

Zagazig University, Faculty of Engineering Midterm Exam
 Academic Year: 2016/2017
 Specialization: Computer & Systems Eng.
 Course Name: Elective Course (5)
 Course Name: CSE4316:Robotics
 Examiner: Dr. Mohammed Nour



Date: 27/04/2017
 Exam Time: 75 Minutes
 No. of Pages: 7
 No. of Questions: 3
 Full Mark: [40]

- ▷ Please **answer all questions**. Use **3** decimal digits approximation.
- ▷ Mark your **answers** for all questions **in the Answer Sheet** provided.
- ▷ In last page, some supplementary identities you *may* need.

Question 1. [10 Marks]

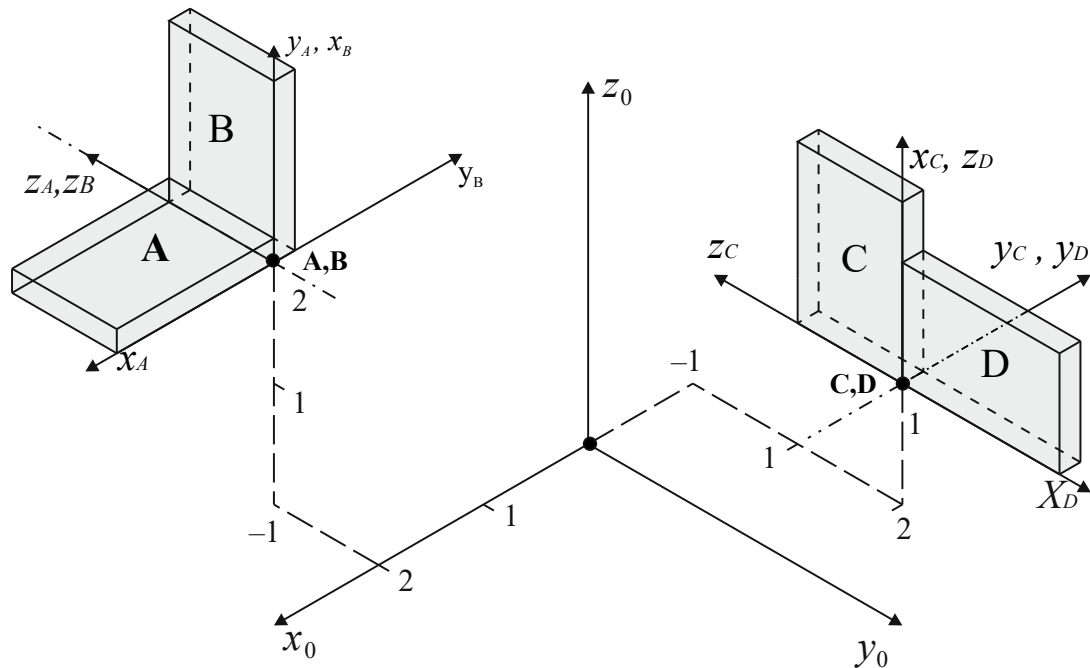
(2 × 5)

1. Which of the following is **false** for a **parallel** manipulator
 - a) it forms a closed-chain.
 - b) Gruebler's formula may be applied to count its DOF
 - c) its configuration space is always planar.
 - d) it is two or more series chains connect the end-effector to the base.
2. Degrees of Freedom of manipulator are Number of
 - a) position variables that have to be specified.
 - b) configuration parameters minimally specified.
 - c) of its joints.
 - d) dimensions of its workspace.
3. One of the robot anthropomorphic characteristics is its
 - a) mechanical arm
 - b) degrees of freedom
 - c) mobility as in rovers
 - d) substitution for humans
4. A suitable robot configuration for high precision pick and place operations is ...
 - a) SCARA
 - b) Articulated-arm
 - c) cylindrical
 - d) Cartesian
5. The world reference frame is
 - a) a universal fixed coordinate frame
 - b) used to specify movements of each individual joint of the robot.
 - c) specifies the movements of the robot tool-tip w.r.t hand frame.
 - d) specifies the movements of work object w.r.t station frame.

Question 2. [15 Marks]

(3 × 5)

Consider the pose of the objects A, B, C and D in space, as shown next:



6. The **relative** rotation matrix that displaces object A into the new pose B is:

- a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. The **relative** transformation that displaces object B into the new pose C is:

- a) $Trans(-1, 3, -3)$ b) $Rot(y_B, 90^\circ) Trans(-1, 2, 1)$
c) $Trans(-1, 2, 1)$ d) $Rot(y_0, -90^\circ) Trans(2, -1, 1)$

8. The homogeneous transformation that relates frame A to frame 0 (i.e. H_A^0) is:

- a) $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

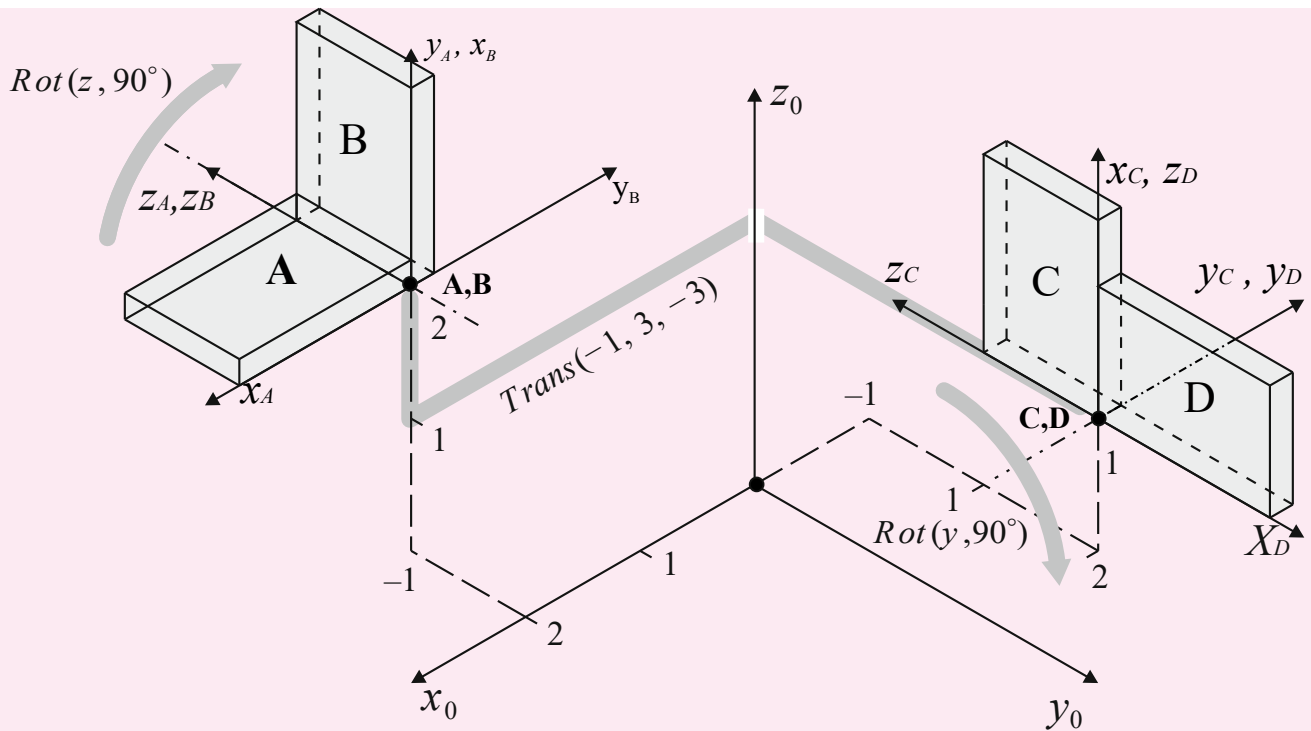
9. The homogeneous transformation that relates frame D to frame A (i.e. H_D^A) is:

- a) $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 2 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

10. The frame transformation H_D^A can be expressed as:

- a) $\begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Solution



to rotate the coordinate frame $\{A\}$ for 90° in the counter-clockwise direction around the z -axis. This can be achieved by:

$$R_B^A = Rot(z_A, 90^\circ) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The displacement resulted in a new pose of the object and new frame $\{x_B, y_B, z_B\}$ shown in Figure. We shall displace this new frame for -1 along the x_B axis, 3 units along y_B axis and -3 along z_B axis, so:

$$T_2 = Trans(-1, 3, -3)$$

This frame will be finally rotated for 90° around the y_C axis in the positive direction

$$R_3 = Rot(y_C, 90^\circ) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$H_A^0 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

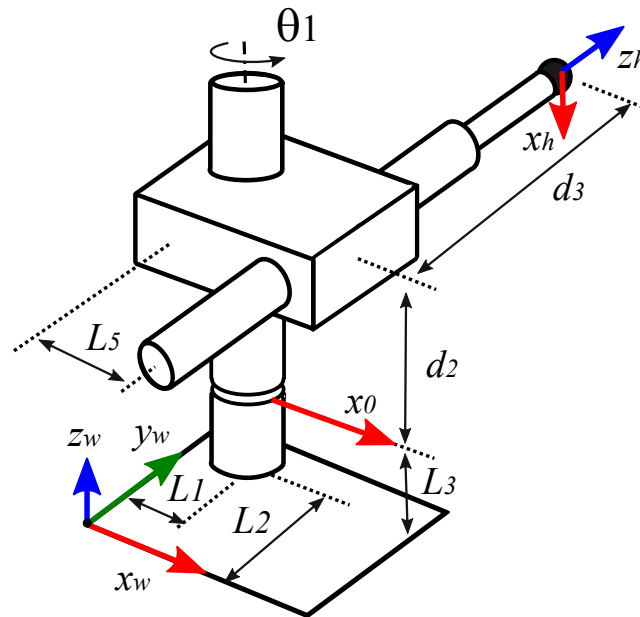
$$\begin{aligned} H_D^A &= R_1 T_2 R_3 = Rot(z_A, 90^\circ) Trans(-1, 3, -3) Rot(y_C, 90^\circ) \\ &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$H_D^0 = H_A^0 H_D^A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

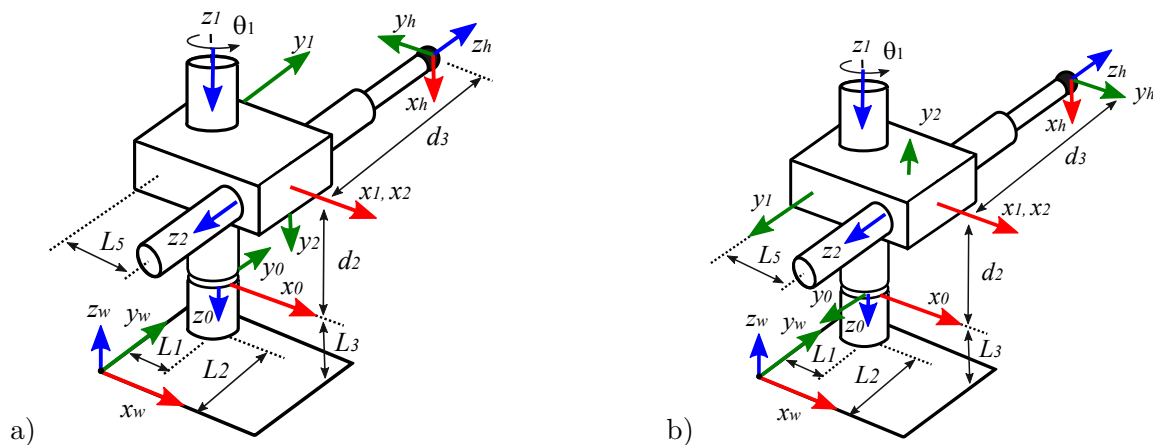
Question 3. [15 Marks]

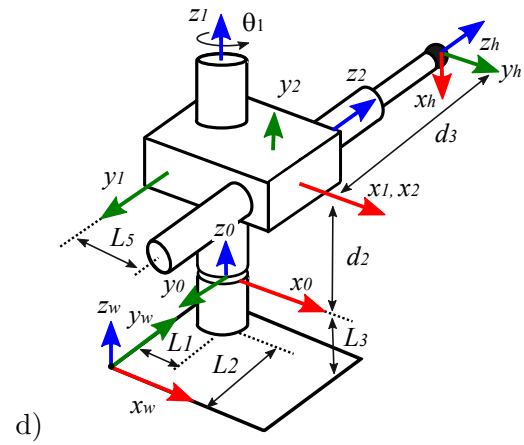
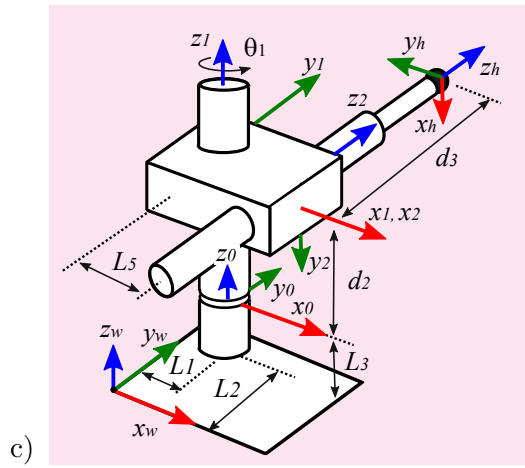
(2 + 3 + 2 + 4 + 4)

Consider the **R2P** robot manipulator mechanism shown next with $\{w\}$, $\{0\}$ and $\{h\}$ as the world, base and tool frame, respectively:



11. According to the DH conventions, we can assign frames to the joints as:





12. The DH parameters of the **spherical** wrist joints:

a)

Link	a_i	α_i	d_i	θ_i
1	L_2	0	0	θ_1^*
2	L_5	0	d_2^*	0
3	0	0	d_3^*	0

c)

Link	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1^*
2	0	0	d_2^*	90
3	L_3	0	d_3^*	0

b)

Link	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1^*
2	L_5	90	d_2^*	0
3	0	0	d_3^*	-90

d)

Link	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1^*
2	L_5	-90	d_2^*	0
3	0	0	d_3^*	90

13. the homogeneous transformation matrix A_2 is found as:

a)
$$\begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 0 & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)
$$\begin{bmatrix} 0 & -1 & 0 & L_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & L_3 \\ 0 & 1 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

14. The robot tool-tip coordinates can be expressed in the base frame using T_h^0 that is calculated as:

a)
$$\begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_3 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_3 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 0 & c_1 & s_1 & -s_1 d_3 + c_1 \\ 0 & -s_1 & -c_1 & c_1 d_3 + s_1 \\ 1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)
$$\begin{bmatrix} 0 & s_1 & c_1 & c_1 d_3 + s_1 + L_1 \\ 0 & -c_1 & -s_1 & s_1 d_3 + c_1 + L_2 \\ -1 & 0 & 0 & d_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d)
$$\begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

15. At the configuration $\mathbf{q} = [0^\circ, L_4, L_6]^T$, the hand **pose** at this configuration is calculated as:

a)
$$\begin{bmatrix} 0 & 1 & 0 & L_1 \\ 0 & 0 & -1 & L_6 \\ -1 & 0 & 0 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

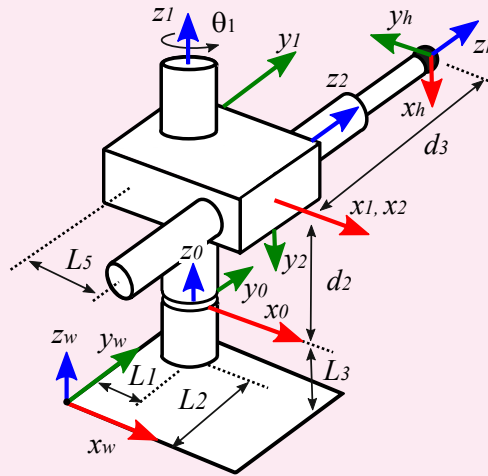
b)
$$\begin{bmatrix} 0 & 1 & 0 & L_3 + L_4 \\ 0 & 0 & 1 & L_2 + L_5 \\ 1 & 0 & 0 & L_1 + L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)
$$\begin{bmatrix} 0 & -1 & 0 & L_3 \\ 0 & 0 & 1 & L_2 \\ -1 & 0 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d)
$$\begin{bmatrix} 0 & -1 & 0 & L_5 + L_1 \\ 0 & 0 & 1 & L_6 + L_2 \\ -1 & 0 & 0 & L_4 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

According to the DH conventions, we can assign frames to all joints as:



The DH parameters of the robot joints:

Link	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1^*
2	L_5	-90	d_2^*	0
3	0	0	d_3^*	90

The matrices describing the relative poses of the neighboring coordinate frames:

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The geometric model of the robot arm is represented by the product of first three matrices:

$$H_3^0 = A_1 A_2 A_3 = \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that $T_h^0 = H_3^0$ i.e. this transformation expresses the robot tool tip coordinates relative to the robot base frame.

The homogeneous transformation from the robot tool tip to the world frame:

Note that the transformation from the robot base frame to the world frame is a pure translation $Trans(L_1, L_2, L_3)$.

Therefore we can get T_h^w by **simple addition** of the translation vector $[L_1 \ L_2 \ L_3 \ 1]^T$ to the last column of H_3^0 :

$$T_h^w = \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 + L_1 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 + L_2 \\ -1 & 0 & 0 & d_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Another way to **systematically** calculated the homogeneous transformation from the robot tool tip

to the world frame as:

$$T_h^w = H_0^w H_3^0 = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -c_1 & -s_1 & -s_1 d_3 + c_1 L_5 + L_1 \\ 0 & s_1 & c_1 & c_1 d_3 + s_1 L_5 + L_2 \\ -1 & 0 & 0 & d_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The given configuration $\mathbf{q} = [0, L_4, L_6]^T$ corresponds to $\mathbf{q} = [\theta_1, d_2, d_3]^T$. So, the hand pose at this configuration is calculated by direct substitution of q in T_h^w :

$$T_h^w = \begin{bmatrix} 0 & -1 & 0 & L_5 + L_1 \\ 0 & 0 & 1 & L_6 + L_2 \\ -1 & 0 & 0 & L_4 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Supplementary Material

Note: you *may* need some or none of these identities:

$$R_Z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_Y = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}, \quad R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$A_i = \begin{bmatrix} \cos \theta_i & -\cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \alpha_i \cos \theta_i & -\sin \alpha_i \cos \theta_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(a \pm b) = \cos(a) \cos(b) \mp \sin(a) \sin(b) \quad \sin(\theta) = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ)$$

$$\sin(a \pm b) = \sin(a) \cos(b) \pm \cos(a) \sin(b) \quad \cos(\theta) = \cos(-\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ)$$

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

if $\cos(\theta) = b$, **then** $\theta = \text{atan2}(\pm\sqrt{1-b^2}, b)$

For a triangle: $A^2 = B^2 + C^2 - 2BC \cos(a)$,

if $\sin(\theta) = b$, **then** $\theta = \text{atan2}(b, \pm\sqrt{1-b^2})$

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$$

if $a \cos(\theta) + b \sin(\theta) = c$, **then** $\theta = \text{atan2}(b, a) + \text{atan2}(\pm\sqrt{a^2 + b^2 - c^2}, c)$

if $a \cos(\theta) - b \sin(\theta) = 0$, **then** $\theta = \text{atan2}(a, b) + \text{atan2}(-a, -b)$