CMPS 3560 Artificial Intelligence Sugeno and Mamdani Inference – Revised Pi Day 2018

PROBLEM:

(1) IF A is a1
   AND B is b1
   THEN C is c1
(2) IF A is a2
   OR B is b2
   THEN C is c2

$$\mu_{b1}(x) = \begin{cases} 0, & x \le 0 \\ -x + 1, & 0 \le x < 1 \\ 1, & x > 1 \end{cases}$$

$$\mu_{a1}(x) = \begin{cases} 1, & x \le 0 \\ -\dfrac{1}{10}x + 1, & 0 \le x < 10 \\ 0, & x > 10 \end{cases}$$

$$\mu_{b2}(x) = \begin{cases} 0, & x \le 0 \\ x, & 0 \le x < 1 \\ 1, & x > 1 \end{cases}$$

$$\mu_{c1}(x) = \begin{cases} 0, & x \le 0 \\ x, & 0 \le x < 1 \\ -x + 2, & 1 \le x < 2 \\ 0, & x > 2 \end{cases}$$

$$\mu_{a2}(x) = \begin{cases} 0, & x \le 5 \\ \dfrac{1}{5}x - 1, & 5 \le x < 10 \\ 1, & x > 10 \end{cases}$$

$$\mu_{c2}(x) = \begin{cases} 0, & x \le 1 \\ x - 1, & 1 \le x < 2 \\ -x + 3, & 2 \le x < 3 \\ 0, & x > 3 \end{cases}$$

Given the following parameters:

- Scaling
- Min/max operations for and/or
- Aggregation with summation
- Inputs: A is crisp 7 and B is crisp 0.2

MAMDANI

1) Step 1: Fuzzification. Plug in crisp variable values to determine the membership value of **all** fuzzy values. Only do this for the <u>input</u> values, not the <u>output</u> values. The input values in this case are a1, a2, b1 and b2. These are the fuzzy values in the antecedent rules.

$$\mu_{a1}(7) = -\frac{7}{10} + 1 = 0.3$$

$$\mu_{a2}(7) = \frac{7}{5} - 1 = 0.4$$

$$\mu_{b1}(0.2) = -0.2 + 1 = 0.8$$

$$\mu_{b2}(0.2) = 0.2$$

2) Step 2: Carry out rules. Determine the fuzzy membership of the antecedent.

(1) μ_{a1}AND μ_{b1}= min$[\mu_{a1}(7), \mu_{b1}(0.2)]$ = min$[0.3, 0.8]$ = 0.3

(2) μ_{a2}OR μ_{b2}= max$[\mu_{a2}(7), \mu_{b2}(0.2)]$ = max$[0.4, 0.2]$ = 0.4

3) Step 3: Carry out clipping or scaling. For this problem we chose scaling as an experimental parameter.

$$0.3 * \mu_{c1}(x) = \begin{cases} 0, & x \le 0 \\ 0.3x, & 0 \le x < 1 \\ -0.3x + 0.6 & 1 \le x < 2 \\ 0, & x > 2 \end{cases}$$

$$0.4 * \mu_{c2}(x) = \begin{cases} 0, & x \le 1 \\ 0.4x - 0.4, & 1 \le x < 2 \\ -0.4x + 1.2, & 2 \le x < 3 \\ 0, & x > 3 \end{cases}$$

4) Step 4: Add them together with aggregation. Note from the lab, it is possible to use product or Probor here instead.

$$C(x) = \begin{cases} 0, & x \le 0 \\ 0.3x, & 0 \le x < 1 \\ (-0.3x + 0.6) + (0.4x - 0.4), & 1 \le x < 2 \\ -0.4x + 1.2 & 2 \le x < 3 \\ 0, & x \ge 3 \end{cases}$$

$$= \begin{cases} 0, & x \le 0 \\ 0.3x, & 0 \le x < 1 \\ 0.1x + 0.2, & 1 \le x < 2 \\ -0.4x + 1.2 & 2 \le x < 3 \\ 0, & x \ge 3 \end{cases}$$

5) Defuzzification of the output. Take the shape generated in Step 4 and calculate the centroid. The Equation is:

$$\text{CRISP} = \frac{\int C(x) * x \, dx}{\int C(x) \, dx}$$

If you've taken calculus, it's possible for you to solve for 'CRISP' directly by evaluation the integral. This is not required, however, because the book demonstrates how to carry this out by sampling a finite number of points. Let the set of finite points be Φ. Carry out:

$$\text{CRISP} = \frac{\sum_\Phi C(x) * x}{\sum_\Phi C(x)}$$

For this example let $\Phi = \{0, 0.5, 1, 1.5, 2, 2.5, 3\}$. In practice, you will want to sample x more frequently.

The corresponding values are:

x	$C(x)$	$C(x) * x$
0	0	0
0.5	0.3(0.5)=0.15	0.3(0.5)(0.5)=0.075
1	$0.1(1) + 0.2 = 0.3$	0.3*1 = 0.3
1.5	0.1*1.5 + 0.2 = 0.35	0.35*1.5 = 0.525
2	$-0.4 * 2 + 1.2 = 0.4$	0.8

2.5	-0.4*2.5+1.2 = 0.725	0.725 * 2.5 = 1.8125
3	0	0
Sum	1.925	3.5125

CRISP = 3.5125 / 1.925 = 1.82

SUGENO

Sugeno is the exact same procedure, except:

1) During problem formulation, you and the expert replace the **OUTPUT** membership functions with singletons (spikes):

$$\mu(x) = \begin{cases} 1 & x = x_0 \\ 0 & Otherwise \end{cases}$$

2) During Step 5), let Φ be the points where the singletons are non-zero.

One way to think of it: Mamdani is high accuracy because you will want to sample as much as possible. Sugeno is streamlined for speed.

Additional problems:

- Revisit this example with Sugeno inference. C1 is a singleton at 1. C2 is a singleton at 2. Does the answer change?
  - Recall that rule 1 is firing with 0.3. Rule 2 is firing with 0.4.

$$C(x) = 0.3\mu_{c1}(x) + 0.4\mu_{c2}(x)$$
$$\frac{\sum_\Phi C(x) * x}{\sum_\Phi C(x)} = \frac{C(1) * 1 + C(2) * 2}{C(1) + C(2)} = \frac{0.3 + 0.4 * 2}{0.3 + 0.4} = \frac{1.1}{0.7} = 1.571$$

- Repeat this for the following parameters:
  - A = 3
  - B = 0.9
  - Use ProbOR and Product instead.
  - Continue to use sum as aggregator.
- Repeat this again, but use ProbOR as the aggregator.