Hwk5&6

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Q.1

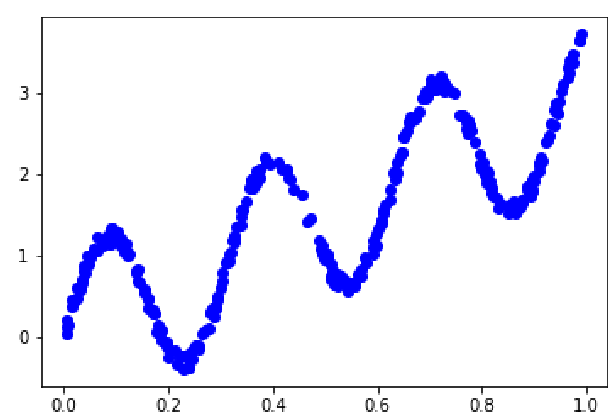
(a) Draw n = 300 real numbers uniformly at random on [0, 1], call them x1, . . . , xn.

(b) Draw n real numbers uniformly at random on [1 , 1], call them 1,...,n.

```
In [1]: import os
        import struct
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import matplotlib as mpl

        np.random.seed(42)
        n=300
        X = np.random.uniform(0,1,size = n).reshape((n,1))
        MU = np.random.uniform(-.1,.1,size = n).reshape((n,1))
        Y = np.zeros(n).reshape((n,1))
```

(c) Let di = sin(20xi) + 3xi + i, i = 1, . . . , n. Plot the points (xi, di), i = 1, . . . , n.

```
In [2]: D = np.sin(20*X) + 3*X + MU
```

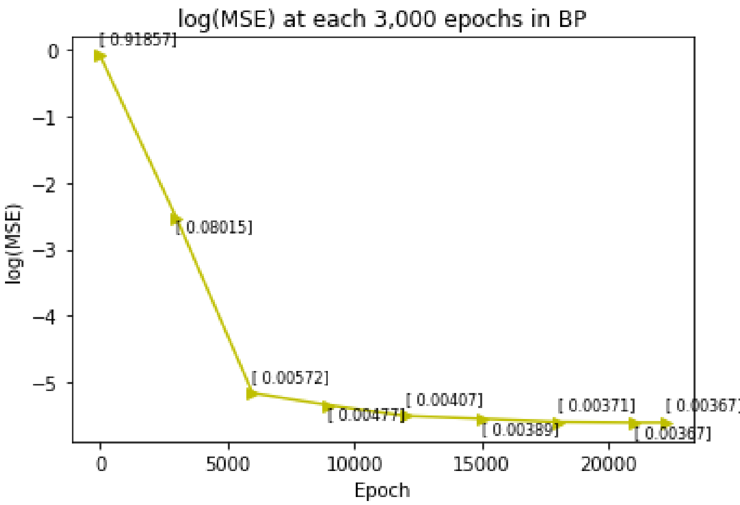


We will consider a 1by N by 1 neural network with one input, N = 24 hidden neurons, and 1 output neuron. The network will thus have 3N + 1 weights including biases. Let w denote the vector of all these 3N + 1 weights. The output neuron will use the activation function (v) = v; all other neurons will use the activation function (v) = tanhv. Given input x, we use the notation f(x,w) to represent the network output.

In [3]:
```
### Step 1 to 4

# Number of hidden layer's neurons
N = 24
W = np.random.uniform(-.1,.1,size = (3*N+1)).reshape((3*N+1,1))
eta = 4
epsilon = 1e-8
mse = list()
mse.append(1e2)
mse.append(1e1)
l=0
m = 0

epoch = 2
U = np.zeros(N)
Z = U
A = W[0:N]
B = W[N:2*N]
C = W[2*N:3*N]
e = W[-1]
```

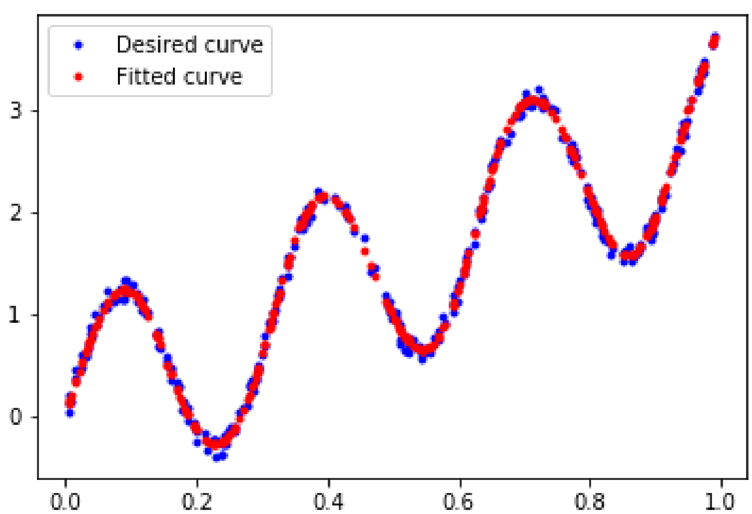
In [4]:
```
### Step 5
while (np.abs(mse[-1]-mse[-2])>=epsilon):
    m += 1
    index = np.random.choice(np.arange(n), size= n,  replace=False)
    # Step 5.A and C
    for i in index:
        U = X[i]*A + B
        Z = np.tanh(U)
        Y[i] = np.transpose(Z).dot(C) + e
        # update A:
        A += 2 / n * eta * X[i] * (D[i]-Y[i]) * (1-Z) * (1+Z) * C
        # update B:
        B += 2 / n * eta * (D[i]-Y[i]) * (1-Z) * (1+Z) * C
        # update C:
        C += 2 / n * eta * (D[i]-Y[i]) * Z
        # update e:
        e += 2 / n * eta * (D[i]-Y[i])
    # Step 5.B
    mse.append(np.sum((D-Y)**2)/n)
    epoch +=1

    if (epoch-2) % 2000 == 0:
            eta *= 0.5

# delete the 2 dummpy points
epoch -= 2
del(mse[0:2])
```

(d) Use the backpropagation algorithm with online learning to find the optimal weights/network that minimize the mean-squared error (MSE) 1 n (di f(xi,w))2. Use some of your choice. Plot the number of n_i=1 epochs vs the MSE in the backpropagation algorithm.



(e) Plot the curve f(x, w0) as x ranges from 0 to 1 on top of the plot of points in (c). The fit should be a "good" fit.

In [72]:



(f) Pseudocode of implementation.

1. Given n, N, and ϵ.
2. Initialize $\eta \in \mathbb{R}, W \in \mathbb{R}^{(3N+1)\times 1}$ randomly. Note: $W = A, B, C, d$, where $A \in \mathbb{R}^N, B \in \mathbb{R}^N, C \in \mathbb{R}^N, ein\mathbb{R}$.
3. Initialize epoch = 0.
4. Initialize MSE_{epoch} = 0 for epoch = 0, 1, ...
5. Do SGD, so sample n numbers out range(n) without replacement as the index:
   1. for i in index, do (this loop is where we compute the mse of each epoch):
      1. Compute the induced local fields as a vector with the current training sample and weights by:
      $$U = x_i \cdot A + B \in \mathbb{R}^N$$
      2. Then, get the N outputs from the N hidden neurons as a vector by:
      $$Z = tanh(U) \in \mathbb{R}^N$$
      3. So, the output of the last neuron will be:
      $$y_i = Z^T \cdot C + e$$
   2. Compute the MSE for current epoch:
   $$MSE_{epoch} = \frac{1}{n}\sum_{i=1}^{n}(d_i - y_i)^2$$
   Then, update the epoch:
   $$epoch \leftarrow epoch + 1.$$
   3. for i = 1 to n, do (this loop is where we update the weights):

Note:

$$\frac{\partial tanh(U)}{\partial U} = \begin{bmatrix} \frac{\partial tanh(u_1)}{\partial u_1} & \frac{\partial tanh(u_1)}{\partial u_2} & \cdots & \frac{\partial tanh(u_1)}{\partial u_{24}} \\ \frac{\partial tanh(u_2)}{\partial u_1} & \frac{\partial tanh(u_2)}{\partial u_2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial tanh(u_{24})}{\partial u_1} & \cdots & \cdots & \frac{\partial tanh(u_{24})}{\partial u_{24}} \end{bmatrix} = \begin{bmatrix} \frac{\partial tanh(u_1)}{\partial u_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\partial tanh(u_{24})}{\partial u_{24}} \end{bmatrix} = \begin{bmatrix} 1 - tanh^2(u_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - tanh^2(u_{24}) \end{bmatrix}$$

3. update the A here:
   1. According to the backpropogation algorithm, we have:

$$\frac{\partial MSE}{\partial A} = -x_i \cdot \frac{2}{n}(d_i - y_i) \cdot \frac{\partial tanh(U)}{\partial U} \cdot C = -x_i \cdot \frac{2}{n}(d_i - y_i) \cdot (1 - tanh(U)) \bigodot (1 + tanh(U)) \bigodot C$$

2. The update would be:

$$A \leftarrow A - \eta \frac{\partial MSE}{\partial A} = A + \eta x_i \cdot \frac{2}{n}(d_i - y_i) \cdot (1 - tanh(U)) \bigodot (1 + tanh(U)) \bigodot C$$

4. update the B here:
   1. According to the backpropogation algorithm, we have:

$$\frac{\partial MSE}{\partial B} = -1 \cdot \frac{2}{n}(d_i - y_i) \cdot \frac{\partial tanh(U)}{\partial U} \cdot C = -1 \cdot \frac{2}{n}(d_i - y_i) \cdot (1 - tanh(U)) \bigodot (1 + tanh(U)) \bigodot C$$

   2. The update would be:

$$B \leftarrow B - \eta \frac{\partial MSE}{\partial B} = B + \eta \frac{2}{n}(d_i - y_i) \cdot (1 - tanh(U)) \bigodot (1 + tanh(U)) \bigodot C$$

5. update the C here:
   1. According to the backpropogation algorithm, we have:

$$\frac{\partial MSE}{\partial C} = -tanh(U) \cdot \frac{2}{n}(d_i - y_i)$$

   2. The update would be:

$$C \leftarrow C - \eta \frac{\partial MSE}{\partial C} = C + \eta \frac{2}{n}(d_i - y_i) \cdot tanh(U)$$

6. update the e here:
   1. According to the backpropogation algorithm, we have:

$$\frac{\partial MSE}{\partial e} = -\frac{2}{n}(d_i - y_i)$$

   2. The update would be:

$$e \leftarrow e - \eta \frac{\partial MSE}{\partial e} = e + \eta \frac{2}{n}(d_i - y_i)$$

7. for every kth (k = 6000) epoch:

$$\eta = 0.5 * \eta$$

4. Loop to A, if $abs(MSE_{epoch} - MSE_{epoch-1}) > \epsilon$.

Reamrk: Could consider to reduce the η after a certain epochs systematically.

Q.**2.**

(150pts) In this computer project, we will design a neural network for digit classification using the backpropagation algorithm (see the notes above). You should use the MNIST data set (see Homework 2 for details) that consists of 60000 training images and 10000 test images. The training set should only be used for training, and the test set should only be used for testing. Your final report should include the following:

0. Input the data files

```
In [2]: # save the original binary MNIST data files in 0-255
```

1. First try

1. Architecture of NN: 784 input neurons, 1 hidden layer with N_1 = 100 neurons, 10 output neurons with 0 or 1 as their outputs to be compatible with the use of 0-1 vector outputs.

2. Output: With [1 0 ̊.. 0] representing a 0, [0 1 0 ̊.. 0] representing a 1 and so on, it is denoted as $f(x_i, w)$.

3.1 Activation functions:, tanh() from input to 1st hidden layer and softmax from 1st hidden layer to output.

3.2 Leanring rate: 0.1.

3.2 Weight initialization: all weights are intialized randomly from unifrom(-0.5,0.5).

4. Energy function: MSE in the form of: $\frac{1}{n}\sum_{i=1}^{n}||d_i - f(x_i, w)||^2$.

5. Other tricks: reduce the eta by 50% when at plateau.

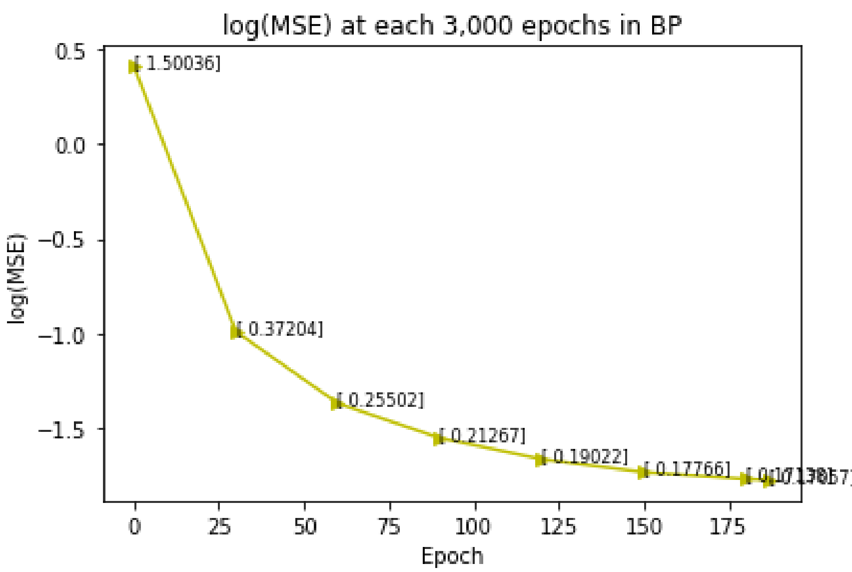
In [6]:
```
## initialization
np.random.seed(2)
# use first n samples from training data to train the NN
n = 60000
# number of neurons in 1st hidden layer, N1
N1 = 100
# learning rate
eta = 1e-2
# convergence threshold
epsilon = 5e-8
# epoch number
epoch = 2
# batch size
bs = 1
# initialize errors
mse = list()
mse.append(1e3)
mse.append(5e2)
# initialize real outputs given the current w
out = np.zeros((n,10))
# initialize weights from input to 1st hidden layer
u = np.random.uniform(-0.05,0.05,size = (784*N1)).reshape(784,N1)
# initialize weights from 1st hidden layer to output
v = np.random.uniform(-0.2,0.2,size = (10*N1)).reshape(N1,10)
# initialzie the bias
b = np.random.normal(-0.1,0.1,size = (bs*N1)).reshape(bs,N1)

# define softmax actiavtion function
def softmax(w):
```

```
        e = np.exp(np.array(w) - np.max(w))
        dist = e / np.sum(e)
        return dist
    # define a continuous vector to 0-1 vector functin
    def vec_to_01(x):
        s1 = np.zeros(x.shape[0])
        q = np.argmax(x)
        s1[q] =1
        return s1
    # define softmax's Jacobian matrix function
     def softmax_jacobian(s):
         return np.diagflat(s) - np.outer(s, s)
     # define tanh's Jacobian matrix function
     def tanh_jacobian(t):
         return np.diagflat(1-t**2)
     def cross_S(out, Label):
         return - np.sum(np.multiply(out, np.log(Label)) + np.multiply((1-out), np.log(1-I
```

```
In [10]: while (np.abs(mse[-1]-mse[-2])>=epsilon):
             index = np.random.choice(np.arange(n), size= n,  replace=False)
             X_train_rndm = X_train[index]
             # this loop is where we update the weights
             for i in range(0,n, bs):
                 lf1 = np.dot(X_train_rndm[i:(i+bs)],u) + b
                 z0 = np.tanh(lf1)          #compute the input layer's out
                 lf2 = np.dot(z0,v)             #compute the 1st H layers' induced local fiel
                 out[i:(i+bs)] = softmax(lf2)        #compute the 1st H layer's out

                 # update u:
                 temp1 = 2 / n * eta *(Y_train[i:(i+bs)] - out[i:(i+bs)]).dot(softmax_jacobia
                 temp2 = np.dot(temp1,v.T)
                 temp2 = temp2.dot(tanh_jacobian(lf1))
                 u -= np.outer(X_train_rndm[i:(i+bs)],temp2)
                 # update b:
                 b -= temp2
                 # update v:
                 v -= np.outer(z0, temp1)
             #  calculate mse for the current epoch
             mse.append(np.sum((out-Y_train)**2)/n)
             epoch +=1

             if mse[epoch-2] <= mse[epoch-1]:
                 eta *= 0.5
             #if (epoch-2) % 2 == 0:

         # delete the 2 dummpy points
         epoch -= 2
         del(mse[0:2])
```

```
In [12]: u0 = u
         b0 = b
         v0 = v
         #  Initialize errors = 0.
         error_test = 0
         # loop on all testing samples
         for i in range(10000):
             lf1 = np.dot(X_train_rndm[i:(i+bs)],u) + b  #compute the input layers' induced local

             z0 = np.tanh(lf1)           #compute the input layer's out
             lf2 = np.dot(z0,v)              #compute the 1st H layers' induced local field
             out[i:(i+bs)] = softmax(lf2)        #compute the 1st H layer's out
             # Find the largest component of v0 = [v0', v1', ...v9']^T^
             predic_out = np.argmax(out[i,:])
             #  If the prediceted output is different to the testing label, error +=1
             diff = predic_out - np.argmax(Y_test[i,:])
             if diff != 0:
                 error_test += 1
         error_test/10000
```

```
Out[12]: 0.122
```



Clearly, we at not satisfied with a 12.2% errorrate on the test data. And I concluded that it was due to the undernumber of the neurons in the hidden layer, careless initialization of parameters and/or hyperparameters, and lack of other tricks. Thus, I had my 2nd try:

2. Second try

1. Architecture of NN: 784 input neurons, 1 hidden layer with N_1 = 150 neurons, 10 output neurons with 0 or 1 as their outputs to be compatible with the use of 0-1 vector outputs.

2. Output: With [1 0... 0] representing a 0, [0 1 0... 0] representing a 1 and so on, it is denoted as $f(x_i, w)$.

3.1 Activation functions:, tanh() from input to 1st hidden layer and softmax from 1st hidden layer to output.

3.2 Leanring rate: Set to 0.03 due to the emperical principle $\eta \sim O(1/\sqrt{m})$, m is 784 here.

3.2 Weight initialization: For hyperbolic tangent units: sample a Uniform(-r, r) with $r = \sqrt{\frac{6}{fan-in+fan-out}}$; For softmax tangent units: sample a Uniform(-r, r) with $r = 4\sqrt{\frac{6}{fan-in+fan-out}}$ (fan-in is the number of inputs of the unit, fan-out is the number of outputs of the unit); For bias: sample the same way as the associated weight.

4. Energy function: MSE in the form of: $\frac{1}{n}\sum_{i=1}^{n}||d_i - f(x_i, w)||^2$.

5. Other tricks: SGD optimizer; reduce the eta by 50% when at plateau; carry 90% of the last epoch's gradient over to the current epoch when updating; Early termination (done mannully)

In [278]:
```
## initialization
np.random.seed(2)
# use first n samples from training data to train the NN
n = 60000
# number of neurons in 1st hidden layer, N1
N1 =150
# learning rate
eta = 0.03
# convergence threshold
epsilon = 5e-7
# epoch number
epoch = 2
# batch size
bs = 1
# initialize errors
mse = list()
mse.append(1e3)
mse.append(5e2)
# initialize real outputs given the current w
out = np.zeros((n,10))
# initialize weights from input to 1st hidden layer
u = np.random.uniform(-0.05,0.05,size = (784*N1)).reshape(784,N1)
# initialize weights from 1st hidden layer to output
v = np.random.uniform(-0.2,0.2,size = (10*N1)).reshape(N1,10)
# initialzie the bias
b = np.random.uniform(-0.05,0.05,size = (bs*N1)).reshape(bs,N1)
# initialize condi
m = 0
l = 0
# intialize momentum
momentum = 0.9
```

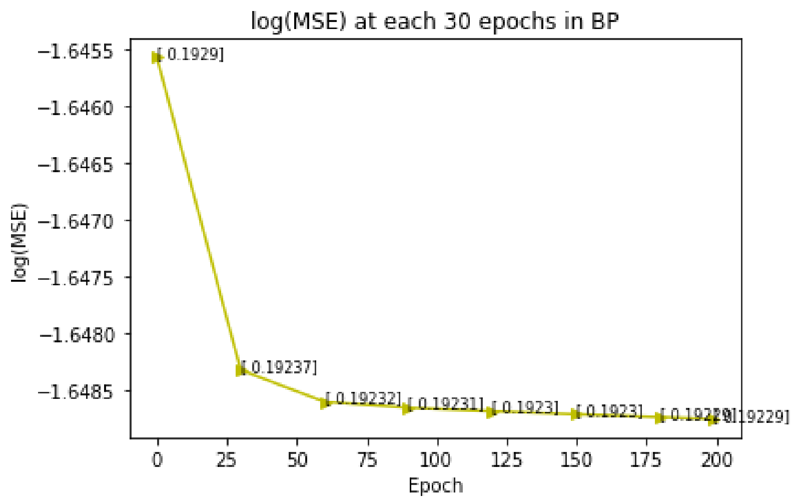
In [279]:
```
while (np.abs(mse[-1]-mse[-2])>=epsilon):
```

```
        m += 1
        index = np.random.choice(np.arange(n), size= n,  replace=False)
        X_train_rndm = X_train[index]
        # re-initialzie last epoch's gradients
        grad_uold = 0
        grad_bold = 0
        grad_vold = 0
        # this loop is where we update the weights
        for i in range(0,n, bs):
            l +=1
            lf1 = np.dot(X_train_rndm[i:(i+bs)],u) + b  #compute the input layers' induced
            z0 = np.tanh(lf1)          #compute the input layer's out
            lf2 = np.dot(z0,v)              #compute the 1st H layers' induced local field
            out[i:(i+bs)] = softmax(lf2)        #compute the 1st H layer's out
            if (True):
                # update u:
                temp1 = 2 / n * eta * (Y_train[i:(i+bs)]-out[i:(i+bs)]).dot(softmax_jacobian(lf2
                temp2 = np.dot(temp1,v.T)
                temp2 = temp2.dot(tanh_jacobian(lf1))
                grad_u= np.outer(X_train_rndm[i:(i+bs)],temp2)
                grad_u = grad_uold*momentum+grad_u
                u -= grad_u
                grad_uold = grad_u
                # update b:
                grad_b = (temp2)
                grad_b = grad_bold*momentum+grad_b
                b -=grad_b
                grad_bold = grad_b
                # update v:
                grad_v = np.outer(z0, temp1)
                grad_v = grad_vold*momentum+grad_v

                v -=grad_v
                grad_vold = grad_v
        #  calculate mse for the current epoch
        mse.append(np.sum((out-Y_train)**2)/n)
        epoch +=1
        if mse[epoch-2] <= mse[epoch-1]:
            eta *= 0.5
    # delete the 2 dummpy points
```

```
In [14]: u0 = u
         b0 = b
         v0 = v
         #  Initialize errors = 0.
         error_test = 0
         # loop on all testing samples
         for i in range(10000):
             lf1 = np.dot(X_train_rndm[i:(i+bs)],u) + b  #compute the input layers' induced local fiel
             z0 = np.tanh(lf1)          #compute the input layer's out
             lf2 = np.dot(z0,v)              #compute the 1st H layers' induced local field
             out[i:(i+bs)] = softmax(lf2)        #compute the 1st H layer's out
             # Find the largest component of v0 = [v0', v1', ...v9']^T^
```

```python
        predic_out = np.argmax(out[i,:])
        #  If the prediceted output is different to the testing label, error +=1
        diff = predic_out - np.argmax(Y_test[i,:])
        if diff != 0:
            error_test += 1
    error_test/10000
```

Out[14]: 0.0672



6.72% to 12.2% error rate on the test data was a big boost. But I concluded that cross-entropy will be a better choice of the energy function of the than good old MSE. Meanwhile, adding more neurons will definitely help me to get into the 5% error rate club. Thus, I had my 3rd try:

3. Third try

1. Architecture of NN: 784 input neurons, 1 hidden layer with N_1 = 250 neurons, 10 output neurons with 0 or 1 as their outputs to be compatible with the use of 0-1 vector outputs.

2. Output: With [1 0̊.. 0] representing a 0, [0 1 0̊.. 0] representing a 1 and so on, it is denoted as $f(x_i, w)$.

3.1 Activation functions:, tanh() from input to 1st hidden layer and softmax from 1st hidden layer to output.

3.2 Leanring rate: Set to 0.03 due to the emperical principle $\eta \sim O(1/\sqrt{m})$, m is 784 here.

3.2 Weight initialization: For hyperbolic tangent units: sample a Uniform(-r, r) with $r = \sqrt{\frac{6}{fan-in+fan-out}}$; For softmax units: sample a Uniform(-r, r) with $r = 4\sqrt{\frac{6}{fan-in+fan-out}}$ (fan-in is the number of inputs of the unit, fan-out is the number of outputs of the unit); For bias: sample the same way as the associated weight.

4. Energy function: Cross-entropy in the form of: $-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{10} d_{ij}log(f(x_i, w)_j)$, where j represents the 10 output classes.

5. Other tricks: SGD optimizer; reduce the eta by 50% when at plateau; carry 82% of the last epoch's gradient over to the current epoch when updating; Early termination (done mannully)

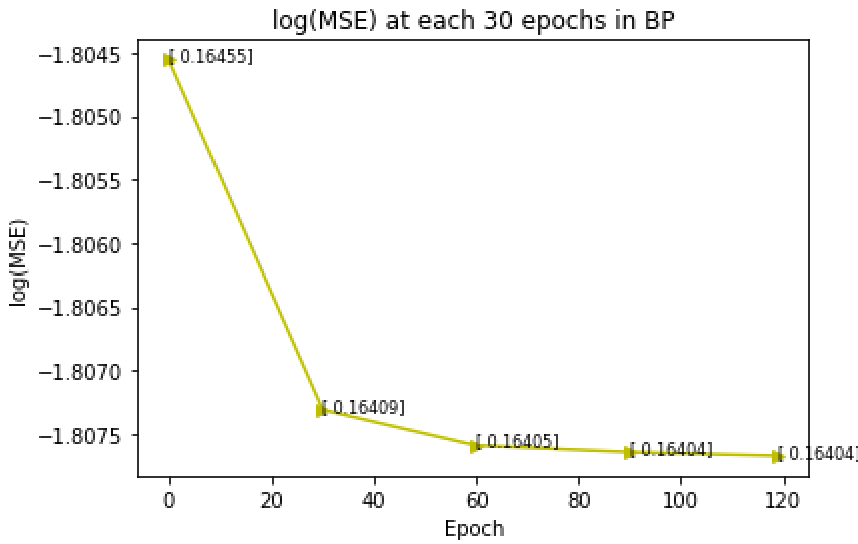
```
np.random.seed(2)
# use first n samples from training data to train the NN
n = 60000
# number of neurons in 1st hidden layer, N1
N1 = 250
# learning rate
eta = 0.03
# convergence threshold
epsilon = 5e-8
# epoch number
epoch = 2
# batch size
bs = 1
# initialize errors
entropy = list()
entropy.append(1e1)
entropy.append(5e0)
# initialize real outputs given the current w
out = np.zeros((n,10))
# initialize weights from input to 1st hidden layer
u = np.random.uniform(-0.05,0.05,size = (784*N1)).reshape(784,N1)
# initialize weights from 1st hidden layer to output
v = np.random.uniform(-0.2,0.2,size = (10*N1)).reshape(N1,10)
# initialzie the bias
b = np.random.normal(-0.1,0.1,size = (bs*N1)).reshape(bs,N1)
# initialize condi
m = 0
l = 0
# intialize momentum
momentum = 0.82
```

In [5]:
```
while (np.abs(entropy[-1]-entropy[-2])>=epsilon):
    m += 1
    index = np.random.choice(np.arange(n), size= n,  replace=False)
    X_train_rndm = X_train[index]
    # re-initialzie last epoch's gradients
    grad_uold = 0
    grad_bold = 0
    grad_vold = 0
    # this loop is where we update the weights
    for i in range(0,n, bs):
        lf1 = np.dot(X_train_rndm[i:(i+bs)],u) + b  #compute the input layers' induced local
        z0 = np.tanh(lf1)          #compute the input layer's out
        lf2 = np.dot(z0,v)              #compute the 1st H layers' induced local field
        out[i:(i+bs)] = softmax(lf2)        #compute the 1st H layer's out
        if (True):
```

```
        # update u:
        temp1 = 1 / n * eta * np.sum(Y_train[i:(i+bs)] / out[i:(i+bs)], axis = 0).dot(softmax_jacobi
        temp2 = np.dot(temp1,v.T)
        temp2 = temp2.dot(tanh_jacobian(lf1))
        grad_u= np.outer(X_train_rndm[i:(i+bs)],temp2)
        grad_u = grad_uold*momentum+grad_u
        u -= grad_u
        grad_uold = grad_u

        # update b:
        grad_b = (temp2)
        grad_b = grad_bold*momentum+grad_b
        b -=grad_b
        grad_bold = grad_b

        # update v:
        grad_v = np.outer(z0, temp1)
        grad_v = grad_vold*momentum+grad_v
        v -=grad_v
        grad_vold = grad_v


       #  calculate mse for the current epoch
       entropy.append(-np.sum(np.log(out) * Y_train)/n)
       epoch +=1
           if entropy[epoch-2] <= entropy[epoch-1]:

               eta *= 0.5

   # delete the 2 dummpy points
   epoch -= 2
   del(entropy[0:2])
```

```
In [157]: u0 = u
          b0 = b
          v0 = v
          #  Initialize errors = 0.
          error_test = 0
          # loop on all testing samples
          for i in range(10000):
              lf1 = np.dot(X_train_rndm[i:(i+bs)],u) + b  #compute the input layers' induced local
              z0 = np.tanh(lf1)           #compute the input layer's out
              lf2 = np.dot(z0,v)              #compute the 1st H layers' induced local field
              out[i:(i+bs)] = softmax(lf2)        #compute the 1st H layer's out
              # Find the largest component of v0 = [v0', v1', ...v9']^T^
              predic_out = np.argmax(out[i,:])
              #  If the prediceted output is different to the testing label, error +=1
              diff = predic_out - np.argmax(Y_test[i,:])
              if diff != 0:
                  error_test += 1
          error_test/10000
```

```
Out[157]: 0.0463
```



Now, with an errorrate of 4.63%. We have accomplised our goal with the 3rd try, where the pseudocode was listed below:

1. Given n, N_1, momentum and ϵ.

2. Initialize $\eta \sim O(1/\sqrt{m}) \in \mathbb{R}, u \in \mathbb{R}^{(784)\times N_1}$ and $b \in \mathbb{R}^{1\times N_1}$ by Uniform(-r, r) with $r = \sqrt{\frac{6}{fan-in+fan-out}}$, then $v \in \mathbb{R}^{N_1\times 10}$ by Uniform(-r, r) with $r = 4\sqrt{\frac{6}{fan-in+fan-out}}$. (fan-in is the number of inputs of the unit, fan-out is the number of outputs of the unit)

3. Initialize epoch = 0.

4. Initialize $loss_{epoch}$ = 0 for epoch = 0, 1, ...

5. Do SGD, so sample n numbers out range(n) without replacement as the index:

    1. for i in index, do (this loop is where we compute the mse of each epoch):
        1. Compute the 1st induced local fields as a vector with the current training sample and weights by:
        $$K = x_i \cdot u + b \in \mathbb{R}^{1\times N_1}$$
        2. Then, get the outputs from the N_1 hidden neurons as a vector by:
        $$Z = tanh(K) \in \mathbb{R}^{1\times N_1}$$
        3. Do the same for the rest, the outputs of the 10 output neurons will be:
        $$L = Z^T \cdot v$$
        $$y_i = softmax(L)$$
    2. Compute the Cross-entropy for current epoch:
    $$loss_{epoch} = -\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{10} d_{ij} log(f(x_i, w)_j)$$
    Then, update the epoch:
    $$epoch \leftarrow epoch + 1.$$

for i = 1 to n, do (this loop is where we update the weights):
Note:

$$\frac{\partial tanh(U)}{\partial U} = \begin{bmatrix} \frac{\partial tanh(u_1)}{\partial u_1} & \frac{\partial tanh(u_1)}{\partial u_2} & \cdots & \frac{\partial tanh(u_1)}{\partial u_q} \\ \frac{\partial tanh(u_2)}{\partial u_1} & \frac{\partial tanh(u_2)}{\partial u_2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial tanh(u_q)}{\partial u_1} & \cdots & \cdots & \frac{\partial tanh(u_q)}{\partial u_q} \end{bmatrix} = \begin{bmatrix} 1 - tanh^2(u_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - tanh^2(u_q) \end{bmatrix} = (1 - tanh(U)) \bigodot (1 + tanh(U))$$

$$\frac{\partial softmax(U)}{\partial U} = \begin{bmatrix} \frac{\partial softmax(u_1)}{\partial u_1} & \frac{\partial softmax(u_1)}{\partial u_2} & \cdots & \frac{\partial softmax(u_1)}{\partial u_q} \\ \frac{\partial softmax(u_2)}{\partial u_1} & \frac{\partial softmax(u_2)}{\partial u_2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial softmax(u_{24})}{\partial u_1} & \cdots & \cdots & \frac{\partial softmax(u_q)}{\partial u_q} \end{bmatrix} = \begin{bmatrix} u_1(\delta_{11} - u_1) & \cdots & u_1(\delta_{1q} - u_q) \\ \vdots & \ddots & \vdots \\ u_q(\delta_{q1} - u_1) & \cdots & u_q(\delta_{qq} - u_q) \end{bmatrix} = U \cdot I - U \otimes U^T$$

3. update the u here:
   1. According to the backpropogation algorithm, we have:

      $$\frac{\partial loss}{\partial u} = -x_i \cdot \frac{1}{n}(d_i - y_i) \cdot \frac{\partial softmax(L)}{\partial L} \cdot v \cdot \frac{\partial tanh(K)}{\partial K} = -x_i \cdot \frac{1}{n}(d_i - y_i) \cdot (L \cdot I - L \otimes L^T) \cdot v \cdot (1 - tanh(K)) \bigodot (1 + tanh(K$$

   2. The update would be:

      $$u_{(t)} \leftarrow u_{(t-1)} \cdot momentum - \eta \frac{\partial loss}{\partial u} = u_{(t-1)} \cdot momentum - x_i \cdot \frac{1}{n}(d_i - y_i) \cdot (L \cdot I - L \otimes L^T) \cdot v \cdot (1 - tanh(K)) \bigodot (1 + tanh(K$$

4. update the b here:
   1. According to the backpropogation algorithm, we have:

      $$\frac{\partial loss}{\partial b} = -\frac{1}{n}(d_i - y_i) \cdot \frac{\partial softmax(L)}{\partial L} \cdot v \cdot \frac{\partial tanh(K)}{\partial K} = -\frac{1}{n}(d_i - y_i) \cdot (L \cdot I - L \otimes L^T) \cdot v \cdot (1 - tanh(K)) \bigodot (1 + tanh(K))$$

   2. The update would be:

      $$b_{(t)} \leftarrow b_{(t-1)} \cdot momentum - \eta \frac{\partial loss}{\partial b} = b_{(t-1)} \cdot momentum - \frac{1}{n}(d_i - y_i) \cdot (L \cdot I - L \otimes L^T) \cdot v \cdot (1 - tanh(K)) \bigodot (1 + tanh(K))$$

5. update the v here:
   1. According to the backpropogation algorithm, we have:

      $$\frac{\partial loss}{\partial v} = -tanh(K) \cdot \frac{1}{n}(d_i - y_i) \cdot \frac{\partial softmax(L)}{\partial L}$$

   2. The update would be:

      $$v_{(t)} \leftarrow v_{(t-1)} \cdot momentum - \eta \frac{\partial loss}{\partial v} = v_{(t-1)} \cdot momentum - \frac{1}{n}(d_i - y_i) \cdot Z \cdot (L \cdot I - L \otimes L^T)$$

6. for every kth (k = 6000) epoch:

   $$\eta = 0.5 * \eta$$

   if loss_{epoch} - loss_{epoch-1} >0

Loop to A, if $abs(loss_{epoch} - loss_{epoch-1}) > \epsilon$.