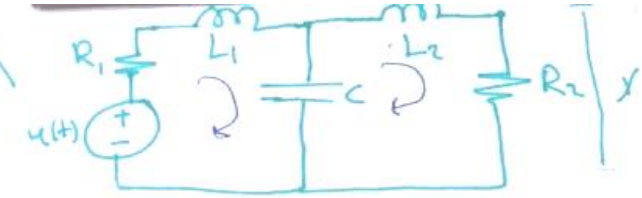
Q1\

(A)



$x_1 = I_{L_1}$
$x_2 = I_{L_2}$
$x_3 = V_C$

* KVL $\quad u(t) = I_{L_1} * R_1 + L_1 \frac{di_{L_1}}{dt} + V_C \Rightarrow L_1 \frac{di_{L_1}}{dt} = u(t) - I_{L_1} * R_1 - V_C$

$$\frac{di_{L_1}}{dt} = \frac{u(t)}{L_1} - I_{L_1} * \frac{R_1}{L_1} - \frac{V_C}{L_1} \Rightarrow \dot{x}_1 = \frac{u(t)}{L_1} - x_1 \frac{R_1}{L_1} - \frac{x_3}{L_1} \quad (1)$$

* KVL $\quad I_{L_2} * R_2 + L_2 \frac{di_{L_2}}{dt} - V_C \Rightarrow L_2 \frac{di_{L_2}}{dt} = V_C - I_{L_2} * R_2$

$$\frac{di_{L_2}}{dt} = \frac{V_C}{L_2} - I_{L_2} * \frac{R_2}{L_2} \Rightarrow \dot{x}_2 = \frac{x_3}{L_2} - x_2 \frac{R_2}{L_2} \quad (2)$$

* KCL $\quad I_{L_1} = I_C + I_{L_2} \Rightarrow I_{L_1} = C \frac{dV_C}{dt} + I_{L_2}$

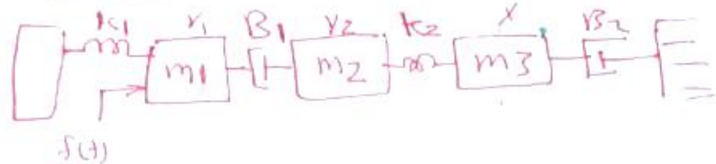
$$C \frac{dV_C}{dt} = I_{L_1} - I_{L_2} \Rightarrow \frac{dV_C}{dt} = \frac{I_{L_1}}{C} - \frac{I_{L_2}}{C}$$

$$\dot{x}_3 = \frac{x_1}{C} - \frac{x_2}{C} \quad (3)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & R & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(B)



$$m_1\ddot{y}_1 + k_1 y_1 + B_1(\dot{y}_1 - \dot{y}_2) = f(t) \quad —(1)$$

$$m_2\ddot{y}_2 + B_1(\dot{y}_2 - \dot{y}_1) + k_2(y_2 - y_3) = 0 \quad —(2)$$

$$m_3\ddot{y}_3 + k_2(y_3 - y_2) + B_2\dot{y}_3 = 0 \quad —(3)$$

$x_1 = y_1$ $\qquad x_3 = y_2 \qquad x_5 = y_3$

$x_2 = \dot{y}_1 \qquad x_4 = \dot{y}_2 \qquad x_6 = \dot{y}_3$

$$x_1 = y_1 \Rightarrow \dot{x}_1 = \dot{y}_1 = x_2$$

$$x_2 = \dot{y}_1 \Rightarrow \dot{x}_2 = \ddot{y}_1 = \frac{-k_1}{m_1} y_1 + \frac{B_1}{m_1}(\dot{y}_2 - \dot{y}_1) + \frac{f(t)}{m_1}$$

$$\dot{x}_2 = \frac{-k_1}{m_1} x_1 + \frac{B_1}{m_1} x_4 - \frac{B_1}{m_1} x_2 + \frac{f(t)}{m_1} \quad —(2)$$

$$x_3 = y_2 \Rightarrow \dot{x}_3 = \dot{y}_2 = x_4 \quad —(3)$$

$$x_4 = \dot{y}_2 \Rightarrow \dot{x}_4 = -\frac{k_2}{m_2} y_2 + \frac{k_2}{m_2} y_3 - \frac{B_1}{m_2}\dot{y}_2 + \frac{B_1}{m_2}\dot{y}_1$$

$$\dot{x}_4 = \frac{-k_2}{m_2} x_3 + \frac{k_2}{m_2} x_5 - \frac{B_1}{m_2} x_4 + \frac{B_1}{m_2} x_2$$

$$= \frac{B_1}{m_2} x_2 - \frac{k_2}{m_2} x_3 - \frac{B_1}{m_2} x_4 + \frac{k_2}{m_2} x_5$$

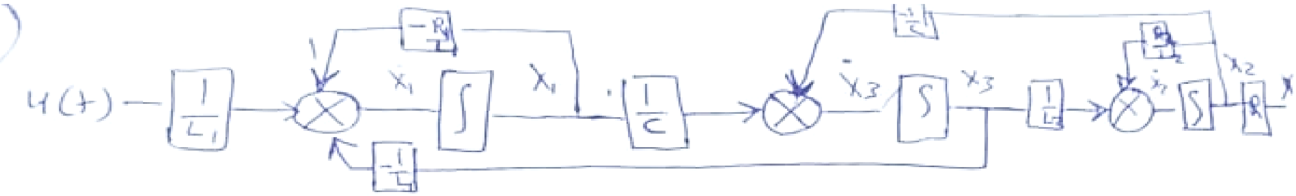
$$x_5 = y_3 \Rightarrow \dot{x}_5 = \dot{y}_3 = x_6 \quad —(4)$$

$$x_6 = \dot{y}_3 \Rightarrow \dot{x}_3 = \ddot{y} = \frac{k_2}{m_3} y_2 - \frac{k_2}{m_3} y_3 - \frac{B_2}{m_3}\dot{y}_3$$

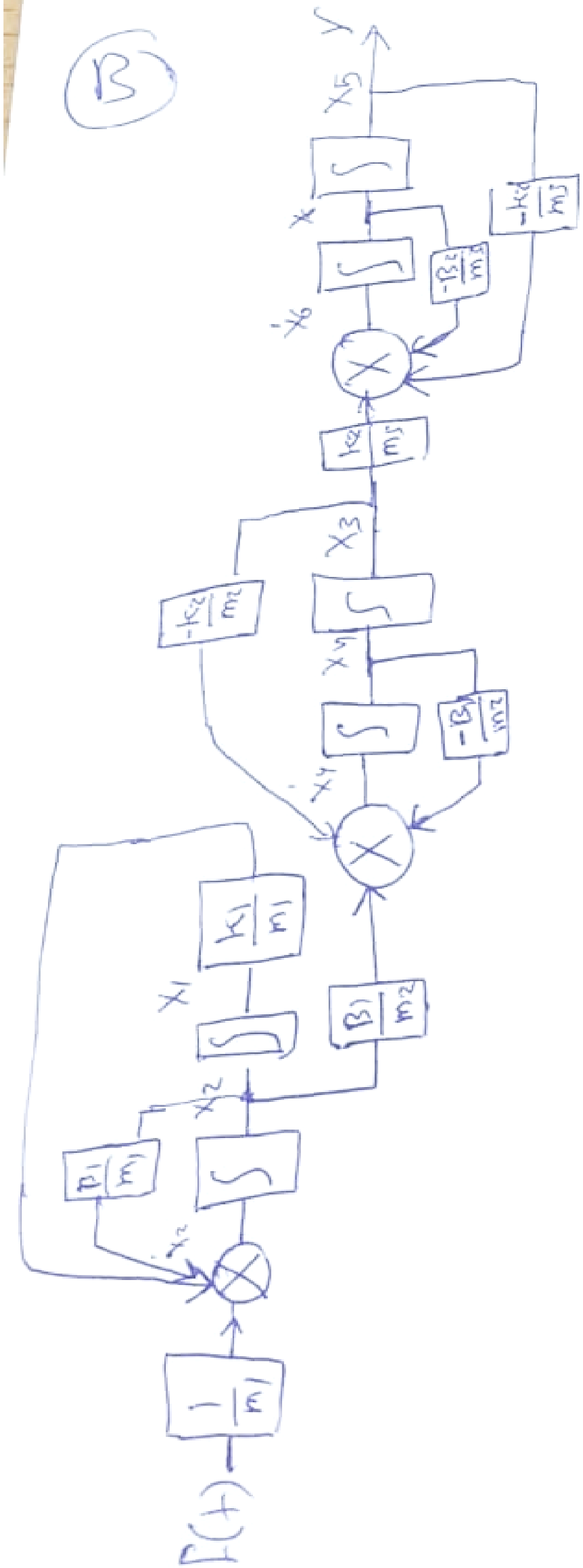
$$\dot{x}_3 = \frac{k_2}{m_3} x_3 - \frac{k_2}{m_3} x_5 - \frac{B_2}{m_3} x_6$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{B_1}{m_1} & 0 & \frac{B_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{B_1}{m_2} & -\frac{k_2}{m_2} & -\frac{B_1}{m_2} & \frac{k_2}{m_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k_2}{m_3} & 0 & -\frac{k_2}{m_3} & -\frac{B_2}{m_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f(t)$$

A



B



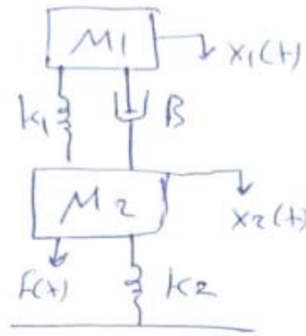
Q2\

$X_1 = x_1(t)$

$X_2 = \dot{x}_1(t)$

$X_3 = x_2(t)$

$X_4 = \dot{x}_2(t)$



$$m\ddot{x}_1(t) + k_1\left(x_1(t) - x_2(t)\right) + B\left(\dot{x}_1(t) - \dot{x}_2(t)\right) = 0$$

$$m_2\ddot{x}_2(t) + k_1\left(x_2(t) - x_1\right) + B\left(\dot{x}_2(t) - \dot{x}_1(t)\right) + k_2 x_2(t) = f(t)$$

$$X_1 = x_1(t) \Longrightarrow \dot{X}_1 = \dot{x}_1(t) = X_2 \quad — (1)$$

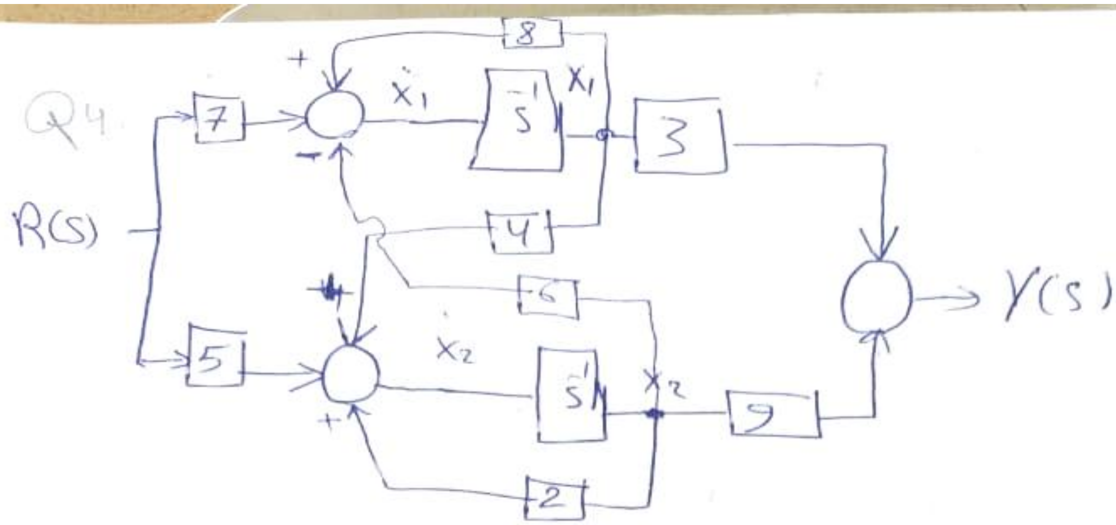
$$X_2 = \dot{x}_1(t) \Longrightarrow \dot{X}_2 = \ddot{x}_1(t) \Longrightarrow \frac{k_1}{m_1}\left(X_3 - X_1\right) + \frac{B}{m_1}\left(X_4 - X_2\right)$$

$$X_3 = x_2(t) \Longrightarrow \dot{X}_3 = \dot{x}_2(t) = X_4$$

$$X_4 = \dot{x}_2(t) \Rightarrow \dot{X}_4 = \ddot{x}_2(t) = \frac{k_1}{m_2}\left(X_1 - X_3\right) + \frac{B}{m_2}\left(X_2 - X_4\right) - k_2 X_3 + f(t)$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{B}{m_1} & \frac{k_1}{m_1} & \frac{B}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{B}{m_2} & \frac{-k_1-k_2}{m_2} & -\frac{B}{m_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

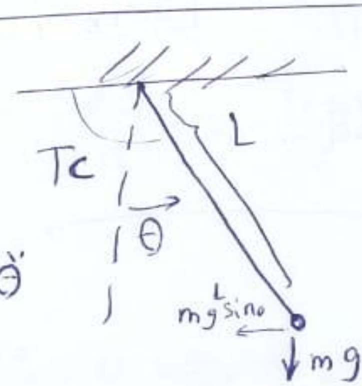
Q4



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 7 \\ 5 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q3



$$T(t) \cong \frac{g}{L} \frac{d\sin\theta}{d\theta}\bigg|_{\theta=0} (\theta - \underset{\substack{\text{equilibrium}\\ \text{condition}}}{\theta_0})$$

$$\Sigma T = I\ddot{\theta}$$

$$T_c + mgL\sin\theta = I\ddot{\theta}$$

$$I = mL^2$$

$$\left[T_c + mgL\sin\theta = mL^2\ddot{\theta}\right] * \frac{1}{mL^2}$$

$$\frac{T_c}{mL^2} - \frac{g}{L}\sin\theta = \ddot{\theta}$$

$$\ddot{\theta} + \frac{g}{L}\underbrace{\sin\theta}_{\text{nonlinear}} = \frac{T_c}{mL^2}$$

$$\ddot{\theta} + \frac{g}{L}\theta = \frac{T_c}{mL^2} \Rightarrow$$

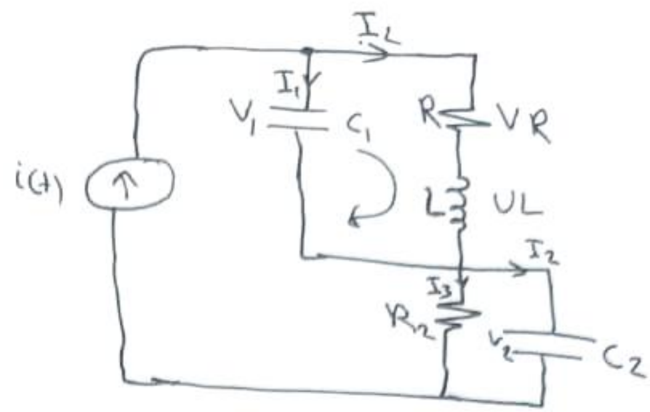
$$x_1 = \theta \Rightarrow \dot{x}_1 = \dot{\theta} \Rightarrow \dot{x}_1 = x_2 \quad (1)$$

$$x_2 = \dot{\theta} \Rightarrow \dot{x}_2 = \ddot{\theta} \Rightarrow \dot{x}_2 = \frac{T_c}{mL^2} - \frac{g}{L}x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{mL^2} \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q\5



* number of state variables = number of storing elemats

$x_1 = V_{C1}$

$x_2 = I_L$

$x_3 = V_{C2}$

(KCL)

$$I(t) = I_1 + I_L \Rightarrow i(t) = C_1 \frac{dv_{c1}}{dt} + I_L \Rightarrow C_1 \frac{dv_{c_1}}{dt} = i(t) - I_L$$

$$\frac{dv_{c1}}{dt} = \frac{i(t)}{c} - \frac{I_L}{c} \Rightarrow \dot{x}_1 = \frac{i(t)}{c} - \frac{x_2}{c}$$

(KVL)

$$V_{R_1} + V_L - V_{C1} = 0 \Rightarrow V_L = V_{C1} - V_{R_1}$$

$$L\frac{di_L}{dt} = V_C - I_L * R \Rightarrow \frac{di_L}{dt} = \frac{V_C}{L} - \frac{I_L R}{L}$$

$$\dot{x}_2 = \frac{x_1}{L} - x_2 * \frac{R}{L} \quad \text{——} (2)$$

$$I_L + I_1 = I_2 + I_3 \Rightarrow i(t) = C_2 \frac{dv_{c2}}{dt} + \frac{V_{C2}}{R_2}$$

$$\frac{dv_{c2}}{dt} = \frac{i(t)}{C_2} - \frac{V_{C2}}{R_2 C_2} \Rightarrow \dot{x}_3 = \frac{i(t)}{C_2} - \frac{x_3}{R_2 C_2} \; (3)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{c} & 0 \\ \frac{1}{L} & -\frac{R}{L} & 0 \\ 0 & 0 & \frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ 0 \\ \frac{1}{C_2} \end{bmatrix} i(t)$$

Q6

$$\dddot{y}+7\ddot{y}+14\dot{y}+8y=r$$

$$x_1=y+\dot{y} \Longrightarrow \dot{x}_1=\dot{y}+\ddot{y} \Longrightarrow \dot{x}_1=x_2 \quad (1)$$

$$x_2=\dot{y}+\ddot{y} \Longrightarrow \dot{x}_2=\ddot{y}+\dddot{y}$$

من المعادلة

$$x_3=y$$

$$\dddot{y}+7\ddot{y}+14\dot{y}+8y=r$$

$$\dddot{y}=r-7\ddot{y}-14\dot{y}-8y$$

$$\dot{x}_2=\ddot{y}+r-7\ddot{y}-14\dot{y}-8y$$

$$=r-6\ddot{y}-14\dot{y}-8y$$

$$x_1=y+\dot{y}$$

$$x_1=x_3+\dot{y} \Longrightarrow \boxed{\dot{y}=x_1-x_3}$$

$$\boxed{y=x_3}$$

$$x_2=\dot{y}+\ddot{y} \Longrightarrow \ddot{y}=x_2-\dot{y}$$

$$\boxed{\ddot{y}=x_2-x_1+x_3}$$

$$\dot{x}_2=r-6(x_2-x_1+x_3)-14(x_1-x_3)-8x_3$$

$$=r-6x_2+6x_1-6x_3-14x_1+14x_3-8x_3$$

$$\dot{x}_2=r-8x_1-6x_2 \quad (2)$$

$$x_3=y \Longrightarrow \dot{x}_3=\dot{y} \Longrightarrow \dot{x}_3=x_1-x_3 \quad (3)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3\end{array} \\ \begin{bmatrix}0&1&0\\ -8&-6&0\\ 1&0&-1\end{bmatrix}\end{array} \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix} r$$

$$Y = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Q7\ $\mathcal{L}\left[\dddot{y}+2\ddot{y}+5\dot{y}+y=3\dot{r}+r\right]$

$$s^3Y+2s^2Y+5sY+Y=3sr+r$$

$$Y_{(s)}(s^3+2s^2+5s+1)=r_{(s)}(3s+1)$$

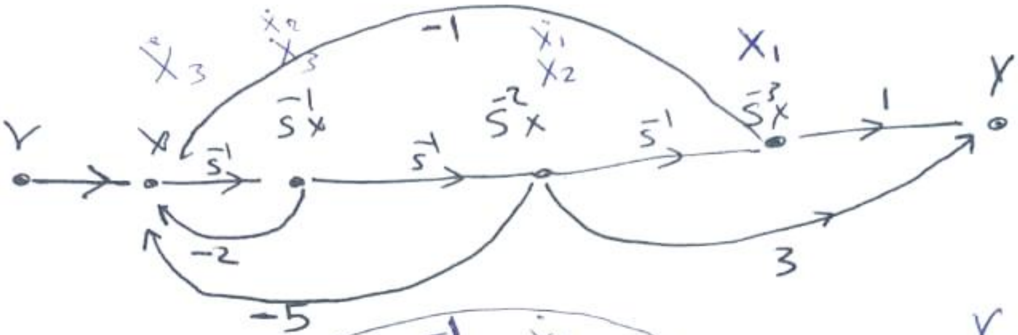
$$\frac{Y(s)}{r(s)}=\left.\frac{3s+1}{s^3+2s^2+5s+1}\right]\frac{s^{-3}}{s^{-3}}$$

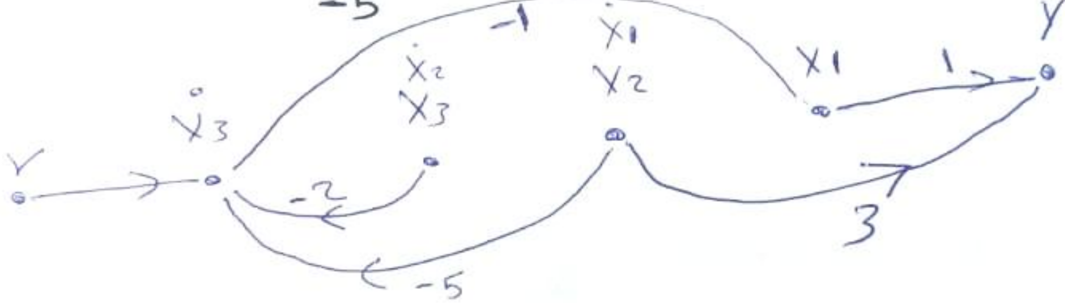
$$\frac{Y}{r}=\frac{3s^{-2}+s^{-3}}{1+2s^{-1}+5s^{-2}+s^{-3}} * \frac{X}{X}$$

$$\frac{Y}{X}=3s^{-2}+s^{-3} \Rightarrow Y(s)=3s^{-2}X(s)+s^{-3}X(s) \text{ — (1)}$$

$$\frac{X(s)}{r(s)}=\frac{1}{1+2s^{-1}+5s^{-2}+s^{-3}} \Rightarrow r(s)=\underline{X(s)}+2s^{-1}X(s)+5s^{-2}X(s)+s^{-3}X(s)$$

$$X(s)=r(s)-2s^{-1}X(s)-5s^{-2}X(s)-s^{-3}X(s) \text{ — (2)}$$





$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -1x_1 - 5x_2 - 2x_3 + r$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -2 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q8\ $\frac{dc_1^2(t)}{dt} + \frac{dc_1(t)}{dt} + 2c_1(t) - 2c_2(t) = r_1(t)$

$\frac{dc_2^2(t)}{dt} + c_2(t) - c_1(t) = r_2(t)$

$x_1 = c_1(t) \Rightarrow \dot{x}_1 = \frac{dc_1}{dt} \Rightarrow \dot{x}_1 = x_2$ — (1)

$x_2 = \frac{dc_1(t)}{dt} \Longrightarrow \dot{x}_2 = \frac{dc_1^2(t)}{dt} \Rightarrow \dot{x}_2 = r_1 + 2x_3 - 2x_1 - x_2$ — (2)

$\frac{dc_1^2}{dt} + \frac{dc_1}{dt} + 2c_1 - 2c_2 = r_1 \Rightarrow \frac{dc_1^2}{dt} = r_1 + 2c_2 - 2c_1 - \frac{dc_1}{dt}$

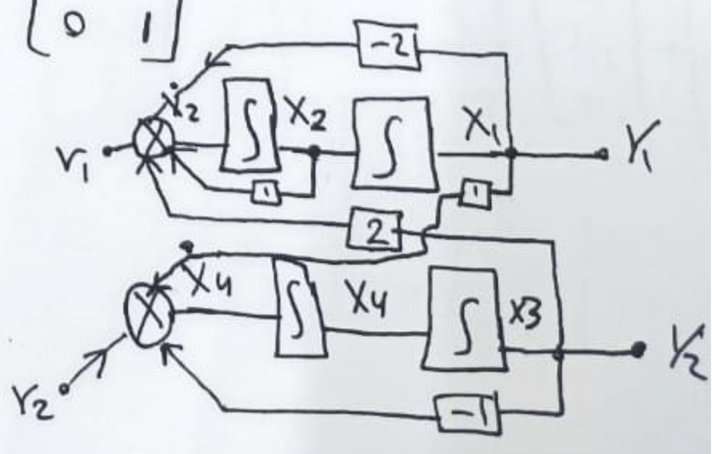
$\dot{x}_2 = r_1 + 2x_3 - 2x_1 - x_2$

$x_3 = c_2(t) \Rightarrow \dot{x}_3 = \frac{dc_2}{dt} \Rightarrow \dot{x}_3 = x_4$ — (3)

$x_4 = \frac{dc_2}{dt} \Rightarrow \cancel{\dot{x}_4} = \frac{dc_2^2}{dt} = r_2 - x_3 + x_1$ — (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$



Q9\

① $\left[\frac{d^2c}{dt^2} + 2\frac{dc}{dt} + c + \int c = r\right]$ Laplace

$$\left[s^2c + 2sc + c + s^{-1}c = r\right] * s$$

$$s^3c + 2s^2c + sc + c = sr \Rightarrow c(s^3 + 2s^2 + s + 1) = sr$$

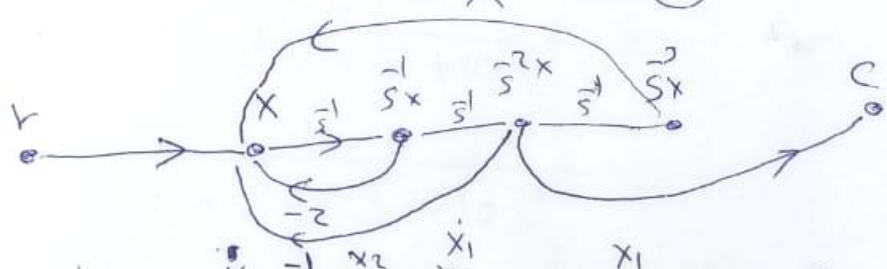
$$\frac{c}{r} = \left[\frac{s}{s^3 + 2s^2 + s + 1}\right]\frac{s^{-3}}{s^{-3}} \Rightarrow \frac{c}{r} = \frac{s^{-2}}{1 + 2s^{-1} + s^{-2} + s^{-3}}$$

$$\frac{c}{r} = \frac{s^{-2}}{1 + 2s^{-1} + s^{-2} + s^{-3}} * \frac{X}{X}$$

$$\frac{X}{r} = \frac{}{1 + 2s^{-1} + s^{-2} + s^{-3}} \Rightarrow r = X + 2s^{-1}X + s^{-2}X + s^{-3}X$$

$$X = r - 2s^{-1}X - s^{-2}X - s^{-3}X \quad \text{—— ①}$$

$$\frac{c}{X} = s^{-2} \Rightarrow c = s^{-2}X \quad \text{—— ②}$$



$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -x_1 - x_2 - 2x_3 + r$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}$$

$$c = \begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

(2)

$$\frac{dc^3}{dt} + 6\frac{dc}{dt} + 5c = 5e$$

$$x_1 = c \Longrightarrow \dot{x}1 = \frac{dc}{dt} \Longrightarrow \dot{x}_1 = x_2 \quad \text{—(1)}$$

$$x_2 = \frac{dc}{dt} \Rightarrow \dot{x}_2 = \frac{dc^2}{dt} \Longrightarrow \dot{x}_2 = x_3 \quad \text{—(2)}$$

$$\dot{x}_3 = \frac{dc^2}{dt} \Rightarrow \dot{x}_3 = \frac{dc^3}{dt} = 5e - 5\dot{x}_1 - 6x_2 \quad \text{—(3)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} e$$

(3)

$$\frac{C}{R} = \frac{2s^2 + 8s + 2}{s^3 + 6s^2 + 12s + 10} * \frac{s^{-3}}{s^{-3}}$$

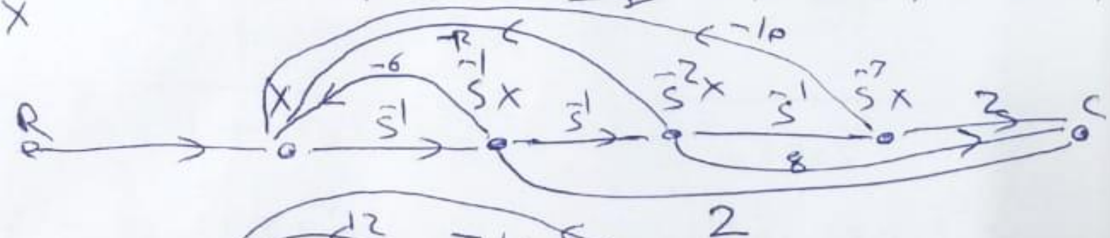
$$\frac{2s^{-1} + 8s^{-2} + 2s^{-3}}{1 + 6s^{-2} + 12s^{-2} + 10s^{-3}} * \frac{X}{X}$$

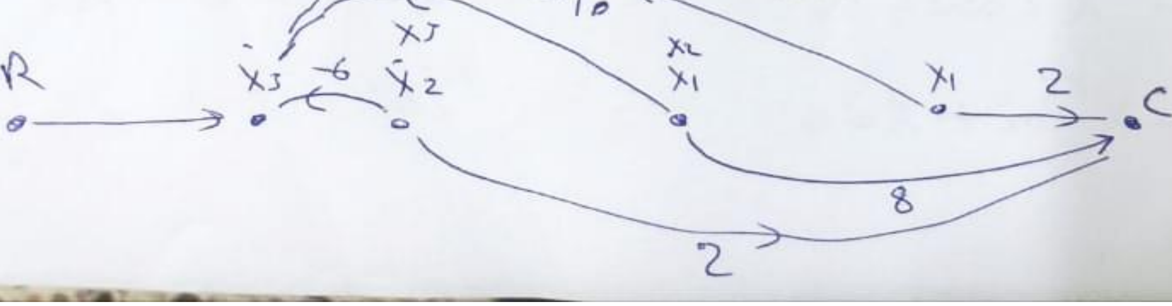
$$\frac{X}{R} = \frac{1}{1 + 6s^{-1} + 12s^{-2} + 10s^{-3}}$$

$$X + 6s^{-1}X + 12s^{-2}X + 10s^{-3}X = R$$

$$X = R - 6s^{-1}X - 12s^{-2}X - 10s^{-3}X \quad \text{—(1)}$$

$$\frac{C}{X} = 2s^{-1} + 8s^{-2} + 2s^{-3} \Longrightarrow C = 2s^{-1}X + 8s^{-2}X + 2s^{-3}X \quad \text{—(2)}$$





$\dot{x}_1 = x_2$ — ①

$\dot{x}_2 = x_3$

$\dot{x}_3 = -10x_1 - 12x_2 - 6x_3 + r$

$c = 2x_1 + 8x_2 + 2x_3$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -12 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$c = \begin{bmatrix} 2 & 8 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

---

(4) $$\frac{Y}{U} = \frac{10(S+3)}{(S+4)^3(S+5)}$$

$$\frac{10S+30}{(S^2+8S+16)(S+4)(S+5}$$

$$\frac{10S+30}{(S^3+12S^2+48S+64)(S+5)}$$

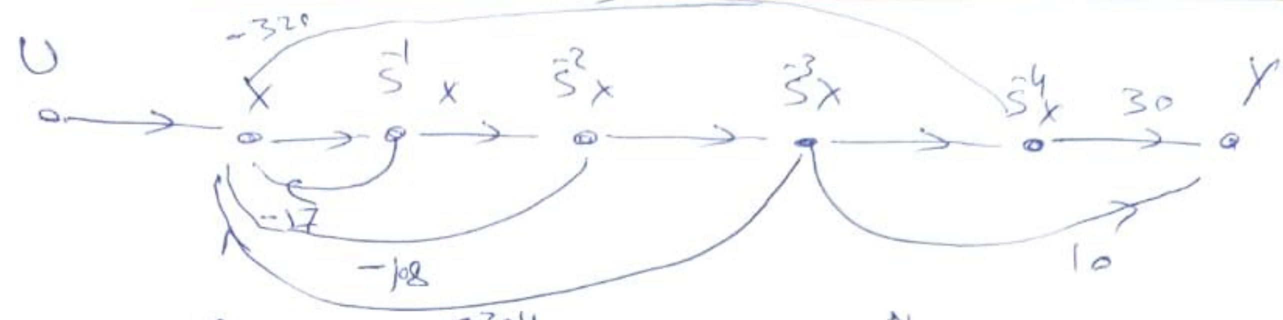
$$\frac{10S+30}{S^4+17S^3+108S^2+304S+320} * \frac{S^{-4}}{S^{-4}}$$

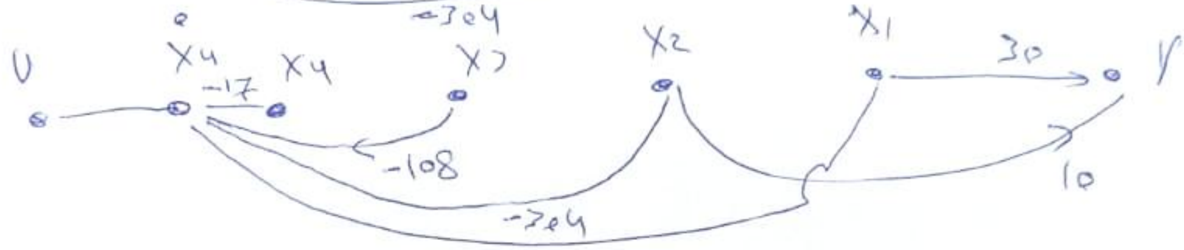
$$\frac{10S^{-3}+30S^{-4}}{1+17S^{-1}+108S^{-2}+304S^{-3}+320S^{-4}} * \frac{X}{X}$$

$$\frac{X}{U} = \frac{1}{1+17S^{-1}+108S^{-2}+304S^{-3}+320S^{-3}}$$

$$X = U - 17S^{-1}X - 108S^{-2}X - 304S^{-3}X - 320S^{-3}X \quad \text{—} ①$$

$$\frac{Y}{X} = 10S^{-3} + 30S^{-4} \Rightarrow Y = 10S^{-3}X + 30S^{-4}X \quad \text{—} ②$$

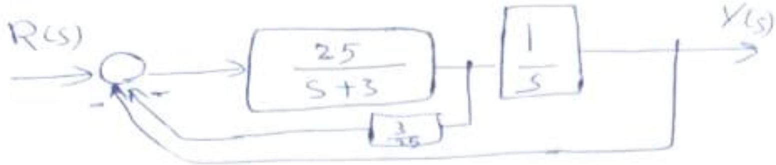




$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} & & & -320 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -320 & -304 & -108 & -17 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 30 & 10 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Q10\



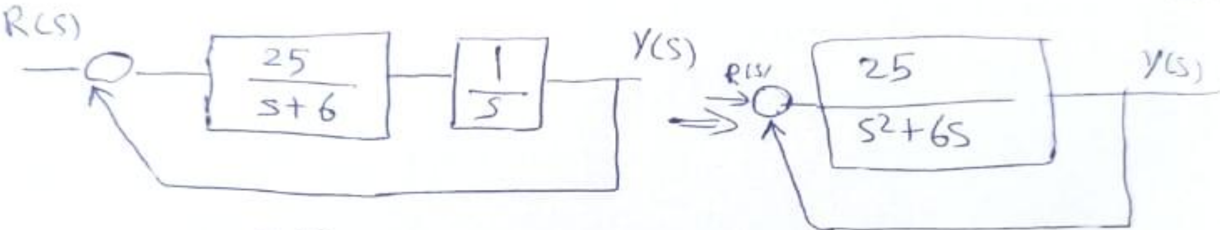
$$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$G(s) = \frac{25}{s+3}$$

$$H(s) = \frac{3}{25}$$

$$\frac{\frac{25}{s+3}}{1+\frac{3}{\cancel{25}} * \frac{\cancel{25}}{(s+3)}} = \frac{\frac{25}{\cancel{(s+3)}}}{\frac{s+3+3}{\cancel{(s+3)}}}$$

$$\frac{25}{s+6}$$



$$\frac{Y(s)}{R(s)} = \frac{\frac{25}{s^2+6s}}{1+\frac{25}{s^2+6s}} = \frac{25}{s^2+6s+25}$$

$$25R(s) = Y(s)\left(s^2+6s+25\right) \Rightarrow 25R(s) = s^2Y(s) + 6sY(s) + 25Y$$

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 25r$$

$$x_1 = y \Rightarrow \dot{x}_1 = \frac{dy}{dt} \Rightarrow \dot{x}_1 = x_2$$

$$x_2 = \frac{dy}{dt} \Rightarrow \dot{x}_2 = \frac{d^2y}{dt^2} = 25r - 25x_1 - 6x_2$$

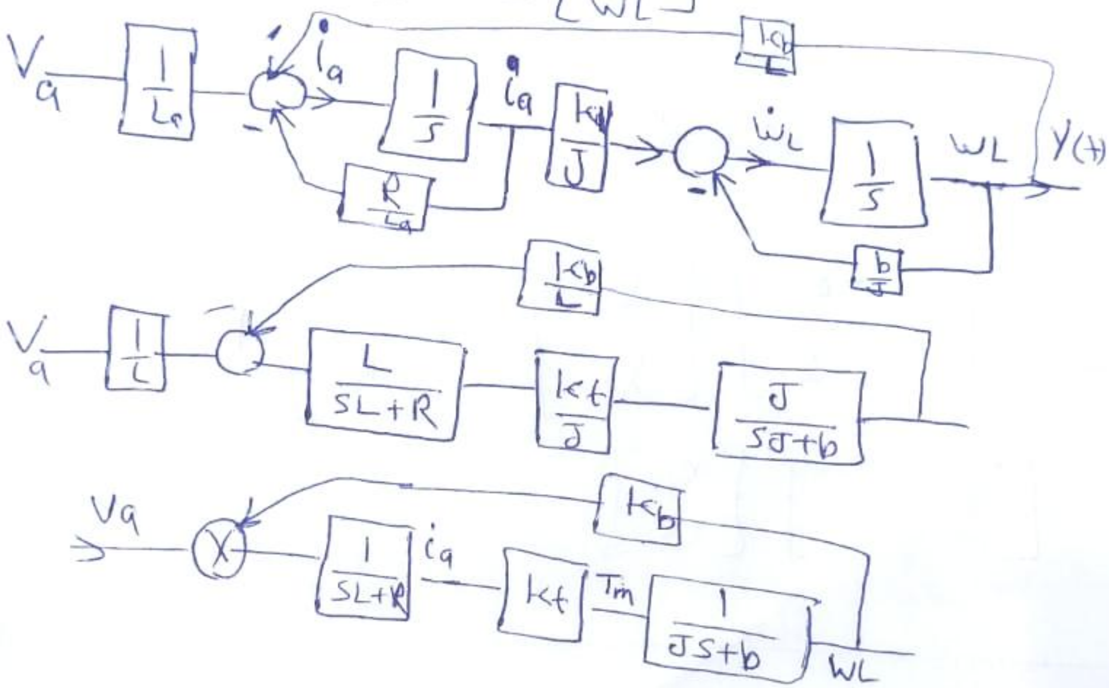
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -25 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 25\end{bmatrix} r$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Q11\

$$\begin{bmatrix}\dot{i}_a\\ \dot{w}_L\end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a}\\ \frac{K_t}{J} & -\frac{b}{J}\end{bmatrix}\begin{bmatrix} i_a\\ w\end{bmatrix} + \begin{bmatrix}\frac{1}{L_a}\\ 0\end{bmatrix} V_a$$

$$Y(t) = \begin{bmatrix} 0 & 1\end{bmatrix}\begin{bmatrix} i_a \\ w_L\end{bmatrix}$$



$$T_m = K_t\, i_a$$