# Joint User / Access Point Selection and Postcoding in Cell-Free Networks

### ABSTRACT

In this paper, the uplink communications in cell-free networks is considered. Access points are connected to a network controller via backhaul links. At the network controller, users' signals are decoded through the postcoders that are designed by a sparsity solver known as sparse group lasso. Sparse group lasso does selection both at the group (i.e., user) and within the group (i.e., access point) levels. Numerical results indicate that the proposed solution achieves the best sum-rate performance as the network and selection sizes are increased.

## I. INTRODUCTION

*Notations*: Throughout the paper,  $(.)^T$ ,  $(.)^H$ ,  $(.)^{-1}$ , and  $\operatorname{tr}\{.\}$  denote the transpose, conjugate transpose, inverse, and trace operations of a matrix, respectively.  $\|.\|_p$  and  $\|.\|_F$  denote the  $\ell_p$  vector and Frobenius matrix norms, respectively.  $\mathcal{CN}(0,x)$  denotes the complex Gaussian distribution with zero mean and variance x.  $E\{.\}$  and |.| are the expectation and absolute value operators, respectively.

# II. SYSTEM MODEL

We consider L single-antenna access points (APs) and K single-antenna users. The received signal at AP l is given as

$$y_l = \sqrt{p} \sum_{k=1}^{K} h_{lk} s_k + n_l,$$
 (1)

where p is the transmit power of a user,  $h_{lk} \sim \mathcal{CN}(0,1)$  is the channel between user k and AP l,  $s_k \sim \mathcal{CN}(0,1)$  is the transmitted symbol from user k, and  $n_l \sim \mathcal{CN}(0,\sigma^2)$  is the noise at AP l. Hence, the received signal at all APs is given as

$$\mathbf{y} = \sqrt{p}\mathbf{H}\mathbf{s} + \mathbf{n},\tag{2}$$

where  $\mathbf{y} = [y_1 \dots y_L]^T$ , s and n are defined similarly, and the channel matrix between the users and APs is given as

$$\mathbf{H} = \begin{bmatrix} h_{11} & \dots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{L1} & \dots & h_{LK} \end{bmatrix}. \tag{3}$$

At the network controller (NC), the received signals from the users are postcoded as follows

$$\mathbf{V}^H \mathbf{v}$$
. (4)

where  $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_K] \in \mathbb{C}^{L \times K}$  is the postcoding matrix whose  $k^{\text{th}}$  column decodes the symbol of user k. A possible postcoding matrix can be given as a sum mean-square error (SMSE) minimizer

$$\underset{\mathbf{V}}{\operatorname{arg\,min}} \ \varepsilon = E\{\|\mathbf{V}^H\mathbf{y} - \mathbf{s}\|_2^2\}. \tag{5}$$

 $\varepsilon$  in (5) can be further rewritten as

$$\varepsilon = E\{\|\sqrt{p}\mathbf{V}^H\mathbf{H}\mathbf{s} - \mathbf{s}\|_2^2\} + E\{\|\mathbf{V}^H\mathbf{n}\|_2^2\}$$
$$= p \operatorname{tr}\{\mathbf{Q}^H\mathbf{Q}\} + \sigma^2 \operatorname{tr}\{\mathbf{V}^H\mathbf{V}\}, \tag{6}$$

where  $\mathbf{Q} = \mathbf{V}^H \mathbf{H} - \mathbf{I}$ .

The NC can be assumed to be power constrained as follows

$$\operatorname{tr}\{\mathbf{V}^{H}\mathbf{V}\} = \|\mathbf{V}\|_{F}^{2} \le P. \tag{7}$$

Assuming maximum M out of K users are selected at the NC, the postcoding matrix can be further constrained with the inequality

$$\|\mathbf{V}\|_{2,0} \le M,\tag{8}$$

where  $\ell_{p,q}$  norm is defined as

$$\|\mathbf{V}\|_{p,q} = \left[\sum_{k=1}^{K} (\|\mathbf{v}_k\|_p)^q\right]^{(1/q)}.$$
 (9)

Relaxing the  $\ell_0$  norm in (8) with the  $\ell_1$  norm, the joint user selection and postcoding problem can be given in an unconstrained form as

$$\underset{\mathbf{V}}{\operatorname{arg\,min}} \ p \|\mathbf{V}^H \mathbf{H} - \mathbf{I}\|_F^2 + \sigma^2 \lambda \|\mathbf{V}\|_F^2 + \mu \|\mathbf{V}\|_{2,1}, \quad (10)$$

where  $\lambda$  and  $\mu$  are the regularizers.

The third summand in (10) induces sparsity at the group level, i.e., the penalty term enforces some users to be silenced. To induce sparsity within the group level, i.e., the penalty term enforces some APs to be silenced, the conventional lasso penalty can be added to (10)

$$\underset{\mathbf{V}}{\operatorname{arg\,min}} \ p \|\mathbf{V}^{H}\mathbf{H} - \mathbf{I}\|_{F}^{2} + \sigma^{2}\lambda \|\mathbf{V}\|_{F}^{2} + \alpha\mu \|\mathbf{V}\|_{2,1}$$
$$+ (1 - \alpha)\mu \|\mathbf{V}\|_{F}, \tag{11}$$

where  $\alpha \in [0,1]$  is for the convex combination of the lasso and group lasso penalties, i.e.,  $\alpha = 0$  and  $\alpha = 1$  give the lasso and group lasso fits, respectively.

### REFERENCES