Xiaohan Lin

1(a).

$P(1G1B) = 1 - P(2G) - P(2B) = 1 - \frac{1}{4} - \frac{1}{4} = \boxed{1/2}$

(b).

$P(\text{at least } 1G) = 1 - P(2B) = 1 - 1/4 = \boxed{3/4}$

(c).

$P(1G1B \mid \text{at least } 1B) = \frac{P(1G1B \cap \text{at least } 1B)}{P(\text{at least } 1B)} = \frac{P(1G1B)}{P(\text{at least } 1B)} = \frac{1/2}{3/4} = \boxed{2/3}$

It's larger than (a), smaller than (b).

(d).

$P(\text{other child is } G \mid \text{one child is } B) = \frac{\frac{1}{2} P(1G1B)}{P(\text{one child is } B)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{1/2} = \boxed{1/2}$

2.

Define events:

I: defendant being innocent

E: evidence shows defendant's blood type match the one found at crime sight.

(a)

We know $P(E|I) = 0.01$ but $P(I|E) = \frac{P(E|I) \cdot P(I)}{P(E)} \neq P(E|I)$.

(b).

If the defender were to be sampled completely randomly from the population, this claim would be correct if there exist no other evidence linking the defender to the crime. So, we can only claim the probability that the defender is guilty is $\boxed{\text{at least}}$ $1/8000$, and it could be very high.

3 (a)

$\boxed{\text{Yes}}$.

Proof:

$$P(X,Y,W|Z) = P(X,W|Z,Y)P(Y) \xlongequal[\downarrow]{\text{b/c. } (X\perp W|Z,Y)} P(W|Z,Y)P(X|Z,Y)P(Y)$$

$$= P(W|Z,Y)P(X,Y|Z) \xlongequal[\downarrow]{\text{b/c } (X\perp Y|Z)} P(W|Z,Y)P(Y|Z)P(X|Z)$$

$$= P(W,Y|Z)P(X|Z) \quad \square.$$

(b).

$\boxed{\text{Yes}}$.

Proof:

$$P(X,Y|Z,W) = P(X|Y,Z,W)P(Y|Z,W) \xlongequal{\text{b/c. } X\perp Y|Z \text{ and } X\perp Y|W} P(X|Z,W)P(Y|Z,W) \square$$

4 (a)

(ii) suffices.

(b)

All (i)(ii) (iii) suffice.

Calculations:

$$P(H|e_1,e_2) = \frac{P(e_1,e_2|H)P(H)}{P(e_1,e_2)}. \quad (\text{so. (ii) suffices})$$

Now, $E_1 \perp E_2 | H$

$$= \frac{P(e_1|H)P(e_2|H)P(H)}{P(e_1,e_2)}. \quad (\text{so (i) suffices}).$$

Note. H is a finite discrete r.v. $\Rightarrow \sum_{j=1}^{k} P(H=j|e_1,e_2) = 1$.

$$= \frac{P(e_1|H)P(e_2|H)P(H)}{\sum_{j=1}^{k} P(e_1,e_2,H=j)}.$$

$$= \frac{P(e_1|H)\,P(e_2|H)\,P(H)}{\sum_{j=1}^{k} P(e_1, e_2|H=j)\,P(H=j)}$$

use $E_1 \perp E_2 \,|\, H$ again

$$= \frac{P(e_1|H)\,P(e_2|H)\,P(H)}{\sum_{j=1}^{k} P(e_1|H=j)\,P(e_2|H=j)\,P(H=j)}$$ (so (iii) suffices).

5.

$k=3$. If we define the tree width, k, as the minimal max-clique size during elimination process minus by one.

To min $|C|$, we'd start the process from nodes w/ less neighbors. meaning a, c, and g. Then, we'd encounter a clique with $|C|=4$ regardless of the relative order of a, c and g. So $k=3$.

For instance,

elimination order $I = \{a, c, g, b, d, f, e\}$.

we'd add 3 edges (b,d), (b,f), (d,f).