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Statistics of Diagnostic Tests

Predictive Values

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Introduction

The section on basic principles showed that we can begin to evaluate diagnostic tests by calculating **sensitivity** and **specificity**.

However, these are defined in terms of probabilities of test results, given disease status. We don't know this in practice:

Wish to go Test  Diagnosis...

		Coronary Artery Disease		Total
		Present (D+)	Absent (D-)	
Exercise Tolerance Test	Positive (T+)	815	115	930
	Negative (T-)	208	327	535
Total		1023	442	1465

Main interest may be in Probabilities “other way round”

Positive Predictive Value $P(D+ | T+) = 815 / 930 = 0.88$

Negative Predictive Value $P(D- | T-) = 327 / 535 = 0.61$

NB PPV and NPV affected by prevalence

Original Table

		Coronary Artery Disease		Total
		Present (D+)	Absent (D-)	
Exercise Tolerance Test	Positive (T+)	815	115	930
	Negative (T-)	208	327	535
Total		1023	442	1465

$$\text{Sensitivity} = 815/1023 = 0.80$$

$$\text{Specificity} = 327/442 = 0.74$$

$$\text{PPV} = 815/930 = 0.88$$

$$\text{NPV} = 327/535 = 0.61$$

Table with Prevalence = 0.3

		Coronary Artery Disease		Total
		Present (D+)	Absent (D-)	
Exercise Tolerance Test	Positive (T+)	350	267	617
	Negative (T-)	90	758	848
Total		440	1025	1465

$$\text{Sensitivity} = 350/440 = 0.80$$

$$\text{Specificity} = 758/1025 = 0.74$$

$$\text{PPV} = 350/617 = 0.57$$

$$\text{NPV} = 758/848 = 0.89$$

Since disease **less** prevalent, +ve test less likely to be real disease & -ve test more likely to indicate absence.

Predictive Values

$$\text{PPV} = \frac{\text{sens} \times \text{prev}}{(\text{sens} \times \text{prev} + (1 - \text{spec}) \times (1 - \text{prev}))}$$

$$\text{NPV} = \frac{\text{spec} \times (1 - \text{prev})}{(\text{spec} \times (1 - \text{prev}) + (1 - \text{sens}) \times \text{prev})}$$

sens - sensitivity;

spec - specificity;

prev – prevalence

Therefore, PPV and NPV can't be compared between populations – apply only to current population/prevalence.

Estimating Precision

Sensitivity & specificity are just proportions

Standard methods allow Confidence Intervals to be calculated.

e.g. CAD example above:

Sensitivity = 0.80 (95% CI: 0.77, 0.82)

Specificity = 0.74 (95% CI: 0.70, 0.78)

Similar considerations for predictive values.

Alternative Approach

Assess test by e.g. comparing probabilities of positive test if disease present to those when disease absent...

$$\text{Positive Likelihood Ratio} = \frac{\Pr(T+|D+)}{\Pr(T+|D-)} = \frac{\text{Sensitivity}}{1-\text{Specificity}}$$

LR gives information on value of test when a + diagnosis.

If Pre-test odds of disease = $\frac{\text{Prevalence}}{1-\text{Prevalence}}$, then post-test odds (+ result) are

$$\text{Post-test Odds} = \text{LR} \times \text{Pre-test odds}$$

Can also define Negative Likelihood Ratio for T-

$$\text{Negative Likelihood Ratio} = \frac{\Pr(T-|D+)}{\Pr(T-|D-)} = \frac{1 - \text{Sensitivity}}{\text{Specificity}}$$

So can then also evaluate Post-test odds (for a – result), as before

Good tests will tend to have **large Positive LR** ($\gg 1$) and **small Negative LR** ($\ll 1$)

Example – Screening for Cervical Cancer

- Carrying out testing in large “healthy” population
- Disease is (relatively) rare (i.e. low prevalence)
- Initially, want high sensitivity & NPV – mainly wish to avoid false negatives
- False positives can be identified at 2nd stage – high specificity & PPV test