphical representation of this region by drawing randomly from the parameter space and checking the order and rank conditions. Add the following code snippet to your mod file:

```
%---------------
%specify parameters for which to map sensitivity
estimated_params;
PSI1, uniform_pdf,,, 0,6; %draw uniformly from 0 to 6
PSI2, uniform_pdf,,,-1,6; %draw uniformly from -1 to 6
RHOR, uniform_pdf,,,-1,1; %draw uniformly from -1 to 1
end;
varobs y pie;
dynare_sensitivity(prior_range=0,stab=1,nsam=2000);
%---------------
```

   which randomly draws $\psi_1 \in [0, 6]$, $\psi_2 \in [-1, 6]$ and $\rho_R \in [-1, 1]$, while keeping all other parameter values at their calibrated values. Focus on the the plots and explain these. What does this imply for the ability of monetary policy to anchor inflation expectations? *Hint: The plots should look similar to Figure* **??**.

8. In your opinion, is the monetary policy strategy conducted by the European Central bank hawkish or dovish? What about the Federal Reserve? Given the insights you gained in this exercise: Would it be desirable for policymakers to be rather hawkish or dovish?

Readings

- An and Schorfheide (2007)

References

An, Sungbae and Frank Schorfheide (2007). "Bayesian Analysis of DSGE Models". In: *Econometric Reviews* 26.2-4, pp. 113–172. DOI: 10.1080/07474930701220071.

Brock, William A and Leonard J Mirman (June 1972). "Optimal Economic Growth and Uncertainty: The Discounted Case". In: *Journal of Economic Theory* 4.3, pp. 479–513. ISSN: 00220531. DOI: 10.1016/0022-0531(72)90135-4.

Hansen, Lars Peter and Thomas J. Sargent (Dec. 2013). *Recursive Models of Dynamic Linear Economies*. Princeton University Press. ISBN: 978-1-4008-4818-8. DOI: 10.1515/9781400848188.

A. Solutions

1 Solution to Brock and Mirman (1972) Model

1. Due to our assumptions , we will not have corner solutions and can neglect the non-negativity constraints. Due to the transversality condition and the concave optimization problem, we only need to focus on the first-order conditions. The Lagrangian for the household problem is

$$L = E_t \sum_{j=0}^{\infty} \beta^j \left\{ \log(C_{t+j}) + \lambda_{t+j} \left(A_{t+j} K_{t+j-1}^{\alpha} - C_t - K_{t+j} \right) \right\}$$

Note that the problem is not to choose $\{C_t, K_t\}_{t=0}^{\infty}$ all at once in an open-loop policy, but to choose these variables sequentially given the information at time t in a closed-loop policy.

The first-order condition w.r.t. C_t is given by

$$\begin{aligned} \frac{\partial L}{\partial C_t} &= E_t \left(C_t^{-1} - \lambda_t \right) = 0 \\ \Leftrightarrow \lambda_t &= C_t^{-1} \end{aligned} \tag{I}$$

The first-order condition w.r.t. K_t is given by

$$\begin{aligned} \frac{\partial L}{\partial K_t} &= E_t(-\lambda_t) + E_t \beta \left(\lambda_{t+1} \alpha A_{t+1} K_t^{\alpha-1} \right) = 0 \\ \Leftrightarrow \lambda_t &= \alpha \beta E_t \lambda_{t+1} A_{t+1} K_t^{\alpha-1} \end{aligned} \tag{II}$$

(I) and (II) yields

$$C_t^{-1} = \alpha \beta E_t C_{t+1}^{-1} A_{t+1} K_t^{\alpha-1}$$

2. First, consider the steady value of technology:

$$\log \bar{A} = \rho_A \log \bar{A} + 0 \Leftrightarrow \log \bar{A} = 0 \Leftrightarrow \bar{A} = 1$$

The Euler equation in steady-state becomes:

$$\bar{K} = (\alpha \beta \bar{A})^{\frac{1}{1-\alpha}}$$

3. Inserting the guessed policy function for C_t inside the the capital accumulation equation yields:

$$K_t = A_t K_{t-1}^{\alpha} - g_C A_t K_{t-1}^{\alpha} = (1 - g_C) A_t K_{t-1}^{\alpha}$$

Therefore, $g^K = (1 - g^C)$. Once we derive g^C, we get g^K.

Inserting the guessed policy function for C_t inside the Euler equation yields

$$\begin{aligned} \frac{1}{C_t} &= \alpha \beta E_t \frac{1}{C_{t+1}} A_{t+1} K_t^{\alpha-1} \\ \frac{1}{g_C A_t K_{t-1}^{\alpha}} &= \alpha \beta E_t \frac{1}{g_C A_{t+1} K_t^{\alpha}} A_{t+1} K_t^{\alpha-1} \\ A_t K_{t-1}^{\alpha} &= \frac{1}{\alpha \beta} E_t K_t \end{aligned}$$

Inserting the decision rule for capital:

$$\begin{aligned} A_t K_{t-1}^{\alpha} &= \frac{1}{\alpha \beta} (1 - g^C) A_t K_{t-1}^{\alpha} \\ \Leftrightarrow g^C &= (1 - \alpha \beta) \end{aligned}$$

Thus the policy function for C_t is

$$C_t = (1 - \alpha\beta) A_t K_{t-1}^{\alpha}$$

and for K_t:

$$K_t = \alpha\beta A_t K_{t-1}^{\alpha}$$

Now inserting $A_t = A_{t-1}^{\rho} e^{\varepsilon_t}$ yields:

$$\begin{aligned} A_t &= A_{t-1}^{\rho} e^{\varepsilon_t} \\ C_t &= (1 - \alpha\beta) A_{t-1}^{\rho} K_{t-1}^{\alpha} e^{\varepsilon_t} \\ K_t &= \alpha\beta A_{t-1}^{\rho} K_{t-1}^{\alpha} e^{\varepsilon_t} \end{aligned}$$

In summary we have found analytically the policy functions. This will not be possible for other DSGE models and we have to rely on numerical methods to approximate the highly nonlinear functions g and h.

4. Here is the main function:

   And this function computes the steady-state:

   If you know Dynare, here is a Dynare mod file:

5. Here is a script that compares the true policy function with the approximated one:

   Note that as long as we stay close to the steady-state the approximated solution is very accurate.

2 Solution to An and Schorfheide (2007)