$$
\begin{aligned}
\mathbf{Y}_{t+1}/\mathbf{Y}_t &= \mathbb{M}\left[\xi_{t+1}\Gamma\psi_{t+1}\mathbf{p}_t\right]/\mathbb{M}\left[\mathbf{p}_t\xi_t\right] \\
&= \Gamma
\end{aligned}
$$

because of the independence assumptions we have made about ξ and ψ.

If all consumers were at their target m where $\mathbb{E}_t[m_{t+1}] = m$, we can show that the growth factor for aggregate assets is approximately Γ:

Aggregate assets are:

$$
\begin{aligned}
\left(\frac{\mathbf{A}_{t+1}}{\mathbf{A}_t}\right) &= \mathbb{M}\left[\frac{a_{t+1,i}\Gamma\mathbf{p}_{t+1,i}}{a_{t,i}\mathbf{p}_{t,i}}\right] \\
&= \Gamma\mathbb{M}\left[\frac{a_{t+1}\psi_{t+1}}{a_t}\right] \\
&= \Gamma\mathbb{M}\left[\frac{(a_t\mathcal{R}_{t+1} + \xi_{t+1} - \mathrm{c}(a_t\mathcal{R}_{t+1} + \xi_{t+1}))\psi_{t+1}}{a_t}\right] \\
& \Gamma\mathbb{M}\left[\frac{(a_t\mathsf{R}/\Gamma + \psi_{t+1}\xi_{t+1} - \mathrm{c}(a_t\mathcal{R}_{t+1} + \xi_{t+1})\psi_{t+1}}{a_t}\right] \\
& \Gamma\mathbb{M}\left[\frac{(a_t\mathsf{R}/\Gamma + \psi_{t+1}\xi_{t+1} - (\mathrm{c}(\mathbb{E}_t[m_{t+1}]) + (\mathrm{c}'(\mathbb{E}_t[m_{t+1}])(m_{t+1} - \mathbb{E}_t[m_{t+1}])))\,\psi_{t+1}}{a_t}\right]
\end{aligned}
$$

but the first term reduces to

$$
\Gamma\mathbb{M}\left[\frac{a_t\mathsf{R}/\Gamma + \psi_{t+1}\xi_{t+1}}{a_t}\right] = \mathsf{R} + 1/a_t
$$

but defining $\bar{\mathcal{R}}_{t+1} = \mathbb{E}_t[\mathcal{R}_{t+1}]$

$$
\Gamma\mathbb{M}\left[\frac{-\mathrm{c}(a_t\mathcal{R}_{t+1} + \xi_{t+1})\psi_{t+1}}{a_t}\right] \approx \Gamma\mathbb{M}\left[\frac{-\left(\mathrm{c}(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1}) + \mathrm{c}'(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1})(\mathcal{R}_{t+1} - \bar{\mathcal{R}}_{t+1})\right)\psi_{t+1}}{a_t}\right] \tag{1}
$$

$$
= \Gamma\mathbb{M}\left[\frac{-\left(\mathrm{c}(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1})\psi_{t+1} + \mathrm{c}'(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1})(\mathsf{R}/\Gamma - \bar{\mathcal{R}}_{t+1}\psi_{t+1})\right)\psi}{a_t}\right. \tag{2}
$$

$$
\Gamma\mathbb{M}\left[\frac{-\mathrm{c}(a_t\mathcal{R}_{t+1} + \xi_{t+1})\psi_{t+1}}{a_t}\right] \approx \Gamma\mathbb{M}\left[\frac{-\left(\mathrm{c}(\mathbb{E}_t[m_{t+1}]) + (\mathrm{c}'(\mathbb{E}_t[m_{t+1}])(m_{t+1} - \mathbb{E}_t[m_{t+1}]))\right)}{}\right. \tag{3}
$$

$$
= \Gamma\mathbb{M}\left[\frac{-\left(\mathrm{c}(\mathbb{E}_t[m_{t+1}]) + (\mathrm{c}'(\mathbb{E}_t[m_{t+1}])(m_{t+1} - \mathbb{E}_t[m_{t+1}]))\right)\psi_{t+1}}{a_t}\right] \tag{4}
$$

while the remainder of the expression is

where $\mathbf{P}_t$ designates the mean level of permanent income across all individuals, and we are assuming that $a_{t,i}$ was distributed according to the invariant distribution with a mean value of a. Since permanent income grows at mean rate Γ while the distribution of a is invariant, if we normalize $\mathbf{P}_t$ to one we will similarly have for any period $n \geq 1$

$$\mathbf{A}_{t+n} = A\Gamma^n + \text{cov}(a_{t+n,i}, \mathbf{p}_{t+n,i}).$$