Quarter wave plates and their potential to increase the accuracy of the Foucault determination of the speed of light.

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The aim of this paper is to demonstrate how the Foucault method can be improved with the use of modern equipment and therefore, a respectable value of c can be obtained in an undergraduate laboratory. To demonstrate this: Two iterations of the Foucault method were carried out, one using an almost identical method to Foucault's in 1850 without a water tube. The second with the addition of a quarter-wave plate. Without the quarter-wave-plate, the best value of c was $1.95 \times 10^8 \pm 1 \times 10^7 ms^{-1}$ and with the quarter wave plate c was found to be $2.56 \times 10^8 \pm 5 \times 10^7 ms^{-1}$.

INTRODUCTION

The nature of light and the implications it has on how we perceive the universe has always been a predominant field of physics. The most profound question for a long period was whether light travelled as a series of particles or as a wave. For example: In 1672, Newton published a paper outlying his investigations into the nature of light, concluding it consisted of particles. It was a great elaboration of the corpuscular theory Descartes originally introduced [1]. This theory contains multiple holes. One of which was the inability to correctly describe what happens to the speed of light when it moves from a less to more optically dense medium. In 1690, Christiaan Huygens suggested a wave model of light [1] and later in 1801, Thomas Young illustrated this wave nature of light using the double slit experiment [2]. Yet, due to the success of Newtons Principia Mathematica, the corpuscular theory of light remained in use.

One implication of Huygens' theory was that light would travel slower in a more optically dense medium. Subsequently, investigations into the speed of light began. Fizeau and Foucault set out to observe its speed but unfortunately ended up parting ways and taking the task on themselves. [3] Foucault carried out two very similar speed of light measurements over the course of his carrier. Both of which utilizing a rotating mirror. The first, in 1850 with a direct intention to finalize the nature of light debate. The second, in 1862, to obtain an accurate value for the speed of light [1]. His final value was $2.98 \times 10^8 ms^{-1}$ [4]. The first method (1850) can be carried out with the correct equipment in a basic laboratory. Unlike the second, which took place in an observatory [1]. In which case it is possible to access the accuracy of Foucault's ingenious method and investigate the improvements modern equipment can make. In the case of this paper, demonstrating the way in which a quarter wave plate can manipulate light for an experimental advantage.

It is important to overview our current understanding of light. In 1905, Einstein published his theory of special relativity. The theory contained two postulates. One of which was light travels at a constant speed c, regardless of the speed of the source [6]. In 1983, an international agreement defined an exact value for c, $299,792,458 ms^{-1}$ [5]. Einstein also, along with many others, played a huge role settling the wave-particle debate. It was found that light exhibited a phenomenon now known as wave-particle duality [7].

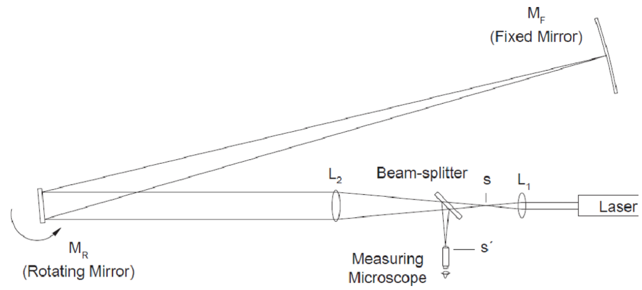

FIG. 1. The Apparatus used by Foucault in his determination of the speed of light. M_R is rotated by a motor, L_1, L_2 are lenses where s is the focus point of L_1 s' is a second focal point when laser beam passes through the beam splitter.

[8]

THEORY

Figure 1 shows the apparatus used by Foucault. The laser beam will pass through L_1, focused on point s then passed through a beam splitter towards L_2. The light strikes M_R as it rotates. Therefore, when returning from fixed mirror M_F, M_R has rotated by an angle of $\delta\theta$. If M_R was fixed, when returning to the beam-splitter the light would focus on a point s'. Since this is not the case and M_R is in fact rotating, the focus point is $(s' + \delta s')$. $\delta s'$ can be related to $\delta\theta$ by the following equation:

$$\delta s' = \frac{2DA\delta\theta}{B + D}. \tag{1}$$

Where A is the distance between L_2 and L_1 minus the focal length of L_1, B Is the distance between L_2 and M_R, D the distance between M_R and M_F.

We can also say that $\delta\theta$ can be described by:

$$\delta\theta = \frac{2D\omega}{c}. \tag{2}$$

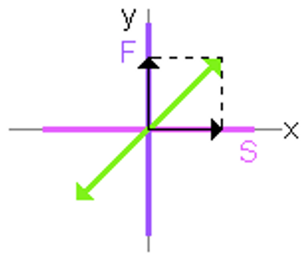

FIG. 2. A front facing view of a quarter-wave plate, where the pink and purple lines represent the slow and fast axis $\hat{\boldsymbol{s}}, \hat{\boldsymbol{f}}$ of the wave plate. The green line represents the axis of polarisation.

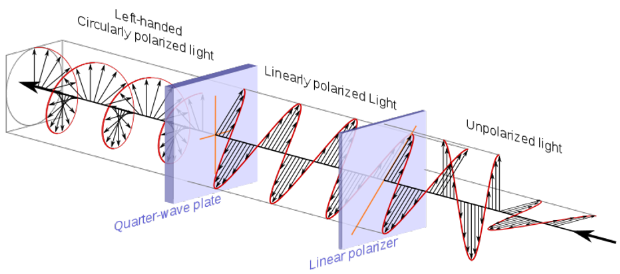

FIG. 3. A visual representation of the nature of light as it passes from a linearly polarised state to a circular polarised state through a quarter-wave plate.

Where c is the speed of light, D is the same as Eq. 1 and w is the rotational frequency of M_R . A combination of equations 1 and 2 will produce a relationship between $\delta s'$ and c:

$$\delta s' = \frac{4AD^2\omega}{c(B+D)} \tag{3}$$

The Quarter-Wave Plate

Quarter-wave plates are often made from crystals and polarize incoming light by an average phase shift. A wave plate will have a fast and slow axis. When the wave plate has an axis of polarisation of 45^o shown in Figure 2.

The electric field of the travelling EM radiation is such that Electric field strength E_f in the fast direction = E_s in the slow direction. Consequently, the wave can be expressed by:

$$f(z,t) = E(\hat{\boldsymbol{f}} + i\hat{\boldsymbol{s}})e^{i(kz-\omega t)} \tag{4}$$

k is wave number, z is location across the z axis, ω is angular frequency. E is electric field strength of incoming EM wave. f and s are the directional vectors for the fast and slow axis (figure 2) respectfully. A wave polarised in this way is known as circularly polarised and is visually presented by figure 3. [9]

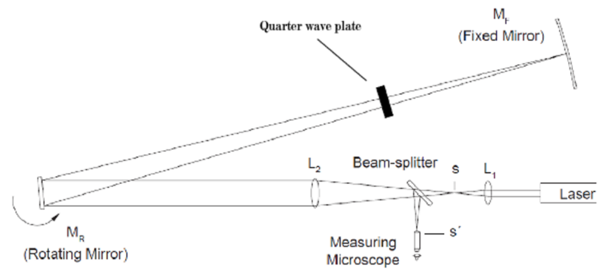

FIG. 4. The exact same as figure 1 with the addition of a quarter wave plate between M_R and M_F.

EXPERIMENTAL DETAILS

Using Eq.3, c can be found from five measurable quantities: A, B, D, ω and $\delta s'$. Once the apparatus as presented in Figure 1 is set up, both A and B can be measured using a meter rule and D, a larger distance, can be measured using a tape measure. These three distances are kept constant throughout.

Eq. 3 presents a linear relationship between $\delta s'$ and ω in which the gradient is described by:

$$m = \frac{4AD^2}{c(B+D)} \tag{5}$$

Since A, B and D are kept constant, plotting ω against $\delta s'$ followed by solving Eq. 5 will leave us with a final measured value of c.

For figure 1 to be set up correctly, the laser beam must be accurately aligned while rotational frequency of M_R (ω) is 0. Once complete the beam will be visible through the measuring microscope. The beam is then brought into focus and a measurement of the beam's location (s') is taken.

The motor can then be switched on; it is warmed up before taking measurements. During the warm-up period a range of values of ω can be recorded and used to estimate error in ω ($\Delta\omega$). A similar method can be used to estimate $\Delta s'$.

Finally, the readings for a linear relationship to be produced can be taken. ω is slowly increased from 100–1000 revolutions per second, recording the point $(s' + \delta s')$ using the measuring microscope. In this case, measurements were taken with both the Vernier and Digital scales. $\delta s'$ against ω can then be plotted, hence c can be calculated (see Eq. 5).

This exact method is repeated, using a quarter wave plate placed between M_R and M_F as shown in Figure 4.

RESULTS

The results for A, B and D were the following: $A = 0.26 \pm 0.001m$, $B = 0.496 \pm 0.001$, $D = 10.2 \pm 0.002m$. In each case error was estimate as the divisions of each measuring device. Three data-sets of deflection ($\delta s'$) were collected for varying values of rotational frequency (ω). Figures 5 and

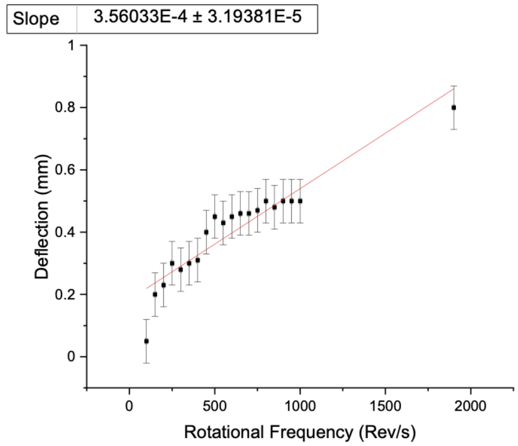

FIG. 5. A graph of deflection $\delta s'$ against revolutions per second ω, recorded using a apparatus shown in figure 1 and the measuring microscopes vernier scale

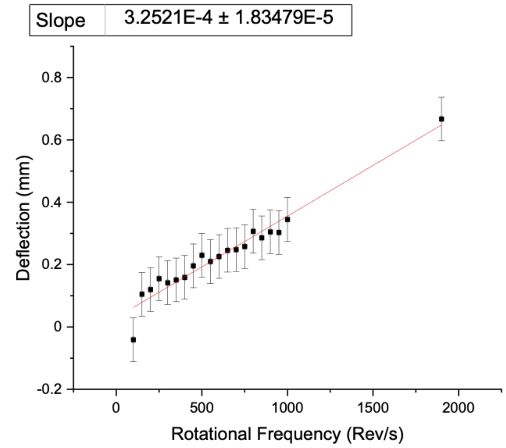

FIG. 6. A graph of deflection $\delta s'$ against revolutions per second ω, recorded using a apparatus shown in figure 1 and the measuring microscopes digital scale

6 show the data collected using the microscopes vernier scale and digital scale respectfully. Both of which are results obtained without a quarter wave plate in place.

Figure 7 is results obtained with the quarter wave plate in place. It is quite noticeable that this data-set is considerably smaller, this is due to the time available with which equipment was accessible. Figure 7 has exactly $\frac{1}{4}$ of the data sets than in each of Figures 5 and 6. Another noticeable differentiation between the data-sets is their maximum values. Where the maximum ω is at approximately 1900 revolutions per second. This is where the motor has been increased to its maximum power output. Again, such a result is not in Figure 7 for the same reason.

All 3 graphs visibly present a relatively linear relationship as expected theoretically (see Eq. 3). Therefore, the results are fitted with a line of best fit. This is the red line in each graph. The gradient of each graph is presented above labelled 'slope'. With the re-arrangement of Eq. 5 for c, the following values of c were obtained. Without a QWP (quarter-wave plate) using the Vernier scale (Figure 5) c was found to be $1.78 \times 10^8 \pm 2 \times 10^7 ms^{-1}$. Without a QWP, using the Digital scale $c = 1.95 \times 10^8 \pm 1 \times 10^7$. Using the QWP along the digital scale, $c = 2.56 \times 10^8 \pm 5 \times 10^7$

Error Analysis

It is important to quickly overview how error was obtained for c,$\delta s'$ and ω. In the case of, $\delta s'$ and ω both used a method taking a small 'sample' data-set and finding a standard deviation with the use of:

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N}} \tag{6}$$

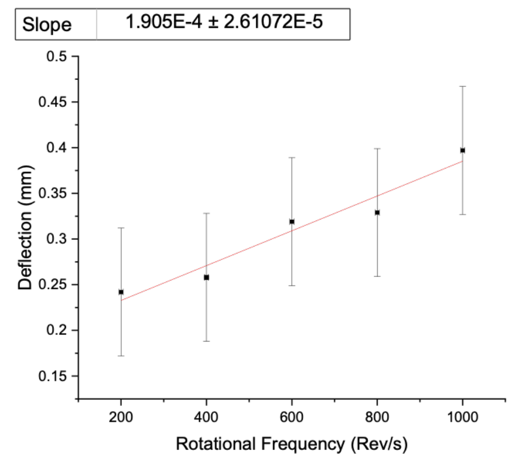

FIG. 7. A graph of deflection $\delta s'$ against revolutions per second ω, recorded using a apparatus shown in figure 4 (Addition of a QWP) and the measuring microscopes digital scale

Errors in the lines of best fit (Figures 5,6,7) are calculated by the graphing software, but to combine this error with errors in A, B and D during the calculation of c from the re-arrangement of Eq. 5 the partial differential method must be used. Thus, Δc can be described by:

$$(\Delta c)^2 = \left(\frac{8AD}{m(B+D)}\Delta D\right)^2 + \left(\frac{4D^2}{m(B+D)}\Delta A\right)^2 + \left(\frac{-4AD^2}{m^2(B+D)}\Delta m\right)^2 + \left(\frac{-4AD^2}{m(B+D)^2}\Delta B\right)^2 + \left(\frac{-4AD^2}{m(B+D)^2}\Delta D\right)^2$$

DISCUSSION

The Effect of The Quarter-Wave Plate

The value of c obtained with the quarter-wave plate is within 15% of the accepted constant from the 1983 agreement

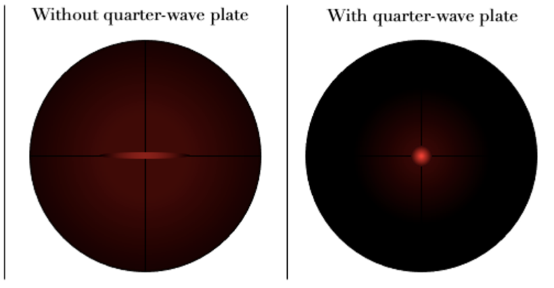

FIG. 8. The view down the microscope whilst collecting data for $\delta s'$, on the left a no QWP is in place (Figure 1. On the right, a QWP is present.4

[5]. Whereas without, the best result is within 35% of this value. This is because of the quarter-wave plates ability to optically isolate the beam which is recorded via the measuring microscope. The Foucault methods accuracy, is highly dependent on the visibility of the distorted beam. Therefore, optical isolation, where unwanted background light and reflections are minimised a more and an accurate result can be obtained [10] [11]. Figure 8, Is a representation of what was observed through the measuring microscope in each case.

Figure 8 accentuates this idea, by visibly showing how the distorted beams appeared without and with a QWP. The method requires the re-arrangement of the microscope's cross-hairs, with a circular point-like beam and less background light, the exact point $(s' + \delta s')$ can be found with far more precision. With a wider beam, shown on the left side of Figure 8, the exact value of $(s' + \delta s')$ is much harder to obtain. Hence, even with a larger data-set (Figures 5 and 6) a value much further from the known constant was found.

Further Evaluation of Results

Lauginie's paper [1], outlines a lot of the motives Foucault had. As previously discussed, the method in which Foucault used in 1850, the same 'modernised' one used in this paper, was originally designed to demonstrate light is slower in water. Since we now know light travels about 25% slower in water [12], to demonstrate this, his design did not need to prioritise accuracy. This could be valid justification for the very inaccurate results we see for the first two data-sets without a QWP. However, this very fact brings the motives of this paper to the foreground. The use of modern technology: The quarter-wave plates ability to perform optical isolation; can take the well-designed Foucault method, with initial motives beyond accuracy to a very accurate measurement of light-speed. The value of course is still 15% away from the constant c. But, with the size of the data-set at present, the results clearly demonstrate what has been discussed.

Possible Improvements

By increasing the number of data-points recorded, particularly when using the QWP error could be reduced from a large 19.5%. One can look at improvements made by Foucault in 1862[1], where the distance D was increased to 20m, and the rotating mirror used a more sophisticated motor. Both changes can improve the results from the method within this paper. In several cases, $(\delta s')$ was extremely small, correctly moving the cross-hairs was difficult and presents human error. With a larger value of D, $\delta\theta$ is larger and so $\delta s'$ is too (Equations 1 and 2). Although, alignment gets harder, results would be improved. In 1926, Michelson carried out the Foucault method with a D of 35km and obtained a value extremely close to the known constant (299796 km/s) [1]. The second factor, the quality of the motor/rotating mirror, also played a large part in producing error in our method. Although through the error analysis, the error in ω was only ± 5 revolutions per second. Further into the experiment such a value was quite a generous one, particularly for results above 600 revs/s. Using a method, in which exact rotational velocities are obtained could certainly improve results. As and example,some modern techniques even use high-speed cameras in combination with an algorithm to correctly measuring rotational velocity [13].

CONCLUSIONS

In conclusion, the results provide clear evidence of how a quarter-wave plate increases the accuracy of the Foucault Method. Since D is kept low, it can be completed inside a basic laboratory. Regardless of the small dataset presented in Figure 7. A more accurate way of calculating the error in ω is an improvement that could have been easily included. However, with this in mind the overall error analysis still provides a value of c, with the QWP, in which its upper bound contains the known constant value for the speed of light, c.

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