Joint User / Access Point Selection and Combining in Cell-Free Networks

Abstract

In this paper, the uplink communications in cell-free networks is considered. Access points are connected to a network controller via backhaul links. At the network controller, users' signals are decoded through the combiners that are designed by a sparsity solver known as sparse group lasso. Sparse group lasso does selection both at the group (i.e., user) and within the group (i.e., access point) levels. Numerical results indicate that the proposed solution achieves the best sum-rate performance as the network and selection sizes are increased.

I. Introduction

Notations: Throughout the paper, $(.)^T$, $(.)^H$, $(.)^{-1}$, and tr$\{.\}$ denote the transpose, conjugate transpose, inverse, and trace operations of a matrix, respectively. $\|.\|_p$ and $\|.\|_F$ denote the ℓ_p vector and Frobenius matrix norms, respectively. a_{ij} is the entry in the i^{th} row and the j^{th} column of the matrix $\mathbf{A}$. $\mathcal{CN}(0,x)$ denotes the complex Gaussian distribution with zero mean and variance x. $E\{.\}$ and $|.|$ are the expectation and absolute value operators, respectively.

II. System Model

We consider L single-antenna access points (APs) and K single-antenna users. The received signal at AP l is given as

$$y_l = \sqrt{p}\sum_{k=1}^{K} h_{lk}s_k + n_l, \tag{1}$$

where p is the transmit power of a user, $h_{lk} \sim \mathcal{CN}(0,1)$ is the channel between user k and AP l, $s_k \sim \mathcal{CN}(0,1)$ is the transmitted symbol from user k, and $n_l \sim \mathcal{CN}(0,\sigma^2)$ is the noise at AP l. Hence, the received signal at all APs is given as

$$\mathbf{y} = \sqrt{p}\mathbf{H}\mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = [y_1 \dots y_L]^T$, $\mathbf{s}$ and $\mathbf{n}$ are defined similarly, and the channel matrix between the users and APs is given as

$$\mathbf{H} = \begin{bmatrix} h_{11} & \dots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{L1} & \dots & h_{LK} \end{bmatrix}. \tag{3}$$

At the network controller (NC), the received signals from the users are combined as follows

$$\mathbf{V}^H\mathbf{y}, \tag{4}$$

where $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_K] \in \mathbb{C}^{L\times K}$ is the combining matrix whose k^{th} column decodes the symbol of user k. A possible combining matrix can be given as a sum mean-square error (SMSE) minimizer

$$\arg\min_{\mathbf{V}} \ \varepsilon = E\{\|\mathbf{V}^H\mathbf{y} - \mathbf{s}\|_2^2\}. \tag{5}$$

The solution to (5) is known as the minimum mean-square error (MMSE) receiver where the vector filters for all APs and users contribute to minimize the SMSE although their contributions can be small. Sparsity inducing terms can be included in (5) to silence APs and users. Hence, the backhaul traffic load can be significantly reduced as well as the total network power consumption and computational complexity.

ε in (5) can be further rewritten as

$$\begin{aligned} \varepsilon =& E\{\|\sqrt{p}\mathbf{V}^H\mathbf{H}\mathbf{s} - \mathbf{s}\|_2^2\} + E\{\|\mathbf{V}^H\mathbf{n}\|_2^2\} \\ =& p\,\text{tr}\{\mathbf{Q}^H\mathbf{Q}\} + \sigma^2\text{tr}\{\mathbf{V}^H\mathbf{V}\}, \end{aligned} \tag{6}$$

where $\mathbf{Q} = \mathbf{V}^H\mathbf{H} - \mathbf{I}$.

Assuming maximum M out of K users are selected at the NC, the combining matrix can be further constrained with the inequality

$$\|\mathbf{V}\|_{2,0} \leq M, \tag{7}$$

where $\ell_{p,q}$ norm is defined as

$$\|\mathbf{V}\|_{p,q} = \left[\sum_{k=1}^{K}(\|\mathbf{v}_k\|_p)^q\right]^{(1/q)}. \tag{8}$$

Relaxing the ℓ_0 norm in (7) with the ℓ_1 norm, the joint user selection and combining problem can be given in an unconstrained form as

$$\arg\min_{\mathbf{V}} \ p\|\mathbf{V}^H\mathbf{H} - \mathbf{I}\|_F^2 + \sigma^2\|\mathbf{V}\|_F^2 + \mu\|\mathbf{V}\|_{2,1}, \tag{9}$$

where μ is the regularizer.

The third summand in (9) induces sparsity at the group level, i.e., the penalty term enforces some users to be silenced. To induce sparsity within the group level, i.e., the penalty term enforces some APs to be silenced, the conventional lasso penalty can be added to (9)

$$\begin{aligned} \arg\min_{\mathbf{V}} \ & p\|\mathbf{V}^H\mathbf{H} - \mathbf{I}\|_F^2 + \sigma^2\|\mathbf{V}\|_F^2 + \alpha\mu\|\mathbf{V}\|_{2,1} \\ & + (1-\alpha)\mu\sum_{l=1}^{L}\sum_{k=1}^{K}|v_{lk}|, \end{aligned} \tag{10}$$

where $\alpha \in [0,1]$ is for the convex combination of the lasso and group lasso penalties, i.e., $\alpha = 0$ and $\alpha = 1$ give the lasso and group lasso fits, respectively.

III. Numerical Results

In this section, the presented numerical results demonstrate the effectiveness of the proposed sparse group lasso (SGL) solution to reduce the backhaul traffic load at the cost of acceptable loss in sum-rate. We assume log-normal shadowing, distance, penetration loss, and power efficiency vary with uniform distribution between 7-9 dB, 35-50 m, and 20-30 dB, and 5-10$\%$, respectively. Finally, we assume the channel center frequency, bandwidth, noise figure, and transmit power are 2 GHz, 10 MHz, 5 dB, and 20 dBm, respectively. The channel gains are drawn i.i.d. from $\mathcal{CN}(0,1)$ distribution. The number of channel realizations is set to 100.

In Table I, the numerical results for $K=16$ users and $L=32,64,128$ APs are presented for the SGL solver (10) and the MMSE combining matrices. For the results in Table I, the sparsity percentage of the network is set to 30%, i.e., nearly one third of the links in a network is significantly weaker than the other links. For the results, SGL solver parameters in (10) are held fixed.

The sum-rates of SGL and MMSE are given in the 2nd and 4th columns, respectively. Their percentage differences are given in the 5th column. The sparsity percentage of the SGL solver is given in the 3rd column. For instance, for the $L=128$ case, 12% indicates that nearly one tenth of the elements in the $\mathbf{V}$ combining matrix is zero. Thus, the backhaul traffic load is dropped by 88%. Whereas the sum-rate loss is 43%.

As seen in Table I, the chosen SGL solver parameters yield the 30% sparsity only for the $L=64$ case. The SGL solver parameters in (10) can be tuned to achieve 30% sparsity for other cases $L=32,128$ as well. As expected, as the degrees of freedom in the network increase, e.g., L is increased, sparser solutions can be obtained for fixed SGL parameters. While sparser solutions can reduce the backhaul traffic load significantly, the sum-rate loss also increases. However, as the network size is doubled, the sparsity is also nearly halved while the sum-rate loss trend is less than halved when $L=64,128$ cases are compared.

The last column in Table I indicates the decision accuracy of the SGL solver. For instance, the link accuracy would be perfect, i.e., 100%, if the zero elements in $\mathbf{V}$ correspond to the weaker links that make up 30% in the channel matrix $\mathbf{H}$.

Note that the conventional application of sparse solver is to estimate the unknown sparse $\mathbf{X}$. Hence, the known measurement matrix, e.g., $\mathbf{\Phi}$ is not necessarily sparse

$$\mathbf{Y}_{\text{ss}} = \mathbf{\Phi}\mathbf{X}. \tag{11}$$

Here, the ss subindex stands for sparse solver. Whereas, in our work, the known channel matrix $\mathbf{H}$ is sparse and we enforce a sparse $\mathbf{V}$ solution by using the SGL solver

$$\mathbf{Y}_{\text{cf}} = \mathbf{V}^H\mathbf{H}. \tag{12}$$

Here, the cf subindex stands for cell-free network. Note that $\mathbf{Y}_{\text{cf}} = \mathbf{I}$ as seen in (10) for our cell-free network problem.

Moreover, in sparse solver applications, some elements of the unknown sparse $\mathbf{X}$ matrix are exactly zero. Whereas in our work, none of the elements of the known $\mathbf{H}$ matrix are zero but as mentioned earlier, some of them have significantly smaller values, i.e., weaker links, than the others. Due to these deviations from conventional sparse solver applications, the link accuracies are moderate as seen in Table I. However, as mentioned earlier, the SGL application in our work is effective since the traffic load trend is halved while the sum-rate loss trend is less than halved as the network size is doubled.

Table I: SUM-RATE AND OTHER RESULTS OF SGL AND MMSE COMBINING MATRICES.

L	SGL sum-rate	Sparsity $\%$	MMSE sum-rate	Sum-rate Diff. $\%$	Link Acc. $\%$
32	96	64	111	14	59
64	99	31	137	29	49
128	89	12	158	43	38

Nevertheless, if the ergodic sparsity of a given network is known, e.g., we assume 30% for all cases in Table I, the SGL parameters can be tuned so that 30% sparse $\mathbf{V}$ solutions are obtained for all cases. As seen in the $L=64$ case, the backhaul traffic load is dropped by 70% while the sum-rate loss is only 30%.

Note that further benefits can be gained by selecting APs and users in the network, i.e., by silencing some of them. For instance, if a user (an AP) has stronger links only to 10% of the APs (users) in the network, that user (AP) can be silenced. Therefore, not only the backhaul traffic load, but also the total power consumption in the network can be reduced by silencing some users. In Table I, the randomly generated channel matrices $\mathbf{H}$ do not have zero columns or rows, i.e., some columns or rows are all zeros. Thus, there is no silenced AP or user in the network, i.e., $\mathbf{V}$ has no zero columns or rows. As mentioned earlier, further benefits, e.g., reduced total power consumption, can be obtained if the channel matrix $\mathbf{H}$ has zero (i.e., weaker links) or nearly zero columns or rows.

Next, we present the bit error rate (BER) results since the problem objective (5) intends to lower the BER of the network. ...

References