Dark Matter Halo Density Profile Calibration

Titus Nyarko Nde

Computing Ph.D.

September 26, 2022

Contents

General Introduction

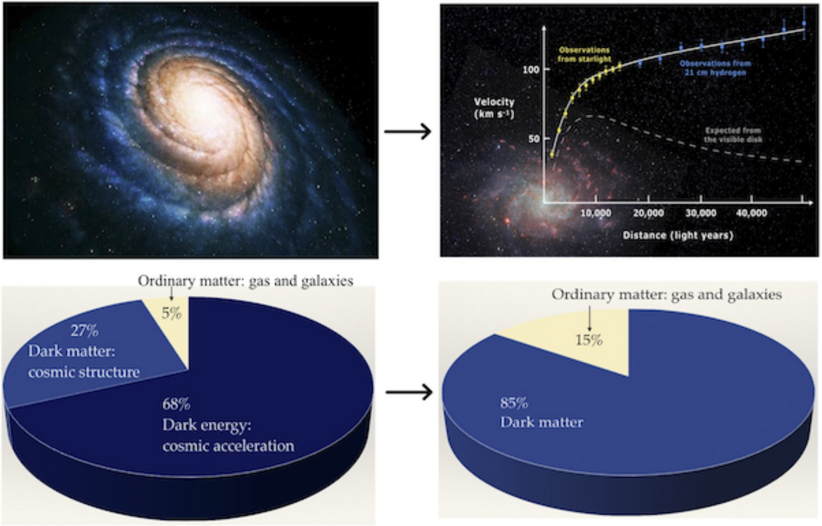

Figure: Dark matter and dark energy in our Universe

The Big Picture

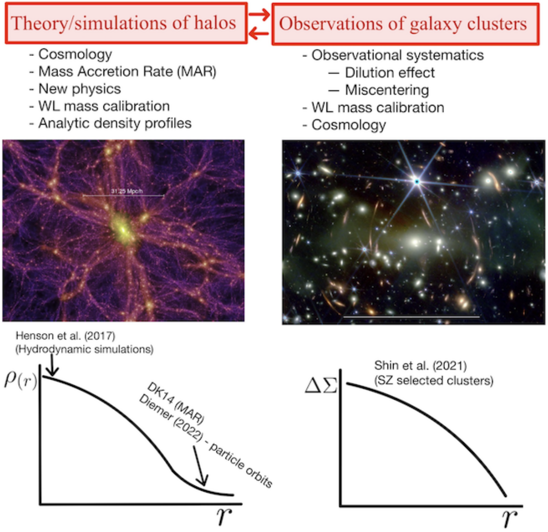

Figure: The big picture

Cosmology and Simulations — The σ_8 Tension

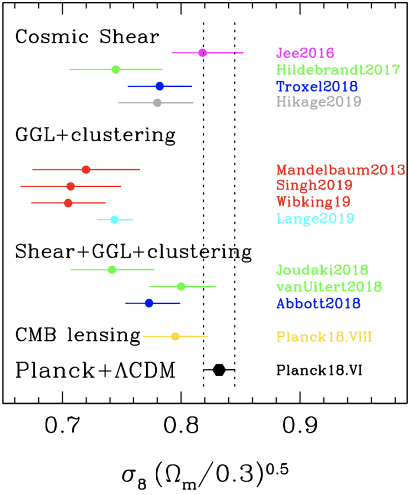

Figure: The σ_8 Tension. (Credit: Dark Energy (Weinberg & White, 2019).)

BOISE STATE UNIVERSITY

Analytic Fitting Functions — 3D Density Profiles

$$\rho_{NFW} = \frac{\rho_{crit}\delta_{crit}}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}, \quad \rho_{crit}\delta_{crit} = \rho_s, \quad r_s = \frac{R_{200}}{c}. \tag{1}$$

$$\rho_{Einasto} = \rho_s \exp\left(-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right). \tag{2}$$

$$\rho_{DK14} = \underbrace{\rho_s \exp\left(-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right)}_{\text{Einasto}} \times \underbrace{\left[1+\left(\frac{r}{r_t}\right)^{\beta}\right]^{-\frac{\gamma}{\beta}}}_{\text{Transition term}} + \underbrace{\rho_{outer}}_{\text{Outer term}}. \tag{3}$$

$$\rho_{outer} = \rho_m \left[b_e \left(\frac{r}{5R_{200}}\right)^{-s_e} + 1\right]; \quad s_e > 0. \tag{4}$$

Analytic Fitting Functions — 2D Density Profiles

The 2D density equation is given as follows:

$$\Sigma(R) = \int_{-l_{max}}^{l_{max}} \rho\left(r = \sqrt{R^2 + l^2}\right) dl, \tag{5}$$

The excess surface density is given as:

$$\Delta\Sigma(R) = \bar{\Sigma}(< R) - \Sigma(R). \tag{6}$$

Observationally, we can compute $\Delta\Sigma$ directly from WL tangential shear, γ_t, via the following equation.

$$\Delta\Sigma(R) = \bar{\gamma}_t(R)\Sigma_{crit}(z_l, z_s). \tag{7}$$

3D Density Plots

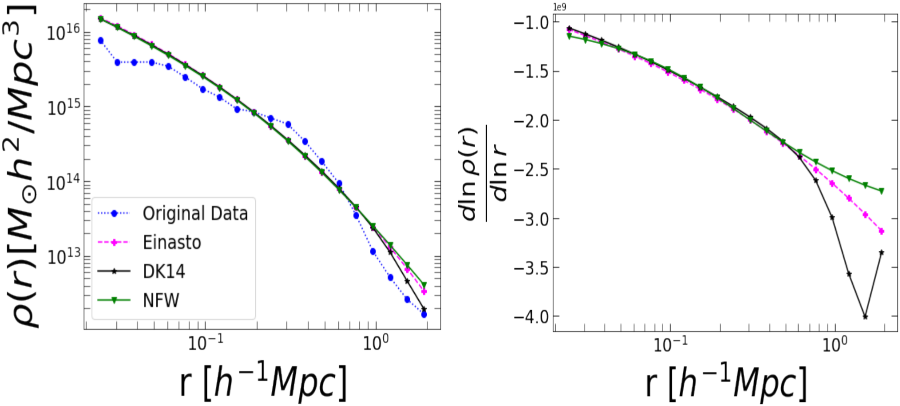

Figure: *Left:* 3-dimensional DK14, NFW, and Einasto density profiles vs a halo from the Buzzard N-body simulation. The halo plotted above has the following properties; $M_{vir} = 3.9 \times 10^{14}, z = 0.39, c = 3.97, R_{vir} = 1.21$, and $= 0.24$. *Right:* The logarithmic slope of the analytic density profiles.

BOISE STATE UNIVERSITY

Current Status of the Field

Observations of Halo Density Profiles

- Weak Lensing (WL) — bending of light from source galaxies
  - inferring halo mass
  - richness-mass relation
  - concentration-mass relation
- Galaxy number density profile — the number density of galaxies per unit area or volume, as a function of distance to halo center
- Model interpretation: Halo Occupation Distribution (HOD) — the number of galaxies per halo of a given mass

Observational data from astronomical surveys come with several systematic effects. These systematic effects contaminate the data to some extent.

Seed Papers

This synthesis considers the following seed papers:

- **Paper I:** Dependence of the outer profiles of halos on their mass accretion rate (Diemer & Kravtsov, 2014).
- **Paper II:** The mass and galaxy distribution around SZ-selected clusters (Shin et al., 2021).
- **Paper III:** A dynamics-based density profile for dark halos – I. Algorithm and basic results (Diemer, 2022).
- **Paper IV:** The impact of baryons on massive galaxy clusters: halo structure and cluster mass estimates (Henson et al., 2017).

BOISE STATE UNIVERSITY

Summary of Results from Seed Papers — Synthesis Matrix

Theme	**Paper I**	**Paper II**	**Paper III**	**Paper IV**
Outer profile	Depends on MAR or PH	Not well calibrated by 2PT and HOD	Depends on MAR. Dominated by infalling matter	Dark matter dominates at large radii
Splashback feature	Steeper in halos with high MAR or PH	Location consistent in WL and galaxy density	Steeper and occurs faster in high-mass halos	—
NFW and Einasto fit	Do not capture the splashback feature and profile dependence on MAR	Do not capture the splashback feature	Do not capture the splashback feature and profile dependence on MAR	Einasto better than NFW; however, both underestimate cluster masses

The Splashback Feature

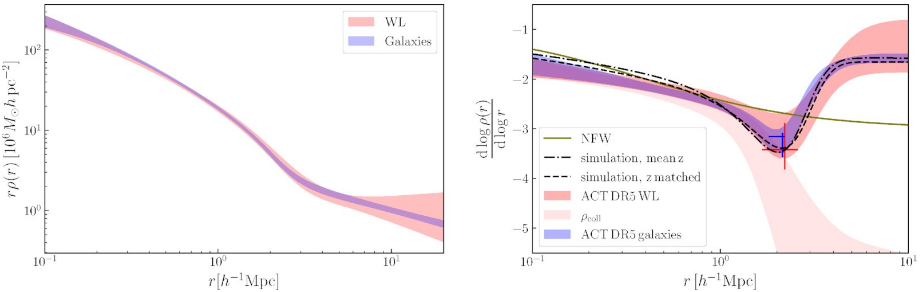

Figure: *Left:* 3D profiles calibrated from galaxy number density and WL. *Right:* Logarithmic slope of the density profiles. The truncation in the orbiting term, ρ_{coll}, occurs before the location of the splashback radius. (Credit: Shin et al., 2021.)

Gaps in the Literature and Future Paths

- **Poor predictions:** As shown in Fig. 4, analytic models usually result in predictions that do not capture the detailed shape of halo density profiles.
- **Uncertainty quantification:** Quantifying prediction uncertainty has been a major concern. None of the analytic models in the literature can do this.
- **Splashback feature:** The mass distribution within the splashback feature is still poorly understood. The location of the splashback radius is also poorly understood.
- **Solving observational systematics:** There is no unambiguous way of handling systematic effects. We need more efficient modeling strategies to fill this gap.

My Ongoing Research: Uncertainty Quantification

The general intuition is to build an army of models whose aggregate performance is significantly better than any of the models in isolation. **Uncertainty quantification techniques help to measure a model's confidence in its predictions.**

Some Uncertainty Quantification Techniques

- Monte Carlo Dropout
- Deep Ensembles
- Bayesian Neural Networks
- Masksembles
- Random Forest

This presentation covers only the first two models in the list above. Note that the list of models is not limited to the one above.

Deep Ensembles (DEs)

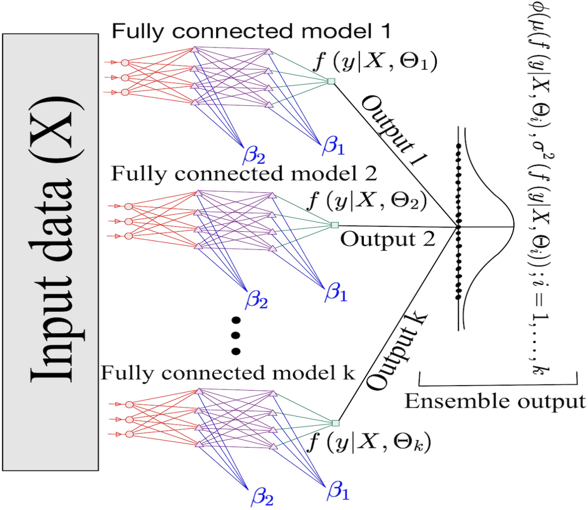

Figure: Illustration of DEs.

Monte Carlo Dropout (MCD)

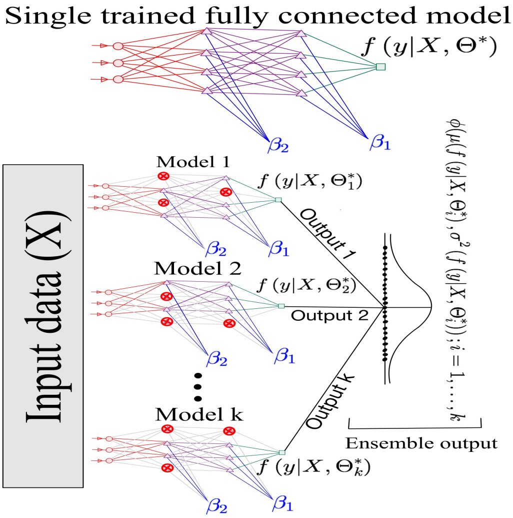

Figure: Illustration of MCD.

Some Results

Sample Distribution of Prediction (MCD)

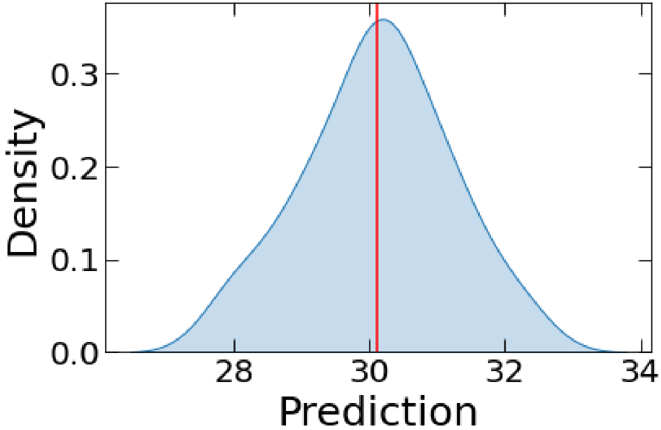

Figure: Sample distribution of a single prediction. Not all the distributions look symmetric. The red vertical line indicates the position of the mean. Halo properties: $z = 0.64078$, $M = 1.85 \times 10^{13}$, $R_{vir} = 0.3834$, $r_{mid} = 0.8455$.

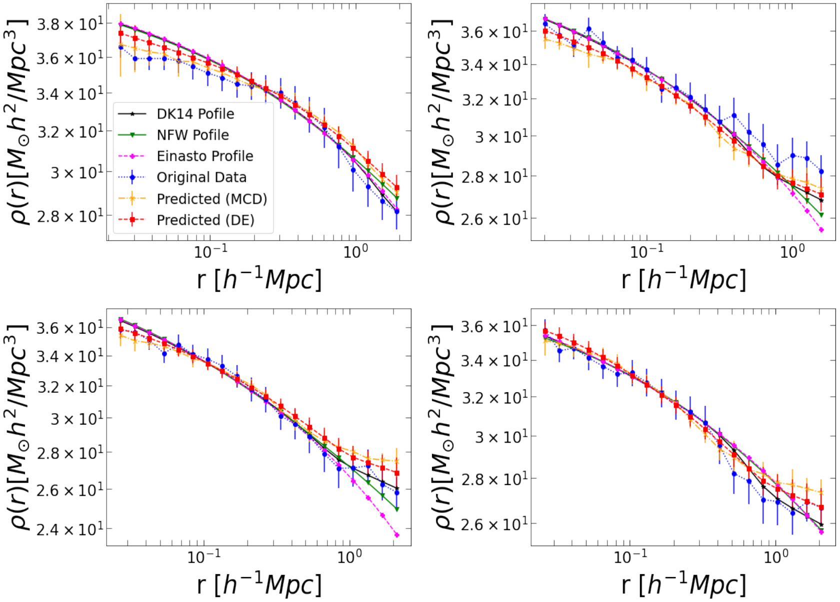

Figure: Some test halos

My Ongoing Research: Actual vs Predicted Values

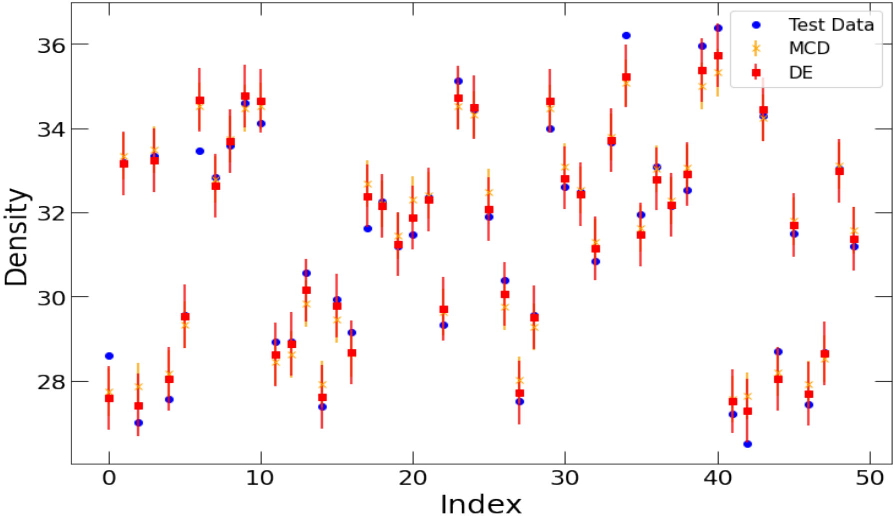

Figure: Predicted vs actual values.

My Ongoing Research: Some Numeric Metrics

Model	MSE (Chi-square)
NFW	0.38053
Einasto	0.5159
DK14	0.3570
MCD	0.3369 (1.1649)
DE	0.2682 (0.8392)

MCD and DE have adjusted R-squared values of 0.9560 and 0.9650, respectively.

Recap

- There is a need to reconcile theory with observations.
- The existing analytic models cannot quantify prediction uncertainties.
- Structure and mass distribution around the splashback feature is still poorly understood.
- As part of a bigger project we are considering more models and more data (such as the 2D density profiles).
- We are building an emulator for calibrating halo density profiles.
- My computing artifact is available on GitHub

Acknowledgement

Many thanks to my supervisor, Dr. Wu, for the great support in my preparation for this comprehensive exam. I also thank Andrius Tamosiunas, Gladys Muthoni Kamau, and the Committee Members for their support.

Thank you!

Backup slide 1: Numeric metrics

Layers	Nodes	NFW MSE	Einasto MSE	DK14 MSE	DEs MSE (Chi-sq)	MCD MSE (Chi-sq)
3	8-8-1	0.3807	0.5160	0.3548	0.2799 (0.8043)	0.2895 (1.2576)
4	8-16-16-1	0.3807	0.5160	0.3548	0.2442 (0.7970)	0.3128 (0.9516)
5	16-30-50-20-1	0.3807	0.5160	0.3548	0.2418 (0.7561)	0.3201 (1.06143)
8	8-64-128-256-256-128-64-1	0.3807	0.5160	0.3548	0.2512 (0.5050)	0.3653 (0.5121)

MCD and DE have adjusted R-squared values of 0.9573 and 0.9664, respectively.

Backup slide 2: Cosmology and Simulations

Table: Cosmological parameters from different astronomical surveys.

Cosmology	$\Omega_{m,0}{}^{a}$	h^{b}	$\sigma_8{}^{c}$	$n_s{}^{d}$
Einstein-de Sitter	1.0000	0.7000	0.8200	1.0000
WMAP1 (2003)	0.2700±0.04	0.7200±0.05	0.9000±0.1	0.9900±0.04
WMAP7 (2011)	0.2743±0.007	0.7020±0.014	0.8160±0.024	0.9680±0.012
Planck 2018	0.3111±0.0056	0.6766±0.0042	0.8102±0.006	0.9665±0.0038

[a] Matter density parameter
[b] Hubble constant
[c] Density fluctuation
[d] Scalar spectral index

BOISE STATE UNIVERSITY

Backup slide 2: Summary of Seed Papers

Paper	Objective	Support	Analysis	Result	Implication
Paper I	Measure density profiles focusing on the outer region	N-body simulation. NFW and Einasto profiles poorly fit outer density profiles	Propose DK14 profile	Detect the splashback feature for the first time in N-body simulations	Observationally detect the splashback feature
Paper II	Measure density profiles	Observational data from ACTDR5 and DESY3. SZ data is more accurate	HOD and DK14 with 2PT	Detect the splashback feature for the first time in SZ data	Splashback feature is not yet well understood
Paper III	Understand the splashback feature	N-Body simulations. Splashback term not well understood	Disentangle orbiting and infalling matter and model them separately	Profiles are dependent on different spatial scales	Single-scale models are insufficient
Paper IV	Measure the impact of baryons on high-mass halos	Hydrodynamic simulations. Ignoring baryons leads to systematic bias in mass estimation	Uses NFW and Einasto profiles	Detects minor baryonic effects on high-mass halos	Baryonic effect remains uncertain in high-mass halos