Epidemic Process Using Hawkes (lab 17)

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Abstract— This report is related to the simulation of epidemic process at early stage using Hawkes process.

I. STRUTURE OF SIMULATOR

In the following, we address different parts of the simulation.

A. Assumption:

For simplicity, we assume that interval times are independent identically distributed so all the events are i.i.d. Another assumption is, the point processes discussed in this article are both simple and non-explosive. The intensity is generated from formula below:

$$\lambda(t|H_t) = \sigma(t) + m \times \sum_{v:T_v<t} h(t - T_v)$$

To produce time-stamp for every children, we use $Tw = Tv + \tau v; w$ where variables $\tau_{v;w} \geq 0$ are i.i.d arbitrarily distributed r.v. with assigned distribution h(t). Finally, for thin process to retain point Ti, we use the probability $\frac{\lambda(T_i|H_{T_i})}{\gamma}$.

To find rho, we use minimize_scalar library that is quadratic optimization technique with constraint in which rho^2 is objective function and maximum number of deaths 20k is the constraint.

B. Inputs:

To simulate Hawkes process, there are several inputs as follows:

1. Time horizon (100 and 365)
2. Distribution of h(t) as uniform and exponential
3. Sigma(t) as $20 \times 1_{0\leq t\leq 10}$
4. M = 2

C. Outputs:

There are two different scenarios based on two distributions with outputs in function of time as below:

1. Conditional intensity
2. Number of infected individuals
3. Number of dead individuals
4. Cost function which is computed when we have intervention

D. Simulation:

Three functions are defined to generate sigma(t) and two distributions for h(t). Since we are dealing with district time (days), cost function is computed as summation of rho^2 for t < T.

In order to simulate the process, we define a function taking parameters which performs all the process for all scenarios. The main data structure is list to store the data. The pseudo code shows the algorithm.

Algorithm: Simulation of a Univariate Hawkes Poisson with Specific Kernel

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Initialize related variables and lists

While t < t_horizon

- compute current intensity $\lambda 1$
- If t > t_start
  - Compute rho
- Else:
  - rho = 1
- compute candidate event time w
- increment t by w
- compute candidate intensity $\lambda 2$
- generate random probability D
- if D < rho* $\lambda 2/\lambda 1$, accepting with probability

  for i in range(current level individuals):

  compute total number of children for next level as temp:

  (temp += generate children of each member in the current level for next level of tree based on Poisson dist. and average number of infected (m))
- update current level individual by temp
- update number of infected by temp
- update number of death as 0.02 of infected

E. Results and Discussion:

An illustrative example of the left-continuous conditional intensity λ(t) when h(t) has uniform and exponential distributions are shown in fig.1 showing the self-exciting feature of intensity.

In case of without intervention, fig2 shows evolution of infected and death numbers as function of day for two distributions. The plots indicate the exponential growth of the evolution as we expect.

In figure 3, we consider non pharmaceutical intervention using uniform distribution in which death numbers do not exceed 20k after 365 comparing to uniform case of fig.2 that death numbers exceed 20k only after 100 days.

In this case, cost function grows after t_start = 20 due to the existence of rho as a reduction coefficient to prevent more infection and less death as a result.

It should be noted that dots in x-axis are candidate event times after thinning.

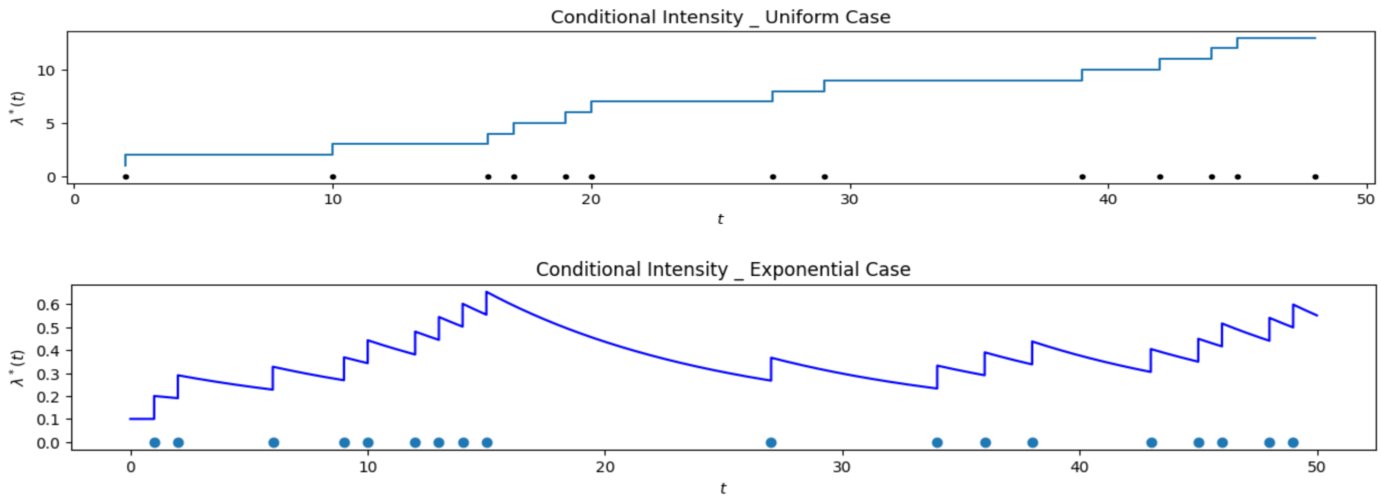

Fig1. left-continuous conditional intensity λ(t)

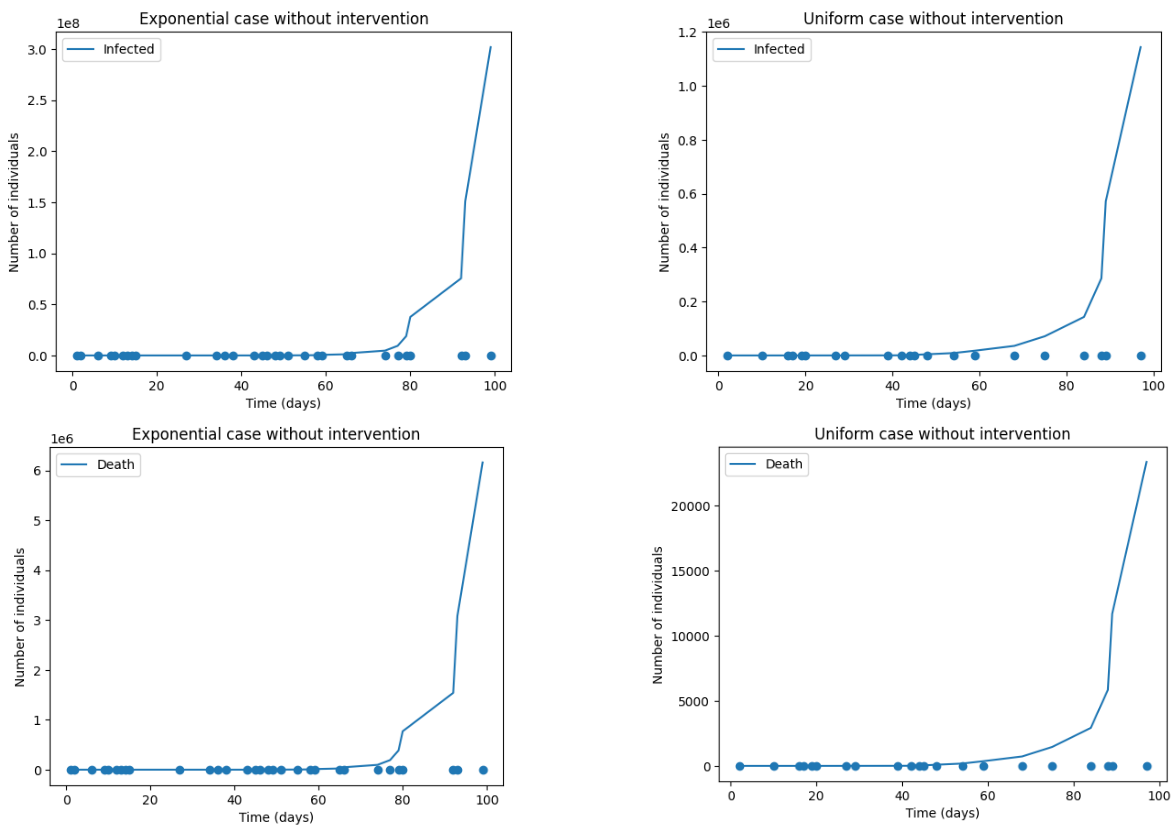

Fig2. Evolution of infected and death numbers

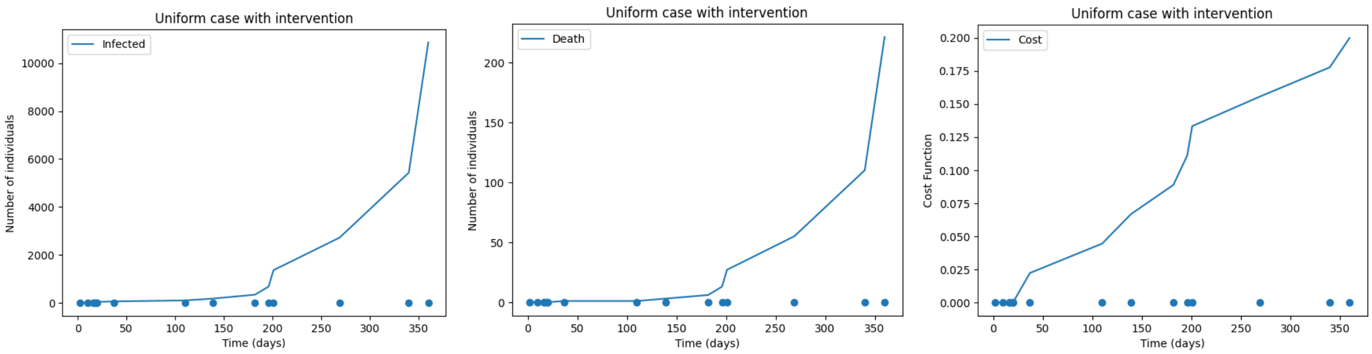

Fig3. Generalized process with non-pharmaceutical intervention and 20k limitation of death number