Lectures 11 **3/7/2018**

Crisp set A of X

$$f_A(x) : X \rightarrow 0, 1,$$

where

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

Fuzzy logic

$$\mu_A(x) : X \rightarrow [0, 1],$$

where

$\mu_A(x) = 1$ if x is totally in A;
$\mu_A(x) = 0$ if x is not in A;
$0 < \mu_A(x) < 1$ if x is partly in A.

Hedges

- 160 – 5'2.4" … 170 -
- Tall = $\{\{0,160\},\{0.5,170\},\{1,182\}\}$
- Short = $\{\{1,160\},\{0.25,170\},\{0,182\}\}$
- Two situations: either we require a new fuzzy variable, or apply a hedge
- Very Tall = $(tall)^2 = \{\{0^2,160\},\{.5^2,170\},\{1^2,182\}\}$

Fuzzy sets, operations on fuzzy sets, monotonic inference

- Fuzzy logic operations
  - Fuzzy logic operations are different from classical logic
  - Recall Boolean logic operations:
    - Compliment
      - Classical logic: invert the variable or expression
      - Fuzzy: similar operation, except we subtract it by one
      - $\mu_{\neg A}(x) = 1 - \mu_A(x)$
    - Intersection (AND)
      - Use min
        - $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$
        - $\mu_{Tall \cap Short}(x) = \{\{0,160\},\{0.25,170\},\{0,182\}\}$
      - Use product
        - $\mu_{A \cap B}(x) = \mu_A(x) \times \mu_B(x)$
        - $\mu_{Tall \cap Short}(x) = \{\{0,160\},\{0.25 * 0.5,170\},\{0,182\}\}$
    - Union (OR)
      - Use max
        - $\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$
        - $\mu_{Tall \cap Short}(x) = \{\{1,160\},\{0.5,170\},\{1,182\}\}$
      - Union with Probabilistic OR
        - $\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x)$

- $\mu_{Tall \cap Short}(x) = \begin{Bmatrix} \{0+1-0*1,160\}, \\ \{0.5+0.25-0.5*0.25,170\}, \\ \{1+0-1*0,182\} \end{Bmatrix} = \begin{Bmatrix} \{1,160\}, \\ \{0.625,170\}, \\ \{1,182\} \end{Bmatrix}$

- Fuzzy rules
  - We want to catch a real big yellow tail!
  - Variables: Temperature, Size of the bait
  - Input: Temperature = {Cold, Ideal, Hot}
    - Ideal = $\{\{0,64F\},\{0.5,72F\},\{1,77F\},\{0.5,80F\},\{0,182F\}\}$
  - Input: Bait Weight = {Small,Big}
    - Big = $\{\{0,2lbs\},\{0.5,4lbs\},\{1,5lbs\}\}$
  - Output: Tuna Length = {Small,Big}
    - Big = $\{\{0,24in\},\{0.5,29in\},\{1,34in\}\}$

IF Temperature is Ideal
AND Bait Weight is Big
THEN Tune Length is Big

IN TESTING:

Being realistic here:

72F and we only got a 4lb

uTemperature(72F) = 0.5
uBaitWeight(4lb) = 0.5

uTemperature(72F) AND uBaitWeight(4lb)

= min(uTemperature(72F), uBaitWeight(4lb))

= min(0.5, 0.5) = 0.5

uBig-1(uTemperature(72F) AND uBaitWeight(4lb)) = uBig-1(0.5) = 29in