Version 4/1/2018
Lecture notes: Measures of belief (MB), measures of disbelief (MD), certainty factors

Recall

- Table enumeration can be very costly
- Alternative uncertainty factor algorithms: LS/LN, MB, MD and certainty factors
- Streamlined uncertainty values that are designed to work with rule-based expert systems from the ground up
    - *Aside: Rule-based expert systems are not as ubiquitous as the text would make it seem. The ability of an algorithm to work well with rules is a positive for the class curriculum, but in production you may want to consult other algorithms. For current rule-based systems consult: decision trees, C4.5 and random forest. For current probability-based algorithms consult: Bayesian nets, hidden Markov models.*

MB, MD and certainty factors

- A certainty factor (CF) is given with each rule
- Human-like system: Expert gives CF values
- Rational approach: Generated CF from MB and MD values based on raw data

$$MB(H,E) = \begin{cases} 1 & \text{if } p(H) = 1 \\ \dfrac{max\,[p(H|E), p(H)] - p(H)}{max\,[1,0] - p(H)} & \text{otherwise} \end{cases}$$

$$MD(H,E) = \begin{cases} 1 & \text{if } p(H) = 0 \\ \dfrac{min\,[p(H|E), p(H)] - p(H)}{min\,[1,0] - p(H)} & \text{otherwise} \end{cases}$$

$$cf = \frac{MB(H,E) - MD(H,E)}{1 - min\,[MB(H,E), MD(H,E)]}$$

- When a rule is asserted, the consequent is asserted with a CF that is a function of the rule's CF and the combination of the antecedent CF values
- Very similar to how fuzzy value are combined with fuzzy logic
- AND operation: min
- OR operation: max
- Conflicts: A rule can attempt to assert something that is already fact, in which case, the CF value is combined

- $$cf(cf_1, cf_2) = \begin{cases} cf_1 + cf_2 \times (1 - cf_1) & \text{if } cf_1 > 0 \text{ and } cf_2 > 0 \\ \dfrac{cf_1 + cf_2}{1 - min\,[|cf_1|, |cf_2|]} & \text{if } cf_1 < 0 \text{ or } cf_2 < 0 \\ cf_1 + cf_2 \times (1 + cf_1) & \text{if } cf_1 < 0 \text{ and } cf_2 < 0 \end{cases}$$

A rational example:

- Consider the rule:
  - IF A is true
    THEN B is true {cf ???}
- Given the following table of raw data:

Sample	A	B
1	0	0
2	0	1
3	1	0
4	1	1
5	1	1
6	0	1
7	0	1
8	0	1

- Generate the table of joint probabilities:

A\B	0	1
0	1/8	4/8
1	1/8	2/8

- Generate MB, MD and finally CF value. A is the evidence (A), B is the hypothesis (H)

P(H) = P(B) = 3/4

Aside: Note that this is short hand … P(B) is P(B=true) or P(B=1)

P(H|E) = P(B|A) = P(A and B) / P(A)
= (2/8) / (3/8) = 2/3

MB(H,E) = MB(B,A)
= (max(P(B|A), P(B)) – P(B)) / (1 – P(B))
= (max(2/3, 3/4) - 3/4) / (1 – 3/4)
= 0 / …
= 0

MD(H,E) = MD(B,A)
= (min(P(B|A), P(B)) – P(B)) / (– P(B))
= (min(2/3, 3/4) - 3/4) / (– 3/4)
= (2/3 – 3/4) / (-3/4)
= -0.08 / -0.75
= 0.11

Consider the raw data. Note that B often happens without A. Thus, if A occurs, it should increase disbelief that B will happen. So, generally, the appearance of A isn't really conclusive evidence for the existence of B, but it does slightly indicate that B is false.

CF = (MB – MD) / (1 – min(MB,MD))
= (0 – 0.11) / (1 – min(0,0.11))
= -0.11 / 1 = -0.11

- After all of that, the rule is updated as follows:
  - IF A is true
    THEN B is true {cf -0.11}
- LET US ASSUME that A is fact with a CF value of 0.5.
  The above rule is fired.
  B is asserted as fact with CF = 0.5 * -0.11 = -0.06

Complex antecedent

- Now consider the following rule:
  - IF C is true
    AND D is true
    THEN B is true {cf 0.75}
    (The expert gave us the CF value for this rule)
- LET US ASSUME that C and D are fact with CF values of 0.2 and 0.6 respectively
- The rule fires … B is asserted with:
  - CF = min(0.2, 0.6) * 0.75 = 0.2 * 0.75 = 0.15

CF value collision

- Note that we asserted B already with -0.06
- Need to use equation to reconcile differences, B can only have one associated CF value
- Use the equation:

CF1 = -0.06
CF2 = 0.15

$$cf(cf_1, cf_2) = \begin{cases} cf_1 + cf_2 \times (1 - cf_1) & \text{if } cf_1 > 0 \text{ and } cf_2 > 0 \\ \dfrac{cf_1 + cf_2}{1 - min\left[|cf_1|, |cf_2|\right]} & \text{if } cf_1 < 0 \text{ or } cf_2 < 0 \\ cf_1 + cf_2 \times (1 + cf_1) & \text{if } cf_1 < 0 \text{ and } cf_2 < 0 \end{cases}$$

CF = (-0.06 + 0.15) / (1 – min(0.06, 0.15)
= 0.09 / (1 – 0.06)
= 0.10

So, it slightly increased certainty