Simulation of a dynamical system

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What is simulation?

Simulating a dynamic system means imitating its behavior in time

- By imitating the system behavior, we artificially recreate its history and evolution
- From the system evolution we can deduce a lot of information on the system itself

What is simulation?

- We may be interested in discovering or verifying system
  - Performance: *how well does it work?*
  - Correctness: *is it working as desired?*
  - Capabilities: *can it reach some objectives?*
  - Reliability: *under which conditions is it working as desired?*
  - …

What is simulation?

- Several simulation objectives:
  - System design
  - Dimensioning of the system components to satisfy requirements
  - Effects of the improvements or changes of system components on the overall system performance
  - Comparison of different design choices
  - …

When do we need to simulate?

- During the design phase
  - no existing implementation or prototype of the actual system
- On already existing systems
  - To study the system in special conditions, like critical or future scenarios
  - To design improvements for the system components

Areas of application

- Manufacturing applications
- Logistics, supply chains and distribution applications
- Transportation and traffic
- Business process simulation
- Health care
- … and many others

Application to ICT

- **Performance analysis**
  - The system can be a (part of a) network, computing element, a storage unit, or a service, an application, an algorithm, a protocol, … as far as it is dynamic system
  - Verification of the correct working of the system under different assumptions of: load, environment (e.g., temperature, radio-channel noise, user behavior), failures, attack, …
  - Verification of conditions to guarantee correct of system working and possibly quality levels
  - System performance under future load assumptions

Application to ICT

- **System dimensioning**
  - Effects due to the increase of the capacity of elements (bandwidth, computing power, no. of cores, storage capacity, …)
  - Effects due to the increase of the number of elements (no. of servers, no. of nodes, no. of transmission channels…)
  - Choice of the number of new resources or the capacity of the resources

Application to ICT

- **System design**
  - Comparison among algorithms
  - Design of new protocols or comparisons of protocols
  - Architectures of communications/computing/…

Simulation model

- To simulate we need to develop a *simulation model*
  - A formal description of how the system works (i.e., evolves with time)
- A simulation model is
  - made by a set of assumptions on the system and its behavior
  - describes a dynamic system as a set of components interacting according to specific rules

Simulation model

- The system and its components are described by *state variables*
  - In any instant, the working conditions of the system are given by the value of the state variables
- To describe the system evolution we need to describe the rules according to which the system changes its own state
- To imitate the system and recreate its history means to rebuild the **sequence of states visited by the system**

Deriving the model

- A simulation model is a formal representation of a system derived from the theoretical knowledge of the system or from empirical observations
- We need to identify the *boundary* between the *system* and its *environment*
  - The system environment, even if it is not the object of our model, has effects on the model and on the system behavior

Some definitions

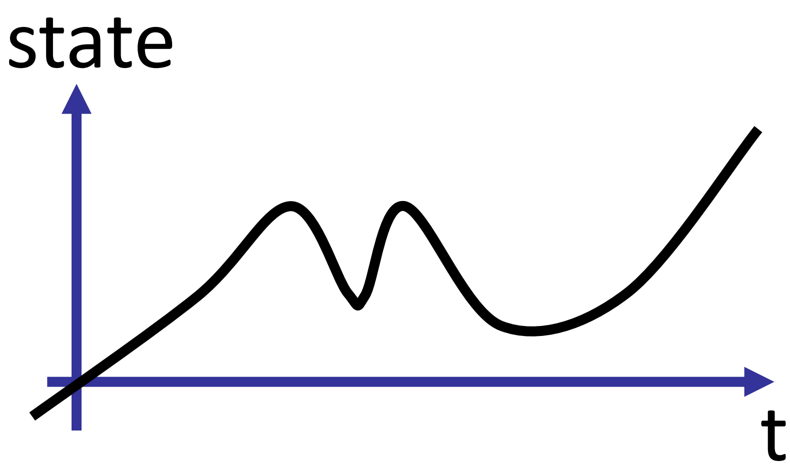
- *State:* describes the system behavior in any time instant
- *Entities:* objects/components of interest in the system
- *Attributes:* properties of an entity
- *Activity:* a time period dedicated to a specific operation
- *Event:* an instantaneous occurrence that changes the system state
- *Endogenous:* events and activities internal to the system
- *Exogenous:* events and activities in the environment that affect the system

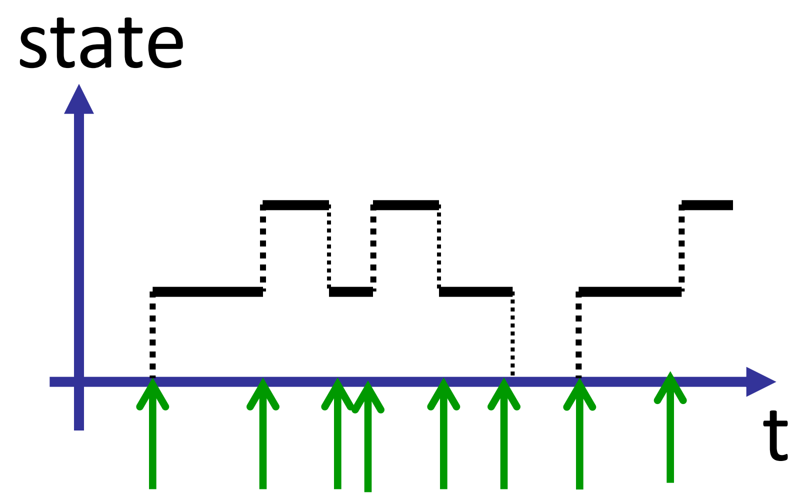
Types of models

- Models can further be classified as
  - *Static:* the system behavior does not change in time (or it is studied in a specific moment in time)
  - *Dynamic:* the system behavior changes in time
- Dynamic models can be classified as
  - *Discrete models*
  - *Continuous models*

Types of models

- Dynamic models can be classified as
  - *Discrete*: the system state changes only in a discrete set of points in time
  - *Continuous*: the system state changes continuously in time (fluid models)





Types of models

- Models can be classified as being
  - *Deterministic*: the model does not contain random variables, therefore a set of input values produces a unique set of outputs
  - *Stochastic*: inputs are represented by random variables, so the outputs are random processes (and estimates of the characteristics of the model)

Observation

- When considering stochastic models, a simulation run consists in
  - Performing a stochastic experiment
  - Observing a single possible sample path of the process describing the evolution of the system state in time
- The simulation output analysis requires the application of statistic techniques on results collected in several simulation runs

Analytical models

- An *analytical model* of the system can be a viable alternative to simulation for simple systems or under special hypothesis
- By using an analytical model, we can obtain indications on the system performance through mathematical expressions
  - Physics laws, differential equations, Markov chains, Stochastic processes

Analytical models

- The analytical models can have
  - *Exact solution:* the solution is represented by a closed form expression (e.g., physics laws, some basic results from queuing theory)
    - x=f(y,z,n)
  - *Numerical solution:* due to its complexity, the solution can be obtained only though numerical approximation (e.g., fixed-point methods, solutions not expressed in closed form, solutions of equation systems)
    - x=f(y,z,n); y=g(z,n); z=(x,y,n)

Analytical vs. simulation models

- Analytical models
  - Emphasize the cause-effect relations between inputs and outputs
  - Limited solution cost (in time and computational power)
  - Help the system comprehension
  - Fewer ‘technical’ errors (bugs)
- Simulation models
  - Better adherence the actual system
  - Easier to introduce incremental improvements in the model
  - Easier to modify some to the underlying model hypothesis
  - Simpler model construction

Analytical vs. simulation models

We will experiment this as lab activity for some system

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Types of models

	Analytical / Numeric	Static / Dynamic	Deterministic / Stochastic
Ohm's Law	**A**	**S**	**D**
Law of Large Numbers	**A**	**S**	**S**
Markovian Processes	**A (N)**	**D**	**S**
Non-linear differential equations	**N**	**D**	**D**
Simulation	**N**	**D**	**S**

Steps in a simulation study

- Problem formulation
- Setting of objectives
- Overall project plan
- Model conceptualization
  - Ability of abstracting the essential features of the system
  - Selection of simplifying hypothesis at a suitable level of details

Steps in a simulation study

- The level of details depends on a trade-off between accuracy (realism) and simplicity
  - A simulation model with too many details
    - Requires longer development time (debugging)
    - Requires a larger running time (long simulations)
    - Produces complex results difficult to be interpreted
  - A simulation model with too few details
    - Is unsuitable to observe some interesting and particular behavior of the system
    - Is unrealistic and produces wrong conclusions on the system behavior

Steps in a simulation study

- Inputs and data collection
  - Selection of the input parameters considered to be important for the model
  - Study and characterization of the parameters (from raw data to statistical distributions)
- Model translation → Coding
- Model verification → Is the code correct?
- Model validation → Is the model correct?
  - Comparison of some model results with the expected model behavior (e.g., with previously collected data)

Steps in a simulation study

- Experimental design
  - Length of the initialization periods
  - Length of simulation runs
  - Number of simulation runs
- Production runs and analysis
- More runs?

Wrap-up

- Simulations are emulations of the behavior of a dynamic system with time
- Simulation is used to understand and design dynamic systems
- Building up a simulation tool is not straightforward, requires carefull definition of elements of the system whose behavior is represented