Lab 16: Epidemic Processes

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I. Introduction

The goal of this laboratory is to simulate epidemic processes using Hawkes processes. Hawkes processes are a mathematical model for self-exciting processes, in which the occurrence of an event increases the probability of another event occurring. Later on the objective is to extend the process by incorporating Non-Pharmaceutical Interventions that reduce the rate of infections, with the aim of keeping the number of deaths within a year to a maximum of 20,000.

II. Preliminary study

As explained before, the Hawkes process is a mathematical model for self-exciting processes, where the occurrence of an event increases the likelihood of another event occurring. As such, it can be used to simulate the spread of an epidemic, and it is expected that the trend of the epidemic will increase over time. This is consistent with real-world observations, such as the COVID-19 outbreak, where contagions continue to rise without intervention. In this laboratory, we will simulate scenarios where no outside intervention is applied to the epidemic process, and anticipate observing an exponential increase in contagions and deaths that will grow in accordance with the parameters characterizing the epidemic process.

The parameters that describe our epidemic process are the following:

- $\sigma(t)$ = 20* $(t \in [0, 10])$ days
- λ = 1/10 days
- m = 2
- h(t) = exponential (with λ = 1/10) or uniform (between 0 and 20)
- death ratio = 2%

The stochastic intensity of a Hawkes process is, indeed, defined by $\lambda(t|H) = \sigma + \text{m} \sum_{i=0}^{t<Ti} h(t - Ti)$, so we should expect that the value of the second part of this expression will be limited during the early stages, and then increase and increasingly affect the progress of the process and we could imagine an exponential trend over time.

III. Simulation of the early stages

To simulate the early stages of an epidemic process I decided to use a tree generation in a time horizon of 100 days. The idea behind this approach is:

1) **Generation of the ancestors**: the ancestors are generated using a Poisson process with rate $\sigma(t)$, defined previously. The results of the Poisson process are the time at which each ancestor is generated.

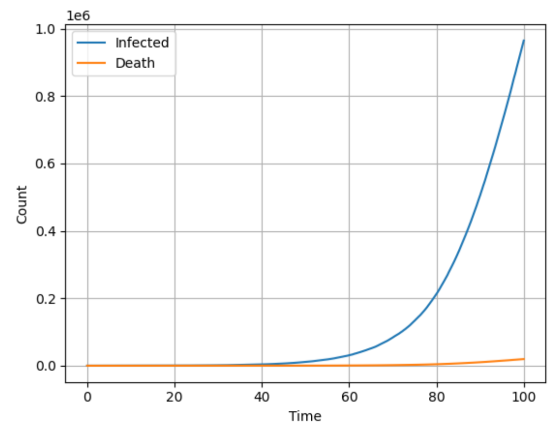

Fig. 1. Infections and deaths using uniform h(t)

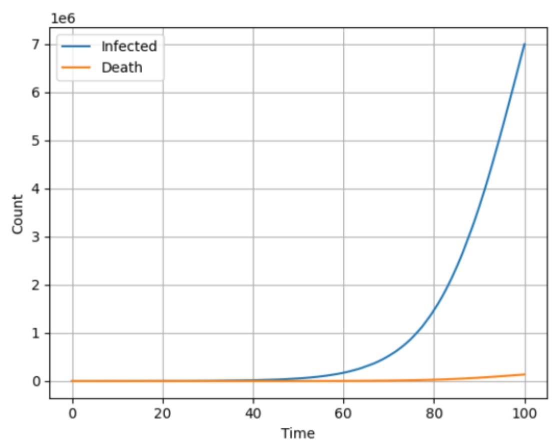

Fig. 2. Infections and deaths using exponential h(t)

2) **Generation of the number of children of each node**: the number of children of each node describes, in the epidemic scenario, the number of people who can be infected starting from a single person. This number is generated using Poisson(m), with m=2 as defined previously.
3) **Generation of the time of each children**: the time at which each children is generated depends on the infection rate of our epidemic process, how quickly the infection is passed from parents to children. This velocity depends on h(t), which we said earlier can be uniform or exponential.

The results of the simulation are showed in figure 1 and

2. As it can be easily see the trend of both the simulation is similar and it is exponential, as predicted. The main difference between the two approaches is that the exponential one turns out to have many more infected than the uniform.

IV. Epidemic over a year

A. Considerations

Keeping in consideration only the uniform generation showed in 1 we discover that in 100 days the epidemic causes about 1 million infected and, as a result, about 20'000 dead. Having it an exponential trend we should expect an incredibly high number of infected and dead over an year without interventions.

In the second part of this laboratory the focus is on extending the process over a time horizon of an year and introduce some Non-Pharmaceutical Interventions with the objective of limit the diffusion. The goal is to minimize the cost of the interventions, with the restriction that the number of deaths must not exceed 20'000 in average over a year.

B. Non-Pharmaceutical Interventions

The Non-Pharmaceutical Interventions introduced are the following:

- **constant**: the intervention as the same magnitude starting from day
- **step function**: an approach that grows arbitrarily over time.
- **exponential function**:an approach that grows exponentially over time.
- **sigmoid function**: an approach that gradually increase over time with a certain steepness.

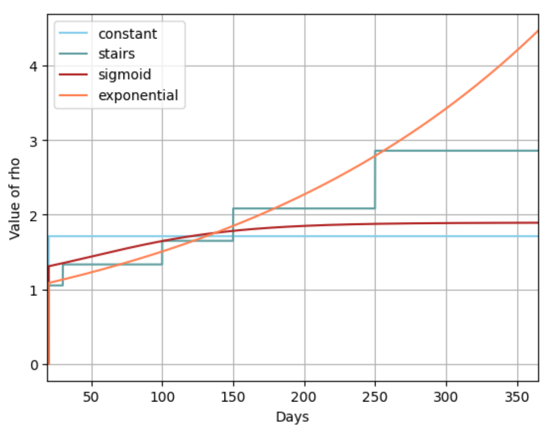

Fig. 3. Impact of the NPI over time

All of the interventions discussed above, with the exception of the constant intervention, were designed to start with a relatively low level of restriction and gradually increase over time. This approach simulates a real-world scenario in which it is difficult to accurately assess the necessary measures during the early stages of an outbreak. This gradual increase in restriction is evident in Figure 3, as the impact of each

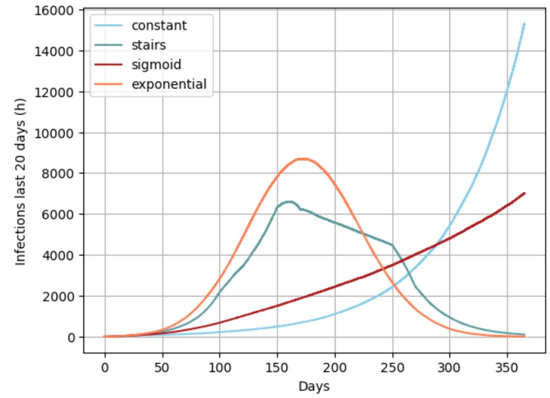

Fig. 4. Impact of the NPI over time

intervention is zero until day 20, at which point the Non-Pharmaceutical Interventions begin. After this point, the interventions follow the trends depicted in the figure, with the exponential intervention having the highest impact at day 365.

It is informative to analyze the trend of h, the number of infected individuals over the past 20 days, during the course of the interventions. This is illustrated in Figure 4. From this figure, it is clear that the epidemic trend using the constant and sigmoid interventions is beginning to rise rapidly at the end of the year, indicating that if the effectiveness of the measures were evaluated over a longer period of time, these interventions would not be effective.

The goal of this lab, however, was to evaluate the measures over a one-year period and assess those with the lowest cost and we will analyze it later.

C. Results

The results of the intervention can be seen in the series of figures (6, 5, 7, 8 that follow.

The simulations indicate that the average number of deaths per year is less than 20,000, however, there is significant variation in the number of deaths from one simulation to another. Despite the similarities in the number of deaths among the various Non-Pharmaceutical Interventions, the costs of these interventions can vary greatly. Upon examination of the cost table I, it is evident that the sigmoid function and the constant intervention are the most cost-effective options for this specific task. This can be attributed to the fact that the cost is defined as the integral over time of the square of rho, and as seen in Figure 3, the cost of the constant and sigmoid interventions is lower.

V. Conclusion

In conclusion, the optimal configuration for the task at hand is to utilize a constant intervention throughout the duration of the task. However, this scenario is not representative of real-world situations. A more realistic approach would be to implement an intervention that varies on a time-based basis. Among the three dynamically adjusted interventions evaluated, the sigmoid intervention performed the best for the task at

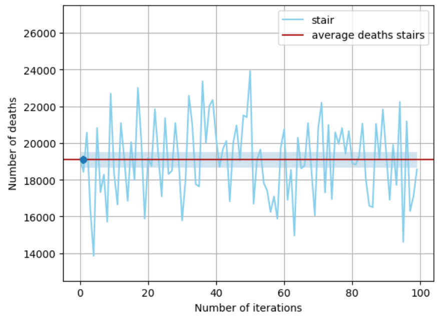

Fig. 5. Impact of the steps NPI on the number of deaths over multiple simulations

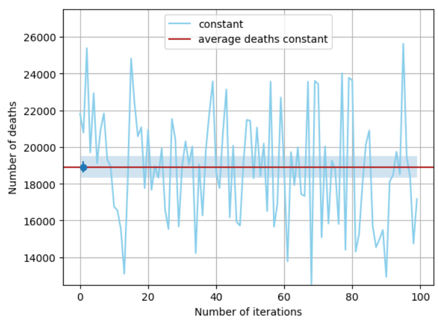

Fig. 6. Impact of the constant NPI on the number of deaths over multiple simulations

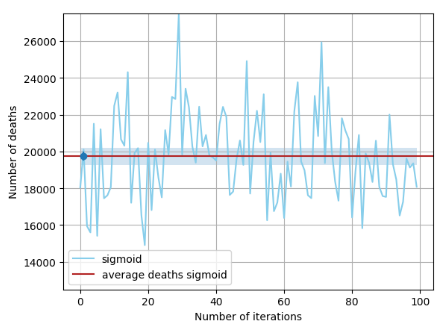

Fig. 7. Impact of the sigmoid NPI on the number of deaths over multiple simulations

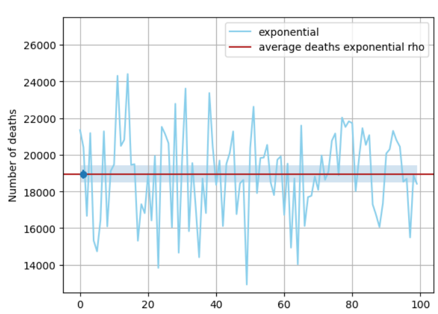

Fig. 8. Impact of the exponential NPI on the number of deaths over multiple simulations

TABLE I
COST AND PARAMETERS TABLE

	Function	Parameters	Cost
Constant	\ (return 1/function)	value = 0.5842	1010.87
Steps	\ (return 1/function)	1 if t <= 20 0.95 if 20 <t <= 30 0.75 if 30 <t <= 100 0.606 if 100 <t <= 150 0.48 if 150 <t <= 250 0.35 if t >250	1644.47
Exponential	eˆ-kt	k = 0.0041	2288.68
Sigmoid	rho_min + (rho_max - rho_min) / (1 + eˆ(-k*x)) (return 1/function)	rho_min = 0.528 rho_max = 1 k(steepness) = 0.020 x(inflection point) = 20 - t	1074.24

hand. It is important to note that the sigmoid and constant interventions, using the parameters utilized in this simulation, may not be effective over extended time horizons.