CMPS 3560: Likelihood of Sufficiency (LS) & Likelihood of Necessity (LN), Certainty Factors

LS/LN

(1) IF 'today' is 'rain' {LS 2.5 LN 0.6}
THEN 'tomorrow' is 'rain' {prior 0.5}

- Bayes rule table enumeration is costly
  - Potentially: $(\text{num. rules})^{(2)^{(\#\text{ evidence vars.})}}$
- LS/LN: streamlined inference
  - Pseudo-probabilities: $O(H)$, $O(H|E)$
  - $LS = P(E|H) / P(E|\neg H)$
  - $LN = P(\neg E|H) / P(\neg E|\neg H)$
  - Use above formulas when calculating rationally, or given by expert
- $O(H) = P(H) / (1 - P(H))$
  - used only if consequent was not previously asserted
- $O(H|E) = LS \times O(H)$
- $O(H|\neg E) = LN \times O(H)$
- $P(H|E) = O(H|E) / (1 + O(H|E))$
- $P(H|\neg E) = O(H|\neg E) / (1 + O(H|\neg E))$
- Consequent asserted w/ appropriate posterior
- When consequent asserted, $O(H)$ is now calculated w/ posterior

Following prev. example:

(1) $O(H) = \dfrac{0.5}{1-0.5} = 1.0$

Lets say it is raining today

$O(H|E) = LS \times O(H)$
$= 2.5(1.0)$
$= 2.5$

$P(H|E) = O(H|E) / (1 + O(H|E))$
$= 2.5 / (1 + 2.5)$
$= 0.71$

$\Rightarrow$ 'tomorrow' is 'rain' asserted w/ <u>0.71</u>

(2) IF 'today' is 'cloudy' {LS 2.0 LN 0.5}
THEN 'tomorrow' is 'rain' $\underbrace{\text{\{Prior 0.5\}}}_{\text{not used b/c rain prev. asserted}}$

$O(H) = \dfrac{0.71}{1-0.71} = 2.45$

Lets say it was <u>not</u> cloudy

$O(H|\neg E) = LN \times O(H)$
$= 0.5(2.45)$
$= 1.225$

$P(H|\neg E) = O(H|\neg E) / (1 + O(H|\neg E))$
$= 1.225 / (1 + 1.225)$
$= 0.55$

$\Rightarrow$ 'tomorrow' is 'rain' now <u>0.55</u>

Certainty Factors

- Even more streamlined
- $cf \in [-1, 1]$

IF <ev>
THEN <hyp.> {cf}

Calc MB/MD → cf → Inference

(Start here w/ expert)

$$MB = \begin{cases} 1, & \text{if } p(H)=1 \\ \dfrac{\max(p(H|E), p(H)) - p(H)}{1 - p(H)}, & \text{otherwise} \end{cases}$$

$$MD = \begin{cases} 1, & \text{if } p(H)=0 \\ \dfrac{\min(p(H|E), p(H)) - p(H)}{-p(H)}, & \text{otherwise} \end{cases}$$

$$cf = \frac{MB(H,E) - MD(H,E)}{1 - \min(MB(H,E), MD(H,E))}$$

$$cf(H,E) = \underbrace{cf(E)}_{\text{cf of evidence that has already been asserted}} \times \underbrace{cf}_{\text{cf of this rule}}$$

IF 'sky' is 'clear'
THEN 'forecast' is 'sunny' {cf 0.8}

Assume 'sky' is 'clear' asserted w/ cf 0.9

$\Rightarrow$ 'forecast' is 'sunny' asserted w/:

$$cf_{forecast,\ sunny} = (0.9)(0.8)$$
$$= 0.72$$