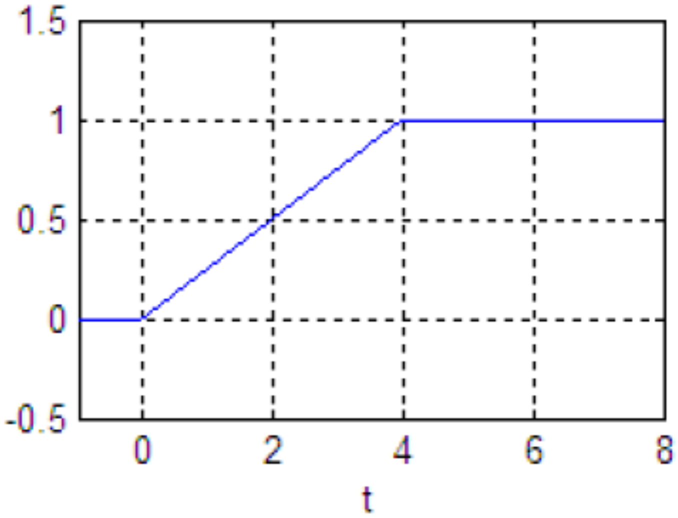
CSE421: Digital Control

Assignment 2

The z-Transform

Q1. Obtain the z-transform for the following curve (assume T=1):



Q2. A function y(t) = 2sin(4t) is sampled every T = 0.1 s. Find the z-transform of the resultant number sequence.

Q3. Find the z-transform of the following function, assuming that T = 0.5 sec:

$$(a)\quad Y(s)=\frac{1}{s^2(s+1)},\qquad (b)\quad Y(s)=\frac{1}{s^2},$$

$$(c)\quad Y(s)=\frac{e^{-Ts}}{s(s+1)},\qquad (d)\quad Y(s)=\frac{(s+3)}{(s+1)(s+2)},$$

$$(e)\quad Y(s)=\frac{(s+1)}{s(s+2)},\qquad (f)\quad Y(s)=\frac{s}{(s+1)^2}$$

Check your answer using the tables of z-transform.

Q4. Determine the z-transform of the following time domain functions.

$$(a) \quad x(k) = k$$

$$(b) \quad x(k) = k^2$$

$$(c) \quad x(t) = 1 - e^{-at}$$

$$(d) \quad x(t) = te^{-at}$$

Hint: you can check your answer with MATLAB command **ztrans**. For example, we can solve (d) using the following commands:

```
>> syms T a k z;
>> xk=(k*T)*(exp(-a*k*T));
>> xz=ztrans(xk,k,z)
```

Q5. For the discrete transfer function G(z) below:

$$G(z) = \frac{1}{z^2 - 0.5z + 0.5}$$

Find:

(a) The unit pulse response.
(b) The unit step response. Verify the DC gain.

Q6. Determine the final value of the sequences whose z-transform is:

$$X(z) = \frac{1}{(1 - z^{-1})} - \frac{1}{(1 - e^{aT} z^{-1})}$$

Q7. Find the inverse z-transform of $X(z)$ using both long division and partial fraction methods. Find also the steady state value of $x(n)$.

$$(a) \quad X(z) = \frac{10z + 5}{(z-1)(z-0.2)}, \qquad (b) \quad X(z) = \frac{1}{1+z}$$

$$(c) \quad X(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}, \qquad (d) \quad X(z) = \frac{z+2}{z(z-2)}.$$

$$(e) \quad X(z) = \frac{1 + 2z + 3z^2 + 4z^3 + 5z^4}{z^4}, \qquad (f) \quad X(z) = \frac{10z + 5}{z^2 - 1.2z + 0.2}.$$

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Assignment 3

Discrete Block diagrams

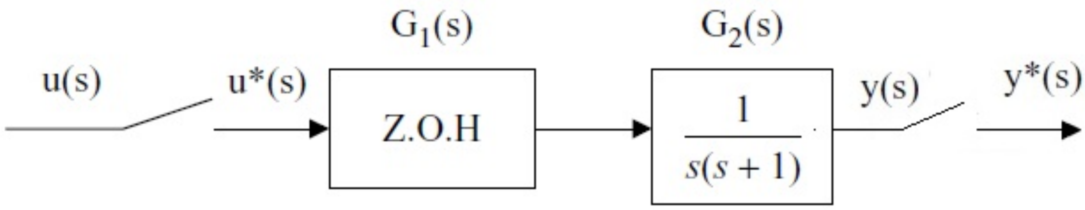
Q1. Assume that the following plant transfer functions are preceded by a zero-order hold. Compute the equivalent discrete transfer function G(z) using a sampling period T = 1. Check you answer with MATLAB.

$$(a)\ G(s) = \frac{1}{s^2}$$

$$(b)\ G(s) = \frac{1}{s(s+1)}$$

$$(c)\ G(s) = \frac{1}{s^2 - 1}$$

Q2. Calculate and plot the pulse response of the following system assuming that the sampling period T = 1sec.



Solution:

The transfer function of the ZOH is

$$G_1(s) = \frac{1-e^{-Ts}}{s},$$

For this system, we can write

$$y(z) = G_1G_2(z)u(z).$$

Now, T=1 and

$$G_1G_2(s) = \frac{1-e^{-Ts}}{s^2(s+1)},$$

Or by partial fractional expansion we can write

$$G_1G_2(s) = (1-e^{-s})\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right).$$

From the z-transform tables

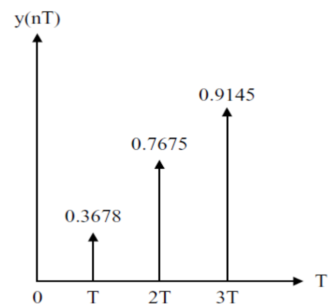
$$G_1G_2(z) = (1-z^{-1})\left(\frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-1}}\right) = \frac{ze^{-1}+1-2e^{-1}}{(z-1)(z-e^{-1})}.$$
$$= \frac{0.3678z+0.2644}{z^2-1.3678z+0.3678}.$$

For a pulse input, u(z) = 1. Therefore the pulse response will be given by

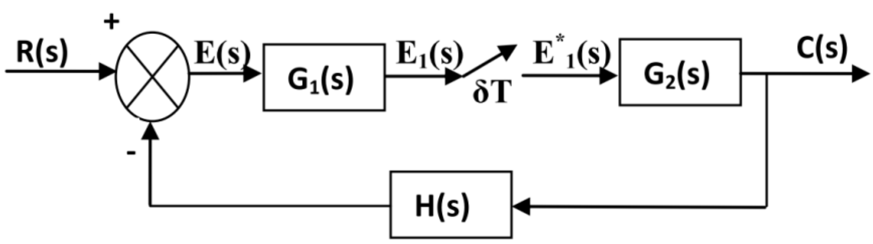
$$y(z) = G_1G_2(z)u(z) = G_1G_2(z) = \frac{0.3678z+0.2644}{z^2-1.3678z+0.3678}.$$

After long division, we obtain the time response

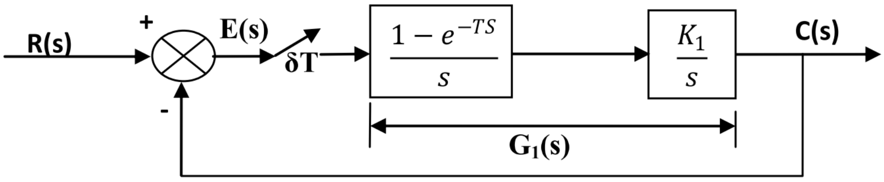
$$y(n) = 0.3678\delta(n-1) + 0.7675\delta(n-2) + 0.9145\delta(n-3) + \cdots$$



Q3. Obtain the output C(z) for the discrete-time control system whose block diagram is shown below:



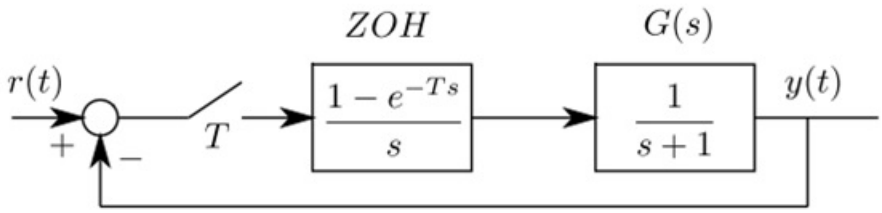
Q4. Obtain the output sequence c(kT) if the input r(t) is a unit step and T= 1sec for the discrete-time control system whose block diagram is shown below:



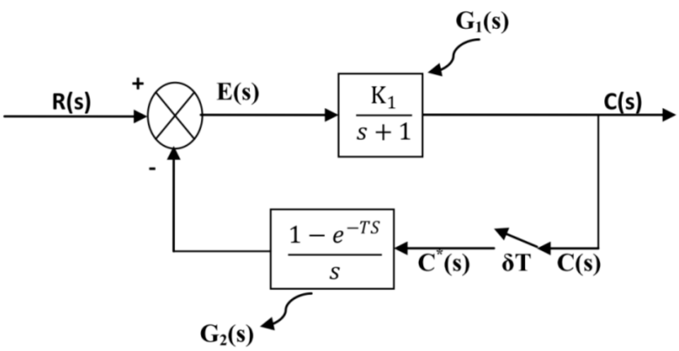
Final answer:

$$c(nT) = 1 - (1 - k_1)^n.$$

Q5. Sketch the step response y(t) of the system shown below for the three samples k = 0, 1, 2. Use sample period T = 1.



Q6. Obtain the output sequence c(kT) if the input r(t) is a unit step, T= 0.2 sec and K_1 = 1 for the discrete-time control system whose block diagram is shown below:



Final answer:

$$c(nT) = \frac{1}{2}[1 - (0.6374)^n] = \{0, 0.181, 0.297, 0.371, 0.418, \ldots\}$$